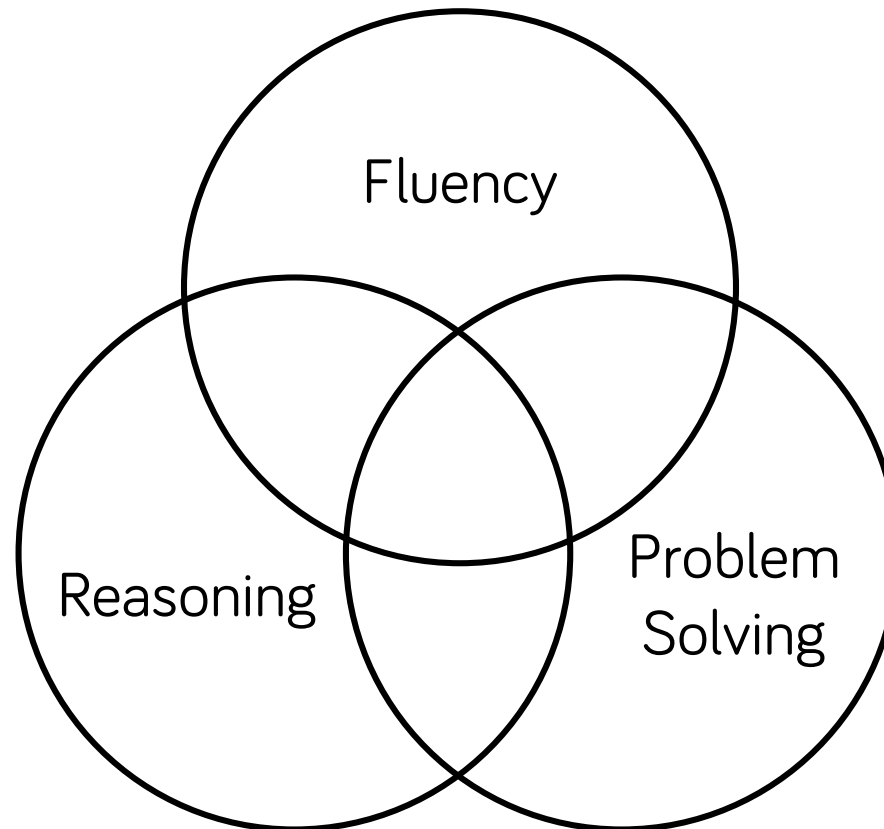


Sort the statements into the strand you feel is being described.

Do any of the statements cross over the strands?



# Reasoning and Problem Solving

## Course Lead

White Rose Maths



@WhiteRoseMaths

# Content

- Why is it important?
- One problem, an integrated approach
- One problem, many solutions
- One problem, many methods
- One problem, too many words?
- Do problems need any words at all?

Why is it important?

*‘Mathematics is a **creative** and **highly inter-connected** discipline... providing the solution to some of history's most intriguing problems.’*

National Curriculum. (2014)

## Characteristics of Effective Learning

### Playing and exploring – engagement

Finding out and exploring  
Playing with what they know  
Being willing to 'have a go'

### Active learning – motivation

Being involved and concentrating  
Keeping trying  
Enjoying achieving what they set out to do

### Creating and thinking critically – thinking

Having their own ideas  
Making links  
Choosing ways to do things

	1970	1999
1	Writing	Teamwork
2	Computational Skills	Problem Solving
3	Reading Skills	Interpersonal Skills
4	Oral Communication	Oral Communication
5	Listening Skills	Listening Skills
6	Personal Career Development	Personal Career Development
7	Creative Thinking	Creative Thinking
8	Leadership	Leadership
9	Goal Setting/ Motivation	Goal setting/ Motivation
10	Teamwork	Writing
11	Organisational Effectiveness	Organisational Effectiveness
12	Problem Solving	Computational Skills

# Why is it important?

1. Ability to work in a team structure
2. Ability to make decisions and solve problems
3. Ability to communicate verbally with people inside and outside an organisation
4. Ability to plan, organise and prioritise work
5. Ability to obtain and process information

Top 5 Skills (2015)



# One problem, an integrated approach

# Calculate $8 \times 5$

Tick (✓) which of the statements below are true.

