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# Long memory in the Egyptian stock market returns

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**Abstract**

This paper examines the presence of long memory in the daily returns of the Egyptian stock market, using parametric and semiparametric methods. Both techniques have their merits and demerits. Accordingly, the Exact Maximum Likelihood (EML) estimation is employed to estimate the ARFIMA model in the time domain; while two main semiparametric techniques, log periodogram (LP) and local Whittle (LW), were applied to estimate the memory parameter in the frequency domain. Unlike the findings for developed equity markets, the results show strong and significant evidence of long memory in the Egyptian stock returns, which refutes the hypothesis of market efficiency. As a result the Egyptian stock returns can be predicted using historical information. The findings of this paper are helpful to regulators, financial managers and investors dealing in the Egyptian stock market.

**JEL Classification:** C14, C22.

**Keywords:** ARFIMA; Egyptian stock market; Exact maximum likelihood; Local Whittle estimation; Log-periodogram regression; Long memory; Semiparametric estimation.

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## 1 Introduction

This paper uses parametric and semiparametric methods to estimate the long memory parameter for the Egyptian stock market returns. A time series has a long memory, whenever the dependence between apart events diminishes very slowly as the number of lags increases. The presence of long memory properties in asset returns has important implications for asset pricing models. Such features can be used to construct a profitable trading strategy. In another words, long memory entails that perfect arbitrage is impossible and contradicts standard derivative pricing models based on Brownian and martingale assumptions (Mandelbrot, 1963).

In the last two decades, it has been of great importance, theoretically and empirically, to study the properties of long memory in financial asset returns which is used as a proxy for analysing market efficiency. The stock market returns are said to exhibit long memory properties, if there is a significant autocorrelation (dependence) between observations widely separate in time. This dependence between apart observations can be utilised to predict future returns, leading to the possibility of consistent speculative profits. Consequently, the existence of long memory in the return series refutes the weak form of the market efficiency hypothesis. The price of an asset determined in an efficient market should follow a martingale process in which each price change is unaffected by its predecessor and has no memory. Therefore, if the returns series display significant autocorrelation between distant observations then past returns can help to predict future returns, thus violating the market efficiency hypothesis which states that, asset prices incorporate all relevant information, where future asset returns are unpredictable, conditioning on past returns.

The development of statistical long memory processes was inspired by Hurst (1951) who was the first to introduce a method for the quantifying of the long memory called rescaled range analysis (R/S). This method involves parameter estimation to capture the scaling behaviour of the range of partial sums of the variable under consideration. Using the rescaled range analysis, Mandelbrot (1971) has found evidence of long memory in the stock returns. However, Lo (1991) pointed out the lack of robustness of the statistical R/S test in the presence of short term memory and heteroskedasticity. Lo (1991) suggested a modified R/S test and tested for long memory in daily US stock market indices and found no evidence of long range dependence. Mills (1993) found weak evidence of long memory in a sample of monthly UK stock returns. Cheung and Lai (1995) provided little evidence of long memory in the Morgan Stanley Capital International stock index data. Ding and Granger (1996) reported evidence of long memory for S&P 500 returns, while Lobato and Savin (1997) saw no evidence of long memory in daily S&P 500 returns over the period July 1962 to December 1994. The majority of the above studies have employed either parametric or semiparametric methods to test and estimate the long memory property. For the parametric method, a complete parametric model to express the autocovariance function as a parametric function of the parameters, $d$, is built, such as ARFIMA model. In contrast, the semiparametric method is only interested in the memory parameter $d$ and does not require the modelling of a complete set of the autocovariances. In the main, both parametric and semiparametric techniques have their merits and demerits. Estimation of fully parametric long memory models is computationally expensive and is subject to misspecification; hence the correct choice of the model is important. On the other hand, the semiparametric estimation considers $d$ as the main parameter of interest. The SPE derives robust estimators since it avoids difficulties over the specification of the short run ARMA parameters; however, the idea of explaining the entire

autocorrelation structure by a single parameter $d$ is highly restrictive. This study employs parametric and semiparametric methods to estimate the long range dependence.

Compared to the world's well-developed financial markets (the U.S. markets), the presence of long memory in emerging capital markets in developing economies has received little attention. Nevertheless, there are various conditions and reasons that contribute to a different dynamics regarding returns in emerging stock markets. Emerging markets are typically much smaller, less liquid, and more volatile than well known world financial markets. Emerging markets may be less informationally efficient. This could be due to several factors such as poor-quality (low precision) information, high trading costs, and less competition due to international investment barriers. Furthermore, the industrial organization found in emerging economies is often quite different from that in developed economies. As a result, it is very important to study the emerging securities markets and the complete characterization of the dynamic behaviour underlying stock returns in these developing economies, in order to attract investors and investment funds seeking to diversify their assets.

The objective of this paper is to examine the presence of long memory in the Egyptian stock returns using parametric and semiparametric methods. The Daily EGX30 price index is considered as a proxy for the Egyptian stock market. There has been very limited research on the behaviour of stocks traded on the Egyptian stock exchange, although the capital market in Egypt is apt to exhibit different characteristics from those observed in developed capital markets. Biases due to market thinness and nonsynchronous, trading should be expected to be more severe in the case of the Egyptian stock market. The Egyptian stock market is not expected to be highly efficient in terms of the speed of information reaching traders compared to the developed capital markets. Furthermore, traders and investors in the Egyptian stock market tend to react slowly and gradually to new information. The existence of long memory will have significant implications in the Egyptian stock market, where future returns can be predicted from past returns, thus violating the market efficiency hypothesis.

This paper is organised as follows. The next section provides an overview of the theoretical and relevant literature review of long memory. Section 3 covers the parametric and semiparametrics methods used to estimate the long memory parameter. Section 4 describes the data and reports the results of the empirical application to the daily Egyptian stock market and finally section 5 offers some concluding remarks.

## 2 Literature Review

### 2.1 Background

Long memory, or **LM**, processes were initially documented in non-economic literature, with interest starting from the empirical examination of data in physical science since at least 1950s. The famous British hydrologist Harold Edwin Hurst (1951), during the engineering of the high Aswan dam, developed an analysis to determine if the yearly flows and inflows into reservoirs of the Nile were random or clustered from year to year using long reliable historical data for the years 622- 1281 recorded at the Roda gauge in Cairo. He determined they were not random and found evidence of dependence over long intervals of time, with stretches when floods are high tending to be above the mean and others when they are low tending to be below the mean. As a result, the data was found to show several cycles; however, these cycles did not exhibit periodicity. Mandelbrot and Wallis (1968) called this

behaviour the *Joseph effect*[1] in reference to the biblical seven good years of abundance and seven bad years of famine[2]. Hurst (1951, 1956 and 1957) also examined 900 records of other natural phenomena (for example, annual river levels, rainfall, temperature and pressure records, tree rings, and sunspot activity) finding non-random positive correlations in most of them (Baillie, 1996).

Since then, LM processes have been investigated by many researchers from very different fields; and because of the diversity of its applications, its literature is generally spread over a large number of journals including Agronomy, Astronomy, Chemistry, Climatology, Engineering[3], Geo-science, Hydrology, Mathematics, Physics and Statistics. Examples of these are presented in, *inter alia*, Hurst (1951, 1956 and 1957), Lawrance and Kottegoda (1977) and Painter (1998) in geophysical data, Mandelbrot and Wallis (1968), Mandelbrot (1972), Hipel and McLeod (1978a, 1978b and 1978c), Bloomfield (1992), Seater (1993) and Kirk-Davidoff and Varotsos (2006) in climatology.

As the data in natural sciences demonstrate a preference towards LM and the source of uncertainty in Economics can be considered as natural phenomena, then we can expect LM to be found in economic data. The importance of LM in economic data was recognised in Mandelbrot (1963), Adelman (1965) and particularly, in Granger (1966), who noticed that for economic time series, the typical shape of the spectral density is a function with a pole at the origin that then decays monotonically at high frequencies. It was not until 1980 that LM models were used by Econometricians and by Financial Researchers *circa* 1995. Granger and Joyeux (1980) propose the use of the fractional differencing to model LM which is related to earlier work by Mandelbrot and Van Ness (1968) describing fractional Brownian motion.

