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Tradability of Goods and Real Exchange Rate Fluctuations

Caroline M. Betts*

University of Southern California

Timothy J. Kehoe*

University of Minnesota
and Federal Reserve Bank of Minneapolis

ABSTRACT

We develop a simple, multicountry, multisector intertemporal general equilibrium model in which the degree of tradability of output differs across sectors. Tradability is determined both by the degree of substitutability in consumption between units of the same good produced in different countries and by the transactions costs that must be incurred to consume goods outside their country of origin. Home bias is endogenously determined. A vector of country specific shocks is realized at each data, and there are complete contingent claims markets. A calibrated version of the model replicates the observed relationship between movements in the bilateral real exchange rate between Mexico and the United States and movements in the relative price of comparatively nontraded goods to traded goods across countries. In addition, the shocks induce movements in trade balances and real exchange rate that are consistent with the data. Finally, the model can also match evidence on sectoral deviations from the law of one price. When the model is adapted to incorporate money as a medium of exchange, and there are no monetary nonneutralities, the same model is capable of replicating the observed relationship in our data between the real exchange rate and the nominal exchange rate.

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1. Introduction

A central assumption in theories of real exchange rate determination and international fluctuations is that all goods in the world economy can be classified according to a strict dichotomy. A good is either perfectly "tradable," being freely exchanged across countries in an internationally integrated market, or it is entirely "nontradable," being produced and consumed in a completely segmented domestic market. In traditional exchange rate theory, tradable goods are exchanged in such highly integrated world markets that their prices satisfy the law of one price at all times. Meanwhile, the prices of nontradable goods are determined entirely by domestic market conditions. In the absence of movements in the relative prices of tradable goods, it is movements in the relative price of nontradable to tradable goods across countries which drive all real exchange rate fluctuations.

Recent evidence from disaggregated price data, however, shows that there are large and persistent deviations from the law of one price for many traded goods. This evidence has been associated with empirical work from the 1980's which showed that very large movements in the U.S. nominal exchange rate during that decade were not reflected in the nominal currency prices of tradable goods. Most importantly, in an empirical analysis of bilateral exchange rates between the U.S. and other industrialized countries, Engel (1999) shows that almost all real exchange rate fluctuations are attributable to fluctuations in the international relative prices of traded goods rather than to any movement in the international relative prices of nontraded to traded goods. As a result of this evidence, most modern theories of real exchange rate determination emphasize segmentation in markets for tradable goods, and the implied potential for law of one price deviations, as the key determinants of the real exchange rate. Specifically, modern theories associate deviations from the law of one price among tradable goods with nominal exchange rate responses to monetary and financial market shocks which are "larger" than the responses of nominal, local currency prices of tradable goods. This differential in the size of responses is attributed to the presence of exogenously specified nominal rigidities in goods markets. The nominal exchange rate change is therefore highly correlated with the "real exchange rate" change for an individual tradable good, as well as at the aggregate level, in modern models of this ilk. These features are consistent with both the evidence from the disaggregated data cited above, as well as with the observation of Mussa (1986) that real and nominal exchange rate changes are highly correlated at the aggregate level.

In this paper, we develop a new quantitative framework for analyzing the sources of real exchange rate fluctuations. The basis for our approach is our observation that the traditional dichotomization of goods into purely "tradable" and "nontradable" is empirically inappropriate. We argue that a more accurate and useful characterization of the data is provided by classifying goods by the *degree* of their tradability. We show, furthermore, that the degree of a good's tradability is well represented by the degree of its actual tradedness in the data, at least in the sense that the size of law of one price deviations vary inversely with the degree of a good's actual tradedness. We illustrate that in data from the U.S. and Mexico, there are, indeed, important deviations from the law of one price for goods with low degrees of tradedness and that – as a result – the real exchange rate is much more variable than the relative price of comparatively nontraded to comparatively traded goods. However, it is also the case that the relative price of comparatively nontraded to comparatively traded goods across countries is highly positively correlated with the real exchange rate. In related work, Betts and Kehoe (2000) have found that these results hold for bilateral exchange rates between the United States and her two other largest trading partners, Canada and Japan.

We then develop an international real business cycle model in which goods are allowed to differ by degree of tradability. When carefully calibrated to data on degrees of actual tradedness of goods, this theoretical model replicates extremely well the observed behavior of both the real exchange rate and relative international prices by sector in data from the U.S. and Mexico. In particular, it is capable of replicating the high correlation of the real exchange rate with the international relative prices of comparatively nontraded to traded goods, the high relative and absolute variability of the aggregate real exchange rate, and deviations from the law of one price by sector. We subsequently show that this, purely real business cycle model can be reinterpreted as a monetary business cycle model in which money is a veil with respect to real activity. In the monetary variant of the model, despite the absence of nominal rigidities, real and nominal exchange rate changes are highly correlated at the sectoral and aggregate level as we observe in our data.

Our model departs from traditional theories of real exchange rate determination in that the outputs of different sectors, rather than being either perfectly tradable or entirely nontradable, have differing degrees of tradability. These degrees of tradability are determined by real transactions costs of trade of the type emphasized by Obstfeld and Rogoff (2000), and by the degree of imperfect substitutability in consumption of the same type of good produced in different countries. In addition, the model incorporates considerable sectoral detail along the lines of multisector static applied general equilibrium models and, as a first cut, we emphasize this sectoral detail rather than more conventional features of real business cycle models, like capital accumulation. The monetary variant of our model differs from recent real exchange rate theories in that there are no exogenous nominal price rigidities which can produce deviations from the law of one price and a high correlation of the real and nominal exchange rate, as in Betts and Devereux

(2000) and Chari, Kehoe and McGratten (2000). In addition, there is no imperfect competition or price-setting ability of the firms in our economy, features which characterize recent work on the role of nominal rigidities and for real exchange rate behavior. In keeping with the international real business cycle literature, as explained by Backus, Kehoe, and Kydland (1992) and Stockman and Tesar (1995), we study the effects of shocks that cause the trade balance to be countercyclical, as it has been found to be in a large body of empirical evidence. Given that we abstract from investment, we model these shocks as affecting the utility of consumption. This modeling shortcut should be thought of as a sort of reduced form, however. We also allow for monetary shocks. To the extent that the experimental results indicate that our approach is worth pursuing, future effort needs to be put into modeling the real shocks and into identifying their underlying sources.

A calibrated version of the purely real business cycle model can match key features of the macroeconomic and sectoral data set that we construct. The model replicates the observed relationship between movements in the bilateral real exchange rate between Mexico and the United States and movements in the relative price of comparatively nontraded goods to traded goods across these two countries. Intuitively, the real exchange rate is much more variable than the relative price of comparatively nontraded to traded goods across countries because the latter variable assumes that there are no relative price movements across sectors of similar degrees of tradability. In addition, the model is capable of matching the variation in and much of the persistence in the real exchange rate when there are sufficiently high degrees of factor immobility across sectors. High degrees of factor immobility limit the extent to which relative sectoral outputs can adjust within countries under real demand shocks. Real demand shocks raise desired consumption of all goods proportionately. However, loosely speaking, the consumption of relatively nontradable goods is more efficiently increased through raising domestic production while the consumption of other more traded goods is more efficiently raised through raising imports. When factors are relatively immobile across sectors, however, this limits the relative increase in comparatively nontraded goods' production, and tends to increase the domestic (and international) relative cross-sectoral price response to the shock.

In addition, the shocks induce movements in trade balances and the real exchange rate that are consistent with the data. Finally, the model can also match sectoral deviations from the law of one price, again conditional on there being a sufficiently large proportion of sector specific factors.

In Section 2, we review evidence on the validity of traditional and modern real exchange rate theory using some new data, and in Section 3 we set out a multicountry, multisector business cycle model that can potentially account for the key features of this data. In Section 4, we develop a decentralized sequential markets representation of the economy, and in Section 5 we collect theorems and proofs establishing the existence of equilibrium, and discuss our method for computing equilibrium. Section 6 is reserved for a description of our calibration of the model to Mexican and U.S. data. Section 7 presents our quantitative results. Section 8 presents a monetary model in which, although money is veil plays no role in determining real variables, we are nonetheless able to analyze the relation between nominal and real exchange rates. Section 9 concludes.

2. Data

In this section we reexamine quantitatively the traditional approach to real exchange rate determination. According to this approach, any good is either perfectly tradable, with an equilibrium price that satisfies the international law of one price, or is entirely nontradable, with a price that is determined by internal conditions. The bilateral real exchange rate at date t between any two countries, and here we consider the case of Mexico and the United States, is given by

$$(1) \qquad RER_t = NER_t \frac{P_t^{us}}{P_t^{mex}}.$$

Here NER_t is the nominal exchange rate, expressed in pesos per dollar, and P_t^{us} and P_t^{mex} are price indices for the United States and Mexico respectively. In traditional models of real exchange rate determination, these aggregate output prices comprise functions of the prices of perfectly tradable and entirely nontradable goods. Hence the real exchange rate can always be thought of as comprising two distinct components: one that measures the relative international prices of perfectly tradable goods and one that measures the relative internal prices of entirely nontradable to tradable goods across countries.

Of course, if traditional real exchange rate theory were correct, then the law of one price would hold instantaneously for all tradable goods. If, in addition, two countries produced exactly the same basket of tradable goods, then the following relationship would hold between aggregate tradable goods price indices:

$$(2) \qquad PT_t^{mex} = NER_t PT_t^{us}.$$

In other words, the prices for tradable goods in Mexico would be the same as in the United States after we use the nominal exchange rate to convert dollars into pesos. Substituting for NER_t from (2) into (1) yields the following equation for the real exchange rate when all tradable goods' prices satisfy the law of one price and the basket of tradable goods is the same in both countries:

$$(3) \qquad \widehat{RER}_t = \left(\frac{RT_t^{mex}}{PT_t^{us}}\right)\left(\frac{P_t^{us}}{P_t^{mex}}\right),$$

We frequently refer to $\widehat{RER}_t$ as the explained real exchange rate, that is, the part of the real exchange rate that is accounted for by movements in the relative internal price of nontradable to tradable goods across countries.

