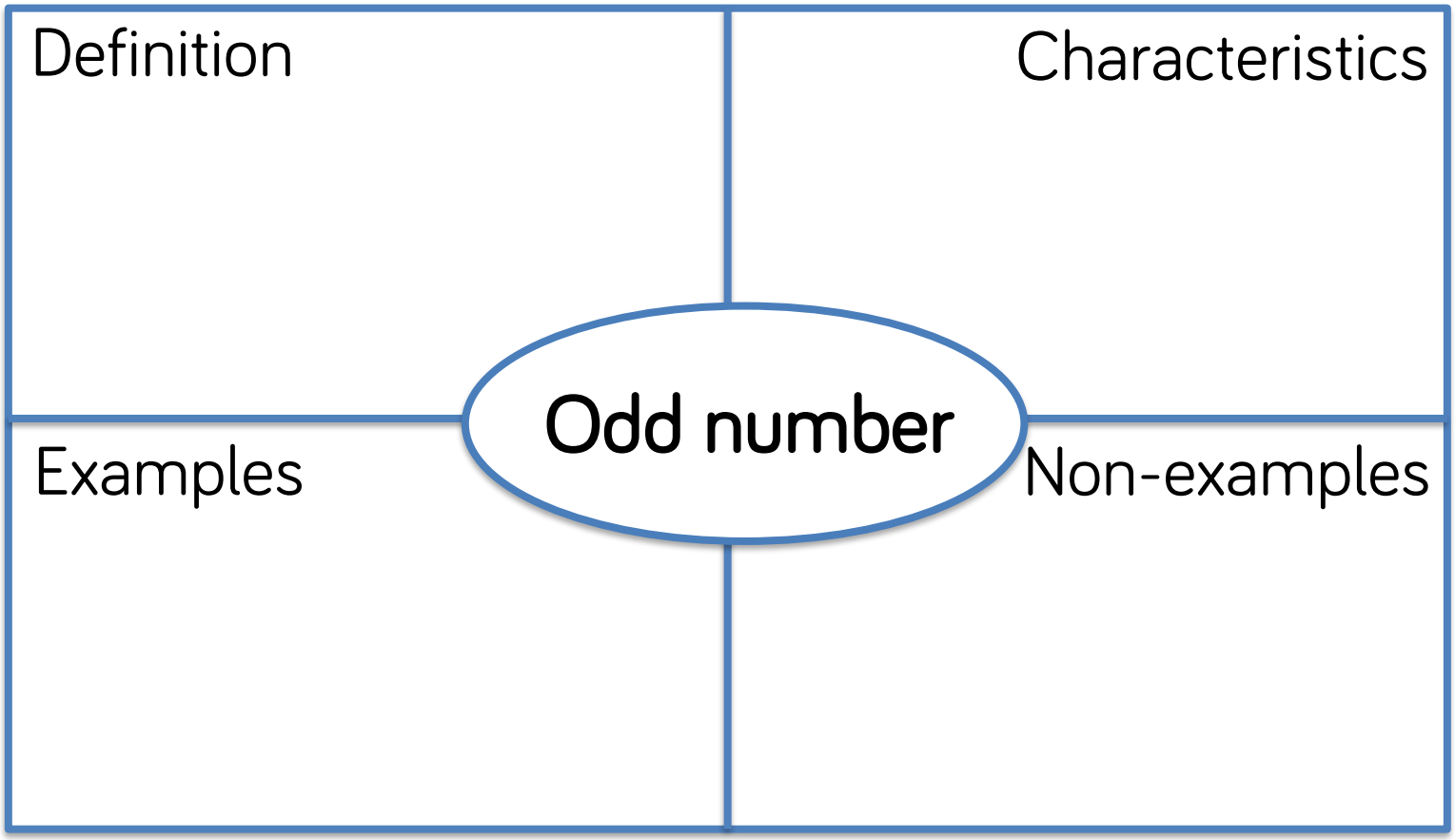
Discuss and complete



Planning for Depth

Course Lead

White Rose Maths

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Content

- Knowledge
- What is variation?
- Conceptual variation
- Procedural variation

Knowledge

Knowledge

- Factual knowledge
- Procedural knowledge
- Conceptual knowledge
- Metacognitive knowledge

Anderson and Krathwohl, (2001)

Knowledge

Factual

Knowledge that is in long term memory. Essential facts and key terminology.

Procedural

The method or procedure. The steps or rules you need to follow to get the correct answer.

Conceptual

The understanding of the meaning of what is happening.

Metacognitive

The awareness of the strategies of learning – 'thinking about thinking.'

'Students are forced into a passive relationship with their knowledge - they are taught only to follow rules and not to engage in sense-making, reasoning, or thought, acts that are critical to an effective use of mathematics.'

Boaler, J. (2015)

Knowledge

Discuss...

$$4 \div \frac{1}{2} = ?$$

Knowledge - Procedural

The KFC method…(Keep it, Flip it, Change it)

