NOTE TO FILE

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A Hypothetical Rethink of Energy and Entropy

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Table of Contents

[1 References 1](#_Toc408578029)

[2 Background 4](#_Toc408578030)

[3 Purpose 4](#_Toc408578031)

[4 Energy 5](#_Toc408578032)

[4.1 Energy as an accounting system applied to a biophysical system 6](#_Toc408578033)

[4.2 Common units 6](#_Toc408578034)

[4.3 Energy is additive across systems, or subsystems 6](#_Toc408578035)

[4.4 Conservation of energy 7](#_Toc408578036)

[4.5 Forms of energy 7](#_Toc408578037)

[4.6 Transformations of energy 7](#_Toc408578038)

[4.7 Localization of energy and the issue of system scale 10](#_Toc408578039)

[4.7.1 Localization of kinetic energy 10](#_Toc408578040)

[4.7.2 Localization of potential energy 10](#_Toc408578041)

[4.8 Grades of energy 11](#_Toc408578042)

[5 Entropy 12](#_Toc408578043)

[5.1 Entropy as an accounting system 13](#_Toc408578044)

[5.2 Entropy as an average 13](#_Toc408578045)

[5.2.1 Entropy of a Histogram – 14](#_Toc408578046)

[5.2.2 Entropy of a static system – 15](#_Toc408578047)

[5.2.3 Entropy of a dynamic system – 15](#_Toc408578048)

[5.3 Forms of Entropy 15](#_Toc408578049)

[5.3.1 Entropy as thermodynamic entropy 15](#_Toc408578050)

[5.3.2 Entropy as informational entropy 16](#_Toc408578051)

[5.3.3 Entropy as economic entropy 16](#_Toc408578052)

[5.3.4 Entropy as ABM entropy 16](#_Toc408578053)

[5.4 Units of entropy 16](#_Toc408578054)

[5.5 Entropy is not additive across systems, or subsystems 16](#_Toc408578055)

[5.6 Non-Conservation of entropy 16](#_Toc408578056)

[5.7 Entropy as the arrow of time 17](#_Toc408578057)

[5.8 Transformations of entropy 20](#_Toc408578058)

[5.9 Localization of entropy 20](#_Toc408578059)

[5.10 Grades of entropy (and energy?) 20](#_Toc408578060)

[6 Summary 21](#_Toc408578061)

[7 Material Added 150101 21](#_Toc408578062)

[8 Final thoughts 23](#_Toc408578063)

# Background

Since 2010 I have been on this journey to understand how economies function, and it has led me to a need to understand the roles of energy and entropy in self-organizing systems, such as ecosystems, and economies. For that, I need to clarify my understanding of the words “energy” and “entropy”. I was taught that energy flows, that energy can be stored, that energy can be consumed, or wasted. I was also taught that energy is conserved, and can be transformed. These are all words that contain an implicit metaphor between energy and a liquid-like substance. The theories that heat was such a liquid-like substance called ‘phlogiston’ or ‘caloric’ were disproven some time ago (circa 1850, see Refs A and B). They were replaced by the ‘mechanical theory of heat’. But those now-discredited theories were proposed, originally, by very smart and enquiring minds due to the aptness of the metaphors. The metaphors live on in our teachings, and in our everyday language, long after the theories have been disproven and discarded. I find, however, in reading about some of these old disproven theories that there are aspects to them that are hidden from view in my understanding of the modern theory.

For example, part of the Caloric Theory of Heat says that caloric was a self-repelling liquid. This was intended to explain why heat spreads out from an initial hot spot. The mechanical theory of heat explains this tendency in terms of diffusion and probabilities. My problem with that is this: it explains how the diffusion of heat happens, but it does little to resolve my desire to understand why it happens. Of course, the caloric theory does not help, either, but it does make the question obvious: “Why is it self-repelling?”

The answer, of course, is in the second law of thermodynamics, and an understanding of entropy. And, “there’s the rub”. Entropy is very difficult to understand. And the language we use around entropy is, again, replete with metaphors or similes referring to substances.

# Purpose

My goal in this note is to rethink what I know about energy and entropy and to try to bring together in one place both the facts that I know and the things that I suspect (hypothesize, speculate, or imagine) to be true, with more conviction in some cases, with less conviction in others.

There is a psychological phenomenon of which I have become aware of late. If we take some complicated difficult-to-understand process or abstract concept, and give it a one-word name, and then we learn how to use that word in a syntactically correct fashion in a sentence, then we think we have mastered understanding of the process or concept. But, all we have done is masked its complexity. We have done this with such common-place words as gravity, time, energy and, less so, entropy.

I am pretty sure that some things I say in this note are somehow not right, though I am doing my best to get it right. At the same time, I am intentionally being somewhat speculative. But, to truly rethink what I know about energy and entropy, I need to be speculative.