$$8 \times 5 = 8 + 8 + 8 + 8 + 8$$

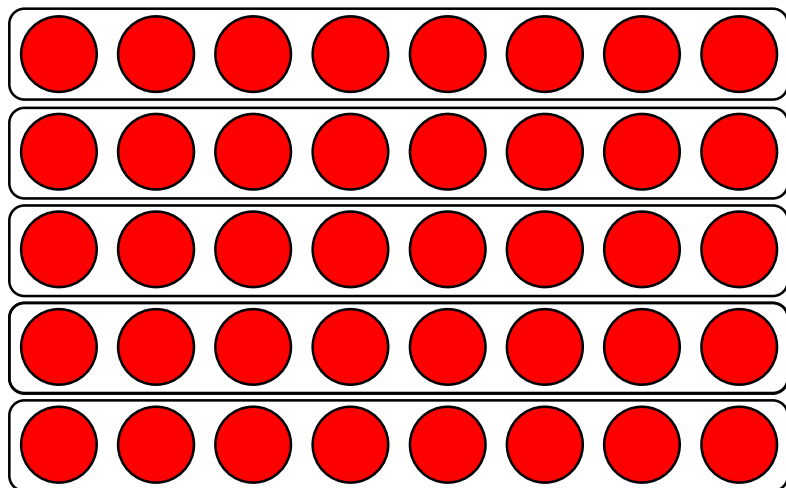
$$8 \times 5 = 16 + 8 + 8 + 8$$

$$8 \times 5 = 16 + 16 + 8$$

$$8 \times 5 = 40$$

Calculate  $8 \times 5$

Tick (✓) which of the statements below are true.



$$8 \times 5 = 8 + 8 + 8 + 8 + 8$$

$$8 \times 5 = 16 + 8 + 8 + 8$$

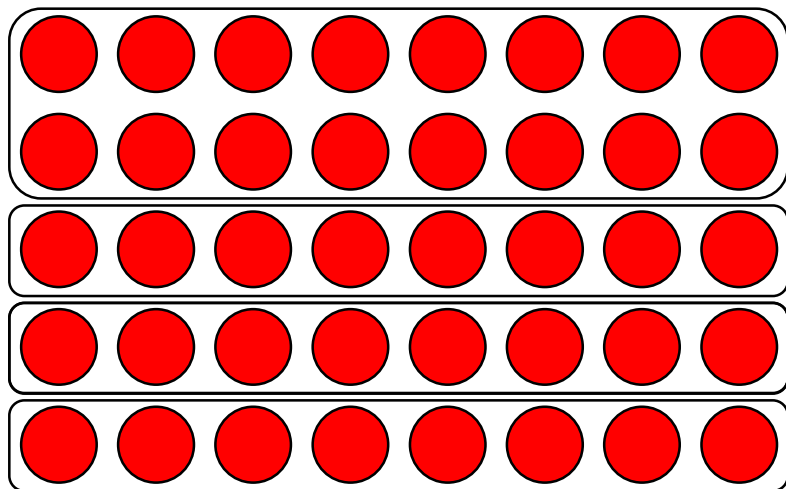
$$8 \times 5 = 16 + 16 + 8$$

$$8 \times 5 = 40$$

If you know this, what else do you know?

Calculate  $8 \times 5$ 

Tick (✓) which of the statements below are true.



$$8 \times 5 = 8 + 8 + 8 + 8 + 8 \quad \square$$

$$8 \times 5 = 16 + 8 + 8 + 8 \quad \square$$

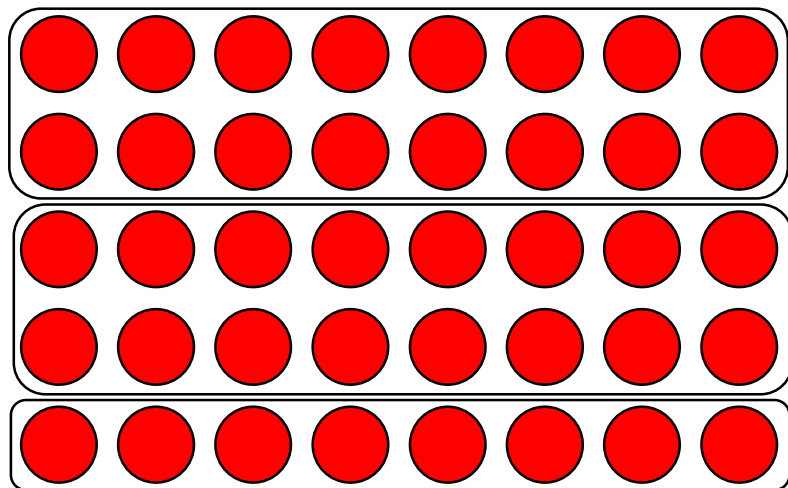
$$8 \times 5 = 16 + 16 + 8 \quad \square$$

$$8 \times 5 = 40 \quad \square$$

If you know this, what else do you know?

