Introduction to Optimization in Energy

DTU Summer School 2018: Modern Optimization in Energy Systems

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Outline

Why Study Energy Systems?

Why Study Energy Systems?

1. **Our life and survival depend on energy**
   - Transportation, health, agriculture, heating / cooling, communications, manufacturing, etc.
   - Dramatic economic and safety consequences when supply fails.

Why Study Energy Systems?

2. Energy acquisition and use impact the environment

- Air pollution: 5.5 million people die each year because of air pollution.

  The main culprit is the emission of small particles from power plants, factories, vehicle exhausts and from the burning of coal and wood.
  https://www.bbc.co.uk/news/science-environment-35568249

- Carbon emissions are a major challenge.
  Canada's Trottier Energy Futures Project http://iet.polymtl.ca/tefp/

  Video of the report's release:
  https://www.youtube.com/watch?v=qGbQgogteTg

Why Study Energy Systems?

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3. Intellectually interesting and challenging!

- North American electricity system is the "biggest machine in the world".
- Distributed generation and demand management increasing the complexity of the system by several orders of magnitude.
- Huge challenge to develop renewable/decarbonising energy supply.
- Incorporate variable and unreliable renewable generation sources
- Use storage to deal with intermittency and variability of supply and demand.
- Long-standing question remains: how to operate a power system at minimum cost.
- Integrate and operate different energy systems: electricity, gas, hydrogen, oil, coal, biofuels, heat.

Oil and Gas

IEA: “While there is no peak oil demand in sight, the *pace of growth* will slow down to 1 mb/d by 2023 after expanding by 1.4 mb/d in 2018.” (my emphasis)
`https://www.iea.org/oil2018/`

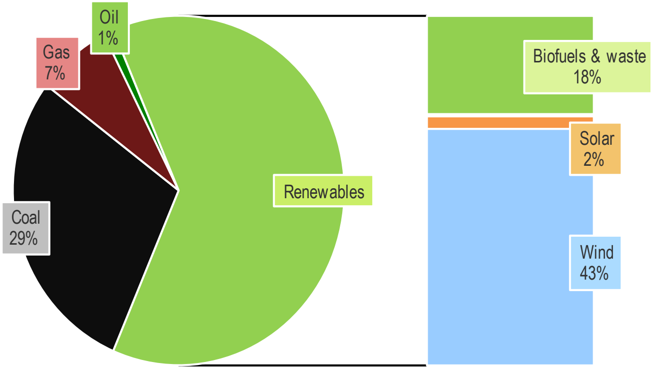
Special issue of *Optimization and Engineering* on “Optimization in the Oil and Gas Industry”:
`https://link.springer.com/journal/11081/18/1/page/1`

Different Countries, Different Realities

Denmark

ELECTRICITY GENERATION: 30.1 TWh

63% renewables (IEA average: 24%)



ⓒ International Energy Agency, Key Energy Data by country

Canada

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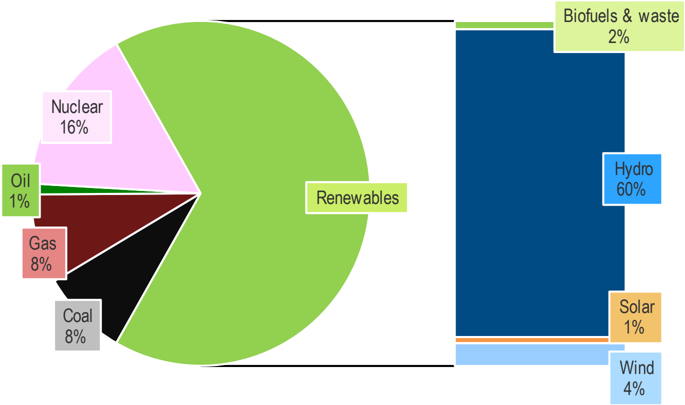
ELECTRICITY GENERATION: 653.1 TWh

67% renewables (IEA average: 24%)

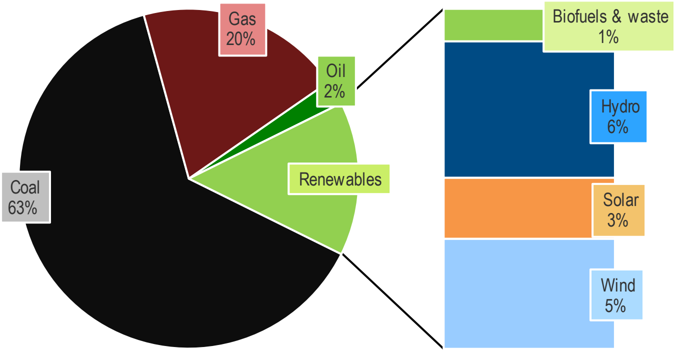


© International Energy Agency, Key Energy Data by country

Australia

ELECTRICITY GENERATION: 257.5 TWh

15% renewables (IEA average: 24%)



Focus today: Electric Energy

- Electricity supply and consumption must match exactly at all times from less that a second to years ahead.
- This makes the problem very hard.
- Storage provides flexibility but it is (still!) difficult to store electricity in large quantities.
- Idea: Divide the "big problem" into interacting subproblems on different times scales.

Electric System Time Scales

$\approx$ 1 second	Dynamic frequency response (normal fluctuations)
$\leq$ 10 min	Reserve (meet unforeseen conditions)
$\leq$ 1 hour	Optimal power flow
1 day	Unit commitment
1 week - 1 year	Maintenance
$\geq$ 1 year	Expansion planning

At what time scales does optimization apply?

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At what time scales does optimization apply?

Optimization Models for Unit Commitment

Optimization Models for Unit Commitment

Reference:

- M.F. Anjos and A.J. Conejo. *Unit Commitment in Electric Energy Systems*, Now Foundations and Trends, 2017 (ISBN 978-1-68083-370-6). http://dx.doi.org/10.1561/3100000014

Unit Commitment (UC)

The purpose of UC is:

- to minimize the system-wide cost of power generation
- while ensuring that demand is met, and
- that the system operates safely and reliably.

Conceptual Formulation

$$\min_{p_j(t),\overline{p}_j(t)} \quad \sum_{t\in T}\sum_{j\in J}\left(c_j(p_j(t)) + c_j^{\mathrm{U}}(t)\right) \tag{1}$$

s.t.

$$\sum_{j\in J} p_j(t) = D(t), \quad \forall t \in T \tag{2}$$

$$\sum_{j\in J} \overline{p}_j(t) \geq D(t) + R(t), \quad \forall t \in T \tag{3}$$

$$p_j(t), \overline{p}_j(t) \in \Pi_j(t), \quad \forall j \in J,\ \forall t \in T \tag{4}$$

where

- $t \in T$ is a time period in the planning horizon
- $c_j^{\mathrm{U}}(t)$ is the cost of starting up unit j in period t
- $p_j(t)$ is the power generated by unit j at time t
- $\overline{p}_j(t)$ is the maximum available power from unit j at time t ($\overline{p}_j(t) \geq p_j(t)$)
- $\Pi_j(t)$ represents the operational constraints on the generators

Power Generation Cost

The cost of producing $p_j(t)$ units of electricity using unit j at time t is denoted $c_j(p_j(t))$.

For ease of computation, it is common practice to model $c_j(p_j(t))$ as a convex piecewise linear, monotonically increasing function:

$$c_j(p_j(t)) \geq \alpha_{js} p_j(t) + \beta_{js}, \quad s = 1, \ldots, C_j,$$

where C_j is the number of linear pieces in the cost function of unit j, and α_{js} and β_{js} are fixed coefficients.

The concavity and strictly monotonic increase of $c_j(p_j(t))$ reflect the increasing marginal cost of generation.

For simplicity, we will assume most of the time that $c_j(p_j(t))$ is given by a single line passing through the origin.

Basic Formulation

Next we model the limitations imposed by the physical characteristics and operating conditions of the generating units. These vary according to the nature of each unit (e.g., coal, gas, nuclear, hydro). In particular,

- units have to operate within a given range of generation limits,
- their power output cannot change too rapidly, and
- when their on/off status changes, it cannot change again before a minimum amount of time has passed.

Binary Decision Variables

We introduce three binary variables per generating unit:

- $v_j(t)$ equals 1 if unit j is on in time period t, and 0 if it is off;
- $y_j(t)$ equals 1 if unit j starts up at the beginning of time period t, and 0 otherwise;
- $z_j(t)$ equals 1 if unit j shuts down at the beginning of time period t, and 0 otherwise.

Binary Decision Variables

A set of constraints is needed to ensure the logical coherence of the values of these variables:

$$v_j(t-1) - v_j(t) + y_j(t) - z_j(t) = 0 \quad \forall j \in J,\ \forall t \in T. \tag{5}$$

For example, suppose that $v_j(0) = 1$ and $v_j(1) = 0$, i.e., unit j is on at time $t = 0$ but not at $t = 1$. Then constraints (5) will require that $y_j(1) - z_j(1) = -1$, which holds only if $y_j(1) = 0$ and $z_j(1) = 1$.

Constraints (5) require knowledge of the values $v_j(0)$ that are the on/off status of the units at $t = 0$, i.e., just before the planning horizon starts.

Startup Cost

A startup cost $c_j^{\mathrm{U}}(t)$ is incurred when unit j starts up at the beginning of time period t.

This cost depends on how long the unit has been inactive. For example, for thermal units this cost is maximum when the boiler is completely cold.

For simplicity, we assume that the startup cost $c_j^{\mathrm{U}}(t)$ is constant for each $j \in J$.

Ramping Constraints

The ramping up of output from t to $t+1$ is limited as follows:

$$p_j(t) - p_j(t-1) \leq R_j^{\mathrm{U}}\, v_j(t-1) + S_j^{\mathrm{U}}\, y_j(t), \forall j \in J,\ \forall t \in T, \tag{6}$$

where R_j^{U} is the maximum ramp-up rate of unit j, and S_j^{U} is its maximum startup rate.

If the unit is on in time period $t-1$, i.e., if $v_j(t-1) = 1$, then the increase in power output at time t cannot be larger than R_j^{U}.

If the unit is turned on in the current time period t, i.e., if $y_j(t) = 1$, then it can output at most S_j^{U} during this period.

Constraints (6) require knowledge of $v_j(0)$ and $p_j(0)$, respectively the on/off status and the generation levels at $t = 0$.

Ramping Constraints

The constraints on ramping down from time period t to $t+1$ are deduced using similar arguments:

$$p_j(t-1) - p_j(t) \leq R_j^{\mathrm{D}}\, v_j(t) + S_j^{\mathrm{D}}\, z_j(t) \;\forall j \in J,\; \forall t \in T, \tag{7}$$

where R_j^{D} is the maximum ramp-down rate of unit j, and S_j^{D} is its maximum shutdown rate.

If unit j is shut down in time period $t+1$, then its output at t cannot be more than S_j^{D}. Otherwise its power output decrease at t cannot be larger than R_j^{D}.

Constraints (7) require knowledge of $p_j(0)$, the generation levels $t=0$.

Uptime and Downtime Constraints

These constraints account for the fact that a unit cannot be turned on or off arbitrarily.

If unit j starts up in time period t then it has to run for at least T_j^{U} time periods before it can be shut down.

Similarly, if it is shut down at t then it has to remain off for at least T_j^{D} periods.

Uptime and Downtime Constraints

The minimum uptime requirement is expressed as:

$$\sum_{k=t-T_j^{\mathrm{U}}+1,\ k\geq 1}^{t} y_j(k) \leq v_j(t) \quad \forall t \in [\, L_j + 1,\ \ldots, |T|\,]\ \forall j \in J, \tag{8}$$

where U_j is the number of time periods that j is required to be on at the start of the planning horizon, and $L_j = \min\{|T|, U_j\}$.

These constraints require knowledge of the operational history of j in the $T_j^{\mathrm{U}} - 1$ time periods before $t = 1$. For example, if $T_j^{\mathrm{U}} = 3$ then:

- If unit j is off at t, then $v_j(t) = 0$, and therefore $y_j(t) = 0$, $y_j(t-1) = 0$, and $y_j(t-2) = 0$ must hold.
- Conversely, if any one of $y_j(t)$, $y_j(t-1)$, or $y_j(t-2)$ equals 1, then $v_j(t) = 1$ must hold.

Uptime and Downtime Constraints

Similarly, we express the minimum downtime requirement as:

$$v_j(t) + \sum_{k=t-T_j^{\mathrm{D}}+1,\ k\geq 1}^{t} z_j(k) \leq 1 \ \forall t \in [\, F_j + 1, \ldots, |\, T\,|\,] \ \forall j \in J \tag{9}$$

where D_j is the number of time periods that j is required to remain off at the start of the planning horizon, and $F_j = \min\{|\,T\,|, D_j\}$.

These constraints require knowledge of the operational history of j in the $T_j^{\mathrm{D}} - 1$ time periods before $t = 1$. For example, if $T_j^{\mathrm{D}} = 4$ then:

- If unit j is on at time t, then $v_j(t) = 1$, and therefore $z_j(t) = 0$, $z_j(t-1) = 0$, $z_j(t-2) = 0$, and $z_j(t-3) = 0$ will hold.
- Conversely, if any one of $z_j(t)$, $z_j(t-1)$, $z_j(t-2)$, or $z_j(t-3)$ equals 1, then $v_j(t) = 0$ must hold.

Generation Limits

The maximum available power at time t from unit j is $\overline{p}_j(t)$.

The additional power over $p_j(t)$ is used to meet the reserve requirements (3).

If unit j is on in time t then $p_j(t)$ must satisfy lower and upper limits $\underline{P}_j$ and $\overline{P}_j$:

$$\underline{P}_j v_j(t) \leq p_j(t) \leq \overline{p}_j(t) \leq \overline{P}_j v_j(t) \; \forall j \in J, \; \forall t \in T \tag{10}$$

Note that if unit j is off at t, then (10) forces $p_j(t)$ and $\overline{p}_j(t)$ to be zero.

Generation Limits

If $v_j(t) = 1$, then the magnitude of $\overline{p}_j(t)$ is constrained by the ramp-up, startup, and shutdown limits.

Constraint (11) limits $\overline{p}_j(t)$ to be no greater than the power output in the previous time period plus the maximum ramp-up rate:

$$\overline{p}_j(t) \leq p_j(t-1) + R_j^{\mathrm{U}} v_j(t-1) + S_j^{\mathrm{U}} y_j(t), \forall j \in J, \ \forall t \in T \qquad (11)$$

The exception is if unit j was started at the beginning of period t, in which case the constraint limits $\overline{p}_j(t)$ to the startup rate.