Figure 01 – Ptolemy’s Model of the Solar System



Orreries/Early Models/Ptolemy’s Model

# Energy

I often hear and use the words ‘energy consumption’ in the faux sense that energy is a substance that can be consumed and the disappears. In common language we talk about energy or electricity as if it is a substance that flows from here to there, that is transformed, that can be generated, that can be used, that can be consumed, or that can be localized in space. These are all metaphors that derive (probably) from an old belief in the substance caloric. As metaphors they imply much that is true, continue to work well, and we continue to use them long after the ‘caloric’ hypothesis has been well and truly discounted. But, as metaphors, they are not the whole story and are sometimes deceiving.

Those “substance” metaphors used to describe energy remind me of Ptolemy’s model of the solar system (e.g. Figure 01 above, and Ref C) in which epicycles upon epicycles described the locations of the planets. Whenever an error was discovered, the careful addition of a new epicycle to the model solved the problem. (See my “Orrery Gravitational Systems” and the Aristotelian and Ptolemaic models of the Universe at Ref D.) Ptolemy’s model of the solar system was very precise, as precise as need be, and in effective use by sailors for over 1,500 years before Brahe, Kepler and Newton overturned it. It was not wrong, but the model did not hold within itself the explanation for why it worked – just the ‘how’.

Similarly, the ‘substance’ metaphors of energy, and the accounting principles around them, have often showed that energy was not conserved, but whenever a problem was found, a new accounting principle was invented to make it work again. The conservation of energy law does not contain the answer within itself as to why energy does what it does, i.e. degrade. Is the answer hidden in the second law?

I am not saying that Ptolemy’s model was wrong. It was precise and useful; it was a kludge, but it worked very well in practice. In the same vein, I wonder if the ‘conservation of energy’ law is a very practical, precise and eminently useful kludge, but a kludge nevertheless.

Setting aside those metaphors of everyday usage is difficult, but the following is a collection of ideas that I believe are true but avoid those metaphors, I think.

## Energy as an accounting system applied to a biophysical system

Science has devised an array of formulae and conversion factors that can be used on a set of measurements of a biophysical system to produce a number we call the total energy of the biophysical system. Energy can be viewed as a number that characterizes certain aspects of that biophysical system or a part of that system that might be considered a biophysical sub-system, and that number is calculated using an arcane accounting system.

At page 155 of Ref E (my old first year physics text) it says that Equations 05 and 06 (below) “are essentially book-keeping statements”. In fact, I note that equation 06 is derived first (being equation 8-4a in the Ref), as they say “from first principles”, and equation 05 is merely the mathematical consequence of 06 (being equation 8-4b in the Ref).

## Common units

The accounting system exists in several versions, as I learned way back in the days of my sojourn at Queen’s – Imperial, MKS, cgs, SI, etc. Now, the accepted SI unit of measure of energy is Joules where one Joule = 1 kilogram meter per second2. So we now have the situation in which, for any given circumstance, there is a set of tools that can be used to compute the total energy in Joules. This tool box contains tools, each of which consists of an accountant’s computational triad of (measurement rules, formulae, conversion or scaling factors).

* The rules are for measurement-making in standards units with appropriate precision.
* The formulae are for combining measurements into a single number, again with appropriate precision.
* Usually the conversion factors are built into the formulae (e.g. the G of G(m1\*m2)/r), but they are in fact qualitatively different from the variables in the formulae. These factors MAKE the energy balance sheets balance.

For short, in this note I will denote such a triad as an energy accounting triad, or an “EA triad”, and when I get to discussions of entropy I will refer to them as SA triads, where the S is for entropy.

If you assume that energy is conserved, then you can find these factors. The big surprise is, you CAN assume that energy is conserved, and therefore you CAN assume that such factors exist and can be found, and you CAN make it work out. And, if it doesn’t work, you CAN find a new EA triad for this new situation, and the balance sheet works again.

In short, the existence of common units of measure for all types of energy is an artifact of the (apparently true) hypothesis that energy can be converted to common units if suitable scaling factors are used, and that is an artifact of the hypothesis that energy is conserved.

## Energy is additive across systems, or subsystems

An extensive property of matter is a property that can be measured as a number, and the measure is additive across many clumps of matter. Examples of extensive properties are mass, weight, and volume.

An intensive property of matter is a property that can be measured as a number, and the measure is not additive across many clumps of matter. Examples of intensive properties are density, and temperature.

Systems are somewhat divisible into non-overlapping subsystems, and the sum of the ‘total energy’ values for the subsystems is the total energy for the aggregate. This means that energy is an extensive property of matter, or of systems.

I view this as an hypothesis that is, so far, apparently true. But, built into this hypothesis is the idea that all of the energy of a sub-system can be localized within a sub-system. I am not sure that this is always true, or even ever true.