$$4 \div \frac{1}{2} = ?$$

Keep it

Change it

Flip it

$$4 \times \frac{2}{1} = 8$$

Knowledge - Factual

$$4 \div 2 = ?$$

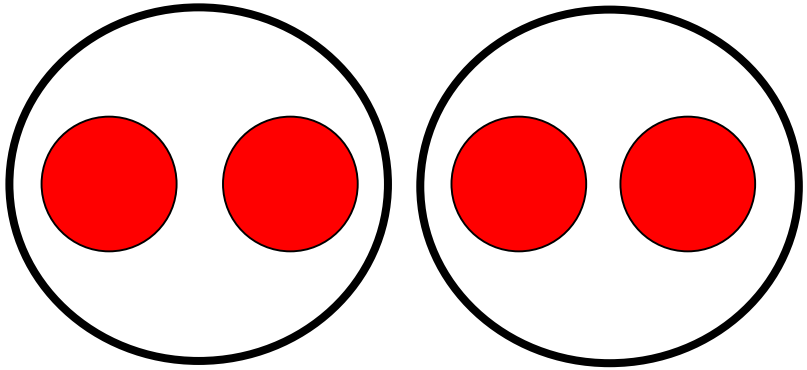
4 ***shared*** into 2 groups

4 ***grouped*** into 2s

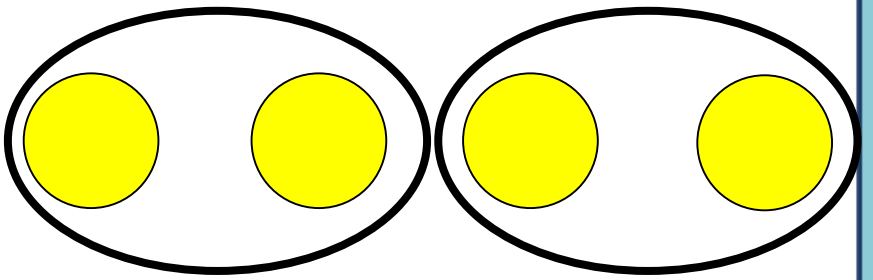
Knowledge - Conceptual

$$4 \div 2 = ?$$

4 *shared* into 2 groups



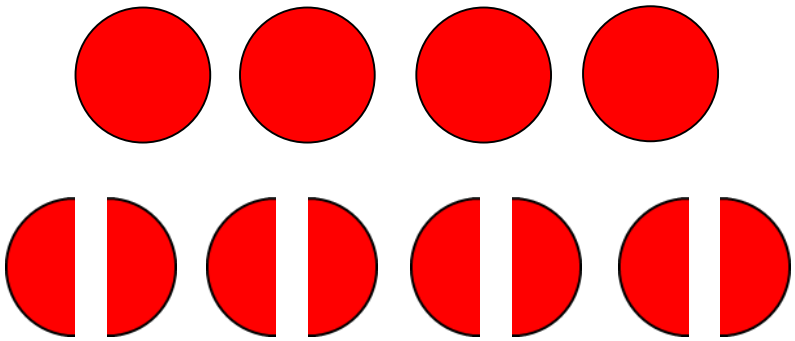
4 *grouped* into 2s



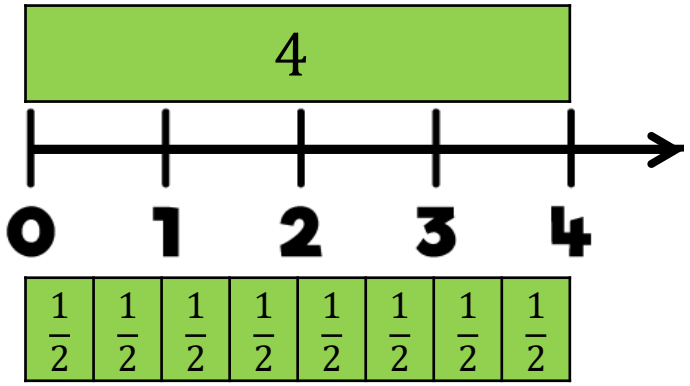
Knowledge - Conceptual

$$4 \div \frac{1}{2} = ?$$

4 *shared* into halves



4 *grouped* into halves



Knowledge

Discuss...

Why is each type of knowledge important?

Which should come first?

Knowledge

Factual

Knowledge that is in long term memory. Essential facts and key terminology.

Procedural

The method or procedure. The steps or rules you need to follow to get the correct answer.

Conceptual

The understanding of the meaning of what is happening.

Metacognitive

The awareness of the strategies of learning – 'thinking about thinking.'

Knowledge

Procedural Knowledge

'Like a toolbox, it includes facts, skills, procedures, algorithms or methods.'

Barr, Doyle et al (2003)

'Learning that involves only memorizing operations with no understanding of underlying meanings.'

Arslan (2010)

'To know how something happens in a particular way.'

Hiebert and Lefevre (1986)

Conceptual Knowledge

'Ideas, relationships, connections or having a sense of something'

Barr, Doyle et al (2003)

'Learning that involves understanding and interpreting concepts and the relationship between concepts.'

Arslan (2010)

'To know why something happens in a particular way.'

Hiebert and Lefevre (1986)

'For most topics, it does not make sense to teach concepts first or to teach procedures first; both should be taught in concert. Gaining knowledge and understanding of one supports comprehension of the other.'

Willingham, D. (2012)

What is variation?

What is variation?

There are different types of variation:

- Conceptual vs. procedural (Gu, Huang & Marton)
- Developmental vs. multi-faceted (Park & Leung)
- Explicit vs. implicit (Huang, Mok & Leung)
- Open-ended vs. closed-ended (Qingpu Curriculum Reform Experimental Group)
- Form vs. solution vs. content (Xiao)

What is variation?

Conceptual Variation

- Examples and non-examples
- Standard and non-standard

What's the same?
What's different?

Procedural Variation

- Spotting patterns
- Making connections
- Generalisations

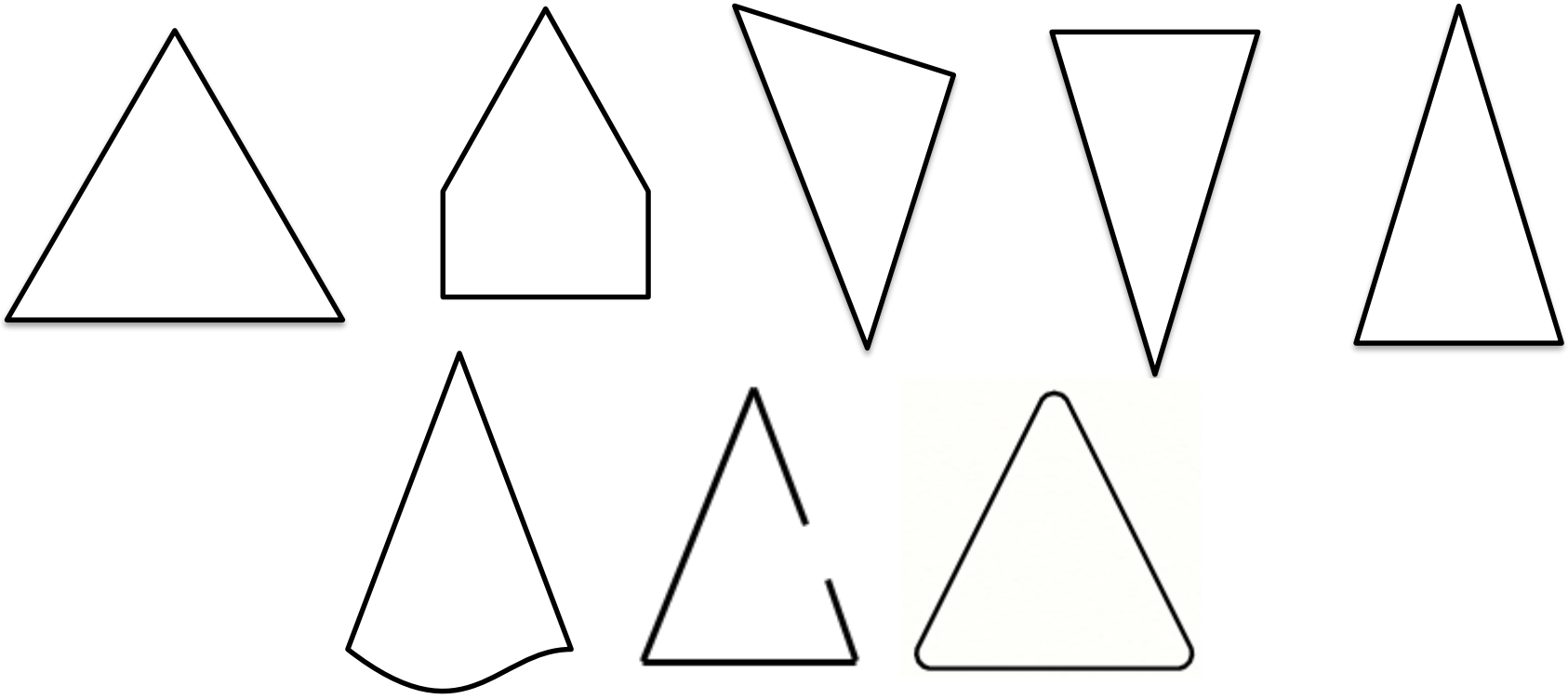
What's stayed the same?
What's changed?