Calculate  $8 \times 5$ 

Tick (✓) which of the statements below are true.



$$8 \times 5 = 8 + 8 + 8 + 8 + 8 \quad \square$$

$$8 \times 5 = 16 + 8 + 8 + 8 \quad \square$$

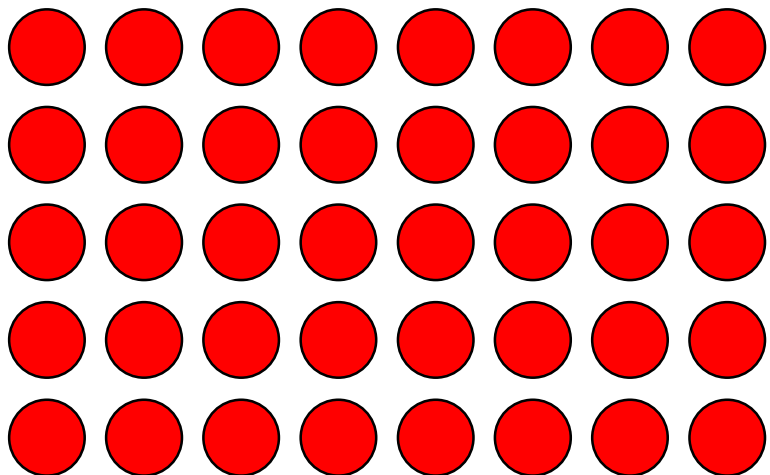
$$8 \times 5 = 16 + 16 + 8 \quad \square$$

$$8 \times 5 = 40 \quad \square$$

If you know this, what else do you know?

Calculate  $8 \times 5$

Tick (✓) which of the statements below are true.



$$8 \times 5 = 8 + 8 + 8 + 8 + 8 \quad \square$$

$$8 \times 5 = 16 + 8 + 8 + 8 \quad \square$$

$$8 \times 5 = 16 + 16 + 8 \quad \square$$

$$8 \times 5 = 40 \quad \square$$

If you know this, what else do you know?

Make the array.

$$9 + 6$$

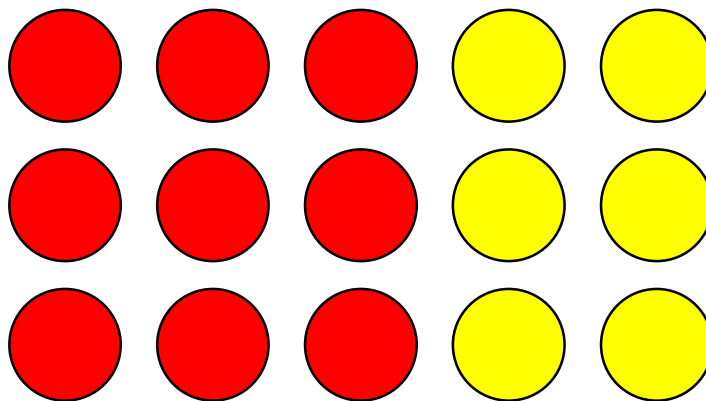
$$3 \times 3 + 2 \times 3$$

$$5 + 5 + 5$$

$$3 \times (3 + 2)$$

$$5 \times 3$$

$$3^2 + 3 \times 2$$



$$3 + 3 + 3 + 3 + 3$$

$$3^2 \left(1 + \frac{2}{3}\right)$$

How many different ways of expressing this numerically can you find?

Calculate  $7 + 3$

What signs are missing?

$$7 + 3 \square 10$$

$$9 \square 3 + 7$$

$$9 > 10 \square 3$$

Explain how you know.



# Envelopes Task

I have 5 envelopes, each containing 2 cards from a set of 0 – 9 digit cards. The sum of the two cards inside each envelope is written on the front.