## Conservation of energy

If the formulae and conversion factors are used correctly, and if the measurements are accurate, and if the total energy in a system does not change over time, we say the system is closed with respect to energy, and we say energy is conserved. I note that the concept of ‘closed system’ implies that energy is localized inside the system, and prevented from ‘flowing out’.

In the experience of scientists, if the system being considered is inclusive enough (large enough), total energy is always conserved. I.e. the total energy in the universe is conserved, we (they) believe. This experience has been tested in small systems in labs, and in larger systems such as our solar system, and shown to be reliably true.

At the level of galactic systems it has proven to be false, and so fictions like ‘dark energy’ are invented, like new epicycles added to Ptolemy’s universe, to explain the failures. We have assumed it to be true in large systems, and this has given us great ability to talk about energy in the biosphere as a whole, in stars, and in galaxies and galactic clusters. That’s a BIG stretch for an assumption, but there has been little evidence that the assumption is invalid, until recently. It seems to be valid within the solar system. That’s pretty cool, and pretty valuable. But, beyond that? Hmm?

## Forms of energy

Common language has identified many ‘forms’ of energy that can ‘transform’ from one to the other. Forms of energy include: radiant energy, heat, kinetic energy, various forms of potential energy, electricity, fossilized fuels, thermal energy, hydro-electric energy, etc. But common language is sloppy. The rules of science are a little more precise. There is kinetic energy associated with moving masses; and there are four types of potential energy, each associated with the four fundamental forces which are gravitational forces, electro-magnetic forces, weak nuclear forces, and strong nuclear forces. As I understand it , these five fundamental forms of energy can exhibit themselves in a myriad of circumstances as translations, as rotations (kinetic), as configurations (potential) and as cyclic changes (potential/kinetic interchanges). For every such circumstance there is an EA triad that can be used to produce a number in Joules. So, I think ‘forms of energy’ merely refers to the particular EA triad used to compute the number.

## Transformations of energy

Transformation of energy happens when one form of energy is converted to another form. I can also say that energy is never ‘transformed’. This is a metaphor. What it really means is that the energy calculated using one EA triad is diminished and the energy calculated using another EA triad is increased by an equivalent amount.

At Ref E, in my old physics text book, on page 161 I get these two definitions:

*... the kinetic energy of a body is the work that a body can do by virtue of its motion.*

and

*The potential energy of a system of bodies is the work that the system of bodies can do by virtue of the relative position of its parts, that is, by virtue of its configuration.*

A little further down the same page it says:

*In general, potential energy is not assigned to any body separately but is considered a joint property of the system.*

If I consider gravitational potential energy between the Earth and an orbiting satellite, for example, it is stored in the gravitational force field found about and between the two objects. Now, a force field does not really exist. It is just a short-hand description of how a mass might be affected if it could be instantaneously transported to a particular spot, for every particular spot, and then knit in our imagination into an integrated field of such potential reactions. It’s a powerful fiction, but a fiction all the same. So, where or how is this not-yet-energy, this ‘potential’ energy stored?

The force FS on the satellite is:

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|  | Equ 01 |

where the negative sign shows attractive forces at work.

On page 407 of Ref E the formula for the potential energy of the satellite/Earth system is developed and is:

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|  | Equ 02 |

where, if r is infinite, US = 0. This is all due to the arbitrary assignment of a reference point of r = infinity in the development of the formula. It is noted that only the difference in such an energy calculation is of any real interest, as r changes from one finite value to another. So, we get the really curious situation in which the location of the energy is unknown and unknowable, other than to say it is a ‘property of the configuration of the system’, and is of totally arbitrary size, and for which only a change of size is of consequence. I recall, as an undergraduate, struggling with this concept, and, finally, giving up.

So, it seems we can ‘transform’ changes of potential energy into changes in kinetic energy, and we can ‘transform’ changes of kinetic energy into changes in potential energy. But these transformations are constrained by the configuration of the system at the time of transformation. As the satellite orbits the Earth, it cannot arbitrarily change all potential energy to kinetic energy, or, in fact, can it make any arbitrary transformations at all. The effect of the constraint is the apparent conservation of the total energy. BUT, and that’s a big but, the total energy is an arbitrary number, being the sum of many numbers of arbitrary size. I can reword the conservation of energy law to say something like:

*Once we decide, in an entirely arbitrary fashion, using some set of EA triads, exactly how much energy is in a system, then in all energy transformations that happen thereafter, the computed total energy of the system using that same set of EA triads will always be the same.*

The total amount of energy in a system is a total fiction if any potential energy is included. And potential energy must always be included. So, only changes in energy are of consequence.

Let Si be a sub-system of the super-system S.

Let PEi be the associated potential energy of sub-system i.

Let KEj be the associated kinetic energy of element Bj of the super-system S.