Constraint (12) ensures that if unit j is shut down at the beginning of time period $t+1$, then $\overline{p}_j(t)$ is bounded above by the maximum shutdown rate.

$$\overline{p}_j(t) \leq \overline{P}_j \left[v_j(t) - z_j(t+1) \right] + z_j(t+1) S_j^{\mathrm{D}}, \forall j \in J, \ \forall t \in T \qquad (12)$$

Otherwise, the constraint becomes redundant, imposing the same upper bound as in (10).

Recap: Basic Formulation of UC

$$
\begin{aligned}
\min_{\Xi} \quad & \sum_{t \in T} \sum_{j \in J} \left(c_j(p_j(t)) + c_j^{\mathrm{U}} y_j(t) \right) \\
\text{s.t.} \quad & \sum_{j \in J} p_j(t) = D(t), && \forall t \in T \\
& \sum_{j \in J} \overline{p}_j(t) \geq D(t) + R(t), && \forall t \in T \\
& c_j(p_j(t)) \geq \alpha_{js} p_j(t) + \beta_{js}, \quad s = 1, \ldots, C_j, && \forall j \in J \\
& v_j(t-1) - v_j(t) + y_j(t) - z_j(t) = 0, && \forall j \in J, \forall t \in T \\
& p_j(t) - p_j(t-1) \leq R_j^{\mathrm{U}} \, v_j(t-1) + S_j^{\mathrm{U}} \, y_j(t), && \forall j \in J, \forall t \in T \\
& p_j(t-1) - p_j(t) \leq R_j^{\mathrm{D}} \, v_j(t) + S_j^{\mathrm{D}} \, z_j(t), && \forall j \in J, \forall t \in T \\
& \sum_{k = t - T_j^{\mathrm{U}} + 1, \; k \geq 1}^{t} y_j(k) \leq v_j(t), \forall t \in [L_j + 1, \ldots, |T|], && \forall j \in J \\
& v_j(t) + \sum_{k = t - T_j^{\mathrm{D}} + 1, \; k \geq 1}^{t} z_j(k) \leq 1, \forall t \in [F_j + 1, \ldots, |T|], && \forall j \in J \\
& \underline{P}_j v_j(t) \leq p_j(t) \leq \overline{p}_j(t) \leq \overline{P}_j v_j(t), && \forall j \in J, \forall t \in T \\
& \overline{p}_j(t) \leq p_j(t-1) + R_j^{\mathrm{U}} v_j(t-1) + S_j^{\mathrm{U}} y_j(t), && \forall j \in J, \forall t \in T \\
& \overline{p}_j(t) \leq \overline{P}_j \left[v_j(t) - z_j(t+1) \right] + z_j(t+1) S_j^{\mathrm{D}}, && \forall j \in J, \forall t \in T
\end{aligned}
$$

where the optimization variables in set Ξ are $p_j(t)$, $\overline{p}_j(t)$, $v_j(t)$, $y_j(t)$, and $z_j(t)$, $\forall j \in J, \ \forall t \in T$.

Example (§ 2.7.1)

Suppose that we have three generators with the following characteristics:

Unit (j)	c_j^{U}	c_j	$\underline{P}_j$	$\overline{P}_j$	S_j^{U}	S_j^{D}	R_j^{U}	R_j^{D}	T_j^{U}	T_j^{D}
1	800	5	80	300	100	80	50	30	3	2
2	500	15	50	200	70	50	60	40	2	2
3	250	30	30	100	40	30	70	50	1	2

- Unit 1 can produce a large quantity of power at low cost, but it incurs a high startup cost and ramps up relatively slowly ("baseload")
- Unit 2 has lower production capacity than unit 1 and a slightly higher cost but costs less to startup ("load following")
- Unit 3 has the lowest production capacity and is expensive, but it has the lowest startup cost and ramps up more quickly ("peaker")

The expectation is that the model will aim to use unit 1 first, then unit 2, and finally unit 3.

Example (§ 2.7.1) (ctd)

Further suppose that at $t = 0$, i.e., just before our planning horizon starts, the operating conditions are:

Unit (j)	v_j	p_j	U_j	D_j
1	1	120	2	0
2	0	0	0	0
3	0	0	0	0

Thus, at $t = 0$, unit 1 is on and producing 120 MW, and units 2 and 3 are off. Furthermore, unit 1 is required to stay on for two time periods at the start of the planning horizon.

Example (§ 2.7.1) (ctd)

Let us now plan for the next six time periods with the following requirements:

Time period t	1	2	3	4	5	6
Demand $D(t)$	230	250	200	170	230	190
Reserve $R(t)$	10	10	10	10	10	10

The optimal solution has total cost equal to 8950:

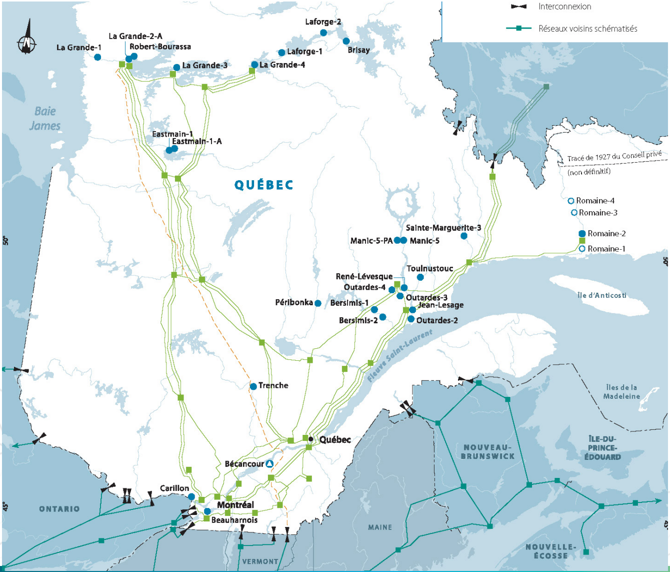
Time period t	1	2	3	4	5	6
$p_1(t)$	170	200	200	170	200	190
$p_2(t)$	60	50	0	0	0	0
$p_3(t)$	0	0	0	0	30	0
$\overline{p}_1(t)$	170	220	250	180	220	250
$\overline{p}_2(t)$	70	50	0	0	0	0
$\overline{p}_3(t)$	0	0	0	0	30	0
Dual of (2)	15	5	5	5	5	5
$\overline{p}_1(t) - p_1(t)$	0	20	50	10	20	60
$\overline{p}_2(t) - p_2(t)$	10	0	0	0	0	0
$\overline{p}_3(t) - p_3(t)$	0	0	0	0	0	0
Dual of (3)	0	0	0	0	0	0

Observations

- Unit 1 is committed throughout, unit 2 is turned on for the first two periods, and unit 3 is turned on at $t = 5$ only.
- Unit 2 is turned on at $t = 1$ because the level of demand in the first time period means that unit 1, which is operating at a level of 120 at $t = 0$, cannot ramp up fast enough to satisfy both demand and reserve.
- The fact that the demand at $t = 1$ is high compared to the production levels at $t = 0$ is reflected in the higher marginal price of energy at $t = 1$.
- Once on, unit 2 must run for two periods before it can be turned off.
- There is a peak in demand at $t = 5$, but it lasts only one period. Therefore, the optimal solution is to turn on unit 3 for only that period and to increase the output of unit 1 to make up the difference.

DC Network-Constrained Deterministic UC

Motivation



Interconnexion
Réseaux voisins schématisés
Laforge-2
Brisay
Laforge-1
La Grande-4
La Grande-3
La Grande-2-A
Robert-Bourassa
La Grande-1
Baie James
Eastmain-1
Eastmain-1-A
QUÉBEC
Tracé de 1927 du Conseil privé (non définitif)
Romaine-4
Romaine-3
Romaine-2
Romaine-1
Sainte-Marguerite-3
Manic-5-PA
Manic-5
Toulnustouc
René-Lévesque
Outardes-4
Outardes-3
Jean-Lesage
Outardes-2
Péribonka
Bersimis-1
Bersimis-2
Île d'Anticosti
Fleuve Saint-Laurent
Trenche
Îles de la Madeleine
Québec
Bécancour
Carillon
Montréal
Beauharnois
ONTARIO
NOUVEAU-BRUNSWICK
ÎLE-DU-PRINCE-ÉDOUARD
MAINE
VERMONT
NOUVELLE-ÉCOSSE
50°
45°

DC Network-Constrained Deterministic UC

- The formulation on slide 28 only allocates sufficient power generation to satisfy the demand plus the spinning reserves.
- It does not account for the transmission system that carries the electricity from the generating units to the loads.
- Because there are limits on the power flow across the lines, our model will provide better information if it ensures that the allocated power can flow through the system.
- The impact of transmission is particularly important in markets where accurate modeling of the congestion of the power lines and the consequent differences in the locational marginal prices (LMPs) of electricity are essential for proper market operation.

Power Flow: AC versus DC

- The flow of power is typically modeled as either alternating current or direct current, and both can be used for UC.
- Alternating current (AC) is an electric current which periodically reverses direction, in contrast to
- Direct current (DC) which is an electric current that flows only in one direction.
- AC is the form that consumers typically use when they access the energy through a power socket.
- DC is the form of electricity provided by batteries, for example.

Modeling Power Flows

- Mathematically, the AC power flow equations are nonlinear and nonconvex, involving sin and cos functions.
- The DC equations are a linearized version of the AC ones.
- The DC approach is more efficient computationally and provides a reasonably good estimate of the system behavior under normal conditions, but it can lead to misleading results in extreme conditions.
- We consider the DC approach now, and the AC approach is presented in Chapter 4.

DC Power Flow

- Let N be the set of nodes in the power system network.
- Let nm be the line connecting nodes n and m, and Λ_n^{L} be the set of nodes directly connected to node n.
- The power flow on a line is proportional to the difference in voltage angles between the nodes at the endpoints of the line:

$$B_{nm}\left(\theta_n(t) - \theta_m(t)\right),$$

where $\theta_n(t)$ is the voltage angle at node n, and B_{nm} is a positive constant equal to the negative of the series susceptance of the line (see Chapter 4).
- Thus, the net power flow (generation minus demand) at node n is given by

$$\sum_{m \in \Lambda_n^{\mathrm{L}}} B_{nm}\left(\theta_n(t) - \theta_m(t)\right).$$

Integration of Network Constraints

Step 1: Enforce power balance at each node independently.
The constraints

$$\sum_{j \in J} p_j(t) = D(t), \quad \forall t \in T$$

are replaced by

$$\sum_{j \in \Lambda_n^{\mathrm{G}}} p_j(t) - \sum_{i \in \Lambda_n^{\mathrm{D}}} D_i(t) = \sum_{m \in \Lambda_n^{\mathrm{L}}} B_{nm} \left(\theta_n(t) - \theta_m(t)\right) \quad \forall n \in N, \forall t \in T, \quad (13)$$

where

- $D_i(t)$ is the load of demand i in period t,
- Λ_n^{G} is the set of generating units located at node n, and
- Λ_n^{D} is the set of demands located at node n.

Integration of Network Constraints (ctd)

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Step 2: Enforce transmission capacity limits for each transmission line.

$$-\overline{P}_{nm} \leq B_{nm}\left(\theta_n(t) - \theta_m(t)\right) \leq \overline{P}_{nm}, \quad \forall n \in N, \forall m \in \Lambda_n^{\mathrm{L}}, \forall t \in T, \tag{14}$$

where $\overline{P}_{nm}$ is the transmission capacity of line nm.

Step 3: Set a reference angle.

$$\theta_{\hat{n}}(t) = 0 \quad \forall t \in T, \tag{15}$$

where $\hat{n}$ is the node of the reference angle.

Integration of Network Constraints (ctd)

Step 4: Enforce appropriate reserve levels in each reserve area.
Constraints (3) are replaced by:

$$\sum_{j \in \Omega_r^{\mathrm{G}}} \overline{p}_j(t) \geq \sum_{i \in \Omega_r^{\mathrm{D}}} D_i(t) + R_r(t) \quad \forall r \in \mathcal{R}, \forall t \in T, \tag{16}$$

where

- $\mathcal{R}$ is the set of reserve areas,
- $R_r(t)$ is the reserve required in reserve area r in time period t,
- Ω_r^{G} is the set of generating units in area r, and
- Ω_r^{D} is the set of demands in area r.

The required reserve in area r, $R_r(t)$, should take into account the interconnections of area r with neighboring areas, since these areas may also provide reserve to area r (and vice versa).

Recap: DC Network-Constrained UC

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$$
\begin{aligned}
\min_{\Xi} \quad & \sum_{t \in T} \sum_{j \in J} \left(c_j(p_j(t)) + c_j^{\mathrm{U}} y_j(t) \right) \\
\text{s.t.} \quad & \sum_{j \in \Lambda_n^{\mathrm{G}}} p_j(t) - \sum_{i \in \Lambda_n^{\mathrm{D}}} D_i(t) = \sum_{m \in \Lambda_n^{\mathrm{L}}} B_{nm} \left(\theta_n(t) - \theta_m(t) \right), && \forall n \in N, \ \forall t \in T \\
& -\overline{P}_{nm} \leq B_{nm} \left(\theta_n(t) - \theta_m(t) \right) \leq \overline{P}_{nm}, && \forall n \in N, \forall m \in \Lambda_n^{\mathrm{L}}, \forall t \in T \\
& \theta_{\hat{n}}(t) = 0, && \forall t \in T \\
& \sum_{j \in \Omega_r^{\mathrm{G}}} \overline{p}_j(t) \geq \sum_{i \in \Omega_r^{\mathrm{D}}} D_i(t) + R_r(t), && \forall r \in \mathcal{R}, \ \forall t \in T \\
& c_j(p_j(t)) \geq \alpha_{js} p_j(t) + \beta_{js}, \quad s = 1, \ldots, C_j, && \forall j \in J \\
& v_j(t-1) - v_j(t) + y_j(t) - z_j(t) = 0, && \forall j \in J, \ \forall t \in T \\
& \text{Ramping constraints (6) and (7)} \\
& \text{Uptime and downtime constraints (8) and (9)} \\
& \text{Generation limits (10), (11) and (12)}
\end{aligned}
$$

where the optimization variables in set Ξ are $p_j(t)$, $\overline{p}_j(t)$, $v_j(t)$, $y_j(t)$, $z_j(t)$, and $\theta_n(t)$, $\forall n \in N$, $\forall j \in J$, $\forall t \in T$.