'Representing ideas in multiple ways both supports low attainers to get a sense of the meaning of the abstract concept, and challenges the high attainers to see a concept differently, gaining a more complete and connected perspective.'

Willingham, D. (2012)

Conceptual variation

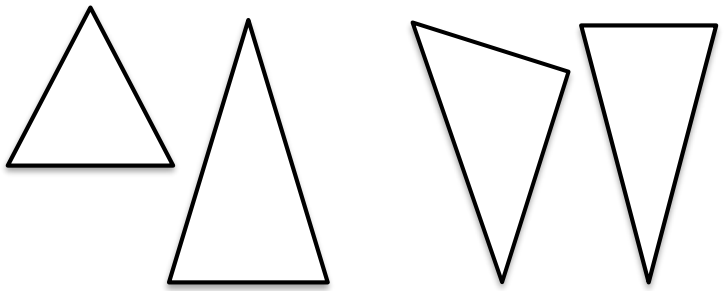
Which shape is a triangle?

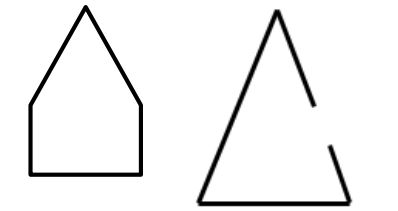


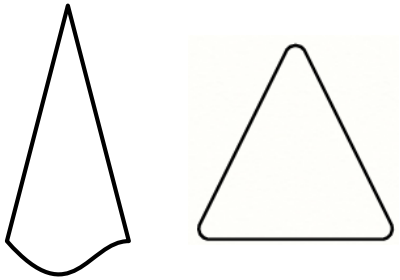
Conceptual variation

What is a triangle?

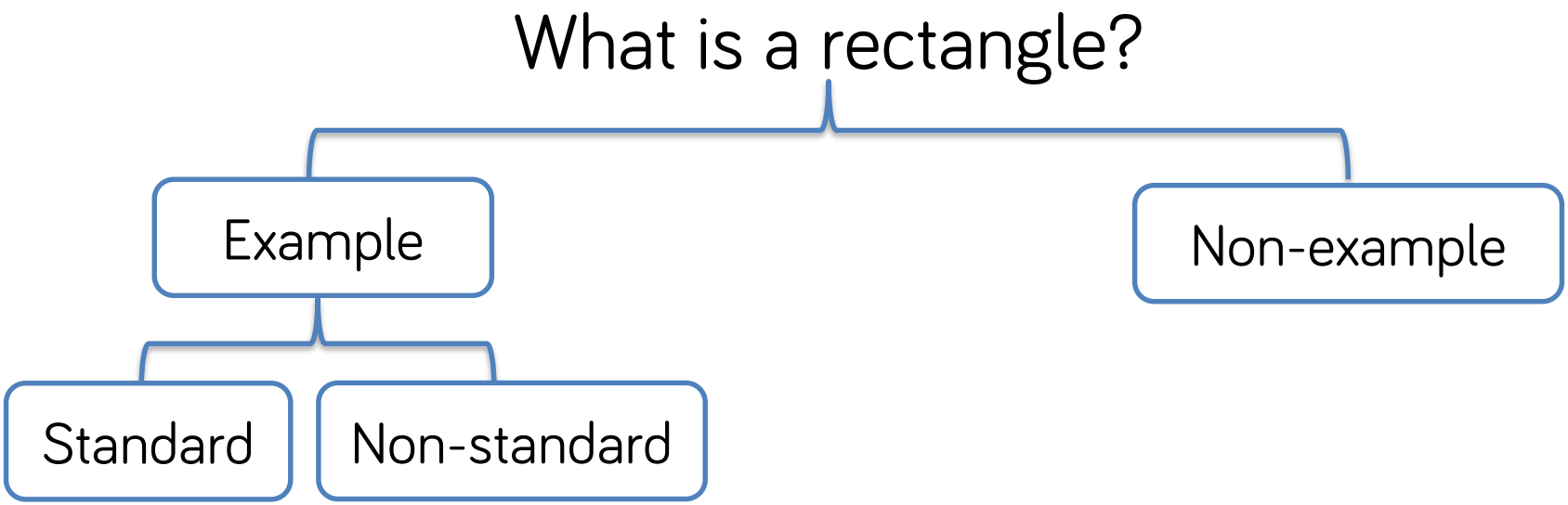
- Example
  - Standard
  - Non-standard
- Non-example







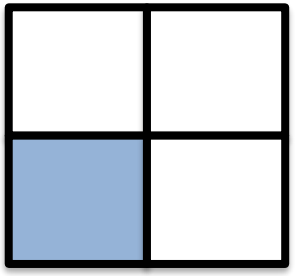
Conceptual variation



Conceptual variation

What does $\frac{1}{4}$ mean?

