Some Stuff about Crystals

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Quartz Crystals are like all other components, each is unique. The parameters vary with each unit. We can measure a sample of a production lot and estimate the performance of the units in the lot. I have a few samples of two different manufacturer's crystals, one low cost in a HC49 package and another in a TO5 package.

I measured the crystals on a Hewlett Packard 8753C network analyzer. The impedance at resonance was measured with and without a series 8.73 pf capacitor.

Table 1

Unit number	Co 1 MHz	f1	rs	with series cap	rs2
1	2.28	9.997818	7.487	10.002032	18991
2	2.28	9.997796	5.74	10.002037	16.834
3	2.28	9.997820	8.57	10.001964	20.272
4	2.28	9.997830	12.93	10.001888	27.715
5	2.28	9.997748	7.537	10.002014	18.49
6	2.28	9.997809	7.57	10.001924	17.983
7	2.28	9.997805	6.88	10.002066	17.27
8	2.28	9.997826	7.27	10.001956	17.217
Tyco sample 10 MHz					
1	3.93	9.997177	9.8018	10.004033	27.01
2	3.93	9.997255	11.729	10.003877	31.419
3	3.93	9.997315	10.745	10.003888	30.412
4	3.93	9.997098	10.347	10.003894	29.474
6	3.93	9.997219	9.1895	10.003852	25.3197

This results in the following crystal model.

Figure 1.

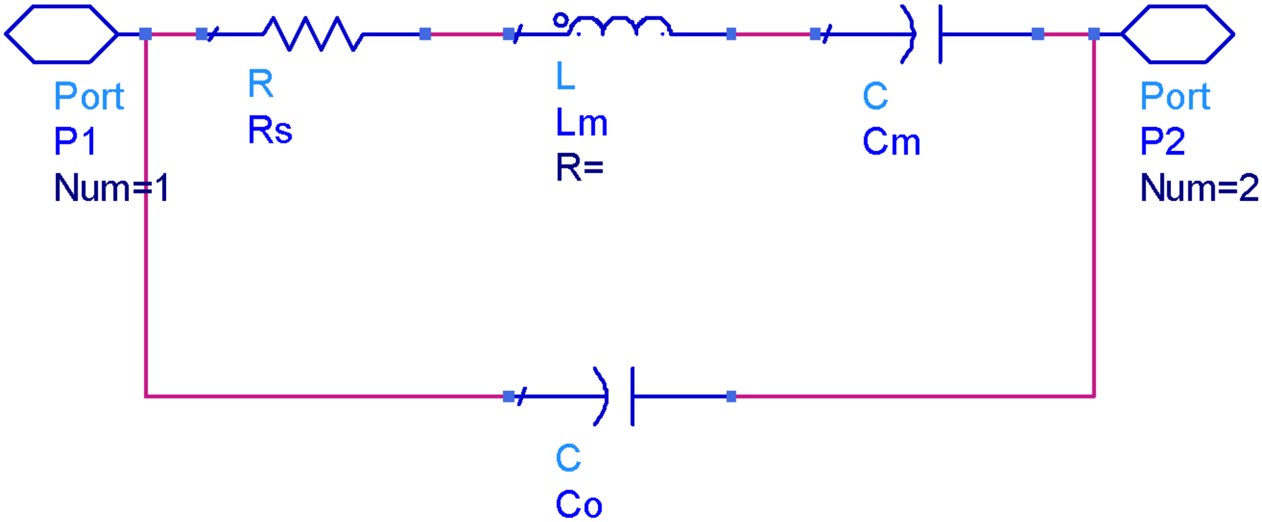

Table 2 Crystal Parameters

Rs	Lm	Cm in pf	Q
7.487	2.72E-02	9.27E-03	2.29E+05
5.74	2.71E-02	9.33E-03	2.97E+05
8.57	2.77E-02	9.12E-03	2.03E+05
12.93	2.83E-02	8.93E-03	1.38E+05
7.537	2.69E-02	9.39E-03	2.25E+05
7.57	2.79E-02	9.06E-03	2.32E+05
6.88	2.69E-02	9.38E-03	2.46E+05
7.27	2.78E-02	9.09E-03	2.41E+05
Tyco Samples			
9.8018	1.67E-02	1.51E-02	1.07E+05
11.729	1.73E-02	1.46E-02	9.30E+04
10.745	1.75E-02	1.45E-02	1.02E+05
10.347	1.69E-02	1.50E-02	1.03E+05
9.1895	1.73E-02	1.46E-02	1.18E+05

The variation in Q is huge! The TO5 crystals while of consistently higher quality show more variation. Yes I repeated the low Q measurement and got a similar result. One of the problems with the calculations is we are looking at the differences in 2 large numbers, the series resonant frequency and the series resonant frequency with a small series capacitor. The series capacitor selected was a silver mica, with a measured capacitance of 8.71 pf (10 pf labeled).

$$C_m := \frac{\left(F_p^{\,2} - F_o^{\,2}\right)\cdot\left(C_o + C_s\right)}{F_o^{\,2}}$$

Another problem is the coupler or bridge directivity. The Hewlett Packard 8753C network analyzer that used has an excellent bridge. The coupler directivity has a significant impact as we get away from a 50 ohm load. At the extremes of the Smith chart the error is greatest.