Then the supersystem S is the union of all of the bodies of mass Bj it contains:

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|  | Equ 03 |

Also, the super-system S is the union of all of the non-overlapping sub-systems Si it contains:

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|  | Equ 04 |

I usually think of the conservation of energy law looking like this:

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|  | Equ 05 |

But, the real conservation of energy law should look like this:

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| --- | --- |
|  | Equ 06 |

In other words, the sum of the changes in potential energies plus the sum of changes in kinetic energy are equal. Mathematically, this is the same as saying the total energy is conserved, but conceptually, for me at least, it’s an entirely different kettle of fish. This is saying that the sum of changes for all EA triads is equal to zero. Or, the sum of all energy transformations is zero. Thus, the law of the conservation of mass is truly an accounting principle, and says little about the nature of energy, but rather, in the substance-implying form I normally understand it (Equ 05), masks the truly abstract nature of energy.

## Localization of energy and the issue of system scale

We talk about energy as if it exists in one place, one locale. This is a simplification of a difficult concept:

### Localization of kinetic energy

Kinetic energy, in all of its various forms, seems to be localized at or near the centre of mass of a moving mass. At first glance, I have no problem with that. But, on thinking about it for a while, I recalled that kinetic energy is dependent on motion, and motion must be defined with respect to an inertial frame of reference. Abusing (somewhat) Einstein’s famous story about observers on trains, a juggler on a moving train would compute the kinetic energy of his juggled objects differently than an observer outside of the train would. To get the same answer, both observers would have to use the same inertial frame of reference.

So, the calculation of the kinetic energy of any object is dependent on the inertial frame of reference used. On the other hand, none of us exist in an inertial frame of reference. Due to our various gravitationally induced orbits, we are only in approximately inertial frames, at best. If you believe the Big Bang hypothesis, then there is a centre of mass of the universe, and, I suppose, that would provide a point of origin, and inertial frame, that would give kinetic energy absolute magnitude and meaning. The Big Bang hypothesis is no longer disputed by scientists, so, I suppose that’s a reasonable assumption.

So, the juggled objects may have immense kinetic energy, or very little. We cannot really tell, until we find the centre of mass of the universe. Must we settle for approximate estimates? It seems we are happy to do so. And yet, it seems to work out ok.

### Localization of potential energy

Potential energy, on the other hand, is not localized to a centre of mass. It seems to be due to the position of two objects within a force field that they generate between themselves, and is not localized to either object, or to their common centre of mass.

My body, for example, has gravitational potential energy with respect to the Earth, the Sun, or the centre of the galaxy, but I cannot say where that energy resides, or how big it is, or if it resides anywhere at all. With three different reference points I can arbitrarily give it three hugely different values. All I can do is define a large enough super-system to be able to apply the accounting principles to the sub elements of the super-system and compute a total energy for the super-system, or for components of it.

On the other hand, I can easily understand that the chemical potential energy of my body is located within my body. The apparent localization of potential energy is purely a function of the spatial dimensions of the super-system needed to compute total energy such that conservation of energy is observed. Too small of a super-system and the concept of potential energy loses meaning, and total energy is not conserved. Too large of a super-system and the numbers are staggeringly large, but strangely inconsequential.

For example, if I compute my gravitational potential energy with respect to the galactic core, I expect the number would be large. But, for any energy transformation that affects me from day to day, the change in such potential would be so small as to be insignificant.

## Grades of energy

Energy may be conserved, but it tends to degrade over time. This is really tricky, but I think this is key to understanding why and how things decay, and how they self-organize.

The best example of degrading energy is the case of a closed box of idealized gas molecules. If the energy is clumpy (some patches of high temperature gas and some of low temperature gas) then it will tend to become less clumpy and more smoothly distributed. We say the energy ‘does work’ as this happens. It is a non-reversible process. Energy is conserved, but the ability of that energy to do more work is lost forever. In the process of becoming smoothly distributed the contained energy degrades in quality.

In this gas-in-a-box example, I am talking about spatial clumpy-ness. Suppose the gas is at temperature Tg. This spatial clumpy-ness can be created by putting an energy hot spot at one end and an energy cold spot at the other. Suppose we open the lid and place in a beaker of warm liquid at one end (Tw=Tg-10) and a beaker of ice at the other (Tc=Tg-20). Heat will ‘flow’ initially from the gas into both the liquid and the solid. But eventually, heat will ‘flow’ from the liquid to the gas and into the ice until some of the liquid evaporates, the ice melts, and there is an equilibrium state of liquid and gas remaining in the box. In this instance, heat really does behave somewhat similar to a self-repellant liquid. Left to its own devices, energy spreads around, spatially, until some thermodynamic equilibrium is established. Once established, that state of equilibrium is stable forever. The energy ‘does work’ to achieve the equilibrium, and then it stalls, and can do no more in that situation.