One quarter of a whole



One quarter of a quantity

A number on a number line

One divided by four

$1 \div 4 = ?$

Examples and non-examples

Which diagram represents $\frac{1}{4}$?

A

B

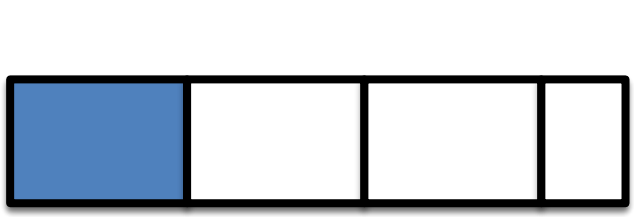
Examples and non-examples

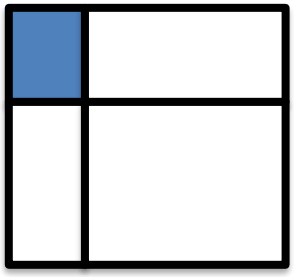
The whole has been divided into ___ equal parts.

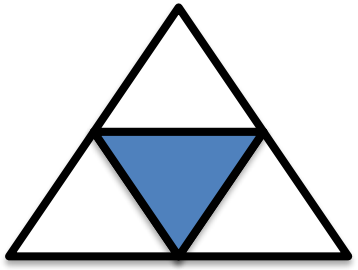
Each part is worth ____

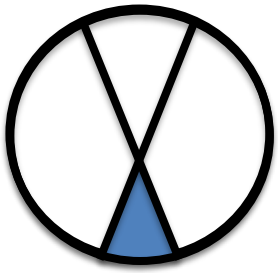
Examples and non-examples

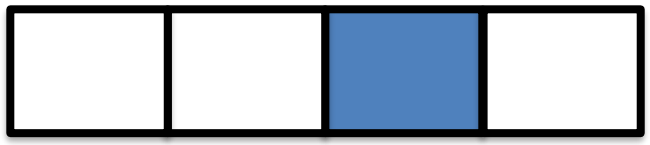
Which diagram represents $\frac{1}{4}$?

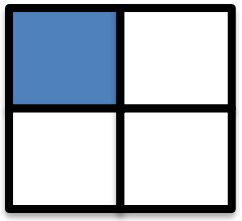










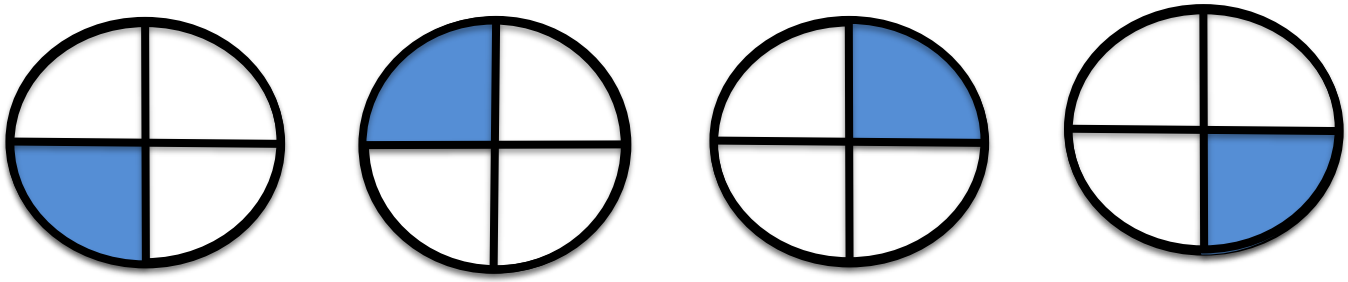


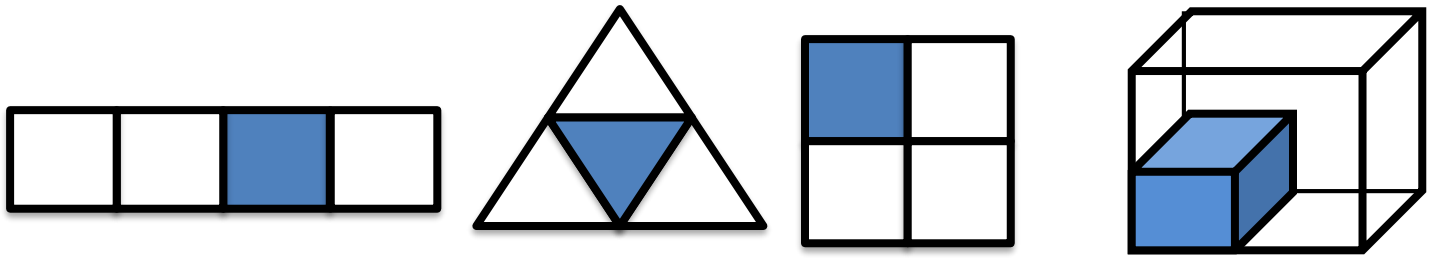
The whole has been divided into ___ equal parts.

Each part is worth ____

Standard and non-standard

How can we represent $\frac{1}{4}$?

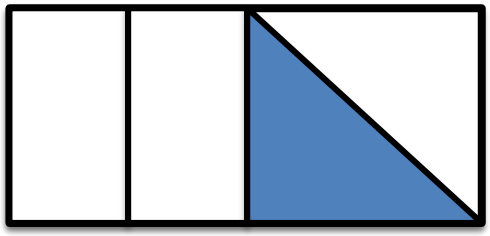




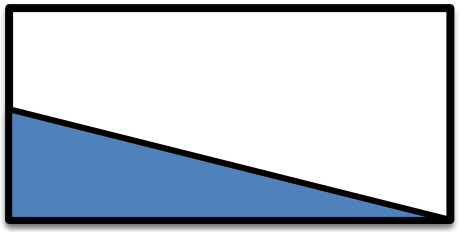
Standard and non-standard

How can we represent $\frac{1}{4}$?

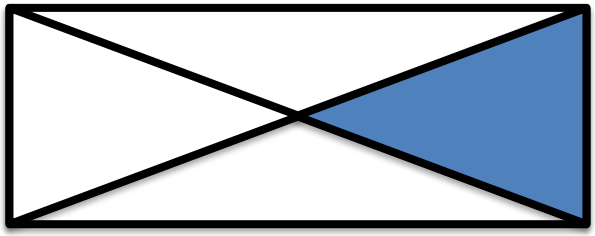
A



B



C



Conceptual variation

How is $\frac{1}{4}$ represented in different areas of maths?