Work with a partner to find out which cards are in each envelope.



# Envelopes Task



- Enabling prompts
  - Where is the best place to start?
  - Shall we list all the possibilities to help us start solving the problem?
- Extension prompts
  - Two cards inside each envelope have been multiplied and the product is displayed showing 0, 18, 21, 30, 32
  - Can you make your own puzzle for a friend to solve?

*'To be fluent in mathematics the pupils must be able to think in mathematical language.'*

*'In order to learn mathematics effectively, pupils primarily need to talk about their mathematical ideas, negotiate meanings, discuss ideas and strategies and make mathematical language their own.'*

Lee, C. (2007)

One problem, many  
solutions

*‘The aim of structured problem-solving is not for students to solve a problem, but **through solving the problem to learn mathematical ways of thinking.**’*

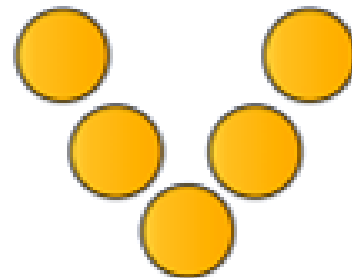
Fuji. (2013)

# One problem, many solutions

## Magic Vs

Place each of the numbers 1 to 5 in the V shape so that the two arms of the V have the same total.

How many different ways can you do it?



# Magic Vs

- Enabling prompts
  - Sort your numbers into odd and even. Pair the odd and even numbers up and place the other in the middle. Does this help?
  - The middle number must be...can't be...might be...because...
- Extension prompts
  - Use the numbers 2 to 6, 12 to 16, 37 to 41, 103 to 107. What do you notice?
  - What happens if your V has arms of length 4?
  - What if it was a magic W?

Problem

Solution

Solution

Solution

Solution

Solution

**Comparing & discussing**

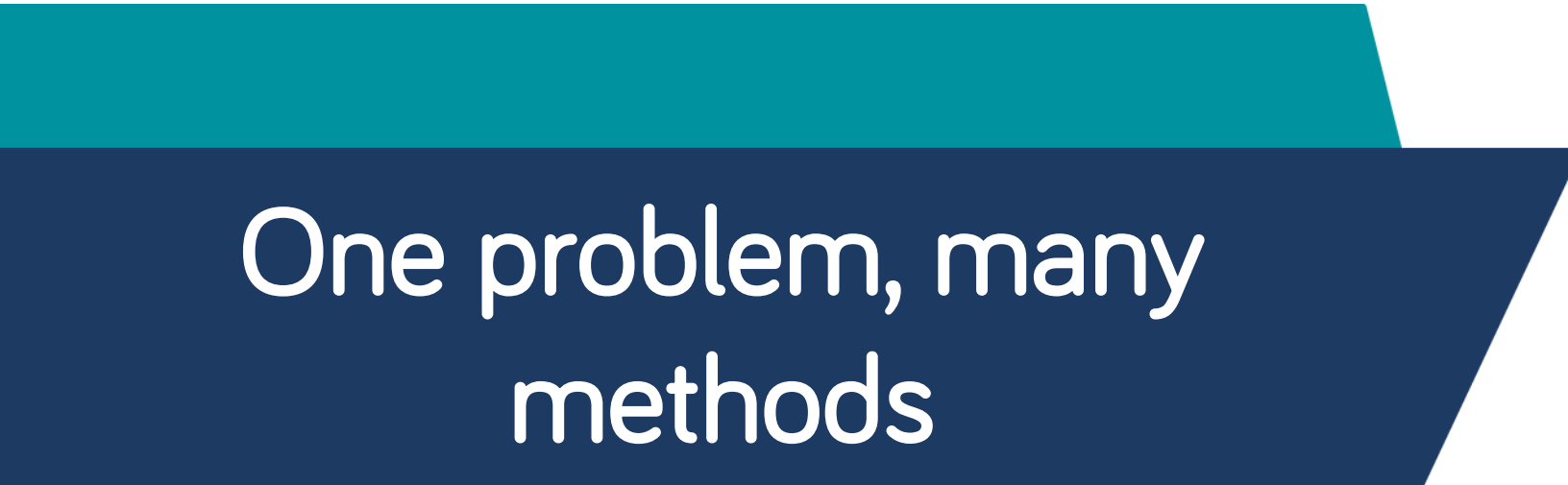
Ideas, questions, generalisations and  
problem solving skills.