Example (§ 3.4)

Consider a small power network with six nodes, and with

- the three generators from § 2.7.1 located at nodes 1, 2, and 3 respectively, and
- demand at nodes 4, 5, and 6.

For simplicity, we assume that there is a single reserve area, corresponding to the whole network.

Recall the characteristics of the three generators:

Unit (j)	c_j^{U}	c_j	$\underline{P}_j$	$\overline{P}_j$	S_j^{U}	S_j^{D}	R_j^{U}	R_j^{D}	T_j^{U}	T_j^{D}
1	800	5	80	300	100	80	50	30	3	2
2	500	15	50	200	70	50	60	40	2	2
3	250	30	30	100	40	30	70	50	1	2

Example (§ 3.4.1)

Suppose that the demands at nodes 4, 5, and 6 are as follows:

Time period t	1	2	3	4	5	6
$D_4(t)$	100	100	80	140	100	80
$D_5(t)$	80	100	80	30	90	60
$D_6(t)$	50	50	40	0	40	50
Total demand	230	250	200	170	230	190
Reserve $R(t)$	10	10	10	10	10	10

that the (negative of) the series susceptance equals 0.8 for every line of the network, and that the lines have the following transmission capacities $\overline{P}_{nm}$:

	1	2	3	4	5	6
1	-	-	-	100	100	100
2	-	-	-	100	100	100
3	-	-	-	-	100	100
4	100	100	-	-	-	-
5	100	100	100	-	-	-
6	100	100	100	-	-	-

Example (§ 3.4.1)

Solving the optimization problem on slide 42, first to find the commitment decisions, and then with the commitment decisions fixed, we obtain the following solution with total cost 8950:

Time period t	1	2	3	4	5	6
$p_1(t)$	170	200	200	170	200	190
$p_2(t)$	60	50	0	0	0	0
$p_3(t)$	0	0	0	0	30	0
Dual at node 1	15	5	5	5	5	5
Dual at node 2	15	5	5	5	5	5
Dual at node 3	15	5	5	5	5	5
Dual at node 4	15	5	5	5	5	5
Dual at node 5	15	5	5	5	5	5
Dual at node 6	15	5	5	5	5	5
$\overline{p}_1(t) - p_1(t)$	0	10	10	10	10	10
$\overline{p}_2(t) - p_2(t)$	10	0	0	0	0	0
$\overline{p}_3(t) - p_3(t)$	0	0	0	0	0	0
$\sum_j (\overline{p}_j(t) - p_j(t))$	10	10	10	10	10	10

Example (§ 3.4.1)

Constraints (13) enforce power balance at each node of the network, and their corresponding dual optimal values can be interpreted as nodal energy prices.

In this example, they are equal to 15 at every node for $t = 1$, and equal to 5 at every node for the subsequent time periods.

Example (§ 3.4.1)

This network-constrained formulation also gives us information about the flows of power in the system.

For instance, at $t = 1$ the power flows are:

	1	2	3	4	5	6
1	-	-	-	-68.33	-55.83	-45.83
2	-	-	-	-31.67	-19.17	-9.17
3	-	-	-	-	-5.00	5.00
4	68.33	31.67	-	-	-	-
5	55.83	19.17	5.00	-	-	-
6	45.83	9.17	-5.00	-	-	-

where positive/negative values are outflows/inflows w.r.t. the column node.

Example (§ 3.4.1)

The flows are calculated using the optimal values of the voltage variables:

Node	$t = 1$	$t = 2$	$t = 3$	$t = 4$	$t = 5$	$t = 6$
1	85.42	93.75	91.67	122.92	104.17	89.59
2	39.58	31.25	8.33	52.08	20.83	10.42
3	21.88	15.63	12.50	78.13	50.00	15.63
4	0	0	0	0	0	0
5	15.63	5.21	4.167	71.88	20.83	13.54
6	28.13	26.04	20.83	84.38	41.67	17.71

and the expression $B_{nm}\left(\theta_n(t) - \theta_m(t)\right)$. At $t = 5$, the flows are:

	1	2	3	4	5	6
1	-	-	-	-83.33	-66.67	-50
2	-	-	-	-16.67	**0**	16.67
3	-	-	-	-	-23.33	-6.67
4	83.33	16.67	-	-	-	-
5	66.67	**0**	23.33	-	-	-
6	50	-16.67	6.67	-	-	-

Example (§ 3.4.2)

Changes in the transmission capacities can significantly affect the UC solution.

For example, if the capacity of the line between nodes 1 and 6 is reduced from 100 to 40, then the total cost is 11500 (28.5% more), and the solution is:

Time period t	1	2	3	4	5	6
$p_1(t)$	150	157.5	147.5	120	162.5	137.5
$p_2(t)$	50	92.5	52.5	50	67.5	52.5
$p_3(t)$	30	0	0	0	0	0
Dual at node 1	5	5	5	5	5	5
Dual at node 2	5	25	5	5	15	15
Dual at node 3	5	30	5	5	17.5	17.5
Dual at node 4	5	15	5	5	10	10
Dual at node 5	5	20	5	5	12.5	12.5
Dual at node 6	5	40	5	5	22.5	22.5
$\overline{p}_1(t) - p_1(t)$	20	10	0	10	0	0
$\overline{p}_2(t) - p_2(t)$	20	0	100	0	10	75
$\overline{p}_3(t) - p_3(t)$	0	0	0	0	0	0
$\sum_j (\overline{p}_j(t) - p_j(t))$	40	10	100	10	10	75

Example (§ 3.4.2)

Observation 1: The commitment decisions are different: units 1 and 2 are turned on throughout, and unit 3 is on in the first time period.

Observation 2: The nodal (dual) energy prices are no longer equal throughout the network in each time period.

- For $t = 2$, $t = 5$, and $t = 6$, the prices between nodes vary significantly.
- The highest prices occur at node 6.
- This is due to the reduced capacity on the line between nodes 1 and 6; the capacity constraint on this line becomes binding in these time periods.
- Key point: A single binding constraint may suffice to cause the prices to be different at every node.

Example (§ 3.4.2)

For period $t = 2$ in which the highest prices are reached, the power flows are:

	1	2	3	4	5	6
1	-	-	-	-60.83	-56.67	**-40.00**
2	-	-	-	-39.17	-35.00	-18.33
3	-	-	-	-	-8.33	8.33
4	60.83	39.17	-	-	-	-
5	56.67	35.00	8.33	-	-	-
6	**40.00**	18.33	-8.33	-	-	-

Contrast them with the flows at $t = 2$ before the capacity reduction on the line:

	1	2	3	4	5	6
1	-	-	-	-75.00	-70.83	-54.17
2	-	-	-	-25.00	-20.83	-4.17
3	-	-	-	-	-8.33	8.33
4	75.00	25.00	-	-	-	-
5	70.83	20.83	8.33	-	-	-
6	54.17	4.17	-8.33	-	-	-

Security-Constrained Deterministic UC

Security-Constrained Deterministic UC

- The security-constrained UC (SCUC) is a formulation of UC that integrates key aspects of system security while scheduling generation.
- The failure of a major system component is called a contingency.
- Our concern here is with contingencies whose impact cannot be handled by the spinning reserves.
- The objective is that the system should be able to continue to meet all the load demands if a contingency happens.

Security-Constrained Deterministic UC

- We consider contingencies pertaining solely to transmission lines. (Contingencies pertaining to generating units can be handled similarly.)
- The formulation we use embodies a corrective view, i.e., the objective is to ensure that the system can transition from the contingency state (where some transmission line limits are violated) to a safe post-contingency state (where no transmission line limit is violated).
- The contingency may occur at any time within the planning horizon, and the optimal schedule ensures that there is sufficient generation capacity available to support the transition to a safe operating state.
- The (more expensive) preventive view that guarantees that all post-contingency states are safe is more common in practice.

Pre- and Post-Contingency Operating Conditions

- Let C denote the set of possible contingencies.
- We identify each of them by the susceptances of the transmission lines under the contingency, denoted B^c_{nm}, where the superscript c indicates contingency c.
- For each contingency $c \in C$, we introduce additional variables:
  - $p^c_j(t)$ denotes the post-contingency c power produced at time t of unit j, and
  - $\theta^c_n(t)$ to denote the post-contingency c voltage angle of node n.

Pre- and Post-Contingency Operating Conditions (ctd)

The post-contingency versions of the network constraints (13), (14), and (15) are respectively:

$$\sum_{j\in\Lambda_n^{\mathrm{G}}} p_j^c(t) - \sum_{i\in\Lambda_n^{\mathrm{D}}} D_i(t) = \sum_{m\in\Lambda_n^{\mathrm{L}}} B_{nm}^c\left(\theta_n^c(t) - \theta_m^c(t)\right), \forall n \in N, \forall t \in T, \forall c \in C \quad (17)$$

$$-\overline{P}_{nm} \leq B_{nm}^c\left(\theta_n^c(t) - \theta_m^c(t)\right) \leq \overline{P}_{nm},\ \forall n \in N, \forall m \in \Lambda_n^{\mathrm{L}}, \forall t \in T, \forall c \in C \quad (18)$$

$$\theta_{\hat{n}}^c(t) = 0,\ \forall t \in T, \forall c \in C \quad (19)$$

Pre- and Post-Contingency Operating Conditions (ctd)

In a similar manner, the post-contingency versions of the ramping constraints (6) and (7) are:

$$p_j^c(t) - p_j^c(t-1) \le R_j^{\mathrm{U}}\, v_j(t-1) + S_j^{\mathrm{U}}\, y_j(t),\ \forall j \in J, \forall t \in T, \forall c \in C \quad (20)$$

$$p_j^c(t-1) - p_j^c(t) \le R_j^{\mathrm{D}}\, v_j(t) + S_j^{\mathrm{D}}\, z_j(t),\ \forall j \in J, \forall t \in T, \forall c \in C \quad (21)$$

Generation limits similar to (10) also apply to the post-contingency power outputs:

$$\underline{P}_j v_j(t) \le p_j^c(t) \le \overline{P}_j v_j(t), \forall j \in J,\ \forall t \in T, \forall c \in C \quad (22)$$

Pre- and Post-Contingency Operating Conditions (ctd)

Finally, it is necessary to link the pre-contingency and post-contingency conditions by bounding their difference:

$$p_j^c(t) - p_j(t) \leq X_j^U,\ \forall j \in J,\ \forall t \in T, \forall c \in C \tag{23}$$

$$p_j(t) - p_j^c(t) \leq X_j^D,\ \forall j \in J,\ \forall t \in T, \forall c \in C \tag{24}$$

where X_j^U (respectively X_j^D) is the maximum increase (resp. decrease) in the power that can be provided by unit j to transition the system to a safe post-contingency state.

Recap: Security-Constrained UC

(For simplicity, minimum up/down time and reserve constraints are omitted.)

$$\min_{\Xi} \quad \sum_{t\in T}\sum_{j\in J}\left(c_j(p_j(t)) + c_j^{\mathrm{U}} y_j(t)\right)$$

s.t.

$$\sum\nolimits_{j\in\Lambda_n^{\mathrm{G}}} p_j(t) - \sum\nolimits_{i\in\Lambda_n^{\mathrm{D}}} D_i(t) = \sum\nolimits_{m\in\Lambda_n^{\mathrm{L}}} B_{nm}\left(\theta_n(t)-\theta_m(t)\right), \qquad \forall n\in N,\ \forall t\in T$$

$$-\overline{P}_{nm} \le B_{nm}\left(\theta_n(t)-\theta_m(t)\right) \le \overline{P}_{nm}, \qquad \forall n\in N, \forall m\in\Lambda_n^{\mathrm{L}}, \forall t\in T$$

$$\theta_{\hat{n}}(t) = 0, \qquad \forall t\in T$$

$$c_j(p_j(t)) \ge \alpha_{js} p_j(t) + \beta_{js}, \quad s = 1,\ldots,C_j, \qquad \forall j\in J$$

$$v_j(t-1) - v_j(t) + y_j(t) - z_j(t) = 0, \qquad \forall j\in J,\ \forall t\in T$$

$$\underline{P}_j v_j(t) \le p_j(t) \le \overline{P}_j v_j(t) \qquad \forall j\in J,\ \forall t\in T$$

Ramping constraints (6) and (7)

$$\sum\nolimits_{j\in\Lambda_n^{\mathrm{G}}} p_j^c(t) - \sum\nolimits_{i\in\Lambda_n^{\mathrm{D}}} D_i(t) = \sum\nolimits_{m\in\Lambda_n^{\mathrm{L}}} B_{nm}^c\left(\theta_n^c(t)-\theta_m^c(t)\right), \qquad \forall n\in N, \forall t\in T, \forall c\in C$$

$$-\overline{P}_{nm} \le B_{nm}^c\left(\theta_n^c(t)-\theta_m^c(t)\right) \le \overline{P}_{nm}, \qquad \forall n\in N, \forall m\in\Lambda_n^{\mathrm{L}}, \forall t\in T, \forall c\in C$$

$$\theta_{\hat{n}}^c(t) = 0, \qquad \forall t\in T, \forall c\in C$$

$$p_j^c(t) - p_j^c(t-1) \le R_j^{\mathrm{U}}\, v_j(t-1) + S_j^{\mathrm{U}}\, y_j(t), \qquad \forall j\in J, \forall t\in T, \forall c\in C$$

$$p_j^c(t-1) - p_j^c(t) \le R_j^{\mathrm{D}}\, v_j(t) + S_j^{\mathrm{D}}\, z_j(t), \qquad \forall j\in J, \forall t\in T, \forall c\in C$$

$$\underline{P}_j v_j(t) \le p_j^c(t) \le \overline{P}_j v_j(t), \qquad \forall j\in J,\ \forall t\in T, \forall c\in C$$

$$p_j^c(t) - p_j(t) \le X_j^U, p_j(t) - p_j^c(t) \le X_j^D, \qquad \forall j\in J,\ \forall t\in T, \forall c\in C$$

where the optimization variables in set Ξ are $p_j(t)$, $v_j(t)$, $y_j(t)$, $z_j(t)$, $\theta_n(t)$, $p_j^c(t)$, and $\theta_n^c(t)$, $\forall n\in N, \forall j\in J,\ \forall t\in T, \forall c\in C$.

Example (§ 4.3)

We use again an example with six nodes, with generators in nodes 1, 2, and 3, and demand at nodes 4, 5, and 6. As in the formulation on slide 59, we do not account for minimum up/down times nor reserve requirements.