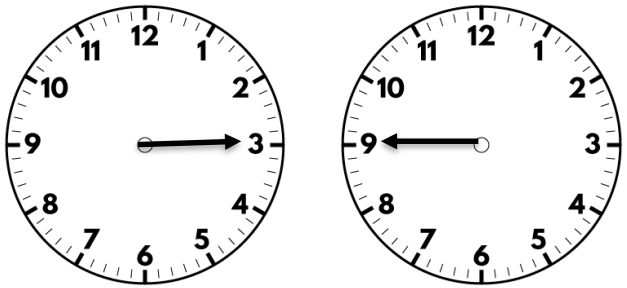
Decimals

0.25

Percentages

25%

Geometry



Time

$250m$ $2.5mm$

Measurement

Procedural variation

'The basic idea of procedural variation is that keeping some things the same and only varying the important concept or idea that you want your children to focus on, gives a greater likelihood of them understanding that concept or idea.'

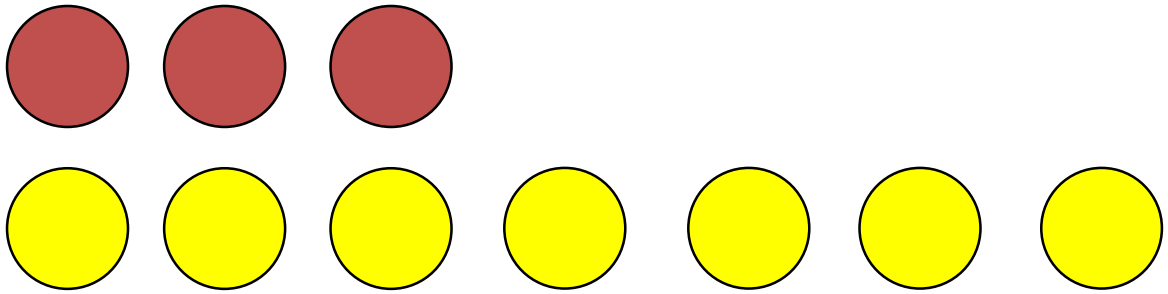
Broadbent Maths (2016)

Procedural variation

What's stayed the same?

$$3 + 4 = 7$$
$$3 + 5 = 8$$
$$3 + 6 = 9$$
$$3 + 7 = 10$$

What's changed?



Procedural variation

What's stayed the same?

$$3 + 4 = 7$$
$$3 + 5 = 8$$
$$3 + 6 = 9$$
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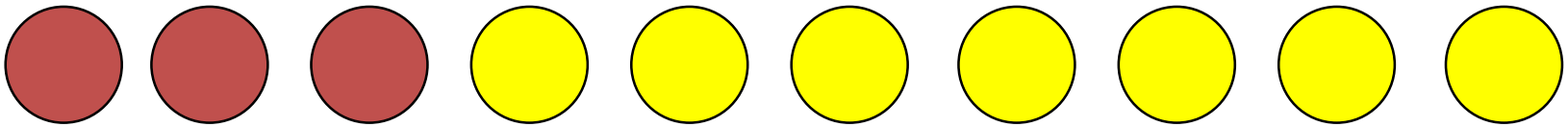
What's changed?



Procedural variation

What's stayed the same?

$$3 + 4 = 7$$
$$3 + 5 = 8$$
$$3 + 6 = 9$$
$$3 + 7 = 10$$

What's changed?

What do you notice?

addend + addend = sum

Why do you think this has happened?
Will this always happen?

Procedural variation

What's stayed the same?

$$3 + 4 = 7$$
$$3 + 5 = 8$$
$$3 + 6 = 9$$
$$3 + 7 = 10$$

What's changed?

Use the number sentences to help you answer:

$3 + 3 =$

$3 + ____ = 11$

$9 + 3 =$

$___ + 10 = 13$

Procedural variation

What's stayed the same?

$$3 + 4 = 7$$
$$3 + 5 = 8$$
$$3 + 6 = 9$$
$$3 + 7 = 10$$

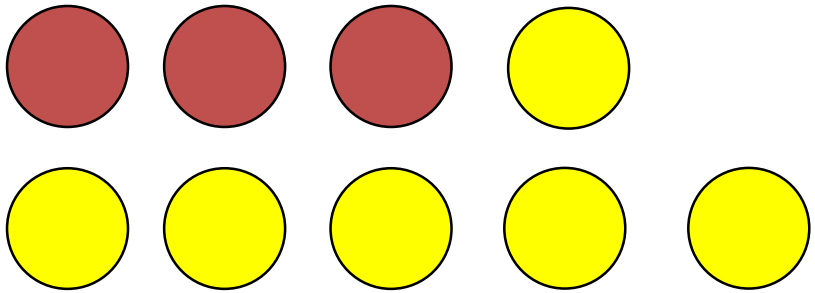
What's changed?

Shade in the **even** numbers green.
Shade in the **odd** numbers blue.

What do you notice?