*'...the focus is **not on one right answer**; rather the goal is to help students understand **how to solve a mathematics problem**...*

*Teachers cover far less material ... but **cover it in depth**: the goal is to **master mathematics concepts**.*'

Hong et al. (2009)

A decorative banner consisting of a teal rectangular bar on top and a dark blue trapezoidal bar below it, both pointing to the right.

One problem, many  
methods

*‘Solving problems is a practical art, like swimming or skiing or playing the piano: you can learn it only by imitation and practice...if you wish to learn swimming you have to go into the water, and if you wish to become a problem solver you have to solve problems.’*

Polya, G. (1960)

# One problem, many methods

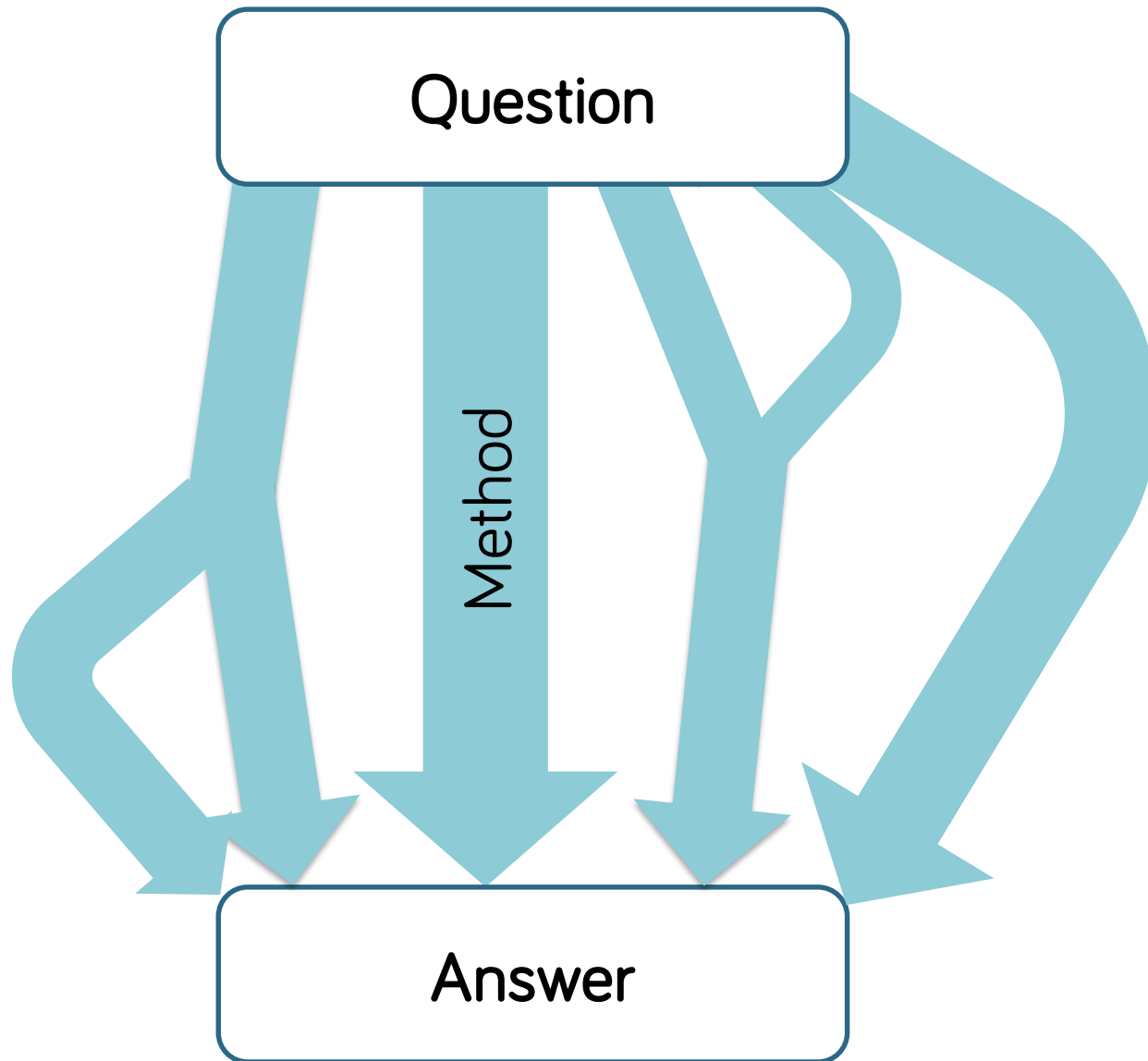
Alice has one 50p coin and four 20p coins.  
She buys one chew bar and a drink.  
How much money does she have left?