This makes sense to me, but it has the implicit assumption of behaviour like the caloric theory would describe. Can I talk about energy being clumpy? Can I talk about energy flowing. That implies localizations of more dense energy and less dense energy. Maybe what I should be talking about is spatial partitions (a breaking into sub-systems) of the volume of the box, and EA triads being appropriately applied to the partitions, and summed. Even as I think of partitions I am tempted to revert to talk of fluxes across boundaries, and that is the old energy-as-substance metaphor re-asserting itself again. It is really insidious, and potent.

There is another kind of ‘clumpy-ness’ that may not be spatial, but rather logical. Suppose we have that insulated box, with a tiny nozzle or hole at one end, and the ability to allow entry of only those idealized atoms that have a speed v. We fill the box to contain N atoms, where N is Avogadro’s number, or some similar very large number. If we look at a distribution of entry speeds of atoms, they would be ‘clumped’ at speed v. However, if we look at the distribution of actual speeds at time T, some time after the last atom is introduced into the box, I am sure it would look like the Maxwell distribution of speeds of ideal gas molecules in a box. This is the kind of ‘clumpy-ness’ that Boltzmann studied, and that lead to his formulation of entropy. Then the clumpy-ness is in terms of classes of atoms based on speed, rather than position.

There is a curious difference between these two examples. In the first, in which there is a spatial partition, energy tends to smooth all of the bumps. In the second, in which there is an energy partition, energy tends to create a particular kind of bump. In both cases, the final distribution represents a maximal level of entropy for that partition.

Whether we partition the atoms by spatial characteristics, or by speed, or by energy, energy has a tendency to change things. We call that business of making changes ‘doing work’. As energy does work, it loses the ability to do further work. It can only do so much work. But, as I saw in the quotes from Halliday and Resnick, our understanding of energy is very closely bound to this concept of ‘doing work’.

The simple answer is, the energy of a system degrades as it does work, and as it degrades:

* The sum of the EA triads stays constant; but
* One measure of its clumpy-ness, the entropy of the system, rises.

My model of the PMM in ModEco lead me to Dr Yakovenko’s work. (E.g. Refs F, G & H). In follow-on studies I developed an approach for computing the entropy of histograms. (See Refs I through N.) In my study of entropy in histograms (Especially Ref I, a draft paper.) I developed the concept of ‘entropic index’ Sindex where Sindex = Sactual / Smaximum. I would hypothesize that a similar concept can be applied to thermodynamic entropy. Then the thermodynamic grade of energy can be defined as GE=(1- Sindex).

I really like this concept of the grade of energy, but I am having difficulty, working it out with ideas like emergy, exergy, or Gibb’s Free Energy. That would be, perhaps, the topic of another NTF.

# Entropy

Similarly, I am using the words ‘entropy production’ in the faux sense that entropy is a substance that can be ‘produced’. Entropy is more like an index that can be calculated from a variety or suite of measurements. Entropy is ‘produced’ when that index rises. I understand far less about entropy than I do about energy, but I strongly believe that it is the second half of the puzzle, of which energy is merely the first half.

This is, perhaps, reflected in the fact that energy is the topic of the first law of thermodynamics, and entropy is the topic of the second law. Both of those laws are provable only in closed systems, and have been extended to hypothetically work in open systems. The work of people like Odum and Hall has gone a very long way towards increasing the understanding of energy in open systems, but, for me, at least, an understanding of entropy is far more elusive. The system used to compute thermodynamic entropy is arcane, to say the least, and even the most simple application of the concept is beyond the understanding of anybody that has not studied physics at the graduate level, it seems. Unfortunately, that excludes me, so I must work with an environment I understand, which is histograms and ABMs. Here is my list of characteristics of entropy drawn therefrom.

## Entropy as an accounting system

Entropy can be viewed as a number that characterizes certain aspects of a system, and that number is calculated using an accounting system that combines measurements using formulae. It seems we have another system of Entropy Accounting triads (SA triads, where the ‘S’ stands for entropy), but it is in a very primitive form, to date. I would hypothesize that this system can be dramatically expanded, and would be extremely insightful, if we remove the constraint that binds it to ‘thermodynamics’.

## Entropy as an average

There are several relevant formulae that can be used to calculate entropy. Three are of direct interest to me for my goal of computing entropy within ABMs:

* The combinatorial formula;
* Surprisals using Stirling’s Approximation for ln(x!); and
* Surprisals using the GammaLn approximation for ln(x!).

For each, suppose there is a set of K numbers ni that sum to N, where i∈[1,K] is an index number.

The combinatorial formula for S is calculated (See Ref K) as:

|  |  |
| --- | --- |
|  | Equ 07 |

where f is a scaling factor.