There is substantial evidence that LM models can be successfully applied to time series data in both Macroeconomics and Financial Economics data; for example, real national output measures, inflation rates, exchange rates, interest rate differentials, stock prices, commodity prices, market indices and forward premiums. These time series show evidence of being neither $I(0)$ nor $I(1)$. When first differenced, those series appear as being over-differenced. This feature is typical of long memory processes. LM processes has also been used in modelling the volatility of asset prices and power transformations of returns. Investigations for LM in real output measures were first studied in Diebold and Rudebusch (1989) and Haubrich and Lo (1989). Baillie, Chung, and Tieslau (1992) and Hassler and Wolters (1995) apply fractionally integrated $ARMA$, or $ARFIMA$, models to describe the fluctuations of the inflation rates. They provided empirical evidence in favour of LM models. Baillie et al. (1992) examined the relationship between the mean and the variability of inflation rates by means of $ARFIMA - GARCH$ models for 10 countries using monthly observations from 1948 to 1990. Baillie, Bollerslev and Mikkelson (1996) find LM in the volatility of the Deutsche Mark- U.S. Dollar (DM-USD) exchange rate. Long range dependence, in asset price series, was reviewed by Brock and De Lima (1996); yet, LM seems much more likely in asset

[1] The *Joseph effect* involves long stretches of time when the process tends to be above the mean, and long stretches of time when the process tends to be below the mean.

[2] In the Bible (Genesis 41, 29-30): "Seven years of great abundance are coming throughout the land of Egypt, but seven years of famine will follow them", and the same phenomenon was also mentioned in the Koran (Joseph 12, 47-48): "He [Joseph] said, What you cultivate during the next seven years, when the time of harvest comes, leave the grains in their spikes, except for what you eat. After that, seven years of drought will come; this will consume most of what you stored for them". However, there are no records of the water level of the Nile from those times.

[3] In signal and image processing.

volatility than in asset returns themselves. LM was found in the Deutshcer Aktien IndeX (DAX, the German stock index) by Lux (1996). In addition, some research demonstrates the existence of LM in smaller and less developed markets. Tolvi (2003) examined the Finnish stock market; Madhusoodanan (1998) provides evidence on the individual stocks in the Indian Stock Exchange. Barkoulas and Baum (2000) give similar evidence on the Greek financial market; while Cavalcante et al (2002) demonstrate LM in Brazil stock market.

Finally, the monographs by Beran (1994), Robinson (2003), Palma (2007) and Samorodnitsky (2007) provide an excellent introduction to LM processes. Additionally, several survey-type articles on LM have been written, for example Taqqu (1986), Hampel (1987), Beran (1992), Robinson (1994), Baillie (1996), Guégan (2005) and Banerjee and Urga (2005). Recently, research on LM is growing significantly leaving some of these surveys very out-of-date.

## 2.2 Defining Long Memory

The terms long memory, long-range dependence, strong dependence or persistence can be used interchangeably. LM can be defined in several ways. Traditionally, LM has been specified in the time domain in terms of long lag autocovariance, or in the frequency domain in terms of explosion of low frequency spectra. Given a stationary time series process $\{y_t\}$ with an autocovariance function $\gamma(j) = Cov(y_t, y_{t+j})$ at lag $j$ that does not depend on $t$, then the process has LM if,

$$\gamma(j) \sim c_\gamma j^{2d-1} \text{ , as } j \to \infty \qquad (2.2.1)$$

for $0 < d < \frac{1}{2}$, where $d$ is the memory (differencing) parameter, or the fractional difference parameter. The constant $c_\gamma$ is finite positive ($0 < c_\rho < \infty$), and the notation "$\sim$" means that the ratio of the left and right sides tend to one for large $j$. The intuition interpretation for this definition is that the dependence between apart events diminishes very slowly as the number of lags increases (tends to infinity) often called a hyperbolic decay. On the contrary, short-range dependence is characterised by quickly decaying correlations at an exponential rate to zero (e.g. $ARMA$ and Markov processes). The asymptotic behaviour in (2.2.1) indicates that the autocovariance decreases very slowly with long lags, or in other words the autocovariances are not summable so that,

$$\lim_{n\to\infty} \sum_{j=-n}^{n} \gamma(j) = \infty \qquad (2.2.2)$$

On the other hand, LM can be described in the frequency domain using the spectral density structure. It is interesting to see how long-range dependence, or LRD, translates from the time domain to the frequency domain. Suppose that $\{y_t\}$ has absolutely continuous spectral density function, then it has a spectral density $f(\lambda)$ that is

$$f(\lambda) = \frac{1}{2\pi} \sum_{j=-\infty}^{\infty} \gamma(j) e^{-i\lambda j} , \quad -\pi \le \lambda \le \pi \qquad (2.2.3)$$

where $f(\lambda)$ is a non-negative, even function, periodic of period $2\pi$ when extended beyond the Nyqvist[4] range $[-\pi, \pi]$. LM in the time domain is expected to be translated into the behaviour of the spectral density around the origin because low frequencies (frequencies around the origin) account for big lags in the time domain. A process with spectral density $f$ is defined to exhibit long memory if,

$$f(\lambda) \sim c_f |\lambda|^{-2d} \qquad \text{, as } \lambda \to 0 \qquad (2.2.4)$$

[4] Nyqvist range is named after the Swedish-American Engineer Harry Nyqvist (1889-1976).

Both definitions, in the time and frequency domain, are not equivalent but connected (Beran, 1994a and Taqqu, 1986). The spectral density in (2.2.4) implies that the spectral density will be unbounded at low frequencies. Hence long range dependence corresponds to the blow-up of the spectral density $f(\lambda)$ at the origin so that it has a pole at frequency zero,

$$f(0) = \frac{1}{2\pi}\sum_{j=-\infty}^{\infty}\gamma(j) = \infty \qquad (2.2.5)$$

Thus if $\gamma(j)$ behaves like a power function at infinity, so does $f(\lambda)$ at zero and the relation can be remembered by the "add one, change sign" rule, where the exponent $2d-1$, is the asymptotic behaviour of $\gamma(j)$.

## 2.3 Long Memory Models

During the World War II, a massive momentum in time series research has evolved as a result of advances in many engineering applications, including spectral analysis and radio signals. Afterwards, a flexible group of models, known as $ARMA$, also called short-range dependent models involving correlation functions that decrease exponentially fast over time, was developed in the time domain. Although short-memory models were used widely, by economists, these models had a number of shortcomings and could not be applied to all fields. Some data seemed to require models, whose correlation functions would decay much less quickly.

Kolmogorov (1940) discovered the fractional Brownian motion, or $fBm$, which was used along with its increments by Mandelbrot to generate long-range dependence. The characteristic of LRD in economic and financial data is lately described by a number of models. This includes the fractional differencing model, the autoregressive fractionally integrated moving average models ($ARFIMA$) and fractional cointegration models. Among these models, the focus would be on the $ARFIMA(p,d,q)$ models introduced by Granger and Joyeux (1980).

In 1971, Box and Jenkins introduced the $ARIMA(p,d,q)$ model,

$$\Phi(L)(1-L)^d y_t = \Psi(L)\varepsilon_t \qquad (2.3.1)$$

where $d$ is an integer, $\Phi(L)$ and $\Psi(L)$ are the polynomials $\Phi(L) = 1 - \sum_{j=1}^{p}\phi_j L^j$ and $\Psi(L) = 1 + \sum_{j=1}^{q}\psi_j L^j$ involving autoregressive and moving average coefficients of order $p$ and $q$ respectively and $\varepsilon_t$ is a white noise process. To ensure the stationarity and inverstibility conditions, the roots of $\Phi(L)$ and $\Psi(L)$ must lie outside the unit circle. Granger and Joyeux (1980) managed to extend the set of $ARIMA$ models by considering instead fractional $d \in (-0.5, 0.5)$ in (2.3.1) which introduces a fractional autoregressive integrated moving average model orders $p$, $d$ and $q$, or $ARFIMA(p,d,q)$ or $FARIMA(p,d,q)$. It has spectral density,

$$f(\lambda) = \frac{\sigma^2}{2\pi}\left|1-e^{i\lambda}\right|^{-2d}\left|\frac{\phi(e^{i\lambda})}{\psi(e^{i\lambda})}\right|^2, \quad -\pi \le \lambda \le \pi \qquad (2.3.2)$$

A fractional white noise process is a particular case which is equivalent to an $ARFIMA(0,d,0)$ process. $ARFIMA$ processes are covariance stationary for $-0.5 < d < 0.5$, mean reverting for $d < 1$ and weakly correlated for $d = 0$. For $d > 0.5$ these processes have infinite variance. For $d \ge 0.5$ the processes have infinite variance but in the literature it is more usual to impose initial value conditions so that $y_t$ has changing, but finite, variance. Granger and Joyeux (1980) and Hosking (1981) considered $ARFIMA(0,d,0)$ and $ARFIMA(1,d,0)$ respectively, which based on Adenstedt's (1974) model. Further

information on $ARFIMA(p,d,q)$ models was give by Sowell (1992), Chung (1994) and others.