The TO5 crystals seemed more sensitive to temperature. It took several minutes for the zero degree S11 measurement to stop changing after taking the crystal in my hand to place it in the test fixture. The HC19(Tyco) settled quicker. It is possible that the TO5 was intended for an oven and the frequency shift per degree at room temperature is higher than the other crystal.

The variation in motional inductance is quite small; one standard deviation is 1.74% of the average value. The oscillators probably can be adjusted to frequency easily. One standard deviation in Q is 18% for the TO5 and almost 9% for the Tyco.

A common way to determine limits that we expect is +/- 3 sigma. If we assume the distribution is Gaussian or normal this should include 99.73% of the population. Motorola has popularized Six Sigma which includes all except 3.4 parts per million or 99.99966%. Six Sigma is mentioned to show an alternative. To learn more Google, Bing or Ixquick Six Sigma.

With the small sample there is a chance that we don't have the true mean, our sample might be biased. In reality, the sample was probably taken on units made the same day, possibly in serial order. There are many reasons for units made at nearly the same time to be similar. All of the manufacturing steps used chemicals from the same bottle, the line voltage was the same, and wear on the saw was the same. While at different times, maybe by years, the process may have changed, new vendors for chemicals, new operators, summer versus winter. Based on the sample our mean is actually an estimate. We can create a confidence interval about our estimate. Typical estimates of the confidence interval are 90%, 95% and 99%. Larger samples reduce the interval by the square root of the number of samples.

$t := 1.645$ t at infiinty for 90,95,99% confidence

$\sigma := 2.0058$ Standard Deviation

$n := 8$ Sample size

Confidence limits

$$cflimits := t\cdot\frac{\sigma}{\sqrt{n}}$$

$$cflimits = 1.167$$

This calculation was for the series resistance, Rs,

It is common for the lot distribution mean to vary from lot to lot, even when the standard deviation is small.

Outliers

Some engineers would argue that the highest and lowest values should be excluded because they are outliers, extreme values that deviate from the other measured values. I would not do so because it is either 25 or 40% of the data. The extreme values, on each data set, is well within +/-3 sigma of the mean estimate. These values look reprehensive. There are techniques (Tukey ref 1) to eliminate outliers.

The Final Models

Models tend to show distributions that look like the chosen distribution, usually normal. Actual production lots are lumpy. Reels of parts come from the same lot or similar in time lots. For example a 90% normal distribution could have 20 production lots with 10 of 100 % yield, 9 of about 95% and one with 0% yield. Often the design yield is less than 90% or even 75%. The design managers don't ask what the anticipated yield will be or what the assumptions for the yield are. This is done to reduce immediate engineering time and cost (there is a future unknown cost).

Typically the crystals are used in oscillators with three major characteristics, frequency, output power and phase noise. Since frequency is somewhat sensitive to temperature it can be controlled in an oven (OCXO), or measured and a variable reactance adjusted to compensate for the shift in the crystal and oscillator parameters. I don't have a temperature chamber to measure the crystal parameters so this part of the model has not been explored.

From the measured data shifts in the motional inductance, Lm, are accompanied by shifts in the motional capacitance, Cm. Making the frequency the variable seems to be appropriate rather than the motional parameters.

Output power is important and we would like to be somewhat stable. Since the oscillator must have a limiting function this is usually fairly small. If we take power out through the crystal we would expect it to be larger than if taken from the output of the limiter.

Phase noise is dependent upon the crystal loaded Q. The loaded Q is a function of both the crystal (unloaded Q) and the oscillator. Variations in the

Crystal	Frequency	Lm	Cm	Rs	Co	Q
T05	$9.997807 \cdot 10^6$+/-78	$L_m := \frac{1}{4 \cdot \pi^2 \cdot f^2 \cdot C_m}$	9.2 femto farads	8+/-1.8+/-6	2.28 pf	226,000
Tyco	$9.997213 \cdot 10^6$ +/-219	$L_m := \frac{1}{4 \cdot \pi^2 \cdot f^2 \cdot C_m}$	14.7 femto farads	10.4+/-1_/-3	3.93 pf	104,000

I've fixed Cm and allowed the motional inductance to vary with the frequency. This should give answers which are useful to an oscillator designer.

The loss Rs should be modeled as lognormal instead of normal (Gaussian). Lognormal is the log of the normal distribution. Lognormal is bounded on one side by zero. It seems likely that the loss will never go negative! Lognormal distributions are readily available in good simulators.

For the purist:

p density of quartz
e thickness of quartz blank
A area
E piezoelectric stress constant
r damping constant. This includes all dissipative sources, contaminants of the quartz, mounting losses, air damping, and surface friction. This looks very manufacturing process dependent.
E_0 dielectric constant of quartz
c quartz stiffness
k permittivity of free space

$$C_o := k \cdot E_o \cdot \frac{A}{e}$$

$$R := \frac{e^3 \cdot r}{8 \cdot A \cdot E^2}$$

Getting working values for the plate area A and its variation is quite

$$L_m := \frac{e^3 \cdot \rho}{8 \cdot A \cdot E^2}$$

$$C_m := \frac{8 \cdot A \cdot E^2}{\pi^2 \cdot e \cdot c}$$

difficult. Vendors are not interested in providing such details of their manufacturing process. These equations (ref 8) appear to be approximate. I have witnessed crystals being "put on frequency" by increasing the plating thickness or applying a pencil eraser to the metal to reduce its thickness. It is possible it does this by changing the quartz stiffness.

References

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http://www.itl.nist.gov/div898/handbook/index.htm Free and online!

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