However, N! is difficult to compute for large N. So there are other approaches to calculating entropy that try to get around this difficulty. Stirling’s approximation for ln(N!) is:

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| --- | --- |
| -N | Equ 08 |

At Ref L I have worked out the steps that use equation 8 to transform equation 7 to come up with the formula for entropy proposed by Boltzmann for thermodynamic entropy, and also proposed by Shannon for informational entropy. It looks like this, and the terms being summed are called, by informational theorists, surprisals:

|  |  |
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|  | Equ 09 |

But, after discussions with Dr Yakovenko, I have come across two problems with the formula based on Stirling’s approximation:

* Stirling’s approximation is great for all values of N∈[170, ∞], but less accurate for the interval N∈[1,170]. Unfortunately, this is exactly the interval in which most agent based models function, with less than 100 agents, so computations using Stirling’s approximation may be suspect.
* The factorial formula is discrete and only defined on positive integers, i.e. on the natural numbers. A calculation of entropic index (- Sactual/Smax) requires that N! accept non-integral arguments N.

There is a continuous function called the Gamma function where ‘gamma’ is the Greek letter ‘Γ’. Γ(N+1)=N!, if N is a natural number. There is also a function called the GammaLn() function where GammaLn(N+1)=ln(Γ(N+1))=ln(N!), again, if N is a natural number. But GammaLn is also defined for real numbers between the naturals, and I need that for my ‘entropic index’ as defined at Ref I and developed further at Ref N. Finally, there is a formula called ‘Lanczos approximation for GammaLn(). If I use that to compute entropy for ABMs in which the number of agents is between 1 and 100 I get very accurate results. Here is the exact formula for entropy using GammaLn(), which can be computed in practice using Lanczos’ approximation for ln(Γ(N+1)), as developed at Ref M:

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| --- | --- |
|  | Equ 10 |

There are simple circumstances in which you can calculate entropy, and more complicated circumstances, each with more/different layers of interpretation. I see at least three such levels.

### Entropy of a Histogram –

At the most basic simple layer, we can calculate the entropy of any histogram. This concept is developed at Ref I.

* Let xi be any number selected from the domain D=[xmin, xmax].
* Let X be a set of A such numbers xi.
* Divide D into K contiguous non-overlapping intervals that tile D.
* Use that ‘partition’ of D to form a histogram of counts nj of elements of X falling within bin j.
* Using the notation, developed at Ref I, where h(n1, ... ,nK) is the histogram, we can use any of the above three formulae to compute the entropy of the histogram.

Based on that, I hypothesize that, for any measureable characteristic for which you can form a **histogram**, you can calculate a type of average that we can call the entropy of that histogram. We can view a histogram as a partition of the domain of the characteristic.

### Entropy of a static system –

To be useful, though, the numbers xi must have some restrictions:

* xi must measure some **meaningful** characteristic of a system. I am uncertain about this, but I think they must, in fact, measure meaningful characteristics of a set of A similar objects within a system (atoms, people, agents, planets, etc.), one reading for each of the A objects.
* xi must be an **extensive** characteristic, i.e. one for which the measurement of the characteristic at the level of the whole system is equal to the sum of the measurements of each of the parts. Examples would be atoms per cc of volume, energy per atom, mass per person, wealth per person.

At this second level of complexity, we can apply any of the three formulae to a system and compute a number we can call entropy of the system associated with a **histogram** of that particular **meaningful** and **extensive** characteristic.

### Entropy of a dynamic system –

I am not sure that entropy of a static system, however, would be of interest to anybody. The real interest is in dynamic systems. To be useful in this context we would need the following additional restrictions:

* xi must measure a **conserved** characteristic of a system, i.e. conserved over time.

At this third level of complexity, we can apply any of the three formulae to a system and compute a number we can call entropy of the system associated with a **histogram** of that particular **meaningful**, **extensive** and **conserved** characteristic.

Entropy, then, in its most general terms, is a kind of weighted average measure (according to Stirling’s approximation) of the probability field that can be associated with any partition of the domain of any extensive variable. (My words.)

[ I have just discovered that this view seems to be in line with the views developed in the study of “Ergodic Systems”. I really need to explore that field of studies more. !!! )

## Forms of Entropy

Like energy, it appears that entropy comes in several forms. There is, of course, the biophysical form first discovered by Clausius and Boltzmann, called thermodynamic entropy. Then there is the form discovered by Shannon called informational entropy. Then there are variations of thermodynamic entropy that come out of Ergodic theory that are highly mathematical, called Kolmogorov-Sinai entropy and Tsallis entropy. For my purposes, I want to hypothesize the existence of economic and ABM entropy as additional forms.

### Entropy as thermodynamic entropy

Energy is an extensive variable. Energy can be assigned to elements of a system, and the sum of the energy of all of the elements is equal to the total energy. We have designed the accounting system of EA triads to ensure this is always true. (The real source of amazement is that we can do this.)