Consider the following nodal demands over the six time periods:

Time period t	1	2	3	4	5	6
$D_4(t)$	100	100	100	100	100	100
$D_5(t)$	0	40	40	40	40	0
$D_6(t)$	0	0	20	0	30	0
Total demand	100	140	160	140	170	100

and lines with susceptances $B_{nm} = 0.8$, and transmission capacities $\overline{P}_{nm}$:

	1	2	3	4	5	6
1	-	-	-	100	-	100
2	-	-	-	100	100	100
3	-	-	-	-	100	100
4	100	100	-	-	-	-
5	-	100	100	-	-	-
6	100	100	100	-	-	-

Example (§ 4.3.1)

Solving the optimization problem on slide 59 with no contingencies ($C = \emptyset$) and no power change limits, first to find the commitment decisions, and then with the commitment decisions fixed, we obtain the following solution with total cost 4850:

Time period t	1	2	3	4	5	6
$p_1(t)$	100	140	160	140	170	100
$p_2(t)$	0	0	0	0	0	0
$p_3(t)$	0	0	0	0	0	0
Dual at node 1	5	5	5	5	5	5
Dual at node 2	5	5	5	5	5	5
Dual at node 3	5	5	5	5	5	5
Dual at node 4	5	5	5	5	5	5
Dual at node 5	5	5	5	5	5	5
Dual at node 6	5	5	5	5	5	5

Observe that all the power is provided by the generator at node 1, which is already running at $t = 0$ with $p_1(0) = 120$ (unchanged initial conditions).

We also note that all nodal energy prices are equal to 5 for every time period.

Example (§ 4.3.2)

Now consider the possibility of a single contingency c corresponding to the loss of the line between nodes 1 and 6.

We set X_j^U and X_j^D equal to half of the corresponding one-hour ramping (up/down) rate of unit j:

Unit (j)	R_j^{U}	R_j^{D}	X_j^U	X_j^D
1	50	30	25	15
2	60	40	30	20
3	70	50	35	25

Example (§ 4.3.2)

Solving the formulation on slide 59 with the highlighted constraints corresponding to this contingency and the power change limits, we obtain the following generation schedule with a (much higher) total cost of 7550:

Time period t	1	2	3	4	5	6
$p_1(t)$	100	90	110	90	90	100
$p_2(t)$	0	50	50	50	50	0
$p_3(t)$	0	0	0	0	30	0
Dual at node 1	5	5	5	5	5	5
Dual at node 2	5	5	5	5	5	5
Dual at node 3	5	5	5	5	5	5
Dual at node 4	5	5	5	5	5	5
Dual at node 5	5	5	5	5	5	5
Dual at node 6	5	5	5	5	5	5

Example (§ 4.3.2)

Observations:

- If the line between 1 and 6 stops operating, then there is no longer a feasible solution with all the generation taking place at node 1 because there would no longer be enough transmission capacity out of node 1 to satisfy all the demand.
- Therefore the SCUC optimal schedule outputs less at node 1 and turns on one or both of the more expensive generators, resulting in a higher total generation cost.

In the event of the contingency, the generation schedule would be:

Time period t	1	2	3	4	5	6
$p_1^c(t)$	100	90	**95**	90	90	100
$p_2^c(t)$	0	50	**65**	50	50	0
$p_3^c(t)$	0	0	0	0	30	0

where the power levels in bold are those where $p_j^c(t)$ differs from $p_j(t)$.

AC Network-Constrained Deterministic UC

AC Network-Constrained Deterministic UC

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- The DC linearization has many advantages, but it provides no information about several key quantities, among them the changes in voltage magnitude and the reactive power flows.
- For instance, losses in transmission cannot be accurately computed, and this can significantly affect the conclusions drawn from the models.
- We will now incorporate the AC power flow equations into UC, but the resulting formulation is a mixed-integer nonlinear optimization problem that is computationally challenging.

AC Power Flow

- An electric current produces a magnetic field around it.
- When the current is alternating, this magnetic field is constantly changing as a result of the oscillations of the current.
- This change in the magnetic field induces another electric current to flow in the same wire, in a direction opposite to the flow of the original current.
- This phenomenon is called reactance, and unless the voltage and current are perfectly in phase, it limits the power that can be effectively transferred.

AC Power Flow

- The resistance of the line limits the power flow as well, and with the reactance forms the impedance Z_{nm} of the line nm:

$$Z_{nm} = R_{nm} + \jmath X_{nm}$$

where R_{nm} is the resistance of the line, X_{nm} is its reactance, and $\jmath = \sqrt{-1}$.

- The inverse of the impedance is the admittance Y_{nm}, a measure of how easily the current is allowed to flow:

$$Y_{nm} = \frac{1}{Z_{nm}} = G_{nm} - \jmath B_{nm} = \frac{R_{nm}}{(R_{nm}^2 + X_{nm}^2)} - \jmath \frac{X_{nm}}{(R_{nm}^2 + X_{nm}^2)},$$

where G_{nm} is the conductance (inverse of the resistance), and B_{nm} is the susceptance from DC power flow (imaginary part of the admittance).

Active and Reactive Power

- Reactive power accounts for the fact that power is not completely transferred as active (or real) power when voltage and current are not in phase.
- The resulting apparent power is equal to the magnitude of the vector sum of active and reactive power.
- In principle, generating units can provide both active and reactive power, and loads can have demand for reactive power.
- We will have the following as variables:
  - $q_j(t)$ denote the reactive power output of generating unit j at period t, and
  - $Q_i(t)$ denote the reactive power load of demand i at period t, and
  - $V_n(t)$ denote the voltage magnitude of node n.
- Also let $\overline{S}_{nm}$ be the apparent power capacity of line nm, and b_{nm}^{shunt} be half of the shunt susceptance of the line (shunt means "in parallel").

Flows on a Line

Denote the difference between the voltage angles at the endpoints of line *nm* by

$$\theta_{nm}(t) = \theta_n(t) - \theta_m(t),$$

so that

- $\theta_{nm}(t) > 0$ means that real power flows from n to m,
- $\theta_{nm}(t) < 0$ means that real power flows from m to n.

In general:

- Real power flows from higher voltage angle to lower voltage angle, and
- Reactive power flows from higher voltage magnitude to lower voltage magnitude.

AC Power Flow Equations–Load Flow

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For all n and for all t,

$$\sum_{j \in \Lambda_n^{\mathrm{G}}} p_j(t) - \sum_{i \in \Lambda_n^{\mathrm{D}}} D_i(t) =$$
$$\sum_{m \in \Lambda_n^{\mathrm{L}}} V_n(t) V_m(t) \Big[G_{nm} \cos \theta_{nm}(t) + B_{nm} \sin \theta_{nm}(t) \Big] - G_{nm} V_n^2(t) \tag{25}$$

$$\sum_{j \in \Lambda_n^{\mathrm{G}}} q_j(t) - \sum_{i \in \Lambda_n^{\mathrm{D}}} Q_i(t) =$$
$$\sum_{m \in \Lambda_n^{\mathrm{L}}} V_n(t) V_m(t) \Big[G_{nm} \sin \theta_{nm}(t) - B_{nm} \cos \theta_{nm}(t) \Big] + V_n^2(t) \left(B_{nm} - b_{nm}^{\mathrm{shunt}} \right) \tag{26}$$

AC Power Flow Equations–Apparent Power

For all n, for all $m \in \Lambda_n^{\mathrm{L}}$, and for all t,

$$\Big(V_n(t)V_m(t)\left(G_{nm}\cos\theta_{nm}(t) + B_{nm}\sin\theta_{nm}(t)\right) - G_{nm}V_n^2(t)\Big)^2 +$$
$$\Big(V_n(t)V_m(t)\Big[G_{nm}\sin\theta_{nm}(t) - B_{nm}\cos\theta_{nm}(t)\Big] + V_n^2(t)\Big(B_{nm} - b_{nm}^{\mathrm{shunt}}\Big)\Big)^2 \leq \overline{S}_{nm}^2, \tag{27}$$

DC as a Simplification of AC

The DC linearization (25)–(27) is obtained by assuming that

- the conductance is negligible, and that
- in all time periods, the variations in voltage angle and voltage magnitude between the two nodes of a line are small.

Under these assumptions, we can make the approximations:

$$G_{nm} \approx 0, \quad \cos\theta_{nm}(t) \approx 1, \quad \sin\theta_{nm}(t) \approx \theta_{nm}(t),$$

and

$$|V_n(t)| = |V_m(t)| = 1 \text{ (in normalized units)}.$$

DC as a Simplification of AC

Substituting these approximations into (25), we obtain:

$$\sum_{j \in \Lambda_n^{\mathrm{G}}} p_j(t) - \sum_{i \in \Lambda_n^{\mathrm{D}}} D_i(t) = \sum_{m \in \Lambda_n^{\mathrm{L}}} B_{nm}\theta_{nm}(t),$$

which is precisely (13).

Because of the assumption that the voltage magnitudes are all unity, the linearization cannot track reactive power flows, so constraint (26) is dropped for DC.

Finally, with the same approximations, (27) is equivalent to (14) in the DC formulation.

Integration of AC Network Constraints

We integrate the AC network into the basic UC formulation on slide 28 by replacing the constraints (2) with constraints (25), (26), and (27).

We also replace the reserve requirements (3) by the requirements per reserve area (16) as done for the DC network-constrained UC.

Similar demand and reserve constraints can be enforced for reactive power, but for simplicity we omit them.

Integration of AC Network Constraints (ctd)

We also add bounds on the voltage magnitude at node n:

$$V_n^{\min} \leq V_n(t) \leq V_n^{\max} \quad \forall n, \forall t, \tag{28}$$

and on the reactive power output of each generating unit j:

$$q_j^{\min} \leq q_j(t) \leq q_j^{\max} \quad \forall j, \forall t. \tag{29}$$

Finally, as in the DC case, we set a reference angle:

$$\theta_{\hat{n}}(t) = 0 \quad \forall t \in T, \tag{30}$$

where $\hat{n}$ is the node of the reference angle.

(For simplicity, minimum up/down time constraints are omitted.)

Recap: AC Network-Constrained UC

$$\min_{\Xi} \sum_{t\in T}\sum_{j\in J}\left(c_j(p_j(t)) + c_j^{\mathrm{U}}\right)$$

$$\begin{aligned}
\text{s.t.}\quad & \theta_{\hat{n}}(t) = 0, && \forall t \in T\\
& \textstyle\sum_{j\in\Omega_r^{\mathrm{G}}} \overline{p}_j(t) \geq \sum_{i\in\Omega_r^{\mathrm{D}}} D_i(t) + R_r(t), && \forall r \in \mathcal{R},\ \forall t \in T\\
& V_n^{\min} \leq V_n(t) \leq V_n^{\max}, && \forall n \in N,\ \forall t \in T\\
& q_j^{\min} v_j(t) \leq q_j(t) \leq q_j^{\max} v_j(t) && \forall j \in J,\ \forall t \in T\\
& \textstyle\sum_{j\in\Lambda_n^{\mathrm{G}}} p_j(t) - \sum_{i\in\Lambda_n^{\mathrm{D}}} D_i(t) = &&\\
& \quad \textstyle\sum_{m\in\Lambda_n^{\mathrm{L}}} V_n(t)V_m(t)\left[G_{nm}\cos\theta_{nm}(t) + B_{nm}\sin\theta_{nm}(t)\right] - G_{nm}V_n^2(t), && \forall n \in N,\ \forall t \in T\\
& \textstyle\sum_{j\in\Lambda_n^{\mathrm{G}}} q_j(t) - \sum_{i\in\Lambda_n^{\mathrm{D}}} Q_i(t) = &&\\
& \quad \textstyle\sum_{m\in\Lambda_n^{\mathrm{L}}} V_n(t)V_m(t)\left[G_{nm}\sin\theta_{nm}(t) - B_{nm}\cos\theta_{nm}(t)\right] + V_n^2(t)\left(B_{nm} - b_{nm}^{\mathrm{shunt}}\right), && \forall n \in N,\ \forall t \in T\\
& \left\{V_n(t)V_m(t)\left(G_{nm}\cos\theta_{nm}(t) + B_{nm}\sin\theta_{nm}(t)\right) - G_{nm}V_n^2(t)\right\}^2 &&\\
& \quad + \left\{V_n(t)V_m(t)\left[G_{nm}\sin\theta_{nm}(t) - B_{nm}\cos\theta_{nm}(t)\right] + V_n^2(t)\left(B_{nm} - b_{nm}^{\mathrm{shunt}}\right)\right\}^2 \leq \overline{S}_{nm}^2, \quad \forall n \in N,\ \forall m \in \Lambda_n^{\mathrm{L}}, && \forall t \in T\\
& c_j(p_j(t)) \geq \alpha_{js} p_j(t) + \beta_{js}, \quad s = 1, \ldots, C_j, && \forall j \in J\\
& v_j(t-1) - v_j(t) + y_j(t) - z_j(t) = 0, \quad \forall j \in J, && \forall t \in T
\end{aligned}$$

Ramping constraints (6) and (7)

Generation limits (10), (11) and (12)

where the optimization variables in set Ξ are $p_j(t)$, $\overline{p}_j(t)$, $v_j(t)$, $y_j(t)$, $z_j(t)$, $\theta_n(t)$, $q_j(t)$, and $V_n(t)$, $\forall n \in N$, $\forall j \in J$, $\forall t \in T$.

Example (§ 5.5.1)

We use the six-node power network as for the example in the DC case, and with the same simplifying assumption that there is a single reserve area corresponding to the whole network.

We also start with the same nodal demand and reserve requirements:

Time period t	1	2	3	4	5	6
$D_4(t)$	100	100	80	140	100	80
$D_5(t)$	80	100	80	30	90	60
$D_6(t)$	50	50	40	0	40	50
Total demand	230	250	200	170	230	190
Reserve $R(t)$	10	10	10	10	10	10

Example (§ 5.5.1)

To allow for the increased power flow in apparent power due to accounting for reactive power, we increase the line transmission capacity limits $\overline{S}_{nm}$ by 20% compared to the limits in the DC case:

	1	2	3	4	5	6
1	-	-	-	120	120	120
2	-	-	-	120	120	120
3	-	-	-	-	120	120
4	120	120	-	-	-	-
5	120	120	120	-	-	-
6	120	120	120	-	-	-

For every line, we maintain the value of the susceptance $B_{nm} = 0.8$, and set the conductance to $G_{nm} = 0.08$ and the shunt susceptance to zero.