Comparing the properties of RER_t and $\widehat{RER}_t$ can shed some light on the validity of the assumptions underlying traditional real exchange rate theory. The working hypothesis of this paper is that the zero-one tradability distinction over goods drawn in traditional real exchange rate theory is inappropriate. Our view is that a good's tradability is determined by fundamental and potentially measurable features of the economy, such as the costs of trading a given product across national borders and the rate at which individual consumers are willing to substitute in consumption foreign for domestically produced goods. According to this view, the degree to which a good is actually traded reflects exactly the degree to which it is tradable. In addition, goods can be of widely differing degrees of tradability. Consequently, RER_t and $\widehat{RER}_t$ can be very different objects.

To evaluate the validity of the traditional approach, we construct measures of RER_t and $\widehat{RER}_t$. We first develop implicit price indices for tradable goods and for aggregate output for the United States and Mexico and then use these to construct the two real exchange rate measures. To construct price indices, we employ sectoral gross output data drawn from each country's national accounts. Specifically, we obtain annual data for the period 1980–1998 on real and nominal gross output for three sectors; agriculture and mining, which we refer to as primaries; manufactures; and services and construction. We choose data on gross output rather than on value added since gross output is the measure for which price indices are most appropriate for the traditional theory discussed so far. Dividing a nominal gross output number by the corresponding real gross output number yields the desired implicit price index for that output, and this can be done either at the level of an individual sector or at more aggregated levels.

We also collect data on the value of total trade in Mexico for each of our three sectors, that is, the value of imports plus the value of exports. Dividing a total trade number by the corresponding gross output for a given sector yields a measure of tradedness of that sector's output. This allows us to identify the sectors that produce relatively traded types of goods and so to develop an implicit price index over all such traded goods, PT_t^i. In other words, we operationalize the abstract concept of tradability used in traditional real exchange rate theory by identifying relatively traded sectoral outputs as being relatively tradable sectoral outputs.

Table 1

Tradedness and Deviations From
Law of One Price in Mexico, 1980–1998

Sector	Tradedness	Deviation
Primaries	20.37	17.63
Manufactures	36.37	15.10
Services	4.30	24.26

The first column of Table 1 presents the percentage of the gross output of each sector that was traded in 1993 in Mexico,

$$\text{(4)} \qquad \text{tradedness}_i = 100\frac{\text{imports}_{i,1993}^{mex} + \text{exports}_{i,1993}^{mex}}{\text{gross output}_{i,1993}^{mex}}.$$

(Here trade is that with the United States.) It is evident that the degree to which the output of each sector is traded varies widely. The output of the manufactures sector is the most highly traded of good types, with primaries and services being the descending ranking of the remaining sectors by degree of tradedness of output. We decide, somewhat arbitrarily, that any sector with a tradedness percentage in excess of 10 percent is a tradable goods sector, so that services is a non-tradable goods sector, while primaries and manufactures are tradable goods sectors.

We compute the implicit price index for tradable goods as

$$(5) \qquad P_t^i = \frac{YN_{pri,t}^i + YN_{man,t}^i}{YR_{pri,t}^i + YR_{man,t}^i}, \quad i = mex,\ us.$$

Here, YN_{jt}^i is the nominal value of gross output for sector j in country i at date t, while YR_{jt}^i is the corresponding real value of gross output in that sector. The aggregate price indices for the United States and Mexico are simply given by

$$(6) \qquad PT_t^i = \frac{YN_{pri,t}^i + YN_{man,t}^i + YN_{ser,t}^i}{YR_{pri,t}^i + YN_{man,t}^i + YN_{ser,t}^i}, \quad i = us, mex.$$

We now compare the properties of RER_t and $\widehat{RER}_t$. Figure 1 plots the two series where NER_t is measured by the annual average nominal exchange rate drawn from the IMF's International Financial Statistics data base. Clearly, the real exchange rate is much more variable than is $\widehat{RER}_t$. In fact, the standard deviation of RER_t is 2.66 times as high as that of $\widehat{RER}_t$. The directional changes in the two series are very similar in Figure 1, however, and the correlation between them is high and positive, at 0.82. Thus, although there are obviously large deviations from the law of one price for the tradable goods baskets that we have used to construct PT_t^{us}, PT_t^{mex}, and $\widehat{RER}_t$, the portion of the real exchange rate that is explained by the relative internal price of nontradable to tradable goods across countries tracks the actual real exchange rate very closely. Figure 3 presents data on first differences of RER_t and $\widehat{RER}_t$. Once again RER_t is more volatile than $\widehat{RER}_t$, with a standard deviation that is larger by a factor of 4.15, but the two series are highly positively correlated, with a correlation coefficient of 0.55.

In Table 1, we see that the varying degrees of tradedness in the indices in the first column should make us suspicious of the traditional approach that identifies goods as being either perfectly tradable or entirely nontradable. Since degrees of tradedness vary widely even across what we have defined as tradable good sectors, the law of one price will likely neither hold perfectly for any sectoral output nor hold to the same degree for different sectoral outputs. In the second column of Table 1, we report the percentage standard deviation of the real exchange rate by sector over the period 1980–1998. Here

$$\text{(7)} \qquad \text{deviation}_i = 100\left[\sum_{t=1980}^{1998} (\log RER_{it} - \overline{\log RER_i})^2 \big/ 18\right]^{1/2},$$

where RER_{it} is the real exchange rate for the price indices of sector i. Observe that, if the law of one price holds for sector i, then the standard deviation of its real exchange rate should be zero. These standard deviations vary widely across what we have defined as tradable good sectors, and none of them are zero.

Thus, two of the basic assumptions underlying traditional real exchange rate theory are grossly violated in this data: not all tradable goods are equally and perfectly tradable, and their prices do not all satisfy the international law of one price. In addition, it is likely that there remain important compositional differences across the two countries in the tradable goods' baskets that we have constructed. This would imply that (2) would fail even if the law of one price held exactly for a subset, if it existed, of identical perfectly tradable goods. Notice, however, that, if we could obtain a fine enough disaggregation of sectors so that compositional differences within a sector were negligible, then the deviation index for a perfectly tradable sector would be close to zero.

Finally, we address the potential problem that we might not have uncovered the degree of tradability of each sectoral output by examining its actual degree of tradedness. Of course, the notion of tradability is abstract in traditional theory, requiring that measurement of some proxy—and tradedness in particular—be used in quantitative analyses. More than this, however, we have seen here that higher degrees of actual tradedness of output by sector correspond to lower average deviations from the law of one price by sector in Table 1. The largest deviations from the law of one price are associated with the services sector. Primaries output exhibits the second largest deviation from the law of one price, manufacturing the smallest deviation. Thus, there is a strong, negative correlation between the degree of tradedness of a sector's output and the deviations from the law of one price that its implicit price index exhibits. Therefore, although the terms "tradability" and "tradedness" are typically thought of as distinct concepts, the evidence that we present here on deviations from the law of one price suggests that they are really interchangeable, at least in terms of the operational distinction between how the equilibrium prices of more and less traded goods are determined.

Our final goal in this section is to examine the relationship between the real and the nominal exchange rate in our data. By contrast to the traditional approach to real exchange rate determination, more recent analyses have emphasized the role of monetary and financial shocks to the nominal exchange rate as a source of fluctuations in the real exchange rate. Much of this recent work is motivated by the observation of Mussa (1986) that changes in real and nominal exchange rates are very highly positively correlated. This observation is widely interpreted as the outcome of nominal price rigidities. Nominal and financial shocks produce fluctuations in the nominal exchange rate while the prices of individual goods do not instantaneously respond.

Large deviations from the law of one price amongst traded goods, and at the aggregate level large and potentially persistent fluctuations of the real exchange rate, arise as a result.

In Figures 2 and 4 we present data on the Mexico-U.S. real and nominal exchange rates, RER_t and NER_t. The correlation between these two series is relatively low, at 0.11, but the correlation of first differences is much higher, at 0.7, consistent with the evidence of Mussa (1986). In addition, the nominal exchange rate appears to be nonstationary in Figure 2, again consistent with the evidence of Mussa. In this paper, we do not assume that the observation of a high correlation between nominal and real exchange rate changes constitutes indirect evidence of nominal price rigidities.

We have made four key observations: First, the degree of tradedness of different sectoral outputs varies widely. Second, the degree of tradedness of a good is negatively correlated with the deviation from the law of one price that its price exhibits, so that tradedness and tradability are actually very similar concepts. These deviations from the law of one price can be thought of as arising from (different) transactions costs of trade, as well as from the degree of substitutability in consumption across goods produced in different countries. Third, there are likely compositional differences in the baskets of traded goods that are used to measure traded goods price indices, and even small differences across countries in the goods produced by any individual sector, both of which invalidate the use of (2) as a basis for thinking about price determination for traded goods. Fourth, changes in the real exchange rate are highly correlated with changes in the nominal exchange rate.

The main implication of the first three of these observations is that quantitative and theoretical real exchange rate analysis in which all goods are treated as either perfectly tradable or entirely nontradable is flawed. The empirical differences in the statistical properties of RER_t and

$\widehat{RER}_t$ measure how far the predictions of this theory are from the data, although they do not allow us to identify whether the key sources of this failure are real transactions costs, compositional differences in traded goods baskets across countries, or international differences in the goods produced by a given sector that generate imperfect degrees of substitutability in consumption.

These implications motivate our development in the next section of a model that incorporates all three potential sources of deviations of the real exchange rate from its traditional value given by $\widehat{RER}_t$. The model is explicitly quantitative in the sense that we can compute the same measures of RER_t, $\widehat{RER}_t$, their correlation and relative standard deviations, degrees of tradedness, and deviations from the law of one price by sector as we have done for the data. In this initial variant of the model, for simplicity, we abstract from monetary features entirely. In Section 9, we address this shortcoming in an extended model in which nominal features are present, yet there are no nominal rigidities, and money is a veil. Again, the model is quantitative, allowing us to directly compute and compare properties of the data generated by our theoretical model with those of the actual data presented above. Specifically, we can evaluate whether a monetary model in which there are no nominal rigidities is capable of replicating the high correlation of real and nominal exchange rate changes observed in our data.

3. Model

We develop a multicountry, multisector general equilibrium business cycle model. There is no money in this model, and capital is a fixed production factor in each sector. The key feature of the model is that we explicitly incorporate a multisectoral structure into an otherwise very simple general equilibrium business cycle framework.

In the model, there are J production sectors. Each sector exclusively produces a single type of good, so there are J types of good, and each sector operates in every country. In addition, there are I countries, and each country is inhabited by an infinitely lived representative consumer-worker. Each of these I consumers consumes all J types of good produced, and consumes units of good type j produced in all I countries. In other words, the output of a given sector produced in two different countries is viewed as imperfectly substitutable in consumption. There are effectively then $I \times J$ differentiated goods in the world economy. The goods are also characterized by different transactions costs of trade, which we describe below.