## 2.4 Estimation Methods

One of the main interests in the literature of long memory is to estimate the unknown parameter $d$ that describes the long memory properties or the low frequency behaviour of the spectral density function $f(\lambda)$. There are two main groups of estimation methods used to test for LM: the parametric estimation ($\boldsymbol{PE}$) and the semi-parametric estimation ($\boldsymbol{SPE}$). For the parametric estimation, a complete parametric model that expresses $\gamma(j)$ for all $j$, or the spectral density function $f(\lambda)$ for all $\lambda$, as a parameteric function of the parameters, $d$ and unknown scale factors, is built, such as $ARFIMA$ model. In contrast, the semi-parametric estimation is only interested in the memory parameter $d$ and do not require the modelling of a complete set of the autocovariances.

In the main, each method has its merits and demerits. Estimation of fully parametric long memory models is computationally expensive, especially in the time domain. Additionally, parametric methods are subject to misspecification. Under- or over-specification of the autoregressive and moving average orders $p$ and $q$, which describes the short range dependent component of $y_t$, can lead to invalidation of the statistical properties and can dangerously bias the estimation of $d$. On the other hand, $SPE$ considers $d$ as the main parameter of interest. *SPE* derives robust estimators since it avoids difficulties over the specification of the short run *ARMA* parameters; however, the idea of explaining the entire autocorrelation structure by a single parameter $d$ is highly restrictive.

### 2.4.1 Parametric Estimation

Several joint estimation methods of the unknown parameters in the $ARFIMA(p,d,q)$ model in equation (2.3.1) were considered. If $y_t$ is assumed Gaussian process, then the log-likelihood function is,

$$\mathcal{L}(\theta) = -\frac{n}{2}\log(2\pi) - \frac{1}{2}\log|\Omega| - \frac{1}{2}y'\Omega^{-1}y \qquad (1.2.4.1.1)$$

The Gaussian maximum likelihood estimate, or MLE, is obtained by maximising $\mathcal{L}(\theta)$ and might be expected to have optimal asymptotic statistical properties. The log likelihood function requires the calculation of the determinant and the inverse of the variance-covariance matrix $\Omega$. These calculations can be done by means of several procedures, for example, Cholesky decomposition method, Durbin-Levinson algorithm and state space techniques. Furthermore, Sowell (1990, 1992) derives the exact MLE of the $ARFIMA$ process with unconditional normally distributed error terms. Sowell's estimator performs poorly if the model is misspecified, like all maximum likelihood. Baillie and Chung (1993) developed a conditional sum of squares estimator in the time domain and show that it performs similarly to Sowell's estimator for the $ARFIMA(0,d,0)$ model. Computer programs for the exact MLE were developed by Doornik and Ooms (2003, 2004), who showed, by building on the work by Sowell (1992), that the exact MLE can be efficiently estimated with storage of order $n$ and computation of order $n^2$. Their ML approach is applied in the Arfima package and therefore also in PcGive (See, Doornik and Hendry (2001)).

The calculation of the exact MLE is complicated and computationally demanding. As a result, alternative procedures have been considered to replace the exact MLE. The use of

approximations to Gaussian ML was developed to speed up the calculation of parameter estimates, without affecting the first order limit distributional behaviour. Estimates maximising such approximations are called Whittle estimates due to Whittle (1951) described in detail in Beran (1994a). Whittle estimates are all $\sqrt{n}$ −consistent and have the same limit normal distribution as the Gaussian MLE. One type of Whittle estimates is the discrete form described in the frequency domain. Suppose the parametric spectral density is $f(\lambda;\theta,\sigma^2)$, where $\theta$ is an $r$ −dimensional unknown parameter vector and $\sigma^2$ is a scalar. Under $ARFIMA(p,d,p)$ specification, the vector $\theta$ is an estimate of the autoregressive, moving average and long memory coefficients. Under a fractional white noise specification, $\theta$ reduces to the long memory parameter $d$. Now suppose the periodogram,

$$I(\lambda) = \frac{1}{2\pi n}\left|\sum_{t=1}^{n} y_t e^{it\lambda}\right|^2 \qquad (2.4.1.2)$$

where $\lambda_j = \frac{2\pi j}{n}$ are the Fourier frequencies. The discrete frequency Whittle estimate, mentioned in Hannan (1973), minimised the Whittle objective function,

$$\mathcal{L}_W(\theta) = \sum_{j=1}^{n-1}\left[\log f(\lambda_j;\theta) + \frac{I(\lambda)}{f(\lambda_j;\theta)}\right] \qquad (2.4.1.3)$$

This form of Whittle estimation has many advantages. One of these is that it is based on the rapid calculation of the periodogram by means of the fast Fourier transform (FFT), even when $n$ is large. Another advantage of Whittle estimate is that their limit distribution is unchanged by many departures from Gaussianity, which means that the same rules of statistical inference can be used without worrying about Gaussianity. Thus the same relatively convenient rules of statistical inference can be used without worrying too much about the question of Gausiannity.

Alternative parametric methods are available but they are less efficient than Whittle estimation when $y_t$ is Gaussian. For example; generalised method of moments, or GMM, has been used to estimate LM models, in both time and frequency domains. However, GMM estimates are not only less efficient in the Gaussian case than the Whittle estimates, but GMM is also computationally less attractive than the Whittle estimation in (2.4.1.3). Beran (1994b) proposed M-estimators for Gaussian long-memory models, while Pai and Ravishanker (1998) use Bayesian analysis to detect changing parameters in $ARIMA$ processes. Hauser (1998) has compared the various maximum likelihood estimators on samples of size 100 using Monte Carlo methods. He concludes that the Whittle estimator with tapered data is most reliable. Hauser, Pötscher and Reschenhofer (1999) are critical of $ARFIMA$ models for estimating persistence in aggregate output. They show that over-parameterisation of an $ARMA$ model may bias the estimates of persistence downwards. In general all the above methods were applied on the stationary LM models, the case $0 < d < 0.5$.

### 2.4.2 Semiparametric Estimation, or SPE

Semiparametric estimation methods were developed to overcome some of the difficulties found in the parametric methods. These methods include the log-periodogram (LP) regression and the local Whittle (LW) estimation, also known as the Gaussian semiparametric estimation. The LP regression is longer established and was the most widely used, but it is less efficient than the LW estimate. The semiparametric estimators of the LM parameter assume the spectral density model,

$$f(\lambda) \sim |\lambda|^{-2d} g(h), \text{ as } \lambda \to 0 \qquad (2.4.2.1)$$

where $g(\cdot)$ is an even function on the Nyqvist range $[-\pi,\pi]$ that determines the short run dynamics of the stationary process $y_t$ and satisfying $0 < g(0) < \infty$.

LP regression estimator was first proposed by Geweke and Porter-Hudak (1983) and also known as GPH estimator. It is considered the first semiparametric estimation of the LM parameter, $d$ in the frequency domain. GPH is based on the characteristic pattern of the periodogram around zero frequencies, which is first estimated from the series, and its logarithm is regressed on the logarithm of a trigonometric function of frequency. This method of estimation has been used comprehensively in macroeconomic and financial time series application because it is easy to implement even before development of any satisfactory theoretical analysis of its asymptotic distributional properties. The performance of this estimator, however, has several drawbacks; one of which concerns the number of values of the periodogram to be used in the regression. Geweke and Porter-Hudak (1983) proposed a heuristic based on the length of the time series. Agiakloglou, Newbold and Wohar (1992) shows that $GPH$ is biased in the presence of strongly autoregressive short memory and in addition does not possess satisfactory asymptotic properties.

Robinson (1995a) has further refined the $GPH$ log-periodogram regression. Using the same notation as $GPH$, the estimator is based on the least-square regression using spectral ordinates $\lambda_1, \lambda_2, \ldots, \lambda_m$ from the periodogram of $y_t$, $I_y(\lambda_j)$, and $j = 1, 2, \ldots, m$, where $m$, a bandwidth or smoothing number, is less than $n$ but is regarded as increasing slowly with $n$ in asymptotic theory.

$$\log[I_y(\lambda_j)] = a + b\log(\lambda_j) + v_j \qquad (2.4.2.2)$$

where $v_j$ is assumed to be $i.i.d$. The least square estimator $\hat{b}$, which gives $\hat{d} = -\frac{1}{2}\hat{b}$, is asymptotically normal and the corresponding theoretical standard error is $\pi(24m)^{-\frac{1}{2}}$. This version is easier to use for actual computation. The value of the estimator $\hat{d}$ depends on the choice of truncation parameter $m$. Diebold and Inoue (2001) showed that the choice of a large value for $m$ would result in reducing standard error at the expense of biasness in the estimator, as the relationship that the $GPH$ regression is based on holds only at low frequencies. On the other hand, consistency requires that $m$ grows with sample size, but at a slower rate. They adapt the rule of thumb of $m = \sqrt{T}$, where $T$ is the number of observations. Additionally, Wright (2000) develops log-periodogram estimators with conditional heavy tails, while Henry (2001) introduced a periodogram spectral estimation for the case of long memory conditional heteroscedasticity.

