Algebra – Simplification of Fractional Expressions and Solving Equation involving Fractional Expressions

TOPIC 1 - Common Mistake

For some reason, when students are working with problems simplifying fractional algebraic expressions, they can become confused and in effect, break the "BOMDAS" rules.

For example, in simplifying the following expression,

$$\frac{x + 6x^3}{x^2}$$

Students will get the first steps right (first factorising the numerator and then dividing numerator and denominator by the common factor "*x*");

$$= \frac{x(1 + 6x^2)}{x^2}$$

$$= \frac{1 + 6x^2}{x}$$

but then try to go another step further and try to simplify the second term by dividing by "*x*" again, and end up with:

$= 1 + 6x$ which is **NOT** correct as it ignores the fact that the "1" also needs to be divided by "*x*".

The **correct** answer to the division by "x" again would be:

$$= \frac{1}{x} + 6x$$

(This is not a lot simpler than the previous correct step anyway so probably isn't worth doing in this case).

To use a somewhat "babyish" example – doing the false step as above would be like saying that if you wanted to divide 2 apples and 6 bananas evenly amongst 3 people then each would get 2 apples and 2 bananas. To express this algebraically, calling the apples "a" and the bananas "b", the problem and incorrect answer would look like:

$$\frac{2a + 6b}{3}$$

$= 2a + 2b$ which again, is obviously **NOT** correct as it has ignored the fact that the apples need to be divided amongst the three people as well.

The correct answer is:

$= \frac{2}{3}a + 2b$ (which in terms of the original problem, means that each person would get 2/3 of an apple, and 2 bananas).

Conclusion and Practice

Students need to resist the temptation to simplify only one of the terms in the numerator of a fractional expression and ignore other terms which cannot be simplified any further. All terms in the numerator are divided by the denominator and if any one of them cannot be simplified further then the whole numerator cannot be simplified further. Textbooks will give plenty of problems for further practice.

TOPIC 2 – Expression Simplification and Equation Solving

Here, we will look at two methods of solving equations involving fractional expressions using the following equation:

$$\frac{2x+1}{7}+\frac{5x-12}{3}=2$$

Method 1 – Full simplification of the fractional expression first before proceeding to solve the equation

$\frac{2x+1}{7}+\frac{5x-12}{3} = 2$ [Next step – bring each term to the lowest common denominator (LCD)]

$\frac{3(2x+1)}{21}+\frac{7(5x-12)}{21} = 2$ [Note – keep "=" signs lined up for good setting out]

$\frac{6x+3}{21}+\frac{35x-84}{21} = 2$ [Expanding terms in brackets]

$\frac{6x+3+35x-84}{21} = 2$ [Bringing the terms together over the common denominator]

$\frac{41x-81}{21} = 2$ [Collecting and simplifying the "like" terms in the numerator]

Note that up to this point we have not done any "solving" as such, we have only simplified the LHS of the equation.

$41x-81 = 42$ [Multiplying both sides by 21]

$41x = 123$ [Adding 81 to both sides]

$x = 3$ [Dividing both sides by 41]

Method 2 – Proceed with "solving" immediately by multiplying both sides of the equation by the LCD

$\frac{2x+1}{7}+\frac{5x-12}{3} = 2$

$\frac{21(2x+1)}{7}+\frac{21(5x-12)}{3} = 42$ [Multiplying both sides by 21]

$3(2x+1)+7(5x-12) = 42$ [Simplifying the numeric fractions generated]

$6x+3+35x-84 = 42$ [Expanding terms in brackets]

$41x-81 = 42$ [Collecting and simplifying the "like" terms]

Now we have reached the same point as in Method 1 above and the solution continues in the same way.

$41x = 123$ [Adding 81 to both sides]

$x = 3$ [Dividing both sides by 41]

Conclusion and Practice

Either method works well, with Method 2 perhaps being a little quicker. The simplification of the LHS used in Method 1 is a useful skill to practice and students should work to become proficient at it. The important thing in both cases is that students understand the steps they are taking along the way. Again, textbooks will give plenty of problems that can be used to practice either method. While students can always work with the one they are most comfortable with, they would be well advised to at least do some from the alternate method.