The notation that we use is as follows. Countries of origin are indexed by $h = 1, ..., I$, countries of destination are indexed by $I = 1, ..., I$, and sectors are indexed by $j = 1, ..., J$. In addition, subscripts denote sectors and country of origin, while superscripts denote country of destination. Hence c^i_{jh} would denote, for example, the consumption in country i of the good produced by sector j in country h.

The representative consumer in country i, $i = 1, ..., I$ is endowed with $\bar{l}^i$ perfectly divisible units of time. This consumer allocates this time between productive labor market activities and leisure. In addition, the portion of time devoted to labor market activities is allocated across the J sectors within country i, so that this labor is not sector specific. Labor is immobile across countries, however.

Representative consumer i has the period utility function $u^i(c^i_1, c^i_2, ..., c^i_J, l^i; z^i)$, where

(8) $$c^i_j = c^i_j(c^i_{j1}, ..., c^i_{jH})$$

is consumption of good j in country i, l^i is leisure in country i, c^i_{jh} is consumption in country i of good j produced in country h, and z^i is a country specific real demand shock that raises the utility derived from a given consumption bundle relative to that derived from a given value of leisure. In addition, c^i_j is the Armington aggregator over consumption of the goods of type (sector) j produced in the I countries. We assume that this aggregator is increasing, concave, and homogenous of degree one. In addition, we assume that the period utility function, u^i, is monotonically increasing and strictly concave, and that $\lim_{c^i_j \to 0} \partial u^i / \partial c^i_j (c^i_1, \ldots, c^i_J, l^i; z^i) = \infty$.

Each good is produced by one sector with a production function of the form begin

(9) $$y_{jh} = f_{jh}(l_{jh}),$$

where y_{jh} is output of good j, f_{jh} is a strictly concave production function that satisfies $f_{jh}(0) = 0$ and $\lim_{j_{jh} \to 0} f'_{jh}(l_{jh}) = \infty$, and l_{jh} is employment in the sector j in country h. This decreasing returns to scale, sector specific production technology implies the existence of sector specific production factors, namely sector specific physical capital and (potentially) sector specific labor or human capital.

With respect to the technology governing the transactions costs of trade, we assume that the cost incurred with trade of a given type of good from country h to country i is proportional to the number of units exported from country h. Specifically, if one unit of good type j is shipped from country h to country i, then δ^i_{jh} units of the good are lost in the shipment process. These costs can be thought of as transportation costs, legal and administrative costs, and the real costs of nontariff barriers to trade, for example. We can also think of them as reflecting tariffs, although in the equilibria that we analyze all tariff revenue must be disposed of by the government through entirely nonproductive activity.

The world economy must satisfy the following feasibility conditions. First, world consumption of good j produced in country h must satisfy

(10) $$\sum_{i=1}^{I}(1+\delta^{i}_{jh})c^{i}_{jh} = y_{jh}, \quad j = 1, ..., J; \quad h = 1, ..., I.$$

Second, we require that the labor supplies and leisure activities undertaken by representative consumer i satisfy

(11) $$\sum_{j=1}^{J} l_{ji} + l^{i} = \bar{l}^{\,i}, \quad i = 1, ..., I.$$

At every date t, there are K possible events, $\eta_t = 1, ..., K$. These K events correspond to K possible values of the vector of idiosyncratic real demand shocks,

(12) $$z_t = (z^1(\eta_t),...,z^m(\eta_t)).$$

We let η be governed by a stationary first order Markov process with transition matrix $\Pi(\eta,\eta')$, where η and η' denote the current period and next period respectively, and

(13) $$\pi_{ij} = \text{prob}(\eta_t = j|\eta_{t-1} = i).$$

Finally, η_0 is given as an initial condition.

Using this notation, we define a state as an event history $s = (\eta_0,\eta_1,...,\eta_{t(s)})$, so that the probability of being in a given state s is

(14) $$\pi(s) = \pi_{\eta_0\eta_1}\pi_{\eta_1\eta_2}\cdots\pi_{\eta_{t(s)-1}\eta_{t(s)}}.$$

Here $t(s)$ is the period in which state s occurs, the length of the vector s minus one. States are simply nodes in a conventional time-uncertainty tree. The set of all possible states, S, is countably infinite.

4. Equilibrium

We first determine the trading opportunities that are available to consumers at any date and state the decision problems of consumers under these trading opportunities. We then define an equilibrium for this decentralized representation of the economy.

We assume that consumers can trade goods and labor in spot markets at any date t = 0, ..., as well as in a market for a complete set of Arrow securities. Consumer i solves the problem of maximizing the utility function

(15) $$\sum_{s \in S} \beta^{t(s)} \pi(s) u^i (c^i_{1s}, ..., c^i_{Js}, l^i_s; z^i(\eta_s))$$

subject to the sequence of one period budget constraints

(16) $$\sum_{j=1}^{J} \sum_{h=1}^{I} (1+\delta^i_{jh}) p_{jhs} c^i_{jhs} + \sum_{\eta'=1}^{K} q_{(s,\eta')} b^i_{(s,\eta')} \leq w^i_s (\bar{l}^i - l^i_s) + r^i_s + b^i_s \quad \text{for all } s,$$

where b^i_s denotes the one period Arrow securities held from $t(s) - 1$ to $t(s)$ that pay off in state s, $q_{(s,\eta')}$ denotes the state s price of an Arrow security that pays off in event η', $b_{(s,\eta')}$ denotes state s purchases of Arrow securities that pay off in event η' at date $t(s) + 1$, and r^i_s are the profits that are the returns to the fixed factors. In addition, the condition

(17) $$b^i_s \geq -\bar{b} \quad \text{for all } i, s,$$

which rules out Ponzi schemes, must be satisfied, where b is a positive constant that is sufficiently large to not otherwise bind in equilibrium. The consumer's initial asset position is

(18) $$b^i_{\eta_0} = 0.$$

Of course,

(19) $$c^i_{js} = c^i_j (c^i_{j1s}, ..., c^i_{jIs}) \quad \text{for all } i, j, s,$$

and profits are given by

(20) $$r_s^i = \sum_{j=1}^{J} (p_{jis} y_{jis} - w_s^i l_{jis}) \quad \text{for all } i, s.$$

The decision problem confronted by sector j in country h is the static problem of maximizing profits subject to the production technology in any state;

$$\max p_{jhs} y_{jhs} - w_s^h l_{jhs}$$

(21) s.t. $$y_{jhs} = f_{jh}(l_{jhs}),$$

where w_s^h is the real wage in country h in state s. Since labor is mobile across sectors, these is a unique competitive real wage in the economy.

We now define an equilibrium for this sequential markets economy.

DEFINITION 1. *A* sequential markets equilibrium *is a sequence of quantities* $(\hat{c}_{js}^i, \hat{c}_{jhs}^i, \hat{l}_s^i, \hat{b}_s^i,$ $\hat{y}_{jhs}, \hat{l}_{jhs})$, *prices* $(\hat{p}_{jhs}^i, \hat{q}_s, \hat{w}_s^i)$ *and profits* $\hat{r}_s^i$, *such that*

1. *Given prices and profits the quantities* $(\hat{c}_{js}^i, \hat{c}_{jhs}^i, \hat{l}_s^i, \hat{b}_s^i)$ *solve the utility maximization problem of consumer i for all i;*
2. *Given prices, the quantities* $(\hat{y}_{jhs}, \hat{l}_{jhs})$ *solve the profit maximization problem of sector j in country h in state s for all j, h, s;*
3. $\hat{r}_s^i$ *are profits in country i in state for all i, s;*
4. *The quantities* $(\hat{c}_{jhs}, \hat{l}_s^i, \hat{y}_{jhs}, \hat{l}_{jhs})$ *satisfy the feasibility conditions for all i, j, s;*
5. *The Arrow securities* b_s^i *satisfy the market clearing conditions*

(22) $$\sum_{i=1}^{I} \hat{b}_s^i = 0 \quad \text{for all } s.$$

5. Computing Equilibrium

Rather than attempting to directly compute the equilibrium allocations and prices of this economy, we apply a number of well known results from general equilibrium theory that allow us to greatly simplify this computation by solving a static social planner's problem. The consumers in the economy whose equilibria are defined in the previous section solve complicated dynamic programming in which the state variables are the current event η and the vector of asset holding b^1, ..., b^I. The social planning problem, however, has only the current event η as a state variable. Consequently we can solve the social planner's problem by solving K static maximization problems, one for each event η, that is, for each vector of consumption shocks $(z^1,...,z^I)$. It is worth stressing, however, that although the solution to the social planner's problem can be broken up into solutions to static problems, the equilibrium that implements it is not at all static: there is a large amount of trade in assets, with corresponding trade deficits and surpluses.

We first state the world planner's problem, which corresponds to the economy that we have described in Section 3. For a given vector of welfare weights for the I countries $a = (a^1,...,a^I)$, the problem is

$$\max \sum_{i=1}^{I} a^i \sum_{s \in S} \beta^{t(s)} \pi(s) u^i(c^i_{1s},...,c^i_{Js}, l^i_s; z^i(\eta_s))$$

(23) s.t. $$\sum_{i=1}^{I} (1+\delta^i_{jh}) c^i_{jhs} \le f_{jh}(l_{jhs}) \quad \text{for } j, h, s,$$

$$\sum_{j=1}^{J} l_{jis} + l^i_s \le \bar{l}^i \quad \text{for all } i, s.$$

$$c^i_{js} = c^i_j(c^i_{j1s},...,c^i_{jIs}) \quad \text{for all } i, j, s.$$

It is easy to show that the solution to this dynamic optimization problem can be derived by solving the K static problems of maximizing the weighted sum of period utilities, subject to constraints only on the allocation in that period,

$$\max \sum_{i=1}^{I} a^i u^i (c_1^i, ..., c_J^i, l^i; z^i(\eta_s))$$

$$(24) \quad \text{s.t.} \quad \sum_{i=1}^{I} (1+\delta_{jh}^i) c_{jh}^i \le f_{jh}(l_{jh}) \quad \text{for all } j, h,$$

$$\sum_{j=1}^{J} l_{jh} + l^i \le \bar{l}^i \quad \text{for all } i$$

$$c_j^i = c_j^i (c_{j1}^i, ..., c_{jI}^i) \quad \text{for all } i, j.$$

The equivalence of the solutions to the dynamic problem and the K static problems is simply the result of the lack of a connection across states in the dynamic problem—a connection that is present in environments where there exists a state variable that is also a choice variable. Here, the planner's decision problem is fundamentally static. In the dynamic problem, for a current state, s, and event η_s, neither the feasible set of allocations nor the choice variables depend on future states and events in any way. Therefore, the planner can simply solve the K static optimization problems associated with the K possible values of η_s for each node s in the time uncertainty tree. This problem is identical across nodes where the current event is η since η follows a stationary Markov process. Therefore, the set of K static computations need be done just once to compute the socially optimal allocations.