As energy does its work, thermodynamic energy rises. If entropy can be viewed as a weighted average of the natural logarithm of probabilities, then thermodynamic entropy can be viewed as weighted average of the natural logarithm of probabilities of a given type of clumpy-ness of energy. That clumpy-ness is normally in terms of a partition of energy. Can it also be in terms of a partition by spatial location? If we think of thermodynamic entropy as just the sum of surprisals using Stirling’s approximation for the combinatorial formula at the heart of entropy, then entropy is the weighted average of the ln(1/P(xi)) where P(xi) is the probability that that clumpy-ness of type Xi is a current description of the system. Here, ‘clumpy-ness’ means ‘shape of histogram on the partition’.

Can thermodynamic entropy be defined in several different ways? Interesting question!

### Entropy as informational entropy

The measure ‘error bits in a message’ is an extensive variable. Error bits per kilobyte is a partition of the message that has clumpy-ness that can be expressed as a histogram. Informational entropy can then be viewed as a weighted average of the natural logarithm of the probability of a given type of clumpy-ness of error bits.

### Entropy as economic entropy

The measure of wealth (in dollars) is an extensive variable. Wealth per person is a suitable partition of wealth that creates a kind of clumpy-ness that can be expressed as a histogram. Economic entropy can then be viewed as a weighted average of a the natural logarithm of the probability of a given type of clumpy-ness of wealth.

### Entropy as ABM entropy

Within an agent based model, find any conserved extensive variable, construct a histogram (a partition) and apply one of the three formulae, and you get a measure of something I can call ABM entropy, for that extensive variable, for that partition. The really cool thing is, if the ABM is a self-organizing closed system, and you manage the partition carefully, then that entropy rises to a maximal value, just the way the second law of thermodynamics says thermodynamic entropy will rise.

I do need to figure out exactly what I mean by “manage the partition carefully”. In progress!

## Units of entropy

Thermodynamic entropy has units of Joules per degree Kelvin, which reduces to Joules per Joule, which reduces to being a dimensionless number. I hypothesize that all entropy of all types is measured in dimensionless numbers.

## Entropy is not additive across systems, or subsystems

Systems that are divisible into non-overlapping subsystems do not have the property that the entropy of each subsystem can be added to give you the entropy of the whole. The sum of the parts does not equal the whole. Entropy is calculated on extensive properties, but, it is not itself an extensive property. It is an intensive property, like density, or temperature.

## Non-Conservation of entropy

The second law of thermodynamics says that thermodynamic entropy in closed systems (i.e. closed with respect to the extensive property energy) rises to a maximal value. It also implies that, within a closed system, entropy does not fall. Entropy is preserved (*does not* fall) but not conserved (*probably* rises). [Note, I have chosen the italicized words very carefully. It can fall, but it doesn’t. It will probably rise, but it doesn’t have to.] S(future) >= S(present).

## Entropy as the arrow of time

The particular circumstances of the non-conservation of entropy implies that time has an arrow that points forwards, *most probably*. So, this is an important thing for me to understand. The following concepts are similar to those developed at Refs I through N for use with ABM entropy.

**The state space** – Imagine an N-dimensional Cartesian space for which each dimension corresponds to one of the N state variables of a system, each dimension having extent and density (i.e. integral or real) determined by the properties of the associated state variable. Call the state space XSPACE. Let X be a point in that state space, being an ordered N-tuple. X∈ XSPACE.

**Transition sphere of states** – Also, imagine that for each state X, a set YSPACE of next possible states that can be achieved, say, exactly one second later, can be assembled (forming a subset that is potentially infinite in number but limited in extent, that I see as spherical, or globular, but fuzzy, in some sense in the state space XSPACE).

**Transition pairs of states within transition sphere** – Also imaging that we can construct an associated set of “transition pairs” of states (X,Yi) for each Yi.

**Probability of transition function** – A probability function P() could possibly be defined such that the probability of transition from state X to state Yi∈ YSPACE can be calculated. Denote such a probability as P(X⇒Yi). The sum over all transition pairs Σi(P(X⇒Yi)) = 1.

**Transition vector** – Finally, imagine that the transition pair (X⇒Yi) is associated with a vector Vi(X⇒Yi) in the transition sphere.

**Weighted average transition vector** – Compute the weighted average transition vector for the state X as:

|  |  |
| --- | --- |
|  | Equ 11 |

**Entropy of a state** – In the state space of any thermodynamic system, thermodynamic entropy SX can be calculated for each and every point X in the state space. This means, e.g. the energies of all component objects in the system (e.g. atoms or sub-systems of atoms) have been measured, tabulated into a partition, and the entropy computed therefrom.

I can now, in my mind, at least, construct a series of functions of X, or fields (sets of numbers or vectors) each element of which is derived from some X.

