Black Hole Arithmetic Practice

①

Determine the Schwarzchild radius (r_{sch})of a stellar black hole with a mass of 10m⊙

Data:

1m⊙ = 2.0 x 10^{30} kg,

10m⊙ = 10 x (2.0 x 10^{30}) = 20 x 10^{30} kg

C = 3.0 x 10^8 m/sec,

G = 6.672 x 10-11 Nm^2/Kg^2

Formula:

$$r_{sch} = \frac{2GM}{C^2}$$

Solution:

$$r_{sch} = \frac{2GM}{C^2} = \frac{2(6.672\ x\ 10^{-11})(20.0\ x\ 10^{30})}{(3.0\ x\ 10^{8})^2}$$

$$= \frac{266.88\ x\ 10^{19})}{9.0\ x\ 10^{16}}$$

= 29.653 x 10^3 m

= 29.653 Km

Answer:
29.653 x 10^3 m
or
29.653 Km

②

Determine the mass (in solar masses) of a black hole with a Schwarzchild radius of 16 million Km

Data:

1m⊙ = 2.0 x 10^{30} kg,

R_{sch} = 16 million Km = 16 x 10^6 Km

= 1.6 x 10^7 Km

= 1.6 x 10^{10} m

C = 3.0 x 10^8 m/sec,

G = 6.672 x 10-11 Nm^2/Kg^2

Formula:

$$r_{sch} = \frac{2GM}{C^2}$$

$$2GM = r_{sch}\ C^2$$

$$M = \frac{r_{sch}\ C^2}{2G}$$

Solution:

$$M = \frac{r_{sch}\ C^2}{2G} = \frac{\left(1.6 \text{ x } 10^{10}\right)\left(3.0\ x\ 10^{8}\right)^2}{2\left(6.672\ x\ 10^{-11}\right)} = \frac{(\ 1.6 \text{ x } 10^{10})(9.0\ x\ 10^{16}\ )}{(13.344\ x\ 10^{-11})}$$

$$= \frac{(14.4\ x\ 10^{26}\ )}{(13.344\ x\ 10^{-11})} = 1.079\ x\ 10^{37} Kg$$

$(1.079\ x\ 10^{37})\ Kg\ \div (2.0\ x\ 10^{30}) = .5396\ x\ 10^{7}\ m_{\odot}$ $\boldsymbol{= 5.396\ x\ 10^{6}\ m_{\odot}}$

③

Determine the mass (in solar masses) of a supermassive black hole that is orbited by a star with a mean/average orbital radius of 1500 AU's and a period of 12 Earth Years

Formula: $P^2 = \frac{4\pi^2 a^3}{Gm}$ (Newton's version of Kepler's Third Law)

Where:

"a" = orbital radius

"p" = orbital period in seconds

"G" = 6.672 x 10-11 Nm^2/Kg^2

"m" = mass of primary in Kg

Rearranged for solving "m": $m = \frac{4\pi^2 a^3}{Gp^2}$

Data:

1 $m_\odot$ = 2.0 x 10^{30} kg

1 AU = 150 million Km = 150 x 10^6 Km = 1.5 x 10^8 Km = **1.5 x 10^{11}m**

Orbital Radius (a) : 1500 AU's

Convert to meters:

1500 x 1.5 x 10^{11}m = 2250 x 10^{11} m = **2.25 x 10^{14} m**

Period (p) = 12 Earth years

Convert to seconds:

$$\frac{12\ \cancel{yr}}{1} \text{ X } \frac{3.65\ x\ 10^2 \cancel{d}}{1\cancel{yr}} \text{ X } \frac{2.4\ x\ 10^1 \cancel{hr}}{1\cancel{d}} \text{ X } \frac{3.6\ x\ 10^3 sec}{1\cancel{hr}} = 378.432 \text{ x } 10^6 \text{ sec}$$

= 3.784 x 10^8 sec

Solution:

$$m = \frac{4\pi^2 a^3}{Gp^2} = = \frac{4(3.14^2)\ x\ (2.25 \text{ x } 10^{14})^3}{(6.672\ x\ 10^{-11})(3.784\ x\ 10^8)^2}$$

$$= \frac{(39.438) x (11.931\ x\ 10^{42})}{(6.672\ x\ 10^{-11})(14.319\ x\ 10^{16})}$$

$$= \frac{470.535\ x\ 10^{42}}{(95.536\ x\ 10^5)} = 4.925 \text{ x } 10^{37} \text{ Kg}$$

$$\frac{4.925\ x\ 10^{37}}{2.0\ x\ 10^{30}}$$

= 2.463 x 10^7 $m_\odot$

= 24.63 million $m_\odot$