We denote the solution to the world planner's problem for countries $i = 1, ..., I$ by the sets of values $(c_j^i(a,\eta), c_{jh}^i(a,\eta), l^i(a,\eta), y_{ji}(a,\eta), l_{ji}(a,\eta))$. We now define the prices $(p_{jh}(a,\eta), w^i(a,\eta),$ $q(a,\eta,\eta'))$ by the following rules:

$$p_{jh}(a,\eta) = a^h \frac{\partial u^h(c_1^h,...,c_J^h,l^h;z^h(\eta))}{\partial c_j^h} \frac{\partial \phi_j^h(c_{j1}^h,...,c_{JH}^h)}{\partial c_{jh}^h}$$

(25) $$w^i(a,\eta) = a^i \frac{\partial u^i(c_1^i,...,c_J^i,l^i;z^i(\eta))}{\partial l^i}$$

$$q(a,\eta,\eta') = \beta\pi_{\eta,\eta'}.$$

The right hand side of the first two of these conditions is simply the marginal social welfare associated with an increment to consumption of good type j produced and consumed in country h, and to leisure in country h, respectively. These terms correspond to the first order conditions for a social optimum in the planner's static problem. We then let the numeraire be units of marginal social welfare in event η. The third price, that of an Arrow security in event η that pays off in the next period in event η', is given by the first order condition for the representative agent's choice of Arrow securities. Since we have selected as the numeraire units of marginal social welfare in event η, no marginal utility terms enter the rule for this price.

We now define transfer functions. The transfer to country i when event η occurs is given by

(26) $$\tau^i(a,\eta) = \sum_{j=1}^{J}\sum_{h=1}^{I} p_{jh}(a,\eta)(1+\delta_{jh}^i)c_{jh}^i(a,\eta) - \sum_{j=1}^{J} p_{ji}(a,\eta)y_{ji}(a,\eta).$$

This function defines the number of units of marginal social welfare that must be assigned by the planner to country i at the optimal allocations, and at the prices that are defined by the rules described above, in order for those allocations to satisfy representative agent h's sequential markets budget constraint in event η. Obviously, $\tau^i(a,\eta)$ is simply the trade deficit of country i when event η occurs, measured in units of marginal social welfare in event η.

It is easy to show that $\tau^i(\lambda a,\eta) = \lambda\tau^i(a,\eta)$, that is, τ^i is homogeneous of degree one in *a*, and that

(27) $$\sum_{i=1}^{I} \tau^i(a,\eta) = 0.$$

It is also straightforward, but not so easy, to show that $\tau^i(a,\eta)$ is continuous in *a* for all *a* that are nonnegative and not all zero. For strictly positive *a* this result follows from the maximum theorem. When some $a^i = 0$ we need to define the prices $p_{ji}(a,\eta)$ using the marginal utilities of some consumer *h* who has $a^h > 0$ and hence $c_{ji}^h(a,\eta) > 0$ in the solution to the social planner's problem.

We also notice that, from agent *i*'s sequential markets budget constraint evaluated at the socially optimal allocations and at the prices defined above, the present value of all current and future savings given utility weights *a* and the current event η is given by

(28) $$b^i(a,\eta) = -\tau^i(a,\eta) + \beta \sum_{\eta'=1}^{K} \pi_{\eta\eta'} b^i(a,\eta').$$

This is a contraction, which has a unique solution since our economy satisfies all of the conditions that are sufficient for Blackwell's sufficient conditions for the contraction mapping theorem to apply. These savings functions $b^i(a,\eta)$ inherit from $\tau^i(\mathrm{a},\eta)$ the properties that they are continuous in *a*, homogeneous of degree one in *a*, and sum to zero.

THEOREM 1. *A sequence of quantities* $(\hat{c}_{js}^i, \hat{c}_{jhs}^i, \hat{l}_s^i, \hat{b}_s^i, \hat{y}_{jhs}, \hat{l}_{jhs})$, *prices* $(\hat{p}_{jhs}, \hat{q}_s, \hat{w}_s^i)$, *and profits* $\hat{P}_s^i$ *is an equilibrium if and only if there exists a vector of welfare weights* $\hat{a}$ *such that* $b^i(\hat{a},\eta_0) = 0$ *for all i.*

Proof. To demonstrate that a solution to the world planner's problem is an equilibrium, we use the solutions to the K static problems to assign values to the equilibrium allocation: $\hat{c}^i_{js} = c^i_j(\hat{a}, \eta_s)$, $\hat{c}^i_{jhs} = \hat{c}^i_{jh}(\hat{a}, \eta_s)$, $\hat{l}^i_s = l^i(\hat{a}, \eta_s)$, $\hat{b}^i_s = b^i(\hat{a}, \eta_s)$, $\hat{y}_{jhs} = y_{jh}(\hat{a}, \eta_s)$, $\hat{l}_{jhs} = l_{jh}(\hat{a}, \eta_s)$. We define equilibrium prices as $\hat{p}_{jhs} = p_{jh}(\hat{a}, \eta_s)$, $\hat{q}_{s,\eta'} = q(\hat{a}, \eta_s, \eta')$, $\hat{w}^i_{js} = w^i_j(\hat{a}, \eta_s)$. It is easy to show that this Pareto efficient allocation and associated prices satisfy all of the necessary and sufficient conditions for a sequential markets equilibrium if $b^i(\hat{a}, \eta_0) = 0$.

To demonstrate that an equilibrium can be computed by solving the planner's K static problems, we rely on standard arguments that imply that any equilibrium allocation is Pareto efficient. Since consumers' utility functions are concave, a Pareto efficient allocation solves the world planners' intertemporal problem, which we have already argued can be separated into the K static problems defined above. O

THEOREM 2. *There exists a vector of weights* $\hat{a}$ *such that* $b^i(\hat{a}, \eta_0) = 0$ *for all i*.

Proof. Notice that the social planner sets $c^i_{jh}(a, \eta) = 0$ if $a^i = 0$ but not all $a^h = 0$. Consequently, $\tau^i(a,\eta) < 0$ for all η if $a^i = 0$, which implies that $b^i(a,\eta_0) > 0$.

Since the savings functions $b^i(a,\eta_0)$ are homogeneous of degree one in a, we can normalize the utility weights to lie in the unit simplex

(29) $$\left\{(a^i, \ldots, a^I) \mid \sum_{i=1}^{I} a^i = 1, a^i > 0\right\}.$$

Define the function $g(a)$ by the rules

(30) $$g^i(a) = \frac{\max[a^i + b^i(a, \eta_0), 0]}{\sum_{h=1}^{I} \max[a^h + b^h(a, \eta_0), 0]}$$

Notice that

(31) $$\sum_{h=1}^{I} \max[a^h + b^h(a,\eta_0), 0] \geq \sum_{h=1}^{I} a^h + \sum_{h=1}^{I} b^h(a,\eta_0) = 1.$$

Consequently, g maps the unit simplex into itself and is continuous since $b^i(a,\eta_0)$ is continuous in a.

Brouwer's fixed point theorem says that there exists a vector $\hat{a}$ in the unit simplex such that $\hat{a} = g(\hat{a})$. Writing out this condition country by country we have

(32) $$A\hat{a}^i = \max[\hat{a}^i + b^i(\hat{a},\eta_0), 0] \quad i = 1, ..., I,$$

where we have multiplied through by A, the denominator in the definition of $g^i(a)$, (30), which is positive. Notice that $\hat{a}^i$ cannot be zero for any i since this would imply that

(33) $$0 = b^i(\hat{a},\eta_0) > 0.$$

Consequently,

(34) $$A\hat{a}^i = \hat{a}^i + b^i(\hat{a},\eta_0), \quad i = 1, ..., I.$$

Summing over i = 1, ..., I, yields $A = 1$, which implies that $b^i(\hat{a},\eta_0) = 0$ for all i. O

Although these theorems, which ensure existence of equilibrium, are comforting, we do not actually compute a vector of welfare weights $\hat{a}^i$ so that $b^i(\hat{a},\eta_0) = 0$ is our numerical experiments. Rather we calibrate the model so that the values of output, consumption, and trade in a benchmark year, 1993 in the case of our Mexico-U.S. model, are an equilibrium. This allows us to calibrate the vector $\hat{a}$.

6. Mexico-U.S. Calibrated Model

In this section we describe a model with 2 countries and 3 sectors calibrated to Mexico-U.S. data. This model has a large number of parameters, giving it the potential to be consistent with a large number of observations. In our calibration procedure, we place severe restrictions on our choice of parameters in order to provide a sharp test of our theory.

We assume that the period utility function of consumer *i* has a nested constant elasticity of substitution form

$$(35)\qquad u^i(c^i_{pri}, c^i_{man}, c^i_{ser}, l^i; z^i)$$

$$= \left(\left[\left(z^i \left(\sum_{j=pri,man,ser} \zeta^i_j (c^i_j)^{\gamma} \right)^{1/\gamma} \right)^{\varepsilon} + (1-z^i)(l^i)^{\varepsilon} \right]^{\psi/\varepsilon} - 1 \right) \Big/ \psi$$

where

$$(36)\qquad c^i_j = (\alpha_{mex}(c^i_{j,mex})^{\rho} + \alpha_{us}(c^i_{j,us})^{\rho})^{1/\rho}, \quad j = pri,\ man,\ ser.$$

Notice that in the limiting case where $\varepsilon = 0$, which is the case that we study,

$$(37)\qquad u^i(c^i_{pri}, c^i_{man}, c^i_{ser}, l^i; z^i) = \left(\left[\left(\sum_{j=pri,man,ser} \zeta^i_j (c^i_j)^{\gamma} \right)^{z^i/\gamma} (l^i)^{1-z^i} \right]^{\psi} - 1 \right) \Big/ \psi .$$

Also notice that, although Mexican and U.S. consumers can have different consumption share parameters ζ^i_j for the 3 types of goods, they have the same preferences for consumption of goods from different countries within a type of good. In our parameterization, the reason why a Mexican representative consumer consumes different proportions of Mexican and U.S. manufactured goods than a U.S. representative consumer, for example, is because he confronts different transaction costs δ^i_{jh}.