79p each



43p each



## One problem, many methods

Kelsey has a pack of number cards 1 – 9

She sorts her cards into three unequal piles.

The numbers on the cards in each pile add up to the same total.

What is the total of each pile and how could this be done?

1

2

3

4

5

6

7

8

9

- Enabling prompts
  - Think about a starting point - how will we know the total in each pile? How could we add up the numbers?
  - Do it practically with number pieces.
  - Work in groups of 3
  - Share strategies
- Extension prompts
  - How many different ways can you find?
  - Can you do it with 3 equal groups?
  - Can you do it where one group has ... cards?

*'Good mathematics is not about how many answers you know, it's about how you behave when you don't know.'*

Author unknown



One problem, too  
many words?

## One problem, too many words?

Tickets for a school play, of which there are 220 in total, cost £5 for adults and £1.50 for concessions.

If 50 children and 130 adults bought a ticket each, how much money would the play make?

How many tickets would be left un-sold?

## One problem, too many words?

Rosie and the other red hens were chased by the fox again last week. They normally lay 24 eggs a week, but because of the scare they only produced three quarters of their normal production. How many eggs did they lay?

What calculation is hidden within this word problem?

$$\frac{3}{4} \text{ of } 24 = 18$$

## One problem, too many words?

Rosie and the other red hens were chased by the fox again last week.

They normally lay 24 eggs a week, but because of the scare they only produced three quarters of their normal production.

How many eggs did they lay?

Still too many words?

## One problem, too many words?

Rosie and the other red hens were chased by the fox again last week.

They normally lay 24 eggs a week, but because of the scare they only produced three quarters of their normal production.

How many eggs did they lay?

# One problem, too many words?

They normally lay 24 eggs a week, but because of the scare they only produced three quarters of their normal production.

How many eggs did they lay?

A hen normally lays 24 eggs a week.

Last week it only laid three quarters of their normal amount.

How many eggs did the hen lay last week?

Is this a problem?

# One problem, too many words?

LO: to solve fraction of amounts word problems.

A hen normally lays 24 eggs a week.

Last week it only laid three quarters of their normal amount.

How many eggs did the hen lay last week?

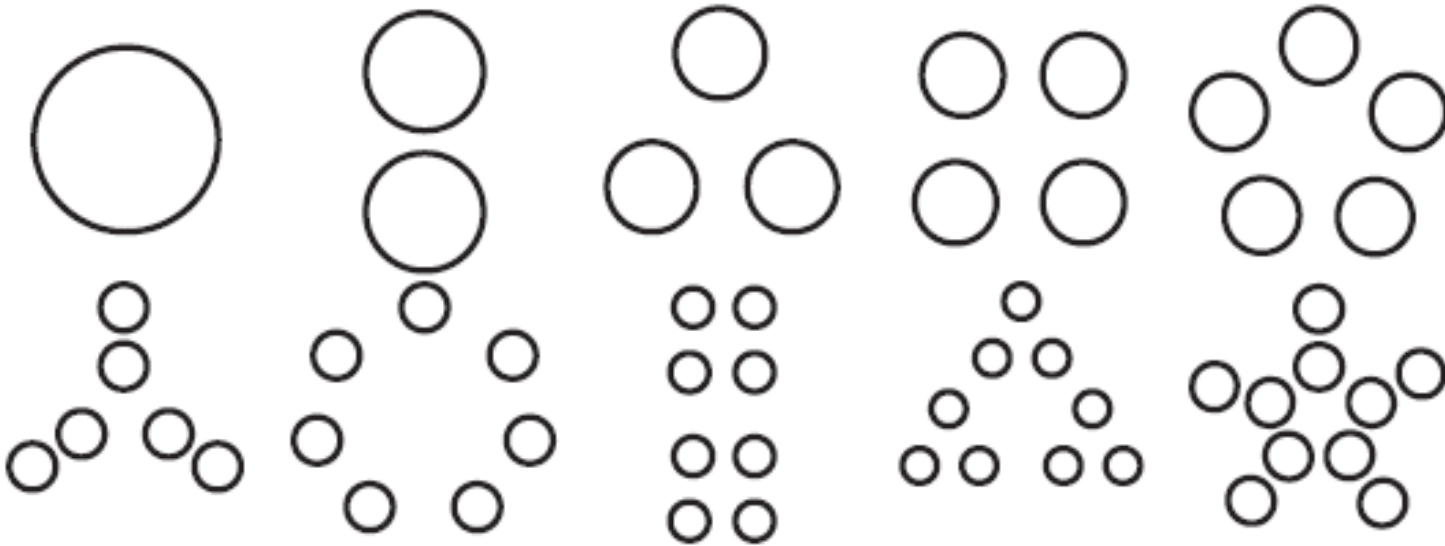
Is it a problem now?