Example (§ 5.5.1)

At each node, we set the minimum and maximum voltage magnitudes (in per-unit) to $V_n^{\min} = 0.95$ and $V_n^{\max} = 1.05$.

We set the nodal reactive power demands according to a typical power factor of 0.75:

Time period t	1	2	3	4	5	6
$Q_4(t)$	75.0	75.0	60.0	105.0	75.0	60.0
$Q_5(t)$	67.5	75.0	60.0	22.5	67.5	45.0
$Q_6(t)$	37.5	37.5	30.0	0.0	30.0	37.5

We set the minimum and maximum reactive power capacity (in absolute value) of the three generating units to 50% of their maximum active power output:

Unit	$q_j^{\min}$	$q_j^{\max}$
1	-150	150
2	-100	100
3	-50	50

Solver for a Nonlinear Formulation

- Unlike the previous formulations, the formulation on slide 77 is a mixed-integer nonlinear optimization problem.
- The KNITRO solver is available on NEOS and can handle this formulation for moderately sized instances of UC.

Example (§ 5.5.1)

Solving the formulation on slide 77, we obtain the following generation schedule for active and reactive power with a total cost is 10238.6:

Time period t	1	2	3	4	5	6
$p_1(t)$	170.00	179.81	149.81	119.81	169.81	139.81
$p_2(t)$	59.74	69.90	50.00	50.00	59.92	50.00
$p_3(t)$	0	0	0	0	0	0
$q_1(t)$	150.00	150.00	125.83	65.86	150.00	148.40
$q_2(t)$	32.65	40.40	26.07	63.53	25.18	-4.01
$q_3(t)$	0	0	0	0	0	0

- We see that unit 1 remains on for all time periods, and unit 2 is now also on for all time periods (instead of only the first two periods).
- Because of the increased commitment of unit 2, unit 3 is never called upon (before it came on at $t = 5$).
- This global increase in the commitment of the units with respect to the DC case is due to the need to provide voltage support.

Example (§ 5.5.1)

The voltage magnitudes are:

Time period t	1	2	3	4	5	6
Node 1	0.9565	0.9564	0.9718	1.0500	0.9567	0.9919
Node 2	0.9520	0.9522	0.9681	1.0500	0.9518	0.9860
Node 3	0.9513	0.9511	0.9674	1.0493	0.9515	0.9867
Node 4	0.9500	0.9500	0.9666	1.0445	0.9500	0.9857
Node 5	0.9507	0.9504	0.9669	1.0490	0.9508	0.9865
Node 6	0.9518	0.9518	0.9680	1.0497	0.9522	0.9868

Example (§ 5.5.1) – Active Power Flows

At $t = 1$ the active power flows are as follows:

	1	2	3	4	5	6
1	-	-	-	-68.43	-55.91	-45.90
2	-	-	-	-31.57	-19.09	-9.10
3	-	-	-	-	-5.00	5.00
4	68.32	31.55	-	-	-	-
5	55.83	19.09	5.00	-	-	-
6	45.85	9.10	-5.00	-	-	-

where positive/negative values are outflows/inflows w.r.t. the column node.

We see that the flows of active power are similar to those in the DC case; the small differences are not significant.

Example (§ 5.5.1) – Reactive Power Flows

Because we are using the AC description for the network, we obtain information about the reactive power flows as well. For $t = 1$ they are as follows:

	1	2	3	4	5	6
1	-	-	-	-56.66	-50.45	-40.52
2	-	-	-	-18.34	-12.03	-2.01
3	-	-	-	-	-5.02	5.03
4	57.74	18.52	-	-	-	-
5	51.22	12.10	5.03	-	-	-
6	41.04	2.022	-5.03	-	-	-

where positive/negative values are outflows/inflows w.r.t. the column node.

- The reactive flows are not necessarily symmetric.
- Consider for instance the line between nodes 1 and 4: there is a injection of 57.74 units at node 1, but only 56.66 units, or 1.9% less, arrive at node 4.
- Similarly, 51.22 units are injected at node 1 into the line connecting it to node 5, but only 50.45 units arrive at node 5, or 1.5% less.
- These differences reflect the transmission losses on the respective lines.

Example (§ 5.5.2)

- Changes in the physical properties of the lines can significantly impact the commitment outcomes.
- As an example, suppose that the value of the line parameter b_{nm}^{shunt} is greater than zero.
- In physical terms, this means that the lines are longer and behave more like capacitors.

Example (§ 5.5.2) (ctd)

For illustration, setting $b_{nm}^{\mathrm{shunt}} = 0.0002$ for all the lines in our example and solving the model, we obtain a total cost is 8939, a decrease of 12.7% compared to the previous cost.

Time period t	1	2	3	4	5	6
$p_1(t)$	170.00	199.68	199.74	169.74	199.68	189.77
$p_2(t)$	59.73	50.00	0	0	0	0
$p_3(t)$	0	0	0	0	30.00	0
$q_1(t)$	150.00	150.00	123.66	97.47	150.00	115.86
$q_2(t)$	3.67	11.64	0	0	0	0
$q_3(t)$	0	0	0	0	-3.33	0

Example (§ 5.5.2) (ctd)

If we look at the reactive power flows for $t = 1$, we have:

1	-	-	-	-61.47	-53.45	-43.54
2	-	-	-	-13.52	-5.40	4.63
3	-	-	-	-	-8.64	1.41
4	58.97	10.07	-	-	-	-
5	50.61	1.84	5.03	-	-	-
6	40.42	-8.24	-5.03	-	-	-

We see that the differences have noticeably increased with the increase in the shunt susceptance.

If we again consider the line between nodes 1 and 4, we see 58.97 units injected at node 1, and 61.47 units arriving at node 4, an **increase** of 4.2%.

Stochastic Unit Commitment

Stochastic Unit Commitment

- A stochastic optimization formulation is relevant if the UC is affected by important uncertainty in the data.
- Handling stochasticity is currently of great and increasing importance because of the uncertainty arising from the variability in the output from sources such as wind- and solar-based generating units.
- Typically these sources are not scheduled/dispatched per se but rather their production is subtracted from the demand, and other units are then scheduled to meet the resulting net demand, i.e., the actual demand minus the stochastic production.
- One consequence is that the net demand curve fluctuations are increasing with the greater penetration of such stochastic generation.

Stochastic Unit Commitment

- Demand fluctuations have traditionally been handled by ensuring a sufficient level of reserve generation.
- This approach can be economically inefficient, and the growing provision of power from stochastic sources is increasing the cost of this inefficiency.
- With multiple jurisdictions around the world experiencing significant increases in the proportion of electricity generated by stochastic sources, the importance of models explicitly incorporating the uncertainty in demand is increasing.

We will consider two approaches to handle the uncertainty in mathematical optimization models:

- Stochastic optimization
- Robust optimization

Two-Stage Stochastic Optimization

To formulate a stochastic UC, we consider two stages:

- The first stage pertains to the optimal scheduling of generation capacity, i.e., the decisions about which units to commit in advance of the actual operation.
- The second stage constitutes a representation (prognosis) of a number of plausible operating conditions that may arise as a result of the uncertainty realization. These possible operating conditions are called scenarios, and for each scenario, an optimal dispatch can be computed based on the commitment decisions made in the first stage.

Reserves are scheduled in the first stage so that the system will be able to accommodate any uncertainty realization, i.e., any operating scenario.

Two-Stage Stochastic Optimization

Philosophy of this two-stage approach:

- In the first stage, scheduling decisions are made using only the information that is available hours or days in advance of real-time operations.
- The uncertainty is then realized in the second stage, and the dispatch adjusts the amount scheduled in the first stage up or down, as required according to the scenario (realization).
- The scenarios take into account the possible net demand realizations over the planning period.
- Each scenario is assigned a probability, and the optimization objective is to minimize the sum of
  - the deterministic cost of the first-stage decisions, and
  - the expected cost of the second-stage decisions.

Limitations of the Stochastic Optimization Approach

- The quality of the solutions obtained critically depends on the choice of the scenarios.
- A broader range of scenarios usually leads to a more accurate model.
- However, increasing the number of scenarios increases the computational cost of the optimization.
- Another issue is that this approach assumes explicit knowledge of the probability distribution of the (uncertain) net demand.
- In practice, this distribution is estimated empirically, using past data and experience and/or using simulation models, and the limitations of the probability estimation may impact the quality of the results.

The robust optimization approach avoids these limitations.

Optimization Setup

The optimization model minimizes the sum of

- the deterministic cost of the first-stage decisions (including the cost of the reserves scheduled), and
- the expected cost of the second-stage decisions (including the cost of the reserve deployment actually called upon according to each scenario).

Some notation:

- Ω is the set of scenarios,
- ω is the index for the scenarios, and
- π_ω is the probability of occurrence of scenario ω.

We assume that $\sum_{\omega \in \Omega} \pi_\omega = 1$.

Optimization Objective

For simplicity, we assume that

- the production and startup costs of all generating units are respectively linear and constant, and
- the cost of deploying reserves is equal to the production cost of the generating unit that deploys the reserve.

Under these assumptions, the objective function is given by:

$$\sum_{t\in T}\sum_{j\in J}\left(c_j p_j(t) + c_j^{\mathrm{U}} y_j(t)\right) + \sum_{\omega\in\Omega}\sum_{t\in T}\sum_{j\in J}\pi_\omega c_j\left(r_{j\omega}^{\mathrm{U}}(t) - r_{j\omega}^{\mathrm{D}}(t)\right) \qquad (31)$$

where $r_{j\omega}^{\mathrm{U}}(t)$ and $r_{j\omega}^{\mathrm{D}}(t)$ are respectively the up-reserve and down-reserve deployed by generating unit j during time period t under scenario ω.

First-Stage Constraints

The first-stage constraints are a subset of those in the basic formulation on slide 28:

- The logical constraints (5).
- The generation limits (10) without the reserve-related variable $\overline{p}_j(t)$.
- The demand constraint (2) to ensure that the total amount of generation in time period t meets the expected net demand $D^{\exp}(t)$ for that period:

$$\sum_{j \in J} p_j(t) = D^{\exp}(t), \ \forall t \in T. \tag{32}$$

The other constraints are not included for the sake of simplicity.

A comment about the ramping constraints:

- We choose to enforce the ramping constraints in the second stage, which is the actual operation stage.
- However, in some electricity markets, the market rules require that ramping constraints be imposed in the first stage.

Second-Stage Constraints

For the second stage, the first set of constraints states that:

- The scenario-dependent amount produced by unit j at time t under scenario ω

must be equal to

- the amount scheduled in the first stage adjusted by the amounts of up-reserve and down-reserve provided by unit j under scenario ω:

$$p_{j\omega}(t) = p_j(t) + r^{\mathrm{U}}_{j\omega}(t) - r^{\mathrm{D}}_{j\omega}(t), \ \forall j \in J, \ \forall t \in T, \ \forall \omega \in \Omega \tag{33}$$

where $p_{j\omega}(t)$ is the actual output of unit j at time t under scenario ω.

Second-Stage Constraints

The next sets of constraints are ω-indexed versions of other constraints from the basic formulation:

- Match production and demand for each scenario:

$$\sum_{j \in J} p_{j\omega}(t) = D_{\omega}(t), \ \forall t \in T, \ \forall \omega \in \Omega \tag{34}$$

- Ramping constraints:

$$p_{j\omega}(t) - p_{j\omega}(t-1) \leq R_j^{\mathrm{U}} \ v_j(t-1) + S_j^{\mathrm{U}} \ y_j(t), \ \forall j \in J, \ \forall t \in T, \ \forall \omega \in \Omega \tag{35}$$

$$p_{j\omega}(t-1) - p_{j\omega}(t) \leq R_j^{\mathrm{D}} \ v_j(t) + S_j^{\mathrm{D}} \ z_j(t), \ \forall j \in J, \ \forall t \in T, \ \forall \omega \in \Omega \tag{36}$$

- Generation limits:

$$\underline{P}_j v_j(t) \leq p_{j\omega}(t) \leq \overline{P}_j v_j(t), \ \forall j \in J, \ \forall t \in T, \ \forall \omega \in \Omega \tag{37}$$

Second-Stage Constraints

The final set of constraints link the commitment decisions and the various operating conditions:

$$0 \leq r^{\mathrm{U}}_{j\omega}(t) \leq M^{\mathrm{U}}_{j} v_j(t), \ \forall j \in J, \ \forall t \in T, \ \forall \omega \in \Omega \tag{38}$$

$$0 \leq r^{\mathrm{D}}_{j\omega}(t) \leq M^{\mathrm{D}}_{j} v_j(t), \ \forall j \in J, \ \forall t \in T, \ \forall \omega \in \Omega \tag{39}$$

where M^{U}_{j} and M^{D}_{j} are respectively the maximum up-reserve and the maximum down-reserve provided by generating unit j.

Recap: Stochastic UC Formulation

(For simplicity, minimum up/down time and network constraints are omitted.)

$$\min_{\Xi} \quad \sum_{t\in T}\sum_{j\in J} c_j p_j(t) + c_j^{\mathrm{U}} y_j(t) + \sum_{\omega\in\Omega}\sum_{t\in T}\sum_{j\in J} \pi_\omega c_j \left(r_{j\omega}^{\mathrm{U}}(t) - r_{j\omega}^{\mathrm{D}}(t) \right)$$

$$\begin{aligned}
\text{s.t.} \quad & v_j(t-1) - v_j(t) + y_j(t) - z_j(t) = 0, && \forall j \in J,\ \forall t \in T \\
& \underline{P}_j v_j(t) \le p_j(t) \le \overline{P}_j v_j(t), && \forall j \in J,\ \forall t \in T \\
& \textstyle\sum_{j\in J} p_j(t) = D^{\mathrm{exp}}(t), && \forall t \in T \\
& p_{j\omega}(t) = p_j(t) + r_{j\omega}^{\mathrm{U}}(t) - r_{j\omega}^{\mathrm{D}}(t) && \forall j \in J,\ \forall t \in T,\ \forall \omega \in \Omega \\
& \textstyle\sum_{j\in J} p_{j\omega}(t) = D_\omega(t), && \forall t \in T,\ \forall \omega \in \Omega \\
& p_{j\omega}(t) - p_{j\omega}(t-1) \le R_j^{\mathrm{U}}\, v_j(t-1) + S_j^{\mathrm{U}}\, y_j(t), && \forall j \in J,\ \forall t \in T,\ \forall \omega \in \Omega \\
& p_{j\omega}(t-1) - p_{j\omega}(t) \le R_j^{\mathrm{D}}\, v_j(t) + S_j^{\mathrm{D}}\, z_j(t), && \forall j \in J,\ \forall t \in T,\ \forall \omega \in \Omega \\
& \underline{P}_j v_j(t) \le p_{j\omega}(t) \le \overline{P}_j v_j(t), && \forall j \in J,\ \forall t \in T,\ \forall \omega \in \Omega \\
& 0 \le r_{j\omega}^{\mathrm{U}}(t) \le M_j^{\mathrm{U}} v_j(t), && \forall j \in J,\ \forall t \in T,\ \forall \omega \in \Omega \\
& 0 \le r_{j\omega}^{\mathrm{D}}(t) \le M_j^{\mathrm{D}} v_j(t), && \forall j \in J,\ \forall t \in T,\ \forall \omega \in \Omega
\end{aligned}$$

where the optimization variables in set Ξ are $p_j(t)$, $v_j(t)$, $y_j(t)$, $z_j(t)$, $p_{j\omega}(t)$, $r_{j\omega}^{\mathrm{D}}(t)$, $r_{j\omega}^{\mathrm{U}}(t)$, $\forall j \in J,\ \forall t \in T,\ \forall \omega \in \Omega$.