The production functions have the form

$$(38)\qquad y_{jh} = \theta_{jh} l^{\lambda_{jy}}_{jh} .$$

Implicitly, there is a fixed factor in each sector with a share parameter $(1-\lambda_{jh})$. The returns to this factor are accounted for in our model as part of the profits r^h.

Table 2

Parameter Values

Parameter	Value	Source
γ	–1.25	Stockman, Tesar (1994)
ε	0.00	Backus, Kehoe, Kydland (1992)
ρ	0.33	Backus, Kehoe, Kydland (1994)
ψ	–1.00	Backus, Kehoe, Kydland (1992)
λ_{ji}	0.33	Chari, Kehoe, McGrattan (1996)

We start the calibration procedure by drawing the elasticity parameters γ, ε, ρ, ψ, and λ_{ji} from the international real business cycle literature. The values of these parameters are reported in Table 2. The rest of the parameters are chosen so that production, consumption, and trade patterns observed in 1993 in Mexico and the United States are equilibrium outcomes of the model. We choose the parameters θ_{jh}, ζ^i_j, and δ^i_{jh} to replicate the values of y_{jh} and $(1+\delta^i_{jh})c^i_{jh}$ reported in Table 3. Labor in each country is normalized to be its 1993 compensation value. The values of the real demand shocks in 1993 are chosen so that in each country one third of labor is dedicated to market activities and two thirds to leisure.

The transaction costs δ^i_{jh} can be thought of as purely psychic: a Mexican values consumption of U.S. goods less than that of Mexican goods because U.S. goods are less familiar. To make this claim concrete, suppose that α_h and δ^i_{jh} are our calibrated parameters. The straightforward calculations using the first order conditions of the static social planner's problem shows that any $\tilde{\alpha}^i_{jh}$ and $\tilde{\delta}^i_{jh}$ that satisfy

$$(39)\qquad \tilde{\alpha}^{mex}_{j,us} = \frac{\alpha_{us}(1+\tilde{\delta}^{mex}_{j,us})}{\alpha_{mex}(1+\delta^{mex}_{j,us}) + \alpha_{us}(1+\tilde{\delta}^{mex}_{j,us})}$$

$$(40) \qquad \tilde{\alpha}^{us}_{j,mex} = \frac{\alpha_{mex}(1+\tilde{\delta}^{us}_{j,mex})}{\alpha_{mex}(1+\tilde{\delta}^{us}_{j,mex}) + \alpha_{us}(1+\delta^{us}_{j,mex})}$$

result in an equivalent model. Our specification where $\alpha^i_{jh} = \alpha_h$ for all i, j is attractive, however, because it results in potentially observable transactions costs δ^i_{jh}.

We parameterize the Markov process on the utility shocks z^1 and z^2 by specifying two independent Markov chains. The shock z^i in country i is specified by a grid of three of the form $(\bar{z}^i - 1/d^i, \bar{z}^i, \bar{z}^i + d^i)$ with the Markov matrix

$$(41) \qquad \Pi^i = \begin{bmatrix} 1-2\pi^i & \pi^i & \pi^i \\ \pi^i & 1-2\pi^i & \pi^i \\ \pi^i & \pi^i & 1-2\pi^i \end{bmatrix}.$$

In the simulations the six parameters $\bar{z}^i, d^i$, and π^i, $i = mex, us$ are calibrated so that the standard deviation of logged output and its autocorrelation match those in the data for each country and so that the base year output has the same distance, in terms of standard deviation, from the mean as does 1993 output in the logged and detrended data. In our original notation, $K = 3 \cdot 3 = 9$.

Table 3

1993 Benchmark Data Set
(Billion U.S. Dollars)

Variable	Mexico	U.S.
y^j_{pri}	42.528	393.037
y^j_{man}	200.469	3082.868
y^j_{ser}	391.132	7820.442
$(1+\delta^{mex}_{pri,j})c^{mex}_{pri,j}$	35.870	2.006
$(1+\delta^{us}_{pri,j})c^{us}_{pri,j}$	6.658	391.031
$(1+\delta^{mex}_{man,j})c^{mex}_{man,j}$	167.197	39.629
$(1+\delta^{us}_{man,j})c^{us}_{man,j}$	33.272	3043.239
$(1+\delta^{mex}_{ser,j})c^{mex}_{ser,j}$	382.778	8.451
$(1+\delta^{us}_{ser,j})c^{us}_{ser,j}$	8.354	7811.991

7. Numerical Experiments

In this section we compare results of some numerical experiments using our calibrated model with the data. Our data consists of both macroeconomic and microeconomic series for Mexico and the United States over the period 1980–998. We have taken logarithms and de-trended the macroeconomic series of total output for both countries, the real exchange rate, and the explained real exchange rate described in Section 2. We have detrended, but not logged, the series on bilateral trade balances as a percent of GDP in both countries.

In reporting moments in our numerical simulations, we rely on analytical formulas based on the calibrated Markov process described in the previous section and numerical solutions of the nonlinear model for each combination of utility shocks. To be specific, we calculate the mean of a variable x^i as

$$(42) \qquad \mu(x^i) = \sum_{\eta^1=1}^{3} \sum_{\eta^2=1}^{3} \hat{\pi}^1_{\eta^1} \hat{\pi}^2_{\eta^2} x^i(\eta^1, \eta^2)$$

where $\hat{\pi}^i_{\eta^i}$ is the invariant distribution of the Markov process on η^i Similarly, we calculate the variance of x^i as

$$\sigma^2(x^i)\sum_{\eta^1=1}^{3}\sum_{\eta^2=1}^{3}\hat{\pi}^1_{\eta^1}\hat{\pi}^2_{\eta^2}(x^i(\eta^1,\eta^2)-\mu(x^i))^2. \tag{43}$$

The correlation of two variables x^i and y^h is

$$\rho(x^i,y^h)=\frac{1}{\sigma(x^i)\sigma(y^h)}\sum_{\eta^1=1}^{3}\sum_{\eta^2=1}^{3}\hat{\pi}^1_{\eta^1}\hat{\pi}^2_{\eta^2}(x^i(\eta^1,\eta^2)-\mu(x^i))(y^h(\eta^1,\eta^2)-\mu(y^h)). \tag{44}$$

The autocorrelation of x^i is

$$\begin{aligned}\rho(x^i,x^{i'})=\frac{1}{\sigma^2(x^i)}&\left\{\sum_{\eta^1=1}^{3}\sum_{\eta^2=1}^{3}\hat{\pi}^1_{\eta^1}\hat{\pi}^2_{\eta^2}(x^i(\eta^1,\eta^2)-\mu(x^i))\right\}\\ &\times\left\{\sum_{\eta^{1'}=1}^{3}\sum_{\eta^{2'}=1}^{3}\hat{\pi}^1_{\eta^1\eta^{1'}}\hat{\pi}^2_{\eta^2\eta^{2'}}(x^i(\eta^{1'},\eta^{2'})-\mu(x^i))\right\}.\end{aligned} \tag{45}$$

Table 4 compares the standard deviations of the real exchange rates by sector in the data with those in the model. Here we use the formula

$$\begin{aligned}\text{deviation}=100\times&\left\{\sum_{\eta^1=1}^{3}\sum_{\eta^2=1}^{3}\hat{\pi}^1_{\eta^1}\hat{\pi}^2_{\eta^2}[(\log p_{j1}(a,\eta^1,\eta^2)/p_{j2}(a,\eta^1,\eta^2))-\right.\\ &\left.\mu(\log(p_{j1}/p_{j2}))]^2\right\}^{1/2}\end{aligned} \tag{46}$$

to calculate the average absolute deviation in the model. Notice that the deviation indices for the different sectors in the model decline monotonically with tradability. On the whole, however, there are more deviations from the law of one price in the data than there are in the model results.

Table 4

Deviations from Law of One Price

	Data	Model
Primaries	17.69	13.45
Manufactures	15.10	12.03
Services	24.18	18.94

Table 5 reports standard deviations and autocorrelations of the macroeconomic variables in the data and in our model. Remember that we have calibrated the Markov process on utility shocks so that the standard deviations and autocorrelations of total output in each country in the model should match that in the data. Looking at the model results for other variables, we see that the trade balances are too volatile and the real exchange rate is not volatile enough. On the whole, however, the results are impressive for such a simple model, especially the relative volatility of the explained real exchange rate compared to the actual real exchange rate.

Table 5

Standard Deviations and Autocorrelations

	Data		Model	
	σ	ρ	σ	ρ
Y^{us}	2.417	0.506	2.417	0.506
TB^{us}	0.060	0.616	0.107	0.563
Y^{mex}	4.950	0.578	4.950	0.578
TB^{mex}	1.448	0.599	2.467	0.563
RER	19.519	0.586	16.276	0.564
$\widehat{RER}$	6.949	0.807	4.008	0.565

Table 6 reports the cross correlations of macroeconomic variables. Here too the results are impressive given the simplicity of the model, especially given that the only source of variation are the two utility shocks. The most noticeable discrepancy in the model results, shown in parentheses, with the data is that in the first column. Over the period 1980–1998 total output fluctuations in Mexico, Y^{mex}, were negatively correlated with those in the United States, Y^{us}. The model, however, even with independent shocks across countries, produces a positive correlation. The positive correlation in the model is more in line with the international transmission that we would expect to see over a larger data series.

Table 6

Correlations
Data/(Model)

	Y^{us}	TB^{us}	Y^{mex}	TB^{mex}	RER
TB^{us}	–0.284 (–0.421)				
Y^{mex}	–0.209 (0.071)	0.286 (0.866)			
TB^{mex}	0.473 (0.421)	–0.984 (–0.979)	–0.361 (–0.870)		
RER	0.343 (0.426)	–0.736 (–0.996)	–0.747 (–0.870)	0.781 (0.992)	
$\widehat{RER}$	0.322 0.421	–0.371 (–0.993)	–0.836 (–0.874)	0.436 (0.996)	0.817 (0.999)

Tables 7 and 8 report the results of numerical experiments in which we vary parameters of the model.