Do problems need any  
words at all?



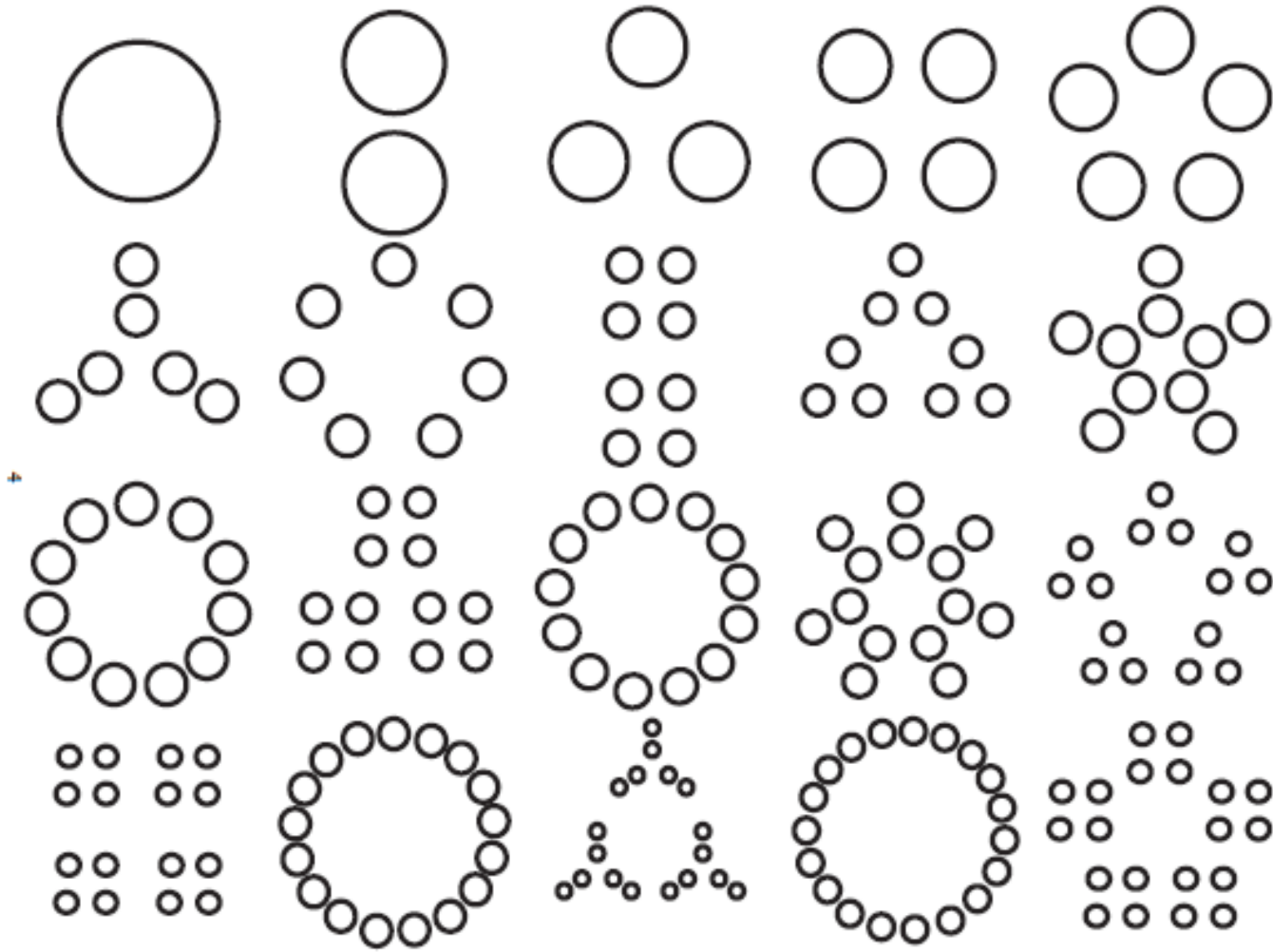
# Do problems need any words at all?