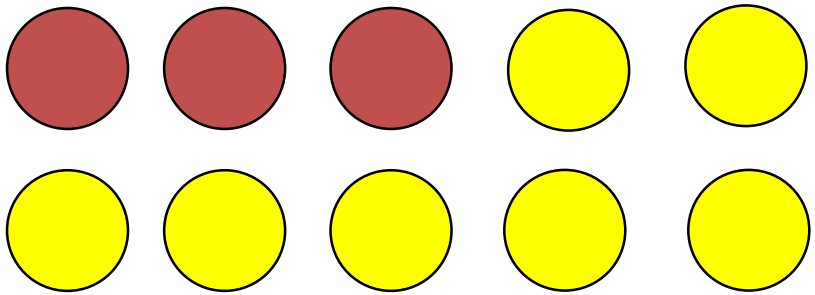
Procedural variation

$$3 + 4 = 7$$
$$3 + 5 = 8$$
$$3 + 6 = 9$$
$$3 + 7 = 10$$



Procedural variation

$$3 + 4 = 7$$
$$3 + 5 = 8$$
$$3 + 6 = 9$$
$$3 + 7 = 10$$



Procedural variation

$$7 + 2 =$$
$$2 + 7 =$$
$$17 + 2 =$$
$$12 + 7 =$$

$$9 - 5 =$$
$$8 - 5 =$$
$$7 - 5 =$$
$$6 - 5 =$$

$$9 - 7 =$$
$$11 - 7 =$$
$$13 - 7 =$$
$$15 - 7 =$$

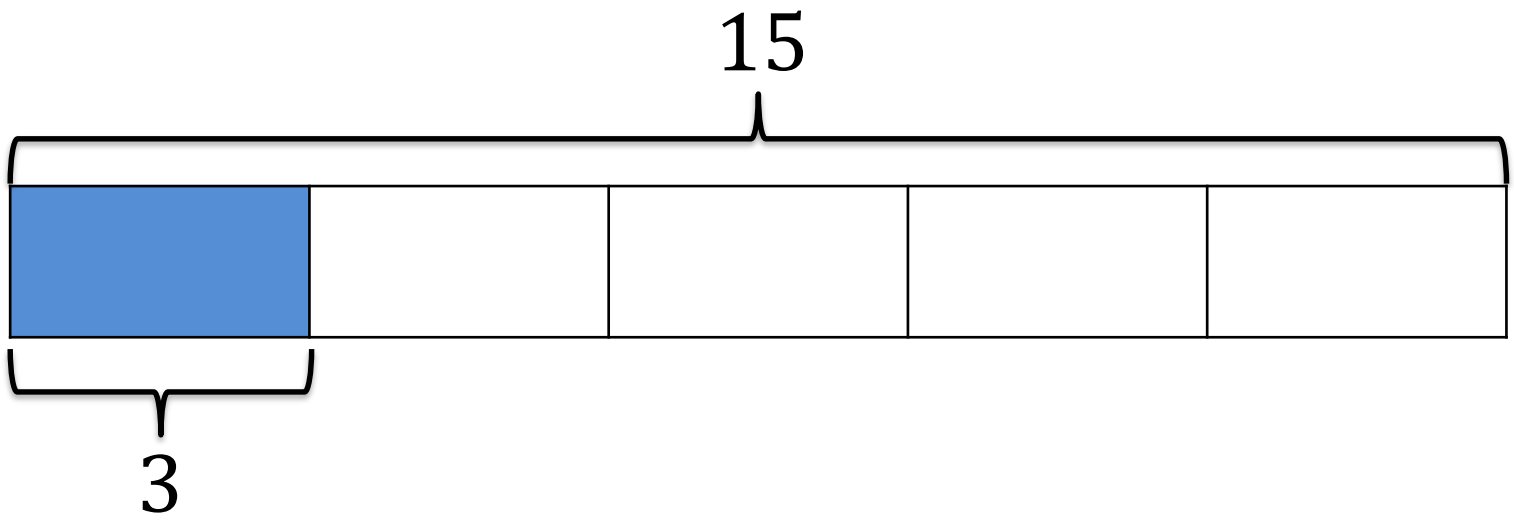
What's stayed the same? What's changed?

What do you notice? If I know this, what else do I know?

*'a well-designed sequence of tasks invites learners to **reflect** on the effect of their actions so that they recognize key **relationships**.'*

Simon and Tzur (2004)

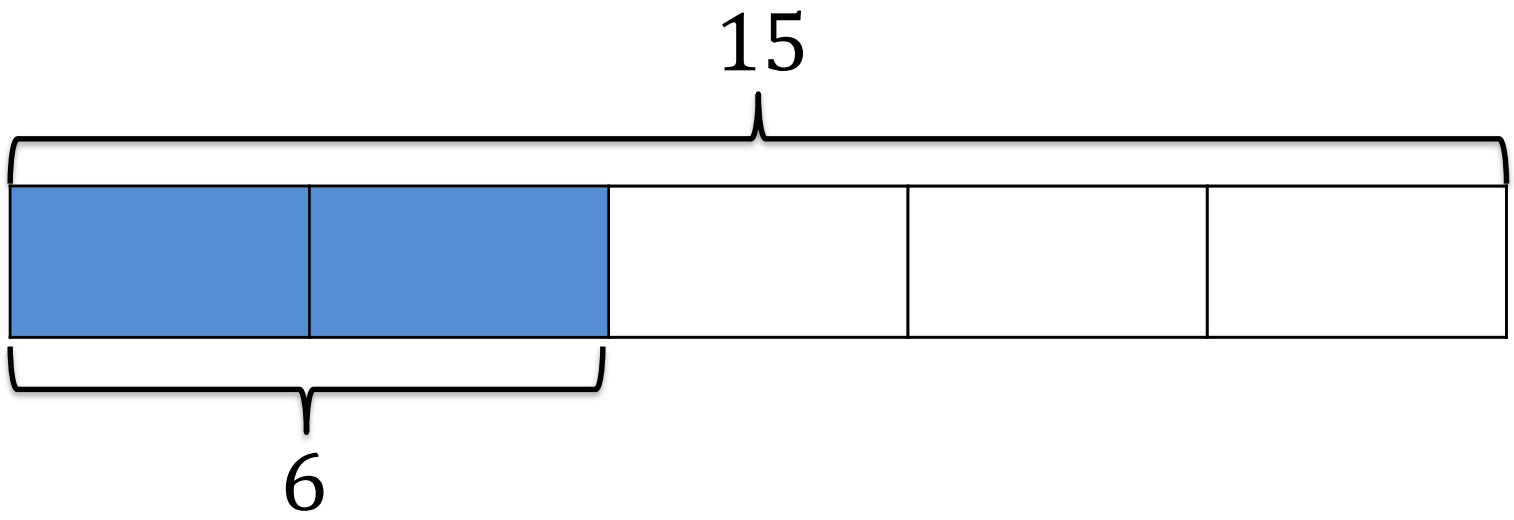
Procedural variation



$\frac{1}{5}$ of $15 = 3$

What's stayed the same? What's changed?