Table 7

Deviations from Law of One Price
Alternative Specifications

	Data	Model	Model	Model	Model	Model	Model
		base case	$\rho = 0.80$	$\lambda_{j,mex} = 0.38$ $\lambda_{j,us} = 0.38$	$\lambda_{ji} = 0.25$	$\gamma = -4.00$	$\psi = -0.50$
Primaries	17.63	13.45	7.72	10.76	18.12	12.88	14.42
Manufactures	15.10	12.03	6.25	9.68	16.03	11.06	12.91
Services	24.26	18.94	16.10	14.43	26.91	19.77	20.30

Table 8

Standard Deviations
Alternative Specifications

	Data	Model	Model	Model	Model	Model	Model
		base case	$\rho = 0.80$	$\lambda_{j,mex} = 0.38$ $\lambda_{j,us} = 0.38$	$\lambda_{ji} = 0.25$	$\gamma = -4.00$	$\psi = -0.50$
Y^{us}	2.417	2.417	2.417	2.417	2.417	2.417	2.417
TB^{us}	0.060	0.107	0.236	0.092	0.136	0.098	0.113
Y^{mex}	4.950	4.950	4.950	4.950	4.950	4.950	4.950
TB^{mex}	1.448	2.467	5.184	1.880	3.390	2.267	2.653
RER	19.519	6.276	12.240	12.642	22.629	16.385	17.435
$\widehat{RER}$	6.949	4.008	5.726	2.783	6.250	5.011	4.269

8. Monetary Model

We have shown that a model in which sectoral outputs differ by degree of tradability, and in which there is a sufficiently high rate of labor immobility across sectors, is capable of replicating many key features of international relative price data, and performs quite well in accounting for other empirical regularities of business cycle data. However, if this framework is to compete seriously with models that rely on nominal rigidities as a model of real exchange rate determination, it must be capable of replicating the observed high correlation of real and nominal exchange rate movements in the post-Bretton Woods period, as well as other empirical regularities in the nominal exchange rate behavior documented by researchers such as Mussa (1986). We now add a monetary dimension to our model.

Specifically, we analyze the simplest possible monetary environment in which we are able to reinterpret the results we have described above as those of a model in which money is a veil. To do so, we consider a framework very similar to that of a multi-country cash-in-advance model in which each country $i = 1, 2, ..., I$ has a national monetary authority which prints units of distinct fiat currency i. As in a cash-in-advance model, the currency of the producer's country must be provided in all international goods market exchange. The timing of transactions is altered relative to a cash-in-advance model, however. (Of course, this description of event timing is merely a physical story which rationalizes a specific set of restrictions on which objects may be exchanged in the economy. In fact, all trade can be thought of as occurring simultaneously.) At the beginning of a period agents are paid for their factor services by firms. They then go to asset markets to trade in Arrow securities and foreign currencies, following which they purchase goods in the international goods market using the appropriate currencies. At the end of each period, firms in country *i* therefore hold the entire outstanding country *i* money stock Finally, when all trades and consumption is completed, overnight injections of new currency occur. Specifically, representative consumer *i*, in his role as owner of country *i* firms, receives lump sum transfers of seignorage revenues resulting from the *i*th monetary authority's injection of new money.

As we assumed above, at every date *t*, there are K possible events, $\eta = 1, ..., K$. Here, however, these *K* possible events correspond to *K* possible values of a vector of idiosyncratic real demand shocks and idiosyncratic money supply shocks which are simply the monetary growth factors by the *I* monetary authorities at the end of state *s*.

(47) $$M^i_{(s,\eta')} = \mu^i(n')M^i_s.$$

Here the monetary shock of country *i* is denoted by $\mu^i(\eta)$. We continue to denote the real demand shock of country *i* by $z^i(\eta)$. As before, we let η be governed by a stationary first order Markov process with transition matrix $\Pi(\eta, \eta')$, while η_0 is given as an initial condition.

The *i*th representative consumer in the monetary world economy maximizes the utility function

(48) $$\sum_{s \in S} \beta^{t(s)} \pi(s) u^i (c_{1s}^i, ..., c_{Js}^i, l_s^i; z^i(\eta_s))$$

where $c_{js}^i = c_j^i(c_{j1s}^i, ..., c_{jIs}^i),$ and l_s^i is leisure. This maximization is subject to the sequence of one period budget constraints,

(49) $$\sum_{j=1}^{J} \sum_{h=1}^{J} (1 + \delta_{jh}^i) E_{hs}^i P_{js}^h c_{jhs}^i + E_{1s}^i \sum_{n'=1}^{K} Q_{(s,\eta')} B^i{}_{(s,\eta')} + M_s^i$$
$$\leq W_s^i (\bar{l}^{\,i} - l_s^i) + R_s^i + \mu^i(\eta) M_{s-1}^i + E_{1s}^i B_s^i \quad \text{for all } s.$$

Here, P_{js}^h is the price of good *j* in country *h* in state *s*, measured in units of currency *h*, E_{hs}^i is the nominal exchange rate measured in units of currency *i* required to purchase one unit of currency *h* in state *s*, and $B_s^i(s, \eta')$ is purchases in state *s* of an Arrow security that pays one unit of currency 1 if event η' occurs at *t*(*s*) + 1, which is measured in units of currency 1 at *t*(*s*) + 1 if event η' occurs. In addition, $Q_{(s,\eta')}$ is the state *s* currency 1 price of a unit of currency 1 delivered at *t*(*s*) + 1 if η' occurs, and R_s^i are profits.

The representative consumer's utility maximization problem is constrained also by the condition

(50) $$B_s^i \geq -\bar{B}_s \quad \text{for all } s,$$

which rules out Ponzi schemes, where B_s is a positive constant that is sufficiently large to not otherwise bind in equilibrium. Finally, the *i*th consumer's initial asset position is

(51) $$B_{\eta_0}^i = 0.$$

Obviously, in equilibrium, the feasibility conditions for the world economy must also be satisfied. Finally, we note that the value of any country's money stock must be equal to the value of world nominal expenditures on that country's goods in equilibrium, or

(52) $$\sum_{j=1}^{J}\sum_{h=1}^{J}(1+\delta_{ji}^{h})P_{js}^{i}c_{jis}^{h}=M_{s}^{i} \quad \text{for } s, i.$$

It is this quantity equation that is essential for deriving relations between prices in the monetary model and those in the model without money.

DEFINITION 2. A sequential markets equilibrium *is a sequence of quantities* $(\hat{c}_{js}^{i},\hat{c}_{jhs}^{i},\hat{l}_{s}^{i},\hat{l}_{jhs},\hat{y}_{jhs},$ $\hat{B}_{s}^{i},\hat{M}_{s}^{i},\hat{T}_{s}^{i}),$ prices $(\hat{P}_{js}^{i},\hat{W}_{s}^{i},\hat{Q}_{s},\hat{E}_{hs}^{i})$ *and profits* $\hat{R}_{s}^{i}$ *such that*

1. *Given prices, profits and seigniorage transfers, the quantities* $(\hat{c}_{js}^{i},\hat{c}_{jhs}^{i},\hat{l}_{s}^{i},\hat{B}_{s}^{i},\hat{M}_{s}^{i})$ *solve the utility maximization problem of consumer i in state s for all i, s;*
2. *Given prices, the quantities* $(\hat{l}_{s}^{i},\hat{y}_{jhs})$ *solve the profit maximization problem of sector j in country h in state s for all j, h, s;*
3. *The money stock* $\hat{M}_{s}^{i}$ *satisfies* $\hat{M}_{s}^{i}=\mu_{s}^{i}\hat{M}_{s-1}^{i}$ *for all i, s;*
4. $\hat{R}_{s}^{i}$ *are profits in country i in state s for all i, s;*
5. *The quantities* $(\hat{c}_{js}^{i},\hat{c}_{jhs}^{i},\hat{l}_{s}^{i},\hat{l}_{jhs},\hat{y}_{jhs})$ *satisfy the feasibility conditions for all i, s;*
6. *The securities* $\hat{B}_{s}^{i}$ *satisfy the market clearing conditions*

(53) $$\sum_{i=1}^{J}\hat{B}=0 \text{ for all } s;$$

It is straightforward to show that there exists an equivalence between the equilibrium that we have just defined and an equilibrium of the nonmonetary economy described in Definition 1.

THEOREM 3. *Suppose that the sequence of quantities* $(\hat{c}_{js}^{i},\hat{c}_{jhs}^{i},\hat{l}_{s}^{i},\hat{l}_{jhs},\hat{y}_{jhs},\hat{B}_{s}^{i},\hat{M}_{s}^{i}),$ *prices* $(\hat{P}_{js}^{i},\hat{W}_{s}^{i},\hat{Q}_{s},\hat{E}_{hs}^{i})$ *and profits* $\hat{R}_{s}^{i}$ *is an equilibrium of the monetary economy. Then there exist sequences of bond holdings* $\hat{b}_{s}^{i},$ *prices* $(\hat{p}_{jhs},\hat{w}_{s}^{i},\hat{q}_{s}),$ *and profits* $\hat{r}_{s}^{i}$ *that, together with the quan-*

tities $(\hat{c}^i_{js}, \hat{c}^i_{jhs}, \hat{l}^i_s, \hat{l}_{jhs}, \hat{y}_{jhs})$ *make up a sequence that is an equilibrium of the economy without money.*

Conversely, suppose that the sequence of quantities $(\hat{c}^i_{js}, \hat{c}^i_{jhs}, \hat{l}^i_s, \hat{l}_{jhs}, \hat{y}_{jhs}, \hat{b}^i_s)$, *prices* $(\hat{p}^i_{jhs}, \hat{w}^i_s, \hat{q}_s)$, *and profits* $\hat{r}^i_s$ *is an equilibrium of the economy without money. Suppose too that* $\mu^i(\eta)$, *i* = 1, ..., *I, are monetary shocks and* $\bar{M}^i_0$, *i* = 1, ..., *I, are initial values of money. Then there exist bond holdings* $\hat{B}^i_s$, *money stocks* $\hat{M}^i_s$, *prices* $(\hat{P}^i_{js}, \hat{W}^i_s, \hat{Q}_s, \hat{E}^i_{hs})$, *and profits* $\hat{R}^i_s$ *that, together with the quantities* $(\hat{c}^i_{js}, \hat{c}^i_{jhs}, \hat{l}^i_s, \hat{l}_{jhs}, \hat{y}_{jhs})$ *make up a sequence that is an equilibrium of the monetary economy.*