Example (§ 6.6)

- We consider the three-generator system from slide 29.
- We additionally set the maximum up-reserve and down-reserve provided by each generating unit j as follows:

Unit (j)	M_j^{U}	M_j^{D}
1	25	15
2	30	20
3	35	25

- The expected net demand is:

Time period t	1	2	3	4	5	6
$D^{\mathrm{exp}}(t)$	220	250	200	170	230	190

- No explicit reserves are required here.

Scenarios

We consider three scenarios defined as follows:

Scenario	Description	Probability
1	Demands 10% lower than $D^{\exp}(t)$ in every period	0.15
2	Demands equal to $D^{\exp}(t)$ in every period	0.60
3	Demands 5% higher than $D^{\exp}(t)$ in every period	0.25

Optimal Commitment

Solving the formulation, we obtain an optimal cost of 10130.75, and the on/off status of the generators is:

Time period t	1	2	3	4	5	6
Unit 1	on	on	on	on	on	on
Unit 2	on	on	on	on	on	on
Unit 3	off	off	off	off	off	off

Optimal Schedules

Time period t	1	2	3	4	5	6
First-stage						
$p_1(t)$	160	175	145	118	166	125
$p_2(t)$	60	75	55	52	64	65
$p_3(t)$	0	0	0	0	0	0
Scenario 1						
$p_{11}(t)$	148	160	130	103	151	121
$p_{21}(t)$	50	65	50	50	56	50
$p_{31}(t)$	0	0	0	0	0	0
Scenario 2						
$p_{12}(t)$	170	180	150	120	170	140
$p_{22}(t)$	50	70	50	50	60	50
$p_{32}(t)$	0	0	0	0	0	0
Scenario 3						
$p_{13}(t)$	170	188	158	128	178	150
$p_{23}(t)$	61	75	52	50	65	50
$p_{33}(t)$	0	0	0	0	0	0

Impact of Changing the Probabilities

Suppose that we change the probabilities to 0.40, 0.50, and 0.10 respectively, so that scenario 1 is more likely, and scenario 3 is less likely.

The result is that, while the commitment decisions are unchanged:

- the total cost decreases by 2.6% to 9867.50,
- the dispatch for the scenarios is unchanged, and
- and the output schedule in the first stage changes slightly:

Time period t	1	2	3	4	5	6
First-stage						
$p_1(t)$	163	170	140	103	166	136
$p_2(t)$	57	80	60	67	64	54
$p_3(t)$	0	0	0	0	0	0

Impact of Changing the Scenarios

Suppose that we change scenario 3 to represent demands 15% higher than the expected demand in every time period, and that the other parameters remain the same.

The commitment decisions are again unchanged, but the total cost increases by 10.6% to 11200.75.

There are no changes in the production schedule for the unchanged scenarios, and the new output schedules for the first stage and for scenario 3 are:

Time period t	1	2	3	4	5	6
First-stage						
$p_1(t)$	163	175	145	118	166	136
$p_2(t)$	57	75	55	52	64	54
$p_3(t)$	0	0	0	0	0	0
Scenario 3						
$p_{13}(t)$	188	200	170	143	191	161
$p_{23}(t)$	65	88	60	53	74	58
$p_{33}(t)$	0	0	0	0	0	0

Robust Unit Commitment

Robust Optimization

- Adaptive robust optimization is a technique for modeling uncertainty that overcomes some of the drawbacks of stochastic optimization at the cost of comparatively more restrictive modeling assumptions.
- Robust optimization avoids the need to define scenarios and make assumptions about their probabilities.
- Instead, a deterministic uncertainty set is defined using limited information about the uncertain quantities:
  - expected value
  - some estimate of their variance (or a range of possible deviations from the expected value).
- If additional information is available, it can often be incorporated to improve the quality of the robust model.

Robust Optimization for UC

- Under the same assumptions as for the stochastic programming formulation, the uncertainty set is defined in terms of possible realizations of net demand.
- Once the uncertainty set is defined, the model computes an optimal solution that protects the system against every possible realization in the set, and in particular the worst case.
- In this sense, the robust approach is more conservative than the stochastic one.

Three-Level Adaptive Robust Formulation

We consider a robust formulation with three levels:

First level The operator schedules the units so as to minimize the generation cost.

Second level For every schedule feasible for the first level, the net demand realizes in the worst possible manner within the uncertainty set, i.e., the production cost is maximized.

Third level Given the schedule and the worst-case demand realization, the operator dispatches the committed units so as to minimize the production cost.

Three-Level Adaptive Robust Formulation

- Note that the robust UC takes into account all possible future demands represented in the uncertainty set.
- Therefore, the optimal robust UC solution will be feasible for any realization of the uncertainty in the second level.
- This is in contrast with the basic formulation on slide 28 that guarantees feasibility only for a single set of demands, and the stochastic optimization formulation on slide 101 that considers only a finite set of preselected scenarios.

Uncertainty Set

- The second-level can realize the net demand $d(t)$ at time t in the range:

$$\underline{D}(t) \leq d(t) \leq \overline{D}(t), \ \forall t \in T. \tag{40}$$

- The budget of uncertainty Γ bounds the total deviation allowed, summed over all $t \in T$, for the realized demand $d(t)$ from $\underline{D}(t)$.
- The uncertainty set is a deterministic set defined in terms of Γ:

$$\sum_{t \in T} \frac{\max\{0, d(t) - D^{\exp}(t)\}}{\overline{D}(t) - D^{\exp}(t)} \leq \Gamma. \tag{41}$$

Uncertainty Set

$$\sum_{t \in T} \frac{\max\{0, d(t) - D^{\exp}(t)\}}{\overline{D}(t) - D^{\exp}(t)} \leq \Gamma.$$

- The main concern is to schedule enough capacity to meet demand, so we want to be protected against unexpected increases in demand.
- The numerator counts the deviation from expected demand only if $d(t)$ lies in the interval $[D^{\exp}(t), \overline{D}(t)]$.
- Except for the max function, (41) is a linear constraint on the variable $d(t)$, and the max function can be linearized using binary variables.

Practical Interpretation of the Budget of Uncertainty

The value of Γ can be chosen anywhere between 0 and $|T|$:

- The value 0 corresponds to the realized demand $d(t)$ being less than or equal to $D^{\mathrm{exp}}(t)$.
- For small values of Γ, the realized demand $d(t)$ cannot deviate too much from D^{exp}, and hence there is limited uncertainty in $d(t)$.
- As Γ increases, the range of values allowed for $d(t)$ increases, and so does the uncertainty.
- The value $|T|$ corresponds to requiring protection for the maximum possible values of $d(t)$.
- The higher the value of Γ, the higher the robust protection.

Optimization Objective

The overall objective is to minimize the total operation cost of the system. The optimization objective is in three parts, each corresponding to one of the levels of the robust formulation:

$$\min_{\Xi_1} \left[\sum_{t\in T} \sum_{j\in J} c_j^{\mathrm{U}} y_j(t) + \max_{\Xi_2} \min_{\Xi_3} \sum_{t\in T} \sum_{j\in J} c_j p_j(t) \right] \tag{42}$$

where the variables for each level of the optimization are:

- $\Xi_1 = \{v_j(t), y_j(t), z_j(t), \forall j \in J, \ \forall t \in T\}$
- $\Xi_2 = \{d(t), \forall t \in T\}$
- $\Xi_3 = \{p_j(t), \forall j \in J, \ \forall t \in T\}$

Optimization Objective

The structure of this objective is aligned with the three levels on slide 111:

- The commitment of the units is determined using the variables in Ξ_1 with the objective that the total cost is minimized.
- The total cost is in two parts:
  - the first part is the startup cost, and
  - the second part is the cost of production.
- The cost of production, given a feasible commitment, is maximized over the demand variables in Ξ_2.
- Then, for each realization of the demand, the cost of dispatch is minimized by the operator who assigns the amount of power produced by each unit via the variables in Ξ_3.

First-Level Constraints

The first-level constraints are the logical constraints (5):

$$v_j(t-1) - v_j(t) + y_j(t) - z_j(t) = 0 \quad \forall j \in J, \ \forall t \in T.$$

that ensure consistency of the values of the binary variables with respect to the decisions to start up or shut down units.

Second-Level Constraints

As second-level constraints, we have (40) and (41) to define the uncertainty set:

$$\underline{D}(t) \leq d(t) \leq \overline{D}(t), \ \forall t \in T$$

$$\sum_{t \in T} \frac{\max\{0, d(t) - D^{\exp}(t)\}}{\overline{D}(t) - D^{\exp}(t)} \leq \Gamma$$

We also enforce ramping constraints to represent the fact that the net demand cannot change too rapidly:

$$d(t) - d(t-1) \leq R_d^{\mathrm{U}}, \ \forall t \in T \quad (43)$$

$$d(t-1) - d(t) \leq R_d^{\mathrm{D}}, \ \forall t \in T \quad (44)$$

Third-Level Constraints

For the third level, the constraints are the fundamental operational requirements:

- balance of supply and demand (2),
- ramping up (6) or down (7) constraints, and
- generation limits as in (10) but omitting the variable $\overline{p}_j(t)$.

For simplicity, we do not account explicitly for dispatching reserves or for uptime and downtime requirements.

Recap: Robust UC Formulation

(For simplicity, minimum up/down time and reserve constraints are omitted.)

$$\min_{\Xi_1} \left[\sum_{t\in T}\sum_{j\in J} c_j^{\mathrm{U}} y_j(t) + \max_{\Xi_2} \min_{\Xi_3} \sum_{t\in T}\sum_{j\in J} c_j p_j(t) \right]$$

$$\text{s.t.} \quad v_j(t-1) - v_j(t) + y_j(t) - z_j(t) = 0, \qquad \forall j \in J, \forall t \in T$$

$$\text{s.t.} \quad \overline{D}(t) \le d(t) \le \underline{D}(t), \qquad \forall t \in T$$

$$\sum_{t\in T} \frac{\max\{0, d(t) - D^{\exp}(t)\}}{\overline{D}(t) - D^{\exp}(t)} \le \Gamma,$$

$$d(t) - d(t-1) \le R_d^{\mathrm{U}}, \qquad \forall t \in T$$

$$d(t-1) - d(t) \le R_d^{\mathrm{D}}, \qquad \forall t \in T$$

$$\text{s.t.} \quad \sum_{j\in J} p_j(t) = d(t), \qquad \forall t \in T$$

$$p_j(t) - p_j(t-1) \le R_j^{\mathrm{U}} v_j(t-1) + S_j^{\mathrm{U}} y_j(t), \qquad \forall j \in J, \ \forall t \in T$$

$$p_j(t-1) - p_j(t) \le R_j^{\mathrm{D}} v_j(t) + S_j^{\mathrm{D}} z_j(t), \qquad \forall j \in J, \ \forall t \in T$$

$$\underline{P}_j v_j(t) \le p_j(t) \le \overline{P}_j v_j(t), \qquad \forall j \in J, \ \forall t \in T$$

where the optimization variable sets for the three levels are respectively:

- $\Xi_1 = \{v_j(t), y_j(t), z_j(t), \forall j \in J, \ \forall t \in T\}$
- $\Xi_2 = \{d(t), \forall t \in T\}$
- $\Xi_3 = \{p_j(t), \forall j \in J, \ \forall t \in T\}$

Computationally Highly Challenging!

- Three-level optimization problems are extremely challenging to solve, even for small instances.
- Indeed, even two-level optimization (usually known as bilevel optimization) is known to be very challenging.
- In view of this, we consider an example with two generators and three time periods.
- The computational methodology we used is described in § 8.2 of Anjos & Conejo.

Data for Examples (§ 7.8)

The two generators are units 1 and 2 with the respective parameters and operating conditions at $t = 0$ as given in slides 29 and 30.

In particular, at $t = 0$, unit 1 is on and producing 120 MW, and unit 2 is off.

The characteristics of the demand for the three time periods are:

Time period t	1	2	3
$\underline{D}(t)$	130	190	150
$D^{\mathrm{exp}}(t)$	170	230	190
$\overline{D}(t)$	210	270	230

We further assume that $d(0) = 200$, and that the ramping limits on the net demand are $R_d^{\mathrm{U}} = 50$ and $R_d^{\mathrm{D}} = 50$.

First Example (§ 7.8.1)

- For this initial case, we set $\Gamma = 2.0$ and solve the formulation on slide 121.
- As part of the solution method, we need to enumerate all the combinations of the commitment decisions v, y, and z.
- Because the first-level constraints (5) completely determine the values of $y_j(t)$ and $z_j(t)$ given the values of the $v_j(t)$ variables, it suffices to enumerate the possible values of $v_j(t)$ for $j = 1, 2$ and $t = 1, 2, 3$.
- Furthermore, because $p_1(0) = 120$ and $S_1^D = 80$, it follows that $v_1(1) = 1$ must hold.
- This leaves us with $2^5 = 32$ combinations to enumerate.
- It turns out that most of them are infeasible, i.e., the binary variables are set in combinations for which there is simply not enough generation capacity committed to satisfy demand.