Procedural variation

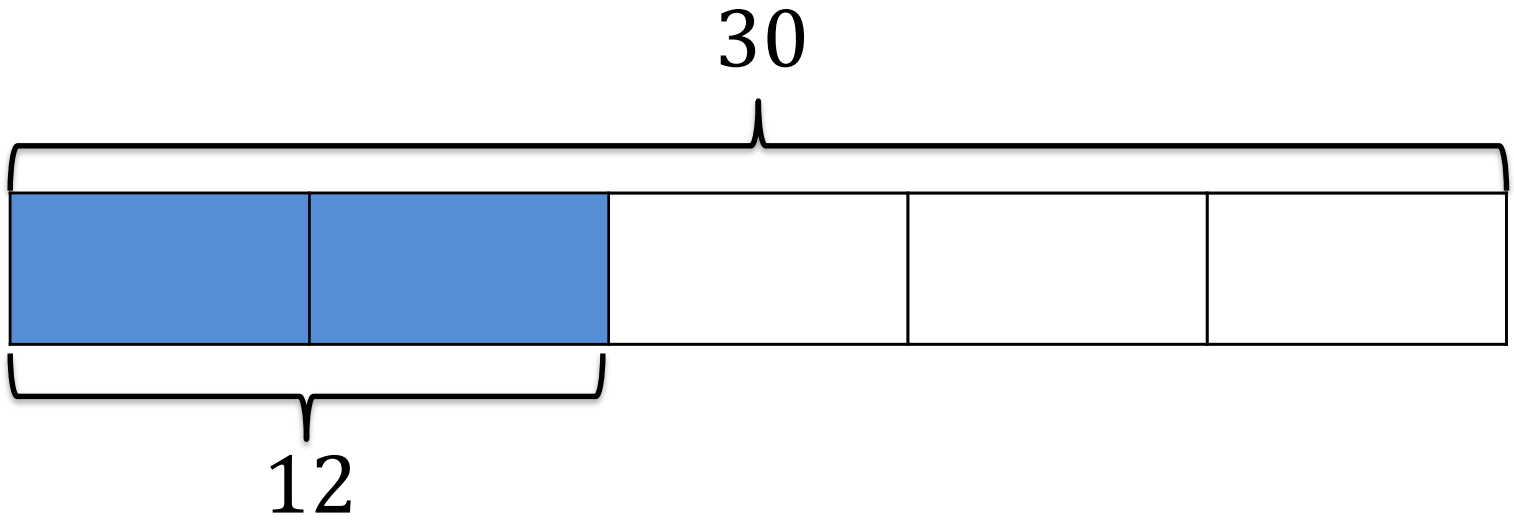


$\frac{1}{5}$ of $15 = 3$

$\frac{2}{5}$ of $15 = 6$

What's stayed the same? What's changed?

Procedural variation

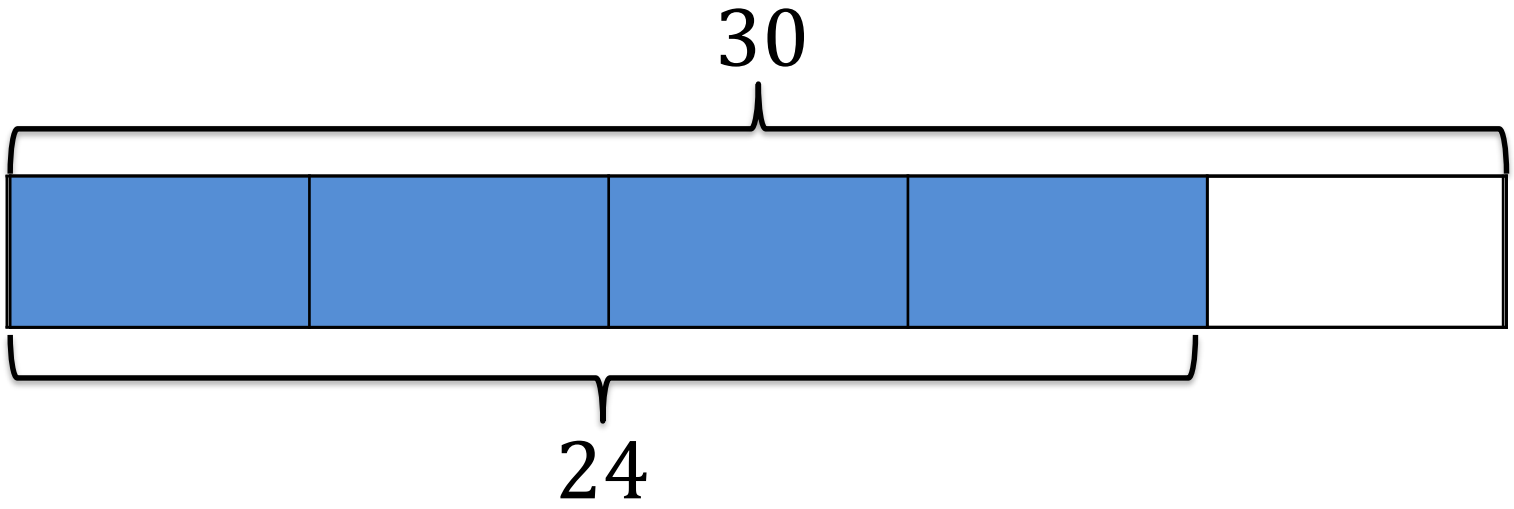


$\frac{2}{5}$ of $15 = 6$

$\frac{2}{5}$ of $30 = 12$

What's stayed the same? What's changed?