Proof. Consider a sequence of quantities $(\hat{c}^i_{js}, \hat{c}^i_{jhs}, \hat{l}^i_s, \hat{l}_{jhs}, \hat{y}_{jhs}, \hat{B}^i_s, \hat{M}^i_s)$, prices $(\hat{P}^i_{js}, \hat{W}^i_s, \hat{Q}_s, \hat{E}^i_{hs})$, and profits $\hat{R}^i_s$ that is an equilibrium of the monetary economy. Normalize $\hat{p}_{11s} = 1$. Define

(54) $$\hat{p}_{jhs} = \hat{E}^1_{hs} \hat{P}^h_{js} / \hat{P}^1_{1s}$$

Now define a nominal price level $\hat{P}^i_s$ using the quantity

(55) $$\hat{P}^i_s = \hat{M}^i_s / \sum_{j=1}^{J} \sum_{h=1}^{I} (1 + \delta^h_{ji}) \hat{p}_{jis} \hat{c}^h_{jis}.$$

It is now easy to assign values to the rest of the variables:

(56) $$\hat{b}^i_s = \hat{B}^i_s / \hat{P}^1_s$$

(57) $$\hat{w}^i_s = \hat{W}^i_s / \hat{P}^i_s$$

(58) $$\hat{q}_{(s,\eta')} = \hat{P}^1_{(s,\eta')} \hat{Q}_{(s,\eta')} / \hat{P}^1_s$$

(59) $$\hat{r}^i_s = \hat{R}^i_s / \hat{P}^i_s.$$

It is now a mechanical matter to verify that all of the equilibrium conditions for the economy without money are satisfied.

Now consider a sequence of quantities $(\hat{c}^i_{js}, \hat{c}^i_{jhs}, \hat{l}^i_s, \hat{l}_{jhs}, \hat{y}_{jhs}, \hat{b}^i_s)$, prices $(\hat{p}^i_{jhs}, \hat{w}^i_s, \hat{q}_s)$, and profits $\hat{r}^i_s$ that is an equilibrium of the economy without money. Given the initial values of the money stock $\bar{M}^i_{\eta_0}$, use the monetary shocks $\mu^i(\eta)$ to generate the sequences of money stocks

(60) $\hat{M}^i_{(s,\eta')} = \mu^i(\eta')\hat{M}^i_s$

Again define the nominal price level $\hat{P}^i_s$ using the quantity equation

(61) $\hat{P}^i_s = \hat{M}^i_s / \sum_{j=1}^{J}\sum_{h=1}^{I}(1+\delta^h_{ji})\,\hat{p}_{jis}\hat{c}^h_{jis}.$

Once again, it is easy to assign values to the rest of the variables:

(62) $\hat{B}^i_s = \hat{P}^1_s \hat{b}^i_s$

(62) $\hat{P}^i_{js} = \hat{P}^1_s \hat{p}_{jis}$

(63) $\hat{W}^i_s = \hat{P}^1_s \hat{w}^i_s$

(64) $\hat{Q}_{(s,\eta')} = \hat{P}^1_s \hat{q}_{(s,\eta')} / \hat{P}^1_{(s,\eta')}$

(65) $\hat{E}^i_{hs} = \hat{P}^i_s / \hat{P}^h_s$

(67) $\hat{R}^i_s = \hat{P}^i_s / \hat{r}^i_s$

Finally, it is again a mechanical matter to verify that all of the equilibrium conditions for the monetary economy are satisfied. O

Since the nominal variables in this model follow random walks, it makes no sense to compare data and model results involving levels. Instead, we compare first differences in the data with the corresponding first differences in the model results. Table 9 compares the standard deviations of first differences in the data with those in the model.

Table 9

Standard Deviations of First Differences

	Data	Model
ΔY^{us}	2.117	2.402
ΔTB^{us}	0.051	0.100
ΔY^{mex}	4.177	4.548
ΔTB^{mex}	1.264	2.306
ΔRER	16.560	15.190
$\Delta \widehat{RER}$	3.988	3.739
ΔM^{us}	2.692	2.692
ΔP^{us}	2.031	3.340
ΔM^{mex}	15.706	15.706
ΔP^{mex}	23.675	17.756
ΔNER	31.964	29.082

Table 10 compares the correlations among first differences in the data with those in the model for the real variables.

Table 10

Correlations of First Differences
Data/(Model)

	ΔY^{us}	ΔTB^{us}	ΔY^{mex}	ΔTB^{mex}	ΔRER
ΔTB^{us}	0.346 (–0.449)				
ΔY^{mex}	0.092 (0.075)	0.651 (0.847)			
ΔTB^{mex}	–0.296 (0.449)	–0.958 (–0.970)	–0.730 (–0.849)		
ΔRER	–0.388 (0.455)	–0.820 (–0.995)	–0.730 (–0.852)	0.856 (0.990)	
$\Delta \widehat{RER}$	0.226 (0.593)	–0.398 (–0.990)	–0.735 (–0.856)	0.518 (0.995)	0.547 (0.999)

Table 11 provides additional comparisons of correlations of first differences in the data with those in the model involving the nominal variables. Notice the high correlations between the first differences in the nominal and the real exchange rate, both in the model and in the data.

Table 11

Additional Correlations of First Differences
Data/(Model)

	ΔM^{us}	ΔP^{us}	ΔM^{mex}	ΔP^{mex}	ΔNER
ΔY^{us}	0.144 (0.144)	–0.346 (–0.603)	–0.244 (0.003)	0.087 –(0.032)	–0.115 (0.288)
ΔTB^{us}	0.130 (–0.066)	–0.052 (0.270)	–0.353 (–0.290)	–0.110 (–0.473)	–0.502 (–0.840)
ΔY^{mex}	–0.181 (0.009)	0.161 (–0.046)	–0.335 (–0.335)	–0.571 (–0.553)	–0.811 (–0.777)
ΔTB^{mex}	–0.062 (0.066)	0.026 (–0.270)	0.282 (0.291)	0.123 (0.475)	0.532 (0.838)
ΔRER	0.239 (0.067)	–0.119 (–0.274)	0.460 (0.292)	0.233 (0.476)	0.699 (0.845)
$\Delta \widehat{RER}$	0.115 (0.066)	–0.226 (–0.271)	0.431 (0.293)	0.593 (0.478)	0.737 (0.845)
ΔP^{us}	–0.477 (0.702)				
ΔM^{mex}	–0.075 (–0.075)	0.247 (–0.056)			
ΔP^{mex}	–0.009 (–0.068)	0.085 (–0.032)	0.637 (0.970)		
ΔNER	0.147 (–0.088)	–0.062 (–0.278)	0.694 (0.751)	0.856 (0.863)	

9. Concluding Remarks

An international real business model with sectoral detail and differing degrees of tradability among sectoral outputs accounts well for both relative price movements by sector and real exchange rate fluctuations in Mexico-U.S. data. In addition, superimposing a quantity theory of money on this real model produces the high correlation between real and nominal exchange rate changes that we observe in the data, even though monetary shocks have no effects on real variables. This exercise has been a first cut however, on the basis of the results which we have documented here, we believe that an approach to modeling international fluctuations in which goods are allowed to differ by degree of tradability is worth further work

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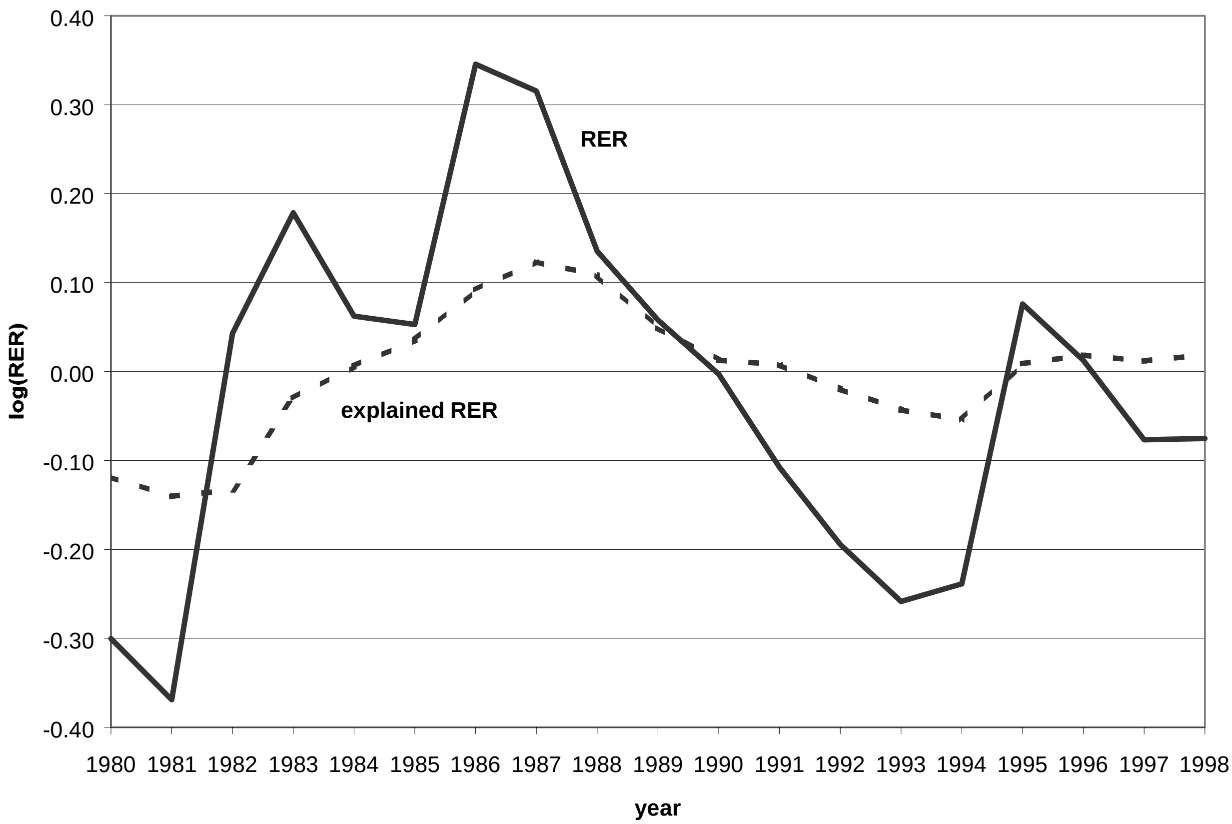