First Example (§ 7.8.1) (ctd)

The optimal values (maximum cost) for the nine feasible combinations are:

$v_1(2)$	$v_1(3)$	$v_2(1)$	$v_2(2)$	$v_2(3)$	Cost
1	1	0	0	0	3100
1	1	0	1	0	4050
1	1	1	0	0	4250
1	1	1	1	0	5050
1	0	1	1	1	7300
1	1	0	0	1	4150
1	1	0	1	1	5450
1	1	1	0	1	5350
1	1	1	1	1	7350

First Example (§ 7.8.1 (ctd))

It follows that the optimal solution to the robust model has a total cost of 3100.

The corresponding generation schedule is:

Time period t	1	2	3
$p_1(t)$	170	220	230
$p_2(t)$	0	0	0

The realized demand is $d(t) = p_1(t)$, t=1,2,3 at optimality.

Impact of the Choice of Γ (§ 7.8.2)

- Let us examine the sensitivity with respect to Γ of the optimal value of the second-level problem for the optimal commitment:

$$v_1(1) = 1, v_1(2) = 1, v_1(3) = 1, v_2(1) = 0, v_2(2) = 0, \text{ and } v_2(3) = 0.$$

- We already know that for $\Gamma = 2.0$, the optimal value is 3100.
- In principle, Γ can take any value between 0 and 3 $(= |T|)$.
- Values of Γ equal to, or close to, zero correspond to fixing, or nearly fixing, $d(t) = D^{\mathrm{exp}}(t)$ for all $t \in T$.

Impact of the Choice of Γ (§ 7.8.2) (ctd)

Solving the formulation for six additional values of Γ, we obtain:

Γ	Optimal (maximum) cost
0	2900
0.5	3000
1.0	3100
1.5	3100
2.0	3100
2.5	3100
3.0	3100

We observe that reducing the budget of uncertainty reduces the optimal cost, as would be expected.

Current Challenges

Current Challenges

- Mixed-integer linear optimization was first proposed for UC problems more than 50 years ago, and it is now used daily by power system operators.
- Plenty of challenges remain when it comes to solving UC problems that incorporate practical requirements such as:
  - security constraints,
  - network effects and power losses, and
  - different uncertainty models.
- There is also great interest in incorporating into UC models the new requirements of smart grids such as:
  - the integration of wind and solar generation,
  - the management of demand-response, and
  - the scheduling of electricity storage devices.
- In summary, even after several decades of research, the UC problem continues to pose challenges to practitioners and researchers alike, and continues to be a very active area of research.

Application of Logic Constrained Equilibria to Power Systems with Storage

Application of Logic Constrained Equilibria to Power Systems with Storage

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- Gabriel, S. A., Siddiqui, S., Conejo, A. J., and Ruiz, C. (2013b). *Discretely-constrained Nash-Cournot games with an application to power markets*. Networks and Spatial Economics, 13, 307-326.
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- Sherali, H. D., Krishnamurthy, R. S., and Al-Khayyal, F. A. (1998). *Enumeration approach for linear complementarity problems based on a reformulation-linearization technique*. Journal of Optimization Theory and Applications, 99, 481-507.

Linear Complementarity Problems

- The linear complementarity problem (LCP) is a well-known problem in optimization. It is characterized by the presence of complementarity constraints:

$$b_i - a_i^T x \geq 0, b_j - a_j^T x \geq 0, \text{ and } (b_i - a_i^T x)(b_j - a_j^T x) = 0$$

- The mixed linear complementarity problem (MLCP) is an LCP in which some of the terms involved in the complementarity constraints are not required to be non-negative.
- The Binary-Constrained MLCPs (BC-MLCP) is an MLCP in which some variables are restricted to be binary.

These classes of problems are NP-hard, and have numerous engineering and economic applications.

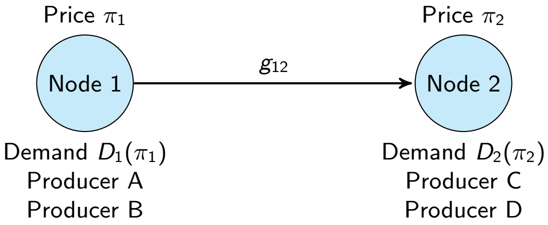
BC-MLCP: Applications and Past Research

The BC-MLCP has not been studied widely in the literature.

- Gabriel et al. (2013a) proposed a mixed integer linear programming (MILP) approach that relaxes the complementarity constraints as well as the integrality. They showed that their approach was suitable to computing equilibria in energy markets.
- That approach was also successful in solving Nash-Cournot games with application to power markets (Gabriel et al. (2013b)), and the electricity pool pricing problem (Ruiz et al. (2012)).

The practical interest of BC-MLCPs can be motivated using a small example from energy economics.

Example:
An Energy Network Equilibrium Problem



At Node 1: Production levels q_1^A and q_1^B and outflows f_{12}^A and f_{12}^B.

At Node 2: Production levels q_2^C and q_2^D.

Formulation as an MLCP

Formulation for producer A:

$$\max_{s_1^A, q_1^A, f_{12}^A} \quad \pi_1 s_1^A + \pi_2 f_{12}^A - c_1^A(q_1^A) - (\tau_{12} + \tau_{12}^{reg}) f_{12}^A$$

$$\text{s.t.} \quad q_1^A \leq \bar{q}^A \quad (\lambda_1^A)$$

$$s_1^A = q_1^A - f_{12}^A \quad (\delta_1^A)$$

$$s_1^A, q_1^A, f_{12}^A \geq 0,$$

where

- $c_1^A(q_1^A) = \gamma_1^A q_1^A$ with $\gamma_1^A > 0$ is the production cost function,
- $\tau_{12}^{reg} \geq 0$ is a (fixed) regulated tariff for using the network from node 1 to node 2, and
- τ_{12} is the (variable) congestion tariff for using the network from node 1 to node 2.

The formulation for Producer B is similar.

Formulation as an MLCP (ctd)

Formulation for producer C:

$$
\begin{aligned}
\max_{s_2^C, q_2^C} \quad & \pi_2 s_2^C - c_2^C(q_2^C) & \\
\text{s.t.} \quad & q_2^C \leq \bar{q}^C & (\lambda_2^C) \\
& s_2^C = q_2^C & (\delta_2^C) \\
& s_2^C, q_2^C \geq 0 &
\end{aligned}
$$

The formulation for Producer D is similar.

Formulation as an MLCP (ctd)

The transmission system operator (TSO) solves:

$$\begin{aligned}
\max_{g_{12}} \quad & (\tau_{12} + \tau_{12}^{reg}) g_{12} - c^{TSO}(g_{12}) & \\
\text{s.t.} \quad & g_{12} \leq \bar{g}_{12} & (\epsilon_{12}) \\
& g_{12} \geq 0, &
\end{aligned}$$

where $c^{TSO}(g_{12}) = \gamma^{TSO} g_{12}$ with $\gamma^{TSO} > 0$ is the network operation cost function.

Formulation as an MLCP (ctd)

KKT conditions for Producer A (similar for Producer B):

$$\begin{aligned} 0 &\le -\pi_1 + \delta_1^A \perp s_1^A \ge 0 \\ 0 &\le \gamma_1^A + \lambda_1^A - \delta_1^A \perp q_1^A \ge 0 \\ 0 &\le -\pi_2 + (\tau_{12}^{Reg} + \tau_{12}) + \delta_1^A \perp f_{12}^A \ge 0 \\ 0 &\le \bar{q}_1^A - q_1^A \perp \lambda_1^A \ge 0 \\ 0 &= s_1^A - q_1^A + f_{12}^A, \qquad \delta_1^A \text{ free} \end{aligned}$$

KKT conditions for Producer C (similar for Producer D):

$$\begin{aligned} 0 &\le -\pi_2 + \delta_2^C \perp s_2^C \ge 0 \\ 0 &\le \gamma_2^C + \lambda_2^C - \delta_2^C \perp q_2^C \ge 0 \\ 0 &\le \bar{q}_2^C - q_2^C \perp \lambda_2^C \ge 0 \\ 0 &= s_2^C - q_2^C, \qquad \delta_2^C \text{ free} \end{aligned}$$

Formulation as an MLCP (ctd)

KKT conditions for the TSO:

$$0 \leq -\tau_{12}^{Reg} - \tau_{12} + \gamma^{TSO} + \epsilon_{12} \perp g_{12} \geq 0$$
$$0 \leq \bar{g}_{12} - g_{12} \perp \epsilon_{12} \geq 0$$

Market clearing conditions:

- Equality of supply and demand:

$$0 = [s_1^A + s_1^B] - (a_1 - b_1\pi_1), \qquad \pi_1 \text{ free},$$
$$0 = [s_2^C + s_2^D + f_{12}^A + f_{12}^B] - (a_2 - b_2\pi_2), \qquad \pi_2 \text{ free}$$

- Congestion tariff:

$$0 = g_{12} - [f_{12}^A + f_{12}^B], \qquad \tau_{12} \text{ free.}$$

The set of all these conditions is an MLCP.

Binary-Constrained Version of the Example

Suppose that the formulation for Producer A is of the form:

$$\begin{aligned}
\max_{s_1^A, q_1^A, f_{12}^A, v_1^A} \quad & \pi_1 s_1^A + \pi_2 f_{12}^A - c_1^A(q_1^A) - (\tau_{12} + \tau_{12}^{reg}) f_{12}^A \\
\text{s.t.} \quad & v_1^A q_{min}^A \leq q_1^A \leq v_1^A q_{max}^A \\
& s_1^A = q_1^A - f_{12}^A \\
& s_1^A, q_1^A, f_{12}^A \geq 0 \\
& v_1^A \in \{0, 1\}.
\end{aligned}$$

Then the resulting model will be a BC-MLCP.

General Form of a BC-MLCP

Let N denote the set of the indices of variables s.t. $N = N_1 \cup N_2$, and

- z_1 denotes the n_1 complementarity variables with indices in N_1, and
- z_2 denotes the n_2 free variables with indices in N_2.

Given $q = \begin{pmatrix} q_1 \\ q_2 \end{pmatrix} \in \mathbb{R}^n$ and $A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \in \mathbb{R}^{n \times n}$,
find $\begin{pmatrix} z_1 \\ z_2 \end{pmatrix} \in \mathbb{R}^{n_1} \times \mathbb{R}^{n_2}$ with $n_1 + n_2 = n$ such that

$$0 \leq q_1 + \begin{pmatrix} A_{11} & A_{12} \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} \perp z_1 \geq 0$$

$$0 = q_2 + \begin{pmatrix} A_{21} & A_{22} \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}, \quad z_2 \text{ free},$$

where some or all of $\begin{pmatrix} z_1 \\ z_2 \end{pmatrix}$ are required to be binary.

The BC-MLCP: A Challenging Problem!

Some of the challenges in solving BC-MLCPs are:

- The feasible region is in general not convex nor connected.
- The complementarity and binary constraints may lead to an exponential number of combinations to check.

There is only one earlier solution method in the literature:
Ruiz et al. (2012) propose to relax both complementarity and integrality, and to trade-off between the two.

Our objective is a solution method that does NOT relax either of them.

Preliminary: Boundedness Assumption

We assume that there exists a finite solution to the BC-MLCP, i.e., all the continuous variables satisfy $-\infty < l_i \leq x_i \leq u_i < +\infty$.

Under this assumption, we can write the set of all solutions of the BC-MLCP (that satisfy the assumption) as:

$$\mathcal{F} = \left\{ x \in \mathbb{R}^n \;\middle|\; \begin{array}{cl} q_i + A_i x \geq 0 & \text{for } i \in N_1 \\ (q_i + A_i x) x_i = 0 & \text{for } i \in N_1 \\ q_i + A_i x = 0 & \text{for } i \in N_2 \\ 0 \leq x_i \leq 1 & \text{for } i \in N \setminus B \\ x_i \in \{0, 1\} & \text{for } i \in B \end{array} \right\}$$

Step 1:
Build an RLT relaxation of BC-MLCP

We relax $\mathcal{F}$ using the well-known Reformulation-Linearization Technique (RLT) developed by Sherali and Adams (1986+, book in 2013).

Our use of RLT differs from the method in Sherali et al. (1998) to solve the LCP (the special case of MLCP with $n_2 = 0$) because no new binary variables are introduced.

The RLT relaxation is constructed following three simple steps:

1. Relax $x_i \in \{0, 1\}$ to $0 \leq x_i \leq 1$ for $i \in B$.
2. Replace the terms $x_i x_j$ with new variable y_{ij} for all $i, j \in N$.
3. Relax the relationship $y_{ij} = x_i x_j$ to

$$y_{ij} \geq 0, x_i - y_{ij} \geq 0, x_j - y_{ij} \geq 0, \text{ and } y_{ij} - x_i - x_j + 1 \geq 0.$$

Step 1:
Build an RLT relaxation of BC-MLCP (ctd)

The RLT relaxation of the set $\mathcal{F}$ is:

$$\tilde{\mathcal{F}} = \left\{ x \in \mathbb{R}^n \;\middle|\; \begin{array}{cl} q_i + A_i x \geq 0 & \text{for } i \in N_1 \\ q_i x_i + \sum_{j=1}^{n} A_{ij} y_{ij} = 0 & \text{for } i \in N_1 \\ q_i + A_i x = 0 & \text{for } i \in N_2 \\ 0 \leq x_i \leq 1 & \text{for } i \in N \\ y_{ij} \geq 0 & \text{for } i \in N \\ y_{ij} \leq x_i & \text{for } i \in N \\ y_{ij} \leq x_j & \text{for } i \in N \\ y_{ij} + 1 \geq x_i + x_j & \text{for } i, j \in N \end{array} \right\}$$

Step 2:
Refine Variable Bounds

The relaxation $\tilde{\mathcal{F}}$ can be improved by tightening the bounds on each variable x_i.

Given the nonconvex nature of $\mathcal{F}$, finding such bounds is non-trivial.

We improve the upper bounds on each complementarity variable as follows:

1. Solve

$$x_{i_0}^{+} = \max_{(x,y)\in\tilde{\mathcal{F}}} x_{i_0}.$$

2. The new upper bound is given by $\min\{x_{i_0}^{+}, \sqrt{y_{i_0,i_0}^{+}}\}$.

This procedure was proposed by different groups of researchers in the 1990s, and is embedded in several global optimization solvers (including αBB, COUENNE, ANTIGONE, SCIP, LaGO).

Step 3: Reduce the Number of Complementarity Constraints

Theorem

For $i_0 \in N_1$, let $x^*_{i_0}$ be an optimal solution value of $\min\limits_{(x,y)\in\tilde{\mathcal{F}}} x_{i_0}$.