There are a plethora of new *SPEs* of the long memory parameter that are more efficient and robust, for example, Kunsch (1987), Robinson (1994b), Lobato and Robinson (1996), Moulines and Soulier (1999), Phillips and Shimotsu (2006) and Phillips (2007). Nevertheless, the most widely used and preferred $SPE$ is the local Whittle estimation proposed by Robinson (1995b), and was further investigated by Dalla, Giraitis, and Hidalgo (2004) and Phillips and Shimotsu (2006), where the objective function is a discrete form of an approximate frequency domain Gaussian likelihood, averaged over a neighbourhood of zero frequency,

$$L(d, C) = \sum_{j=1}^{m}\left[\log(C\lambda_j^{-2d}) + \frac{I(\lambda_j)}{C\lambda_j^{-2d}}\right] \qquad (2.4.2.3)$$

where $m$ is a bandwidth (see, Kunsch (1987)). The argument requires $m$ to be of smaller order than $n$. It is inadvisable to choose $m$ too large as bias can then result. However the longer the series length $n$, the larger we can choose $m$, so that in very long series the extra robustness gained by the semiparameteric approach may be worthwhile. LW estimator is

shown to be asymptotically normal and more efficient than previous estimators. $SPE$ also includes data differing and data tapering methods. Phillips and Shimotsu (2004) propose variant of the local Whittle estimation procedure that does not rely on differencing or tapering and they further extend the range where the estimator of $d$ has standard asymptotic.

## 3 Methodology

This section introduces the methodology used to estimate the long memory parameter using parametric and semiparametric methods. The parametric estimation, analysed in the time domain, is based on the likelihood function as mentioned before in section 1.2.4. The parametric estimation used in this paper is the Exact Maximum Likelihood (EML) estimation method developed by Sowell (1992). On the other hand, the semiparametric estimation for the memory parameter is based on frequency domain. Two semiparametric estimators will be considered in this section, the GPH log-periodogram regression and the local Whittle estimator.

### 3.1 The Exact Maximum Likelihood (EML)

Consider the following $ARFIMA(p,d,q)$ process,

$$\Phi(L)(1-L)^d y_t = \Psi(L)\varepsilon_t$$

where $\Phi(L)$ and $\Psi(L)$ are the polynomials

$$\Phi(L) = 1 - \sum_{j=1}^{p} \phi_j L^j$$

and

$$\Psi(L) = 1 + \sum_{j=1}^{q} \psi_j L^j$$

involving autoregressive and moving average coefficients of order $p$ and $q$ respectively and $\varepsilon_t$ is a white noise process. Now assume $Y = (Y_1, \dots, Y_T)'$ follows a normal distribution with $Y \sim N(0, \Sigma)$. The EML procedure allows for simultaneous estimation of both the long memory parameter and ARMA parameters. The maximum likelihood objective function is expressed as,

$$l_E(\Phi, \Psi, d; Y) = -\frac{T}{2}\log|\Sigma| - \frac{1}{2}Y'\Sigma^{-1}Y \qquad (3.1.1)$$

As a result, the EML estimator of $d$ can be derived as,

$$\hat{d}_{EML} = \arg\max\left[-\frac{T}{2}\log|\Sigma| - \frac{1}{2}Y'\Sigma^{-1}Y\right] \qquad (3.1.2)$$

This estimator can be inconsistent if the AR and MA orders of the ARFIMA model are misspecified, like all maximum likelihood. The ARFIMA model's EML estimate in the OxMetrics 6 package was used to estimate the long memory parameter (see Doornik and Ooms, 2003).

### 3.2 GPH Log-periodogram Regression

This and the next subsections investigate the main semiparametric methods applied to the Egyptian stock market to estimate the long memory parameter $d$ in the frequency domain. The semiparametric estimation used in this paper is carried out in the Time Series Modelling (TSM) 4.32. These methods are not a recommended substitute for maximum likelihood estimation of an $ARFIMA(p,d,q)$ model if there is confidence that the ARMA components

are correctly specified, but they impose fewer assumptions about the short-run. The assumption is that the spectrum of the process takes the form

$$f(\lambda) = \left|1 - e^{-i\lambda}\right|^{-2d} f^*(\lambda) \qquad (3.2.1)$$

where $f^*$ represents the spectral density of an $ARMA(p,q)$; and hence, the short-range component of the dependence. This is assumed smooth in the neighbourhood of the origin, with $f^*(0)' = 0$. Note the alternative representation

$$f(\lambda) = \lambda^{-2d} g(\lambda) \qquad (3.2.2)$$

where $g$ is likewise assumed smooth at the origin with $g(0)' = 0$.

Equation (3.2.1) is a semiparametric model, where the long memory parameter, $d$, is parametrically specified in the frequency domain; on the other hand, the short memory component represented in $f^*(\lambda)$ is not required to obey any parametric model. The two semiparametric estimators discussed in this paper are the GPH and LW estimators. The log-periodogram estimator (i.e. GPH) minimise some distance between the periodogram and the spectral density function at low frequencies represented by the first $m$ Fourier frequencies, $\lambda_j = \frac{2\pi j}{T}$, $j = 1, \dots, m \ll [\frac{T}{2}]$. Estimation is usually between a set frequency band $(0,m]$ to capture the long run component $f(\lambda) = \lambda^{-2d} g(\lambda)$ whilst the remainder of the frequencies capture the local variations.

This method is based on the periodogram of the time series defined by

$$I(\lambda) = \frac{1}{2\pi T} \left|\sum_{t=1}^{T} e^{it\lambda}(y_t - \bar{y})\right|^2 \qquad (3.2.3)$$

A series with long memory has a spectral density proportional to $\lambda^{-2d}$ close to the orıgın (2.4). Assuming the fact that the spectral density of a stationary process can be formulated as $f(\lambda) = f_0(\lambda) 4\sin^2(\frac{\lambda}{2})$ we may consider a regression of the logarithm should give a coefficient of $-2d$. The GPH estimator is based on the log linearization of the periodogram as follows,

$$\log\{I(\lambda_j)\} = C - d\log\left\{4\sin^2\left(\frac{\lambda_j}{2}\right)\right\} + \varepsilon \qquad (3.2.4)$$

The memory parameter is estimated

$$\hat{d}_{GPH} = -\frac{\sum_{j=1}^{m}(y_j - \bar{y})\log\{I(\lambda_j)\}}{2\sum_{j=1}^{m}(y_j - \bar{y})} \qquad (3.2.5)$$

We consider only harmonic frequencies $\lambda_j = \frac{2\pi j}{T}$, $j \in (l, m]$, where $l$ is a trir discarding the lowest frequencies and $m$ is a bandwidth parameter. A necessary condition for consistency which depends on the bandwidth is that $\frac{m}{T} \to 0$ as $T \to \infty$.

### 3.3 Local Whittle Estimation

Kunsch (1987) proposed a local Whittle (LW) estimator and then developed by Robinson (1995). This estimator represents approximately a MLE in the frequency domain, since for larger $T$

$$I(\lambda_j) \sim e^{f(\lambda_j)^{-1}}$$

As a result, the likelihood function is, (3.3.1)

$$L\{I(\lambda_j), \dots, I(\lambda_m), \theta\} = \prod_{j=1}^{m} \frac{1}{f_\theta(\lambda_j)} e^{-I(\lambda_j) f(\lambda_j)^{-1}} \qquad (3.3.2)$$

where $\theta = (C, d)$ is the parameter vector. The log-likelihood function becomes,

$$l(\theta) = \sum_{j=1}^{m} \left[ -\log f_\theta(\lambda_j) - \frac{I(\lambda_j)}{f_\theta(\lambda_j)} \right] \quad (3.3.3)$$

In the neighbourhood of zero frequency we obtain,

$$L(d, C) = \sum_{j=1}^{m} \left[ \log C - 2d \log(\lambda_j) + \frac{I(\lambda_j)}{C\lambda_j^{-2d}} \right] \quad (3.3.4)$$

$$\frac{\partial l(C,d)}{\partial C} = \sum_{j=1}^{m} \left[ \frac{1}{C} + \frac{I(\lambda_j)}{C^2 \lambda_j^{-2d}} \right] \quad (3.3.5)$$

yielding

$$\hat{C} = m^{-1} \sum_{j=1}^{m} \left[ \frac{I(\lambda_j)}{\lambda_j^{-2d}} \right] \quad (3.3.6)$$

Inserting $\hat{C}$ for $C$ in (1.3.3.4) and by minimisation, the local Whittle estimator can be written as,

$$\hat{d}_{LW} = \arg\min \left( \log \left[ m^{-1} \sum_{j=1}^{m} \left[ \frac{I(\lambda_j)}{\lambda_j^{-2d}} \right] \right] - 2dm^{-1} \sum_{j=1}^{m} \log(\lambda_j) \right) \quad (3.3.7)$$

Robinson (1995) showed the LW estimator is consistent for $d \in (-0.5, 0.5)$. However, its consistency depends on the bandwidth $m$, which satisfy $\frac{1}{m} + \frac{m}{T} \to 0$ as $T \to \infty$. The LW estimator is more attractive due to its nice asymptotic properties, the mild assumptions underlying it and the likelihood interpretation. Robinson (1995) also showed that

$$\sqrt{m}(\hat{d}_{LW} - d) \to N(0, \tfrac{1}{4})$$

## 4 Data and Empirical Results

To analyse the Egyptian stock market, the daily EGX30 stock index traded on Cairo Stock Exchange has been used in this paper. The data covers the period from the first transaction, 01 January 1998 to 09 May 2010 for a total of 3,050 observations. The EGX30 Price Index includes the top 30 companies in terms of liquidity and activity in Egypt. It is weighted by market capitalisation adjusted by the free float. This stock index can be considered as a proxy for the Egyptian stock market. The period under analysis is of major importance because it starts with the revival of the stock market after major changes in political and economic reforms in 1990s and before the January Revolution in 2011. All subsequent analysis is done on the daily return series (see Figure 4 for the daily stock returns) by taking the natural logarithmic first-difference on EGX30 price index (see Figure 1),

$$r_t = \ln\left(\frac{P_t}{P_{t-1}}\right) = \ln P_t - \ln P_{t-1}$$

where $P_t$ denotes the stock index in day $t$.

**Figure 1:** The EGX30 daily stock index

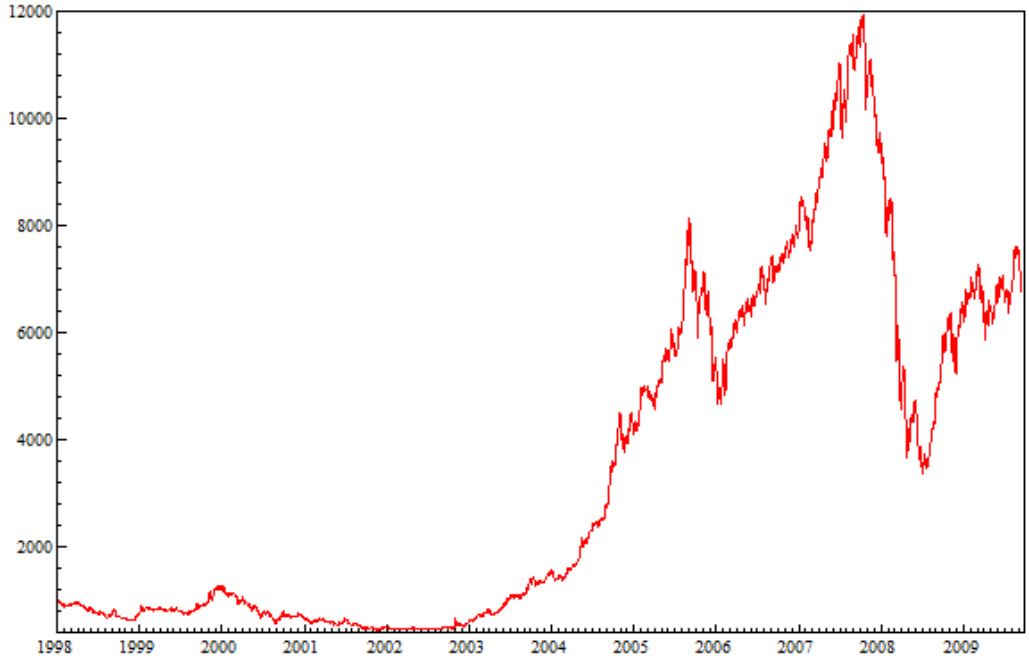



**Figure 2:** The periodogram of the daily EGX30 index

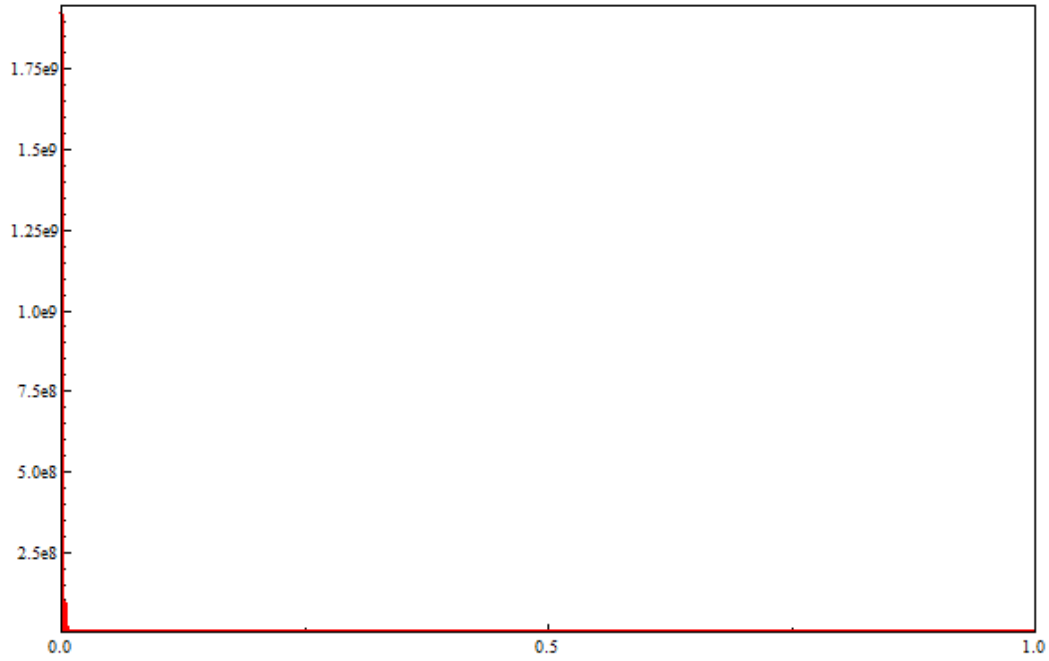


**Figure 3:** The correlogram of the daily EGX30 index

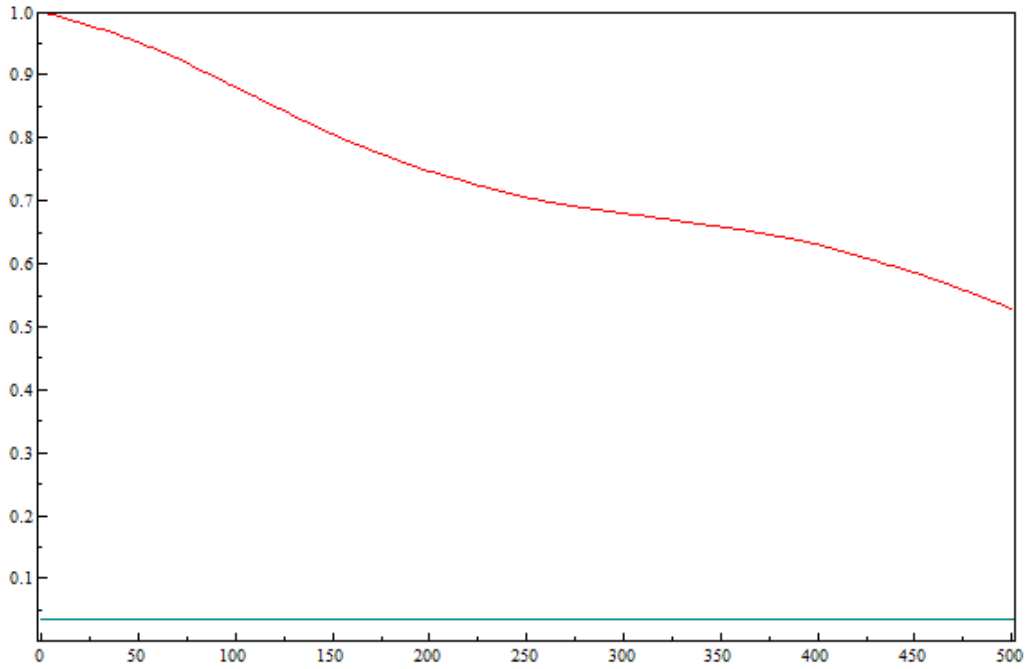



**Figure 4:** The EGX30 daily returns

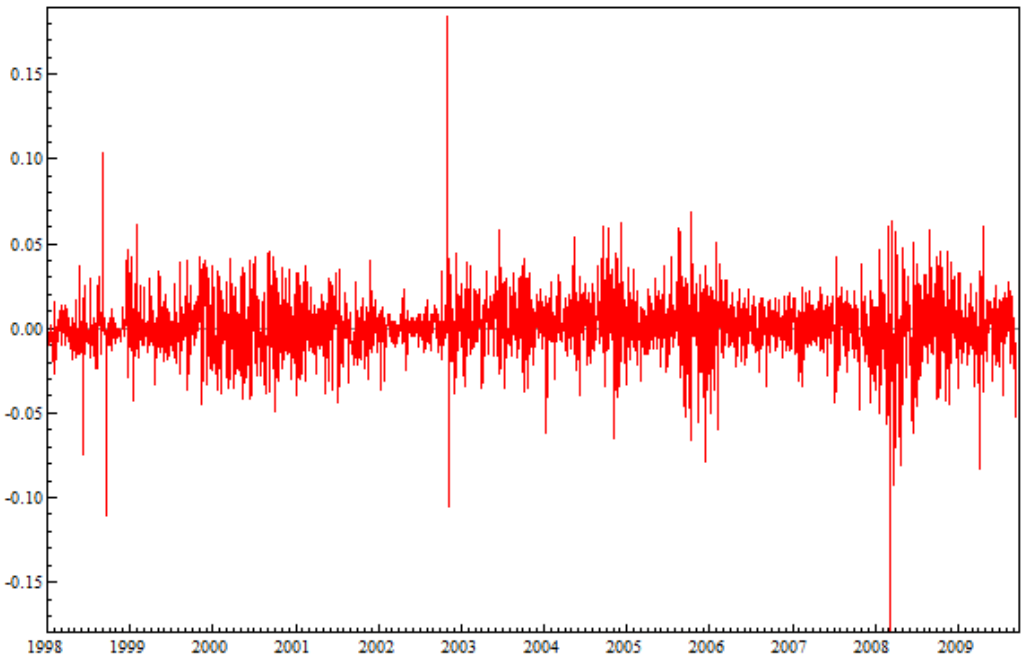


**Figure 5:** The periodogram of the daily EGX30 returns series

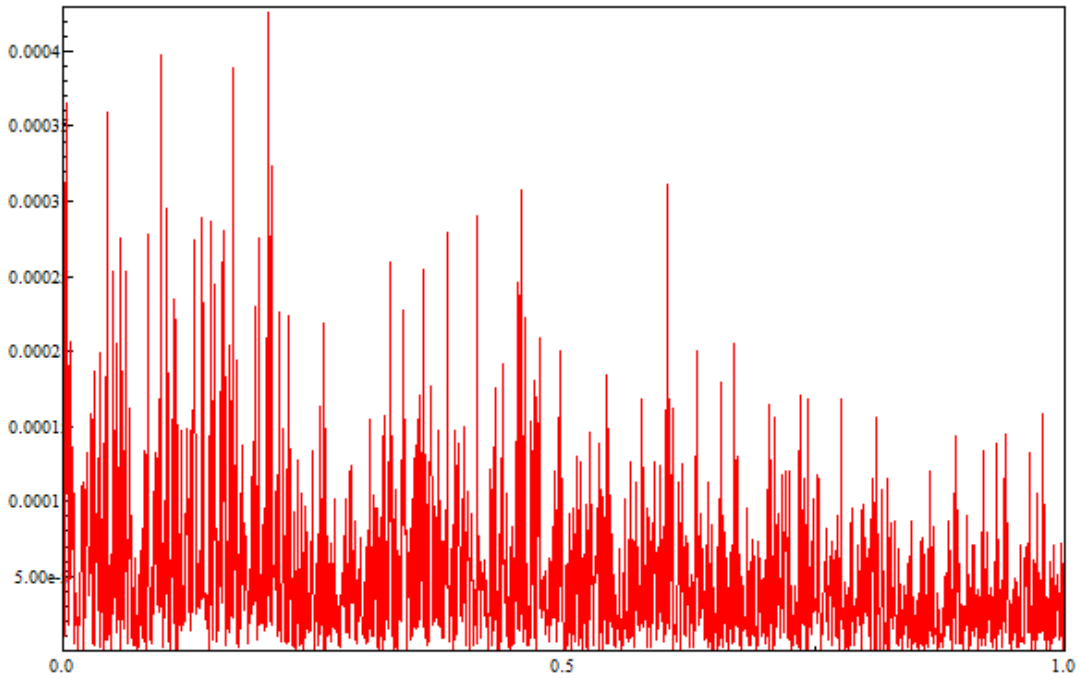



**Figure 6:** The correlogram of the daily EGX30 returns series

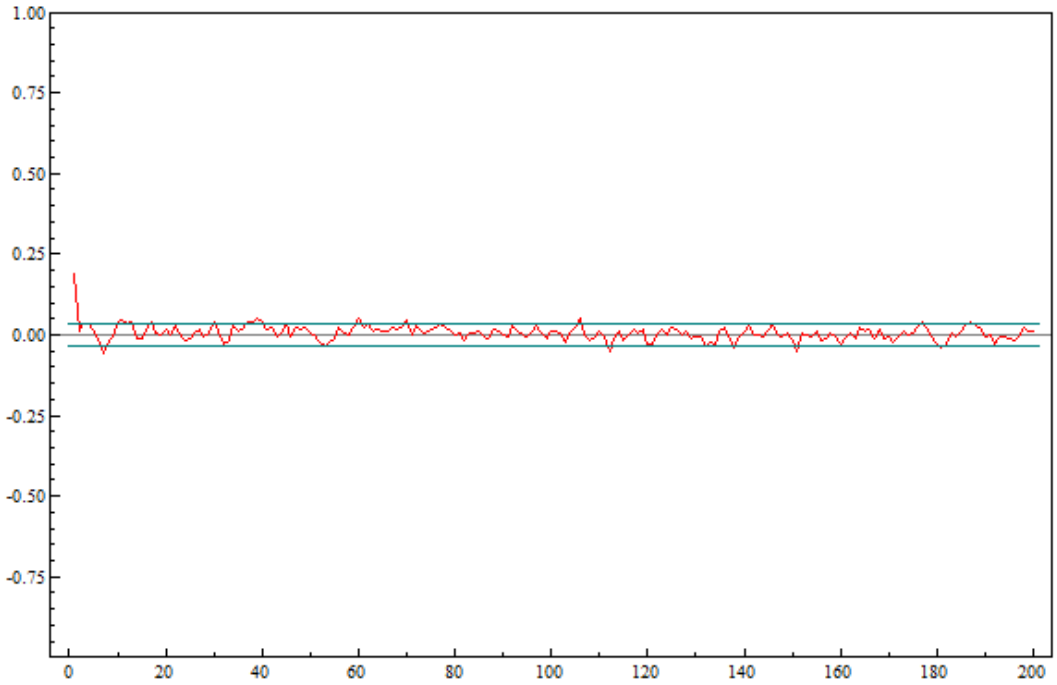


Figure 1 displays plots of the original EGX30 price index series where a nonstationary appearance can be observed. This can be also confirmed through its corresponding periodogram where large values (a large peak) are observed around the zero frequency and also across the correlogram with values decaying very slowly. Plots of the EGX30 daily return series data, with its corresponding periodogram and correlogram are displayed in Figures 4, 5 and 6. The return series may now be stationary (see figure 4). Based on the shapes of the corresponding periodogram and correlogram, the series may be over-differenced suggesting a presence of long memory. Dominant peak areas occurs around low frequencies (figure 5) and the correlogram declines steadily but very slowly and remains positive for many lags, indicating the presence of stationary long memory component (figure 6).

**Table 1:** Descriptive Statistics of EGX30 daily returns (1 Jan. 1989- 9 May 2010)

| Obs. | Mean | S.D. | Skewness | Kurtosis |
|---|---|---|---|---|
| 3049 | 0.0006 | 0.0179 | -0.248 | 12.337 |
| Min. | Max. | Jarque-Bera | ADF | KPSS |
| -0.179 | 0.183 | 11107 | -21.25 | 0.3845 |

Note: The critical values of ADF unit root tests are -2.54, -1.95, -1.61 at 1%, 5%, 10% levels of significance.

Table 1 displays the descriptive statistics for the EGY30 daily returns over the full sample. The sample mean return is positive and very close to zero. There are significant departures from normality as the returns series is negatively skewed possibly due to the large negative returns associated with the financial crisis of 2007- 2009. The unconditional distribution is peaked with fat tails. The data also display a high degree of kurtosis. Such skewness and kurtosis are common features in asset return distributions, which are repeatedly found to be leptokurtic. The data also fail to satisfy the null hypothesis of normality of the Bera-Jarque at the 1% level. Table 1 also includes the implementation of ADF and the KPSS tests. The ADF test shows evidence of non-stationary. The results of the ADF unit root test indicate that the return series are stationary by rejecting the null hypothesis of $I(1)$ at 1% level. For the KPSS test, the critical values are 0.739, 0.463 and 0.347 corresponding to the 1%, 5% and 10% level respectively. The null hypothesis of $I(0)$ against long memory alternatives is rejected (KPSS = 0.38) at the 10% level suggesting that the long memory process can be appropriate representation for the return series.

**Figure 7:** The distribution of the daily EGX30 returns series

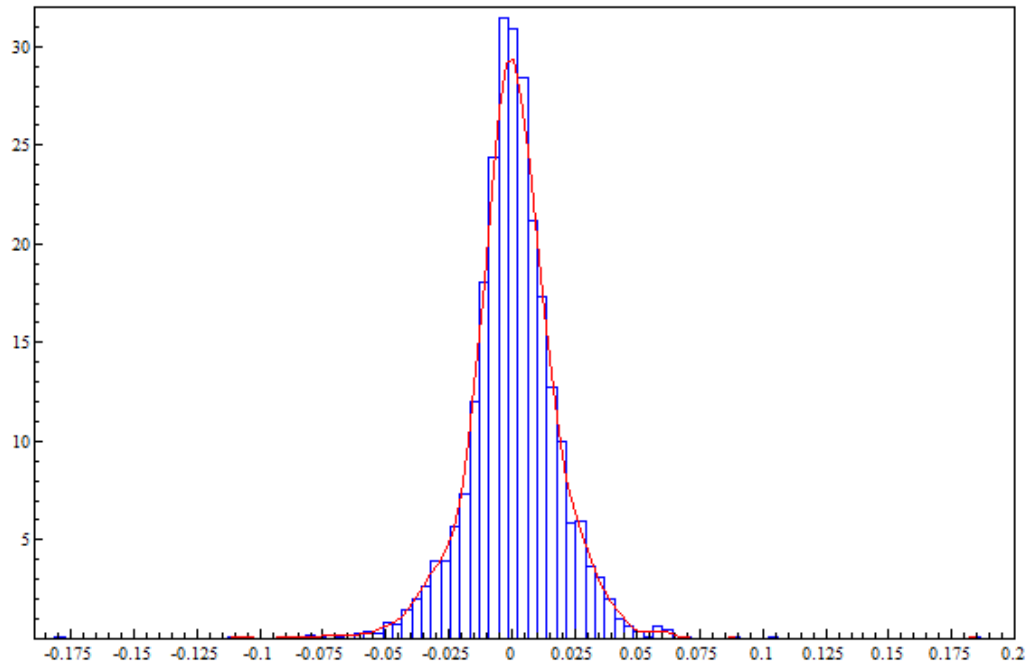



**Figure 8:** The QQ plot of the daily EGX30 returns series

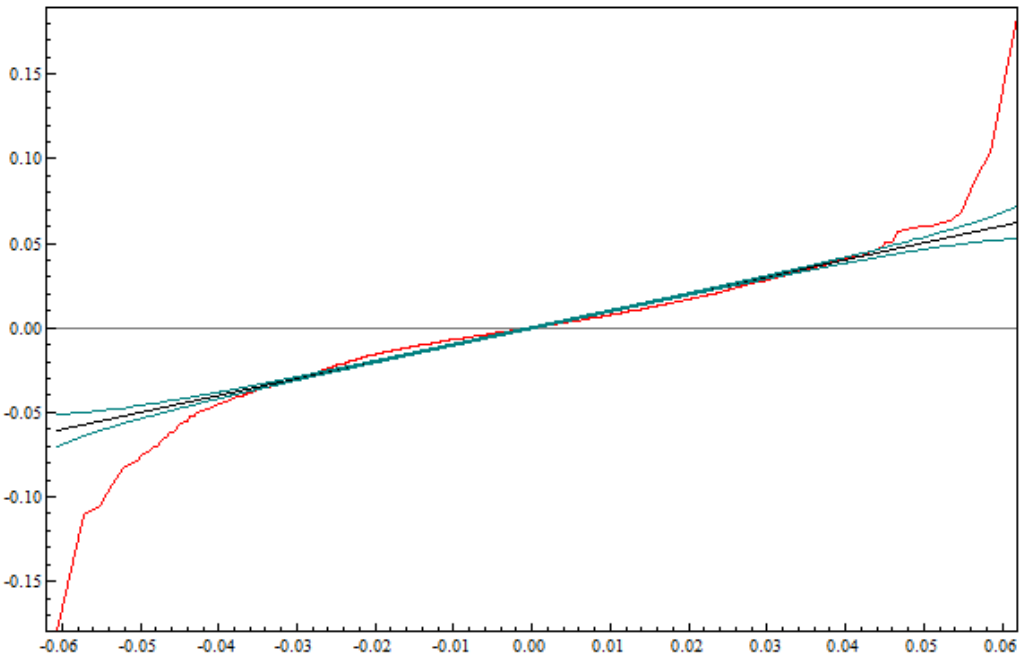


**Table 2:** ML estimation of ARFIMA models using EXG30 returns

| $p$ | $q$ | $\hat{d}$ | S.E. | AIC |
|---|---|---|---|---|
| 0 | 1 | 0.035 | 0.019 | -5.232 |
| 1 | 0 | 0.031 | 0.023 | -5.231 |
| 1 | 1 | 0.055 | 0.023 | -5.232 |
| 0 | 2 | 0.061 | 0.026 | -5.233 |
| 1 | 2 | 0.058 | 0.027 | -5.232 |
| 2 | 0 | 0.068 | 0.028 | -5.232 |
| 2 | 1 | 0.070 | 0.029 | -5.231 |
| 2 | 2 | 0.063 | 00.30 | -5.231 |
| 3 | 0 | 0.051 | 0.032 | -5.232 |
| 3 | 1 | 0.046 | 0.034 | -5.231 |
| 3 | 2 | 0.275** | 0.109 | -5.232 |
| 0 | 3 | 0.056 | 0.030 | -5.232 |
| 1 | 3 | 0.049 | 0.035 | -5.231 |
| 2 | 3 | 0.259** | 0.107 | -5.232 |
| 3 | 3 | 0.031 | 0.032 | -5.235 |
| 0 | 4 | 0.043 | 0.033 | -5.231 |
| 1 | 4 | 0.255** | 0.108 | -5.232 |
| 2 | 4 | 0.032 | 0.029 | -5.235 |
| 3 | 4 | 0.174** | 0.078 | -5.232 |
| 4 | 0 | 0.044 | 0.035 | -5.231 |
| 4 | 1 | 0.258** | 0.123 | -5.234 |
| 4 | 2 | 0.054 | 0.027 | -5.231 |
| 4 | 3 | 0.186 | 0.081 | -5.232 |
| 4 | 4 | 0.416* | 0.062 | -5.236 |

Note: * and ** indicate statistical significance at the 1% and 5% levels respectively.

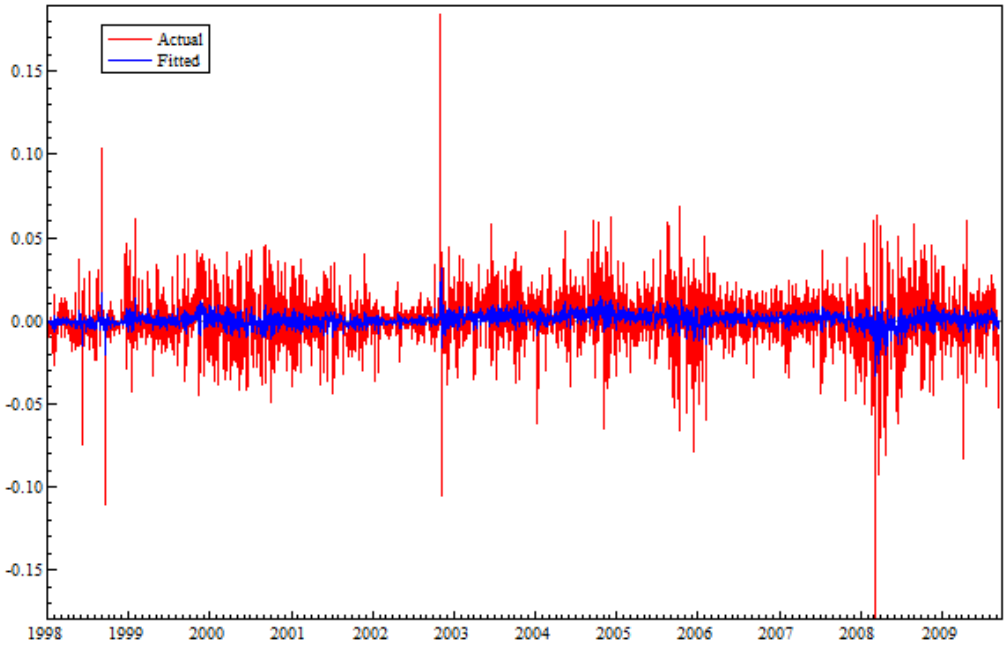
**Table 3:** ML estimation of ARFIMA$(4, d, 4)$ model

| | Coefficient | S.E. | t-value | p-value |
|---|---|---|---|---|
| $\hat{\mu}$ | 0.0003 | 0.035 | 0.09 | 0.925 |
| $\hat{d}$ | 0.461 | 0.062 | 6.67 | 0.000 |
| $\hat{\phi}_1$ | 0.721 | 0.055 | 9.76 | 0.000 |
| $\hat{\phi}_2$ | 0.168 | 0.061 | 3.10 | 0.002 |
| $\hat{\phi}_3$ | -0.933 | 0.058 | -15.9 | 0.000 |
| $\hat{\phi}_4$ | 0.616 | 0.068 | 9.65 | 0.000 |
| $\hat{\psi}_1$ | -0.965 | 0.070 | -15.3 | 0.000 |
| $\hat{\psi}_2$ | -0.159 | 0.063 | -2.92 | 0.004 |
| $\hat{\psi}_3$ | 0.101 | 0.051 | 16.2 | 0.000 |
| $\hat{\psi}_4$ | -0.785 | 0.046 | -15.6 | 0.000 |

Note: All estimators are statistically significant at the 1% level.

The parametric estimation for the returns series were derived by means of the Exact MLE of the OxMetrics 6 ARFIMA package, while the TSM modelling was used to obtain the long memory estimates via semiparametric methods. The ARFIMA model's Exact MLE (Maximum Likelihood Estimate) in the OxMetrics 6 package was used (see Doornik and Ooms, 2003). The models with different orders are estimated for ARFIMA $(p, d, q)$. Table 2 show the results from various ARFIMA models with different specifications where $p + q$ equals and less than 4. The model is selected based on the Akaike's information criterion (AIC) and log likelihood values. The selected ARFIMA model is ARFIMA$(4, d, 4)$[5]. The estimated results show that the memory parameter is 0.41. The evidence of long memory property can be found in the model estimation where the long memory parameter is statistically significant at 1% level (see table 3). Hence, the EGX30 returns series exhibit long memory features. ARFIMA $(4, 0.41, 4)$ model is fitted to the data to capture the long memory characteristics of the returns series as in Figure 9.

**Figure 9:** The fitted ARFIMA$(4, 0.41, 4)$ model




Moreover, the presence of the long memory properties in the Egyptian stock market suggests that ARFIMA models can improve forecasting performance by providing very reliable out-of-sample forecasts for both the long memory and the short run dynamic properties of the return series. The Egyptian stock returns series is forecast by using the ARFIMA model fitted to the EGX30 returns series according to the AIC. This forecast should significantly outperform any others using standard linear models. The period 9 April 2010 to 9 May 2010 is used for out of-sample forecasting. Table 4 reports the ex-ant forecasting performance for the EGX30 returns series (see figure 10).

[5] If the lag polynomials for AR and MA have common roots, a more economical ARMA $(p-1, q-1)$ model suffices and hence written as a lower-order process. Unique roots were found and are either real or in complex conjugate pairs. The $\phi$ 's roots are outside the unit circle, while the $\psi$'s roots are inside the unit circle. So, it is an ARMA (4, 4). Alternatively, a purely autoregressive process can be considered which may typically require a higher number of parameters.

**Table 4:** Out-of-sample Forecasting Performance for the daily EGX30 Returns

| Forecasting Horizon | 1 | 5 | 10 | 15 | 30 |
|---|---|---|---|---|---|
| RMSE | 0.0622 | 0.1145 | 0.1899 | 0.2536 | 0.4019 |
| MAD | 0.0597 | 0.0871 | 0.1540 | 0.2321 | 0.3358 |

Note: The out-of-sample period is from 9 April 2010 to 9 May 2010. The forecasting horizon is reported in *k*-steps ahead. The RMSE stands for the root mean square error, while the MAD is the mean absolute deviation.

**Figure 10:** The ARFIMA$(4, 0.41, 4)$ model forecast

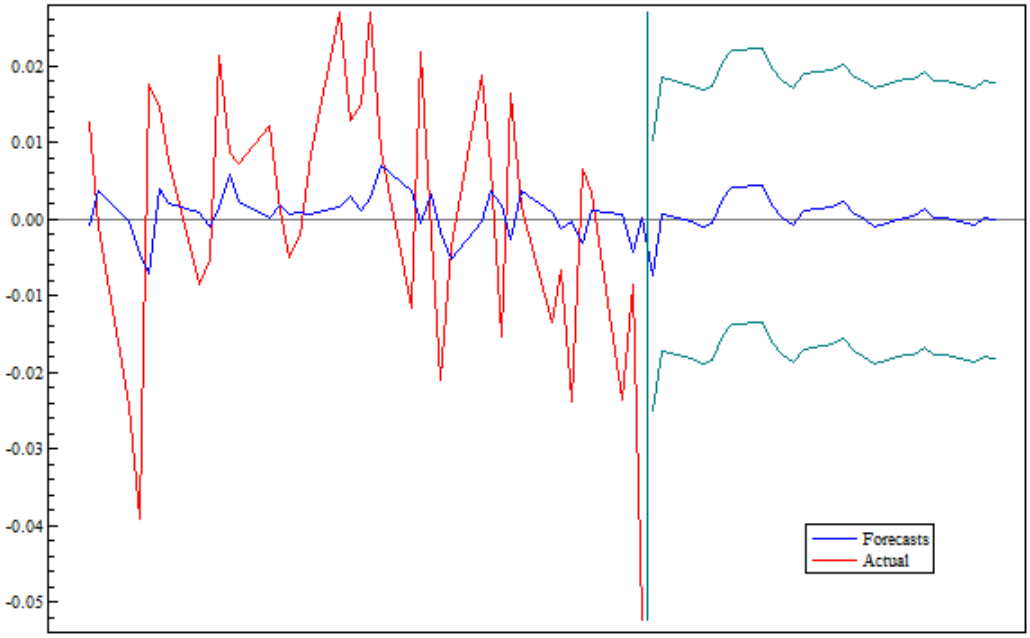



**Table 5:** Semiparametric estimates of $d$ for returns

| | Bandwidth | | | |
|---|---|---|---|---|
| | $m = n^{0.45}$ | $m = n^{0.5}$ | $m = n^{0.55}$ | $m = n^{0.6}$ |
| $\hat{d}_{GPH}$ | 0.2644 | 0.1981 | 0.1979 | 0.1838 |
| | (0.1212) | (0.0954) | (0.0787) | (0.0631) |
| $\hat{d}_{LW}$ | 0.2536 | 0.1758 | 0.1346 | 0.1290 |
| | (0.0833) | (0.0680) | (0.0559) | (0.0458) |

Note: The standard errors are provided in parentheses.

Table 5 reports the semiparametric estimates of the long memory parameter $d$ for the two estimators GPH and LW. The conventional setting of the bandwidth to be equal to the square root of the same size ($m = n^{0.5}$) was adopted. Moreover, $d$ estimates were reported for

different bandwidths $m = n^{0.45}$, $n^{0.55}$ and $n^{0.6}$ in order to evaluate the sensitivity of the results to the choice of the bandwidth. The results are not too sensitive to the bandwidth. Looking at the returns series, both estimators present similar results which show the existence of long memory features. The estimated $d$ values range between 0.1 and 0.3, which is the property of stationary long memory processes. All the estimates of $d$ are significantly positive at the 1% level. This result, due to semiparametric techniques, confirms the presence of long memory in the Egyptian stock returns as that of the parametric method.

## 5 Concluding Remarks

This paper applied the parameter and semiparametric techniques to examine the long memory property in the daily Egyptian stock market returns. The exact maximum likelihood estimation was employed as a parametric method in the time domain to estimate the ARFIMA model, while two semiparametric methods were used to estimate the memory parameter in the frequency domain. The results from the ARFIMA model show evidence of long memory in the EGX30 returns. The results were also confirmed using the semiparametric methods. Both techniques provide strong evidence of long range dependence in the EGX30 returns. This implies that price movements in the Egyptian stock market appear to be related and affected by past and remote observations. The paper's findings suggest that long memory plays an important role in the structure and the dynamic behaviour of the Egyptian stock market returns and hence, influence the investment strategies involving multinational equities portfolios. Moreover, the presence of long memory in the Egyptian stock market may suggest constructing nonlinear econometric models, such as ARFIMA, for improved and more efficient price forecasting performance.

Furthermore, long memory in returns is not consistent with market efficiency. This market inefficiency in the Egyptian stock market can be attributed to the high persistence of risk factors in the market or due to the lack of liquidity. Accordingly, investors can exploit such inefficiency to earn excess returns. In addition, regulators should analyse the sources of the persistence in the Egyptian stock market that takes the form of long memory in order to improve its efficiency.

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