Figure 1: Mexico-U.S. Real Exchange Rate

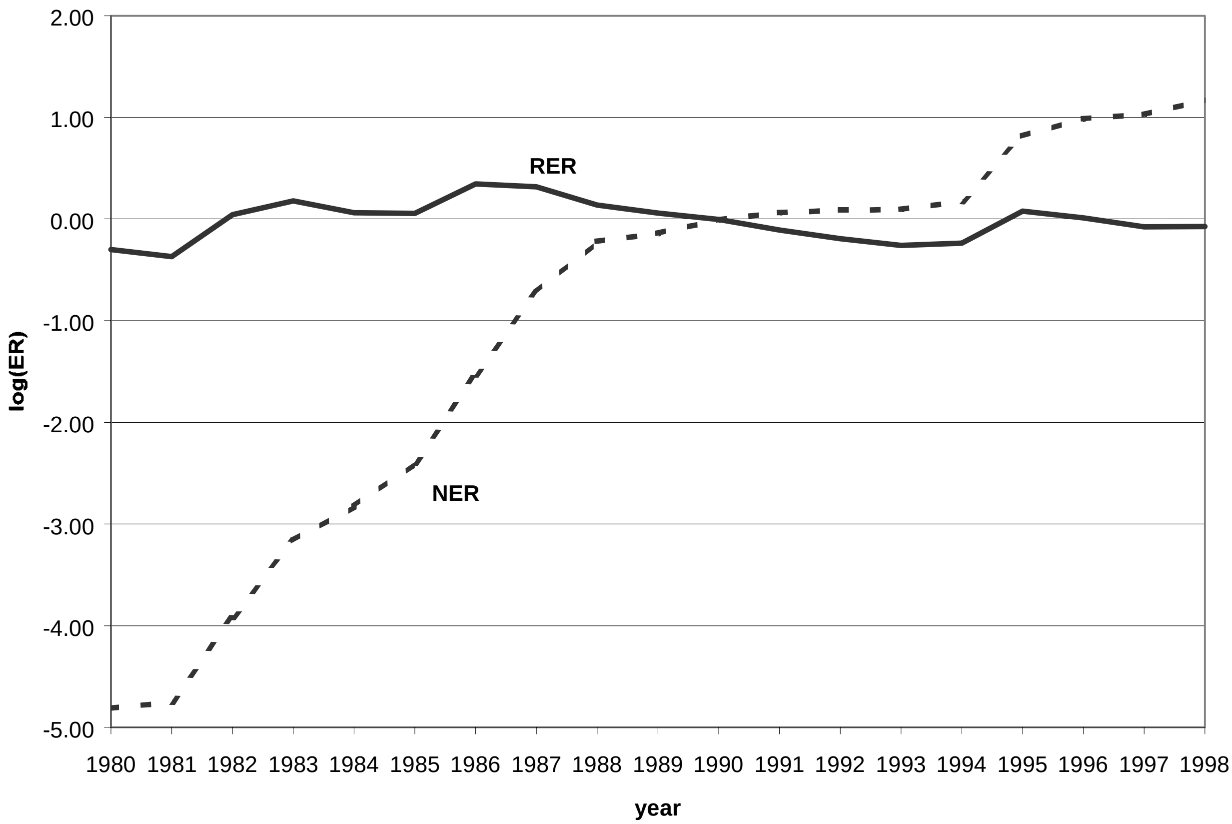

Figure 2: Mexico-U.S. Real and Nominal Exchange Rates

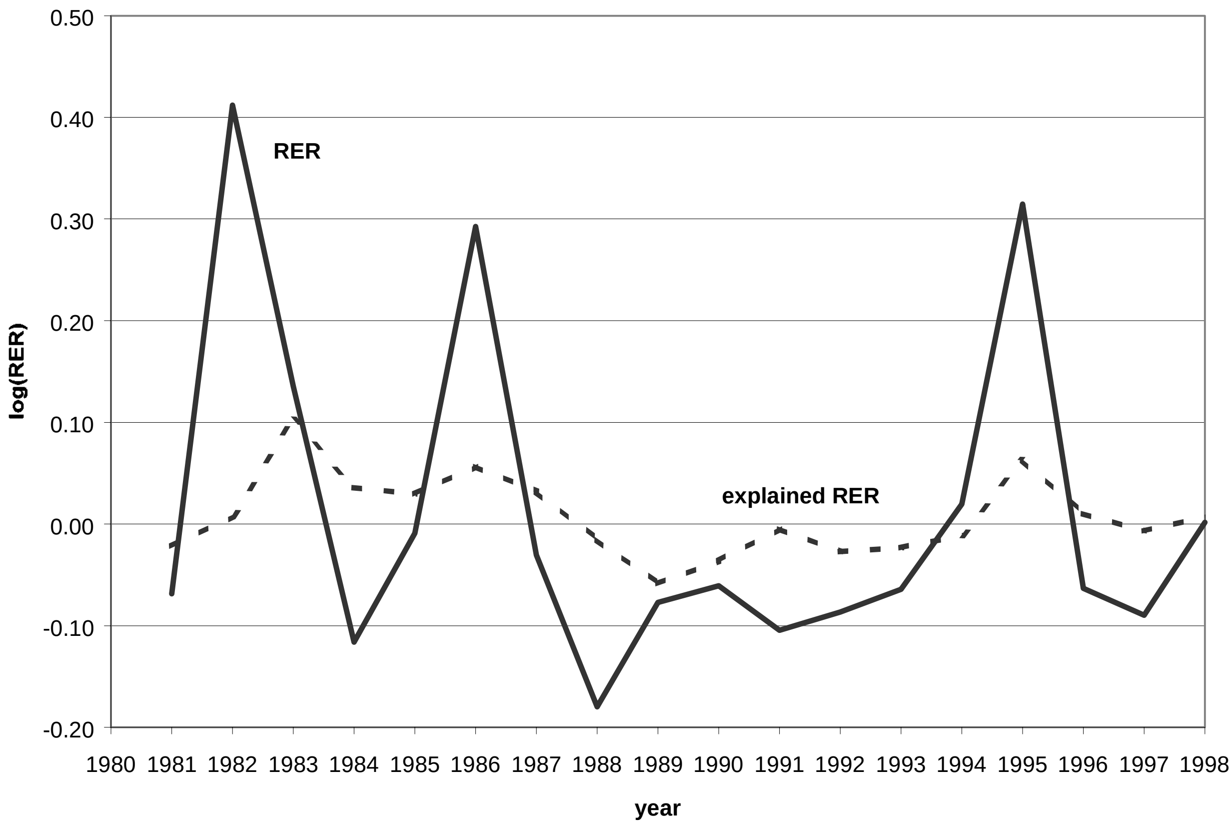

Figure 3: First Differences in Mexico-U.S. Real Exchange Rate

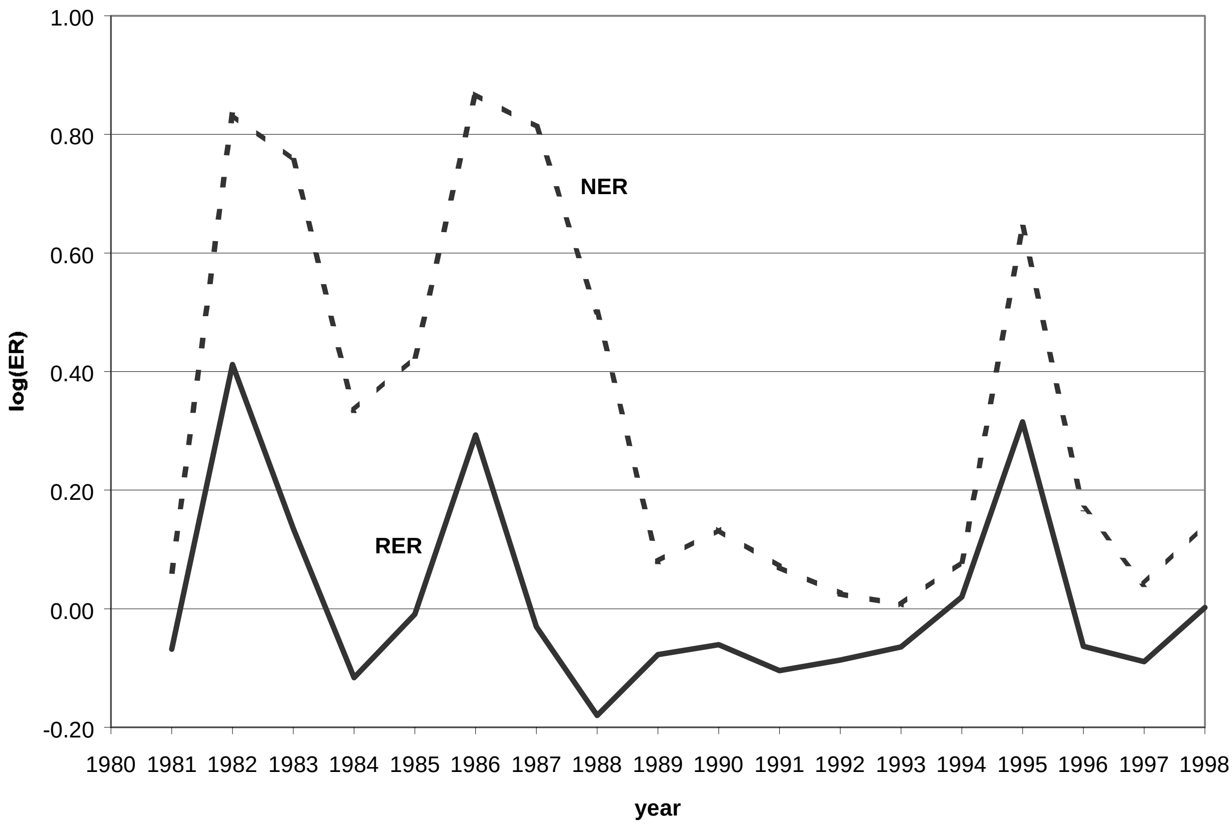

Figure 4: First Differences in Mexico-U.S. Real and Nominal Exchange Rates