What do you notice?



What do you think 12 might look like?

Why?



I know that the next one is ... because ...

I think this because ...

Because .... then I think ....

All...some...might...

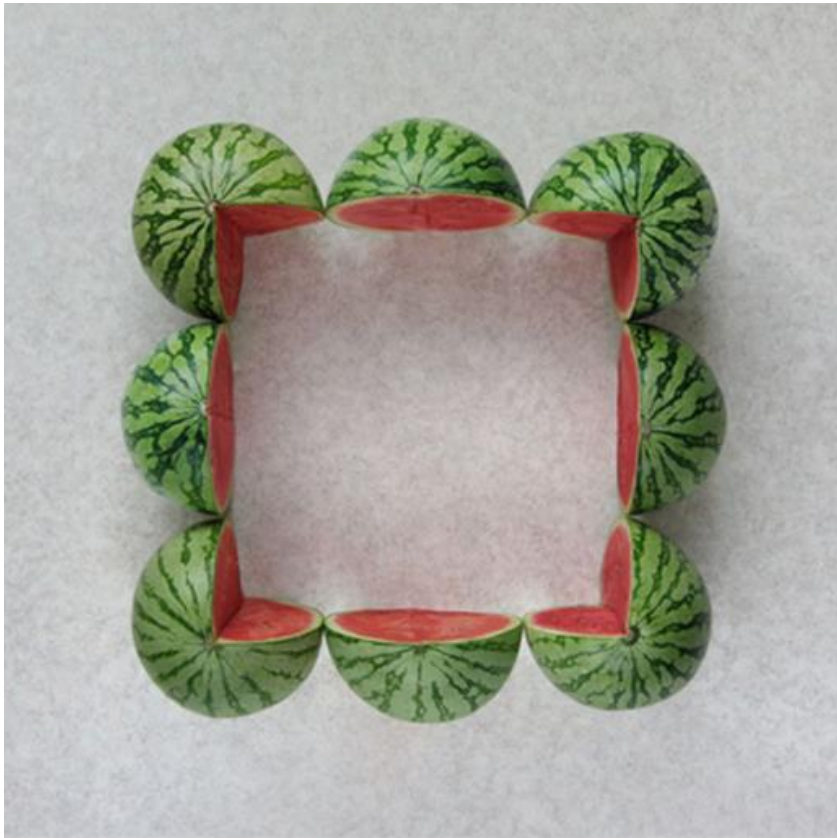
When I tried .... I noticed that ...

This can't work because ...

If this is true then ...

The pattern looks like ...

Do problems need any words at all?



[Ntimages.weebly.com](http://Ntimages.weebly.com)

*‘Becoming a proficient mathematician means **working with all of the proficiencies** – fluency, problem solving, reasoning and understanding – from the beginning. And by mathematician I mean **anyone using mathematics** in her or his life. **Everyone is a mathematician.**’*

Askew, M. (2012)

A decorative banner consisting of a teal rectangular bar on top and a dark blue trapezoidal bar below it, both pointing to the right.

Any Questions?

# Thank you

White Rose Maths



@WhiteRoseMaths

@WRMathsSec

[www.whiterosemaths.com](http://www.whiterosemaths.com)

# References

- Askew, M. (2012) Transforming Primary Mathematics. Routledge, Oxon
- Drury, H. (2015) Mastering Mathematics OUP, Oxford
- Boaler, J. (2016) Mathematical Mindsets Jossey-Bass
- Mason, B. Burton, L. and Stacey, K. (2010) Thinking Mathematically, Pearson Ed. Ltd
- ATM Primary Questions and Prompts