We have the following:

- *If $x^*_{i_0} \neq 0$, then the constraint $(q_{i_0} + A_{i_0}x)x_{i_0} = 0$ can be replaced by $q_{i_0} + A_{i_0}x = 0$.*
- *If $x^*_{i_0} = 0$ and the problem $\min\limits_{(x,y)\in\tilde{\mathcal{F}}^+_{i_0}} x_{i_0}$ is infeasible, then the constraint $(q_{i_0} + A_{i_0}x)x_{i_0} = 0$ can be replaced by $x_{i_0} = 0$.*

where $\tilde{\mathcal{F}}^+_{i_0}$ is obtained from $\tilde{\mathcal{F}}$ by replacing the i_0^{th} equation of the form $q_i x_i + \sum\limits_{j=1}^{n} A_{ij}y_{ij} = 0$ with $q_{i_0} + A_{i_0}x = 0$.

Step 4:
MILP Reformulation of the BC-MLCP

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Let

$$N_1^+ = \{i \in N_1 : (q_i + A_i x)x_i = 0 \text{ is replaced by } q_i + A_i x = 0\}$$

and

$$N_1^0 = \{i \in N_1 : (q_i + A_i x)x_i = 0 \text{ is replaced by } x_i = 0\}.$$

Clearly either $N_1^+ \cup N_1^0 = N_1$ or $N_1^+ \cup N_1^0 \subsetneq N_1$.
In the first case, all the complementarity constraints have been replaced and therefore the set $\mathcal{F}$ is equivalent to

$$\mathcal{F}^1 = \left\{ x \in \mathbb{R}^n \;\middle|\; \begin{array}{cc} q_i + A_i x = 0 & \text{for } i \in N_1^+ \\ x_i = 0 & \text{for } i \in N_1^0 \\ q_i + A_i x = 0 & \text{for } i \in N \setminus N_1 \\ 0 \le x_i \le 1 & \text{for } i \in N \setminus B \\ x_i \in \{0,1\} & \text{for } i \in B \end{array} \right\}.$$

We find a solution to the BC-MLCP by solving an MILP over $\mathcal{F}^1$.

Step 4:
MILP Reformulation of the BC-MLCP (ctd)

In the second case, only some of the complementarity constraints have been replaced and the set $\mathcal{F}$ is equivalent to

$$\mathcal{F}^2 = \left\{ x \in \mathbb{R}^n \;\middle|\; \begin{array}{cc} q_i + A_i x \geq 0 & \text{for } i \in N_1 \setminus (N_1^+ \cup N_1^0) \\ (q_i + A_i x) x_i = 0 & \text{for } i \in N_1 \setminus (N_1^+ \cup N_1^0) \\ q_i + A_i x = 0 & \text{for } i \in N_1^+ \\ x_i = 0 & \text{for } i \in N_1^0 \\ q_i + A_i x = 0 & \text{for } i \in N \setminus N_1 \\ 0 \leq x_i \leq 1 & \text{for } i \in N \setminus B \\ x_i \in \{0, 1\} & \text{for } i \in B \end{array} \right\}.$$

In this case, we follow the work of Sherali et al. (1998) and define an MILP equivalent to finding a solution in $\mathcal{F}^2$.

Step 4:
MILP Reformulation of the BC-MLCP (ctd)

- Introduce new binary variables for each complementarity variables such that:
$$v_i = \begin{cases} 0 \text{ if } x_i = 0 \\ 1 \text{ if } x_i > 0 \end{cases}$$
- Introduce the variables $w_{ij} = v_i x_j$
- Relax the quadratic equation $w_{ij} = v_i x_j$ using the McCormick inequalities.

Final MILP Problem

$$
\begin{array}{lll}
\min\limits_{x,v,w} & q^T v + \sum_{i \in N_1 \setminus (N_1^+ \cup N_1^0)} \sum_{j \in N} A_{ij} w_{ij} & \\
s.t. & \sum_{j=1}^{n} A_{kj} w_{ij} + q_k v_i \geq 0 & \forall i, k \in N_1 \setminus (N_1^+ \cup N_1^0) \\
& \sum_{j=1}^{n} A_{kj} x_j + q_k \geq \sum_{j=1}^{n} A_{kj} w_{ij} + q_k v_i & \text{for } i, k \in N_1 \setminus (N_1^+ \cup N_1^0) \\
& q_i + A_i x = 0 & \text{for } i \in N_1^+ \\
& x_i = 0 & \text{for } i \in N_1^0 \\
& q_i + A_i x = 0 & \text{for } i \in N \setminus N_1 \\
& x_j \in \{0, 1\} & \text{for } j \in B \\
& 0 \leq w_{ij} \leq 1 & \text{for } i \in N_1 \setminus (N_1^+ \cup N_1^0), j \in N \\
& w_{jj} = x_j & \text{for } i \in N_1 \setminus (N_1^+ \cup N_1^0) \\
& v_i \in \{0, 1\} & \text{for } i \in N_1 \setminus (N_1^+ \cup N_1^0) \\
& w_{ij} \geq 0 & \text{for } i \in N_1 \setminus (N_1^+ \cup N_1^0), j \in N \\
& w_{ij} \leq x_j & \text{for } i \in N_1 \setminus (N_1^+ \cup N_1^0), j \in N \\
& w_{ij} \leq v_i & \text{for } i \in N_1 \setminus (N_1^+ \cup N_1^0), j \in N \\
& w_{ij} + 1 \geq x_j + v_i & \text{for } i \in N_1 \setminus (N_1^+ \cup N_1^0), j \in N.
\end{array}
$$

Solution For Two-Node Example

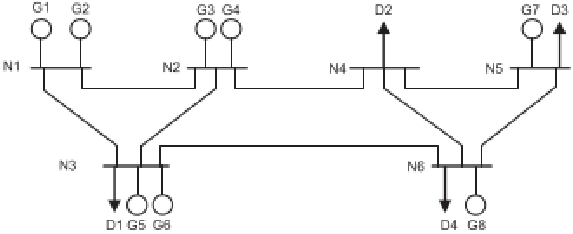
Prod. A		Prod. B		Prod. C		Prod. D		TSO			
s_1^A	5	s_1^B	3	s_2^C	5	s_2^D	0	g_{12}	5	# iter.	11
q_1^A	10	q_1^B	3	q_2^C	5	q_2^D	0	ϵ_{12}	2	time	0.023
f_{12}^A	5	f_{12}^B	0	λ_2^C	0	λ_2^D	0	π_1	12	compl.	Yes
λ_1^A	2	λ_1^B	0	δ_2^C	15	δ_2^D	15	π_2	15	B&B	No
δ_1^A	12	δ_1^B	12					τ_{12}	2.5		

- Solution different from those reported in Gabriel et al. (2013)

BC-MLCP for Market-Clearing

Example from Ruiz et al. (2012)

- 6 nodes, 8 producers, 4 demands
- DC power line model
- Start-up and shut-down costs modelled using binary variables

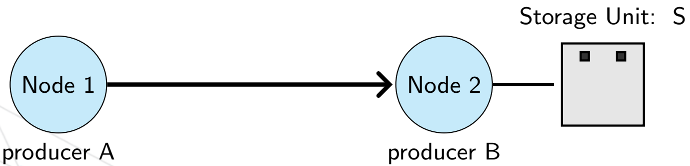


- Our algorithm replaced 99 complementarities (out of 248) by linear equations

Equilibria in Power Markets with Storage

Consider an energy storage operator in a power market as a service provider in a context where the transmission capacity as well as the generation capacities are limited in terms of satisfying the demand at peak times.

We use a small example with a network of two nodes, two producers and a storage operator at node 2:



Equilibria in Power Markets with Storage

- We consider two time periods: an off-peak period followed by an on-peak period.
- During the off-peak period, either producer can send a part of its production to the storage unit which will then be released during the on-peak demand period to be sold on the market at node 2.
- Given the economic opportunity of storage, the producers could respond by expanding their generation capacity.
  We investigate the profitability of the storage operator in two scenarios:
  - Scenario 1 Capacity expansion is not an option for the producers
  - Scenario 2 Producers are willing to expand their production capacities

Formulation as an MLCP

Same as previously with the addition of the problem for the storage operator:

$$\max_{h^S} \quad \omega^S h^S - \gamma^S h^S$$

$$\text{s.t.} \quad h^S \leq \sigma^S \quad (\theta^S)$$

$$h^S \geq 0$$

where

- σ^S is the storage capacity
- h^S is the storage level used
- ω^S is the unit tariff for using the storage facilities,
- γ^S is the unit cost of storage

Additional Data for Example

We use the demand function

$$D_{tn}(\pi_{tn}) = a_{tn} - b_{tn}\pi_{tn},$$

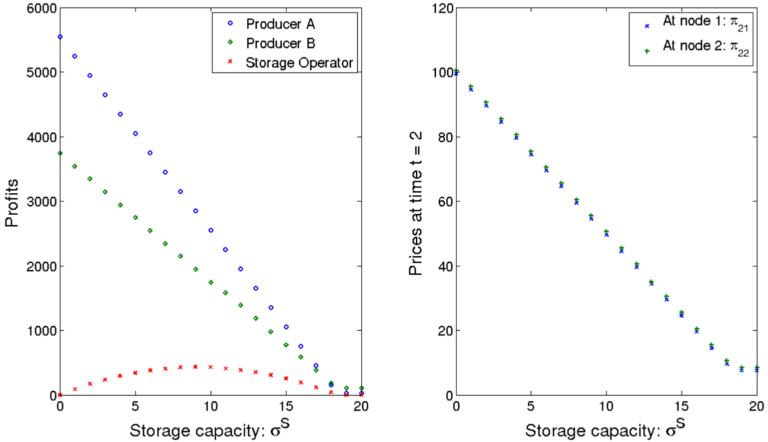
and the values of the parameters are:

$$\begin{array}{llllll}
\gamma^A = 7 & \gamma^B = 7 & \gamma^{TSO} = 1 & \gamma^S = 0.5 & \bar{g}_{12} = 30 & \tau^{reg} = 0.5 \\
a_{11} = 20 & a_{21} = 40 & a_{12} = 30 & a_{22} = 80 & & \\
b_{11} = 0.1 & b_{21} = 0.1 & b_{12} = 0.1 & b_{22} = 0.1 & & \\
q^A_{max} = 60 & q^A_{min} = 5 & q^B_{max} = 40 & q^B_{min} = 5 & & \\
E^A_{max} = 25 & E^A_{min} = 5 & E^B_{max} = 25 & E^B_{min} = 5 & &
\end{array}$$

Observations:

- At $t = 2$, after supplying up to 40 units to node 1, producer A can supply only up to 20 units, which together with the existing capacity of B will not satisfy the demand at a low cost:
  A: 20 MW, B: 40 MW results in a price of 200 \$/MWh $(80 - 0.1\pi_{22})$.
- Therefore, capacity expansion and/or storage may be profitable.

Scenario 1: Impact of Storage Capacity



Scenario 2: Generation Expansion vs Storage

Let ρ^A and ρ^B be the unit costs of the expanded generation.

For our example,

- The unit cost of generation for each producer is 7, the unit cost of storage is 0.5, and the unit cost of transport from 1 to 2 is 1
- It would cost B at least 7.5 to generate and store electricity, and it would cost A at least 8 to sell electricity to node 2.
- Therefore we investigated three cases:

  $\rho^A = \rho^B = 7.4$ less expensive than using storage
  $\rho^A = \rho^B = 7.6$ more expensive than using storage but less than the cost for A to sell directly to node 2
  $\rho^A = \rho^B = 8.1$ more expensive than the previous two cases

Scenario 2: Generation Expansion vs Storage (ctd)

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		$\rho^A = \rho^B = 7.4$		$\rho^A = \rho^B = 7.6$		$\rho^A = \rho^B = 8.1$	
		t=1	t =2	t=1	t=2	t=1	t=2
Producer A	q^A	19.3	39.3	19.3	39.3	26.9	60
	E^A	0	0	0	0	0	0
Producer B	q^B	38.55	40	40	40	40	40
	E^B	5	25	5.0	23.53	0.00	0
Storage	h_t^S	14.25	0	15.71	0	18.4	0
$(\sigma^S = 30)$	ω^S	0.5	0	0.5	0	0.5	0

	Profits $(\sigma^S = 30)$			
	Prod. A	Prod. B	TSO	Storage
$\rho^A = \rho^B = 7.4$	0	20.50	0	0
$\rho^A = \rho^B = 7.6$	0	25.50	0	0
$\rho^A = \rho^B = 8.1$	30.00	100.00	0	0

Scenario 2: Generation Expansion vs Storage (ctd)

Case $\rho^A = \rho^B = 8.1$:

	Profits				Prices				
σ^S	A	B	TSO	Storage	π_{11}	π_{21}	π_{12}	π_{22}	h^S
0	6	44	0	0	7	7.1	7	8.1	0
1	6	44	0	0.6	7	7.1	7	8.1	1
2	6	44	0	1.2	7	7.1	7	8.1	2
3	6	44	0	1.8	7	7.1	7	8.1	3
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
10	6	44	0	6	7	7.1	7	8.1	10
11	6	68	0	0	7	7.1	7.6	8.1	10.76
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
16	6	68	0	0	7	7.1	7.6	8.1	10.76
17	450	380	0	119	7	14.5	8	15.5	17
18	150	180	0	36	7	9.5	8	10.5	18
19	30	100	0	0	7	7.5	8	8.5	18.4
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
30	30	100	0	0	7	7.5	8	8.5	18.4

Summary

- We proposed a novel solution approach for the BC-MLCP.
- It starts with an RLT relaxation of the BC-MLCP, fixes complementarity constraints to linear equations as much as possible, then uses MILP.
- Neither the complementarity nor the integrality are relaxed.
- Applying this solution approach to power markets with the addition of a storage operator, it is possible to quantify the extent to which:
  - the presence of a storage operator is beneficial during on-peak demand periods, and
  - whether it can be profitable for a storage investor to operate as a service provider.

Conclusion

Optimization has Myriad Applications in Energy!

Optimization problems arise in particular from many current challenges in smart grids, including:

- Demand response, smart buildings, and distributed resources
- Electric vehicles
- Energy storage, not restricted to batteries
- Optimal use and maintenance of existing infrastructure
- Isolated / islanded systems
- Economic aspects

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Thank you, and enjoy the week!