**Energy field** – For example, I can define a set of numbers EFIELD where EX is the energy of state X. EX ∈ EFIELD  is computed using the appropriate EA triads in each case of X. If this is a closed system with respect to energy, then we would expect EX to be a constant for all X∈XSPACE.

**Entropy field** – Based on XSPACE, there is a field SFIELD where SX is the thermodynamic entropy of state X. SX ∈ SFIELD  is computed using the appropriate SA triad in each case of X. I don’t have the math skills to prove it, or the time to undertake such proof, but I hypothesize that SX is a smooth continuous function of X. So, the partial derivatives of SX with respect to some state variables is non-zero for most X, maybe. SX may have a gradient in the neighborhood of each X∈XSPACE.

**Weighted Average Transition Vector field** – Vx forms a closely associated vector field VFIELD defined on X.

* I hypothesize that this field is smooth and continuous around each X∈XSPACE.
* I hypothesize it has a gradient in the neighborhood of each X∈XSPACE.

**‘Probability of exhibition’ field** – Now, this idea and the next idea get really tricky and I am not nearly so convinced that this is right. But it is a very enticing set of ideas. Let P(X) be the probability that the system will be in state X. This functions at two levels and I don’t fully understand the implications of the two.

At one level, P(X) might the probability that an atom has energy level X. Then X represents a bin of a histogram, a part of a partition, a so-called macro-state. X then is a state of a component (or atom or agent) of the system, and not a state of the system itself.

At the other level, P(X) might be the probability that the system has configuration X. Then X represents a state of all bins of the histogram or partition. In this section of this note I take it to have the second meaning, though in the formulae for entropy it has the first meaning.

But, what does that mean? Is P(X) the probability that, if an observer happens across the system in a random state, it is in state X? Doesn’t prior history have a huge say in such probabilities? Since most closed systems are at equilibrium, P(X) would be high for states in the limit set, and close to zero for the rest. It seems to me that this understanding of P(X) is fundamental to Boltzmann’s understanding of entropy. It says, all things being equal, in how many indistinguishable ways can you configure the system to get state X, without regard to past history? So, on the face of it, P(X) must be independent of SX and EX, but totally dependent on history, up to the point that the system enters its limit cycle.

History and contingency MUST play a role in the actual momentary state X that a system exhibits prior to entering its limit cycle.

That being said, I can imagine a function P(X) = the probability that the system is in state X∈XSPACE, for which there is a field PFIELD that is *dependent* on history and contingency, which is smooth and continuous, and which has a gradient around X.

**‘Probability of transition’ field** – I developed this concept informally while writing Ref I. But I need to think it through a little more formally, so I will do that here.

Assumption A: I assumed above that one could compute Pi(X⇒Yi) where X⇒Yi is a transition vector associated with a transition pair (X, Yi), and I used that to find the most probable transition vector and called that VX. Let me denote the end point of the vector VX as the point Z.

Assumption B: I also assume that if I can compute P(A⇒B) for transition pair (A, B), then I can compute P(B⇒A). this is a small extension of assumption A.

There is no reason to believe that P(X⇒Z) = P(Z⇒X). I.e. there is no reason to believe that the probability of transition from state X to state Z is the same as the probability of transition from state Z to state X. The probability of transition is asymmetric within a transition pair.

Assumption C: Let me further assume that Z∈Y, i.e. the state Z is an element of the transition ball of X (a reasonable assumption if all of the state variables are real vice integral).

Putting these assumptions together, this means that P(VX) can be computed and P(-VX) can also be computed. Define TFIELD as a function of X∈XSPACE to P(VX)-P(-VX). Since P(VX) >= P(-VX) by definition, TFIELD is a positive probability field.

I hypothesize that TFIELD forms a smooth and continuous field, that it has gradient around X∈XSPACE.

I don’t know why I can imagine this to be true, with all of its layers of assumption, but have difficulty with the P(X) issue. Wouldn’t P(X⇒Yi) be related to P(Yi)-P(X) in some fashion. If P(Yi) is greater than P(X), it stands to reason that the system will tend to migrate towards Yi from X. This would imply that P(X) has meaning.

So, here’s the cherry on the pudding. I hypothesize that the gradient of SFIELD is associated with the gradient of VFIELD and the gradient of TFIELD in the sense that when the partial derivative of SFIELD is positive, for any state variable, then the partial derivatives for VFIELD and TFIELD will also be positive. In other words the most probable direction of change of the system corresponds to the fastest rising entropy and the most probable trajectory.

I wonder whether this is just another way for me to see and understand the state-space diagrams in Strogatz’ book. (See Ref O.)

There is one further thing about which to remind myself, in this compendium of ideas, before I move on. At Ref I, I noted that P(VX) >> P(-VX), except when X is part of the limit cycle of the system. The probability of transition is dramatically asymmetric for undoing any autonomous system change prior to reaching equilibