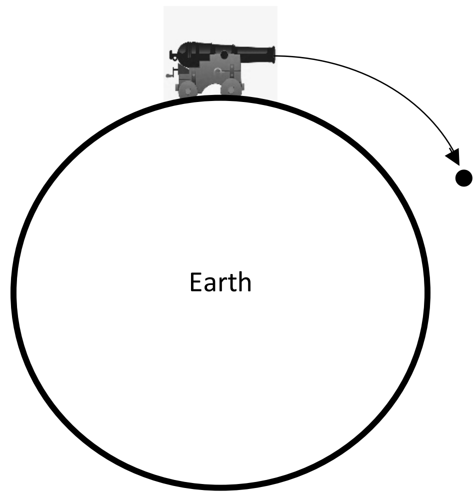
Gravity – Part 1

We need to get a more comprehensive grasp of how gravity works, and the best way to start is with

Sir Isaac Newton.

Recall his thought experiment with the cannon:



Recall also that the upshot was that Newton sought to establish a link between mass and gravity, hence the Universal Gravitational Constant ("G") (6.672×10^{-11} Kg^2/N^2), and he applied it to Kepler's Third Law

Kepler's Third Law:

$P^2 = a^3$

Where:

p = the period (time) of an orbit

a = the average radius of the orbit

Newton's version of Kepler's Third Law

$p^2 = \frac{4\pi^2}{Gm} \times a^3$, which can also be written as $p^2 = \frac{4\pi^2 a^3}{Gm}$

Where **"G"** is the Universal Gravitational Constant and **"m"** is the mass of the primary (the object being orbited)

Here Newton has established the relationship between mass, gravity, and orbits.

That's background we've already covered, but we need to go further. To do that, we first need to be clear on a few concepts:

Part 2: Mass vs weight – *they're not the same!*

Mass:

Newton defines "mass" two ways:

1. A quantity of material
2. A measure of inertia   ("Inertia": Resistance to a change in motion)

Weight:

Weight is "force" (force is defined as "a push or a pull"). Another way to define force: "That which may affect motion."
How do we get "weight"? Gravity!
Here's what Newton says:

F=ma - the second most famous equation in the world!

Where "**F**" is force, "**m**" is mass, and "**a**" is "acceleration", which ***can*** be due to gravity or "a_g". We'll get to the "acceleration" part a little later.

Confused yet? Let's try this:

Imagine floating in space. Don't worry, we gave you a space suit. As you float in space, you are "weightless," but you still have mass. Your carcass is still "a quantity of material." You still have all the atoms and molecules that make up you.

You look to your left, and you see a basketball floating there. If you push the basketball, you may expect the basketball to move away while you stay relatively still, and you'd be right. The ball has less material, less inertia.

But then you look to your right, and you see a US Navy aircraft carrier (also known by Navy types as "100 thousand tons of diplomacy") floating beside you. You push against the barnacle-encrusted hull, and *you* move away – because it's got tons more material than you and hence inertia. Yet all three objects – you, the ball, the aircraft carrier – are **weight**less.

Acceleration:

Defined as "a change in velocity over time." This one can be a little slippery.

If you increase your speed from 65 MPH to 75 MPH over the course of 5 seconds, then your acceleration is "+ 2 miles per hour per second" – your speed increases by 2 MPH every second.

$$\boxed{a = \frac{75\,MPH - 65\,MPH}{5\,sec})}$$

If you hit the brakes and go from 65 MPH to 35 MPH in 5 seconds – a 30 MPH decrease over the course of 5 seconds - then your acceleration is " – 6 MPH per sec"; your speed has decreased by 6 MPH every second.

$$\boxed{a = \frac{35\,MPH - 65\,MPH}{5\,sec}}$$

Here on Earth the planet's surface gravity is such that a freefalling object accelerates at 32 feet per second per second, **written in shorthand as "32 ft/sec^2."** (The metric equivalent is **9.8 m/sec^2**)

Thus, the acceleration of gravity on Earth (a_g) is 32 ft/sec^2 and 9.8 m/sec^2.

What that means is that if you drop an object, its initial speed will be zero, and its speed will increase by 32 ft/sec for each second it drops. After 1 second, its speed is 32 ft/sec; after 2 seconds, its speed is 64 ft/sec; after 3 seconds, the speed is 96 ft/sec and so on.

Part 3: Putting it all together

Going back to **F = ma:**

Going metric, if you have 1 kilogram of mass, then on Earth that means that

F = (1)x(9.8) = 9.8 Newtons of force (weight)

If your body's mass is 5.156 slugs (the US unit of mass), then

F (your weight) = **m** (5.156 slugs) x **a** (32 ft/sec^2) = **164.992 pounds**

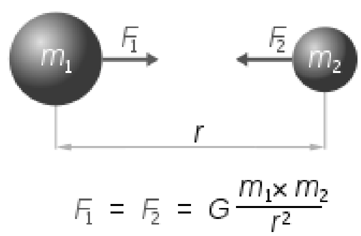
(pounds are measures of weight, not mass!)

Pretty straightforward – if you have more mass, you have more weight, and if you have more gravity (a_g), you have more weight.

Newton's Law of Gravity Formula:

$$F = G\,\frac{m_1 m_2}{r^2}$$

This describes the force of gravity (F) between two masses (m_1, m_2), and how it relates to the Universal Gravitational Constant and the distance between the two objects (r).



This will work for any two objects, such as the moon and the earth, or a neutron star orbiting a stellar black hole.

But, if all we want to figure out is the surface gravity on a given planet or moon,

then there's a hack we can use:

We'll assume that object 2 (m_2) is **on the surface of the planet** (object 1, or m_1) and of insignificant mass and size, so we eliminate it from the equation. "r" becomes simply the radius of the planet, or the distance from the surface to the center.

By doing this we are now setting **F_g numerically equal to a_g**. (Street fighting physics!)

Our simplified hack is now: $F_g = \frac{Gm}{r^2}$

Furthermore, we can rewrite this to figure out the mass of the planet: $m = \frac{Fr^2}{G}$

Ultimately, what this means is that if we can measure the acceleration of gravity on a planet's surface, and if we know the size (radius) of the planet, then we can measure its mass.

We'll do a lab on that:

We're going to measure the mass of the Earth using a string, a rock, a tape measure and a wristwatch.