Procedural variation

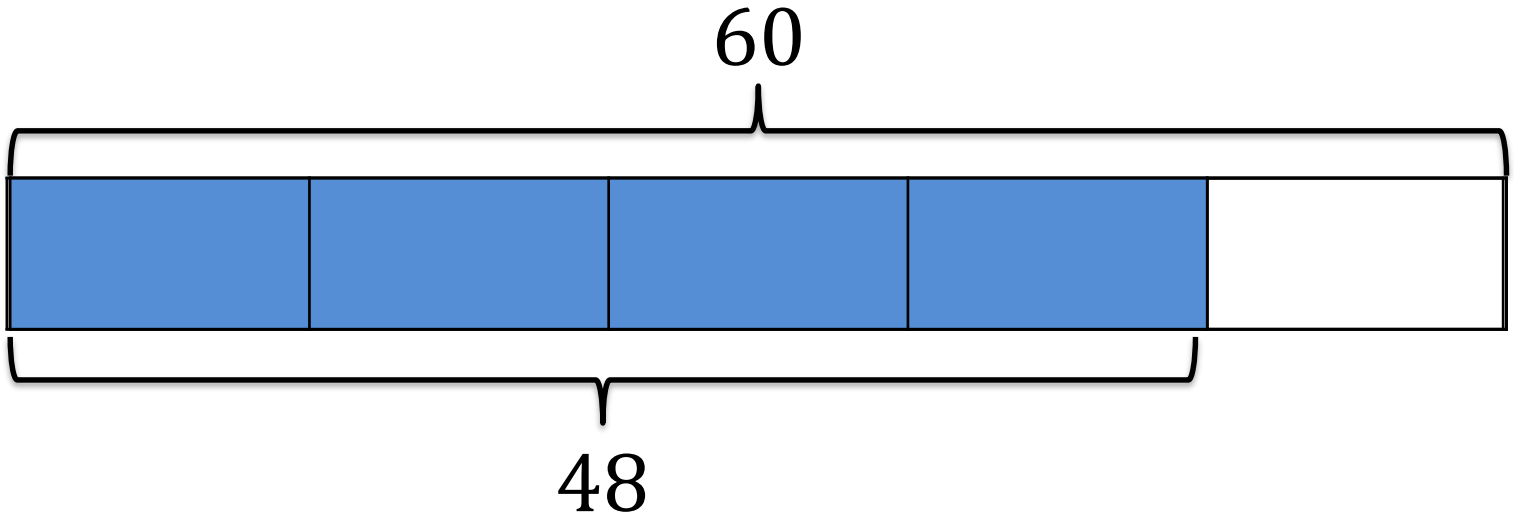


$\frac{2}{5}$ of $30 = 12$

$\frac{4}{5}$ of $30 = 24$

What's stayed the same? What's changed?

Procedural variation

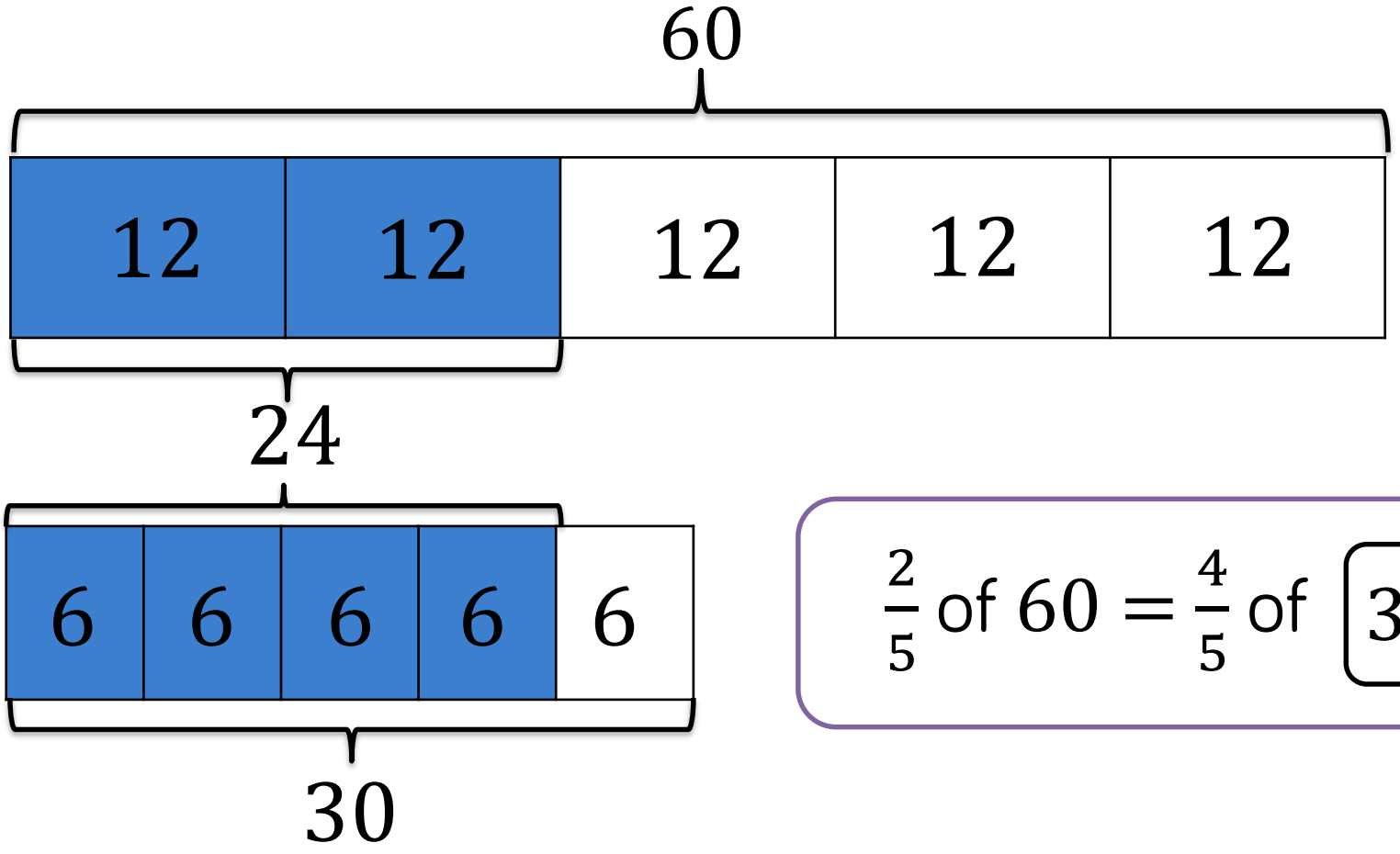


$\frac{4}{5}$ of $30 = 24$

$\frac{4}{5}$ of $60 = 48$

What's stayed the same? What's changed?

Procedural variation



$\frac{2}{5}$ of 60 = $\frac{4}{5}$ of 30

'The central idea of teaching with variation is to highlight the essential features of the concepts through varying the non-essential features.'

Gu, Huang & Marton. (2004)

Procedural variation

$3\overline{)36} = 12$

$4\overline{)49} = 12\text{ r }1$

$5\overline{)56} = 11\text{ r }1$

$4\overline{)87} = 21\text{ r }3$

$4\overline{)85} = 21\text{ r }1$

$3\overline{)64} = 21\text{ r }1$

What's stayed the same?

What's changed?

What is the focus of the learning?

Procedural variation

$$3\overline{)36} = 12$$

$$3\overline{)37} = 12 \text{ r } 1$$

$$3\overline{)38} = 12 \text{ r } 2$$

$$3\overline{)39} = 13$$

$$3\overline{)63} = 21$$

$$3\overline{)64} = 21 \text{ r } 1$$

What's stayed the same?

What's changed?

What generalisations can we make?

Procedural variation

Always, Sometimes, Never

A three-digit number with seven hundreds and six ones is a multiple of four.

'When we look at a sequence of questions, instead of thinking 'are they P or C variation?' the point is to imagine what underlying structures learners will experience, and how and when these can be drawn out to become new mathematical learning.'

Watson, A. (2018)

Any Questions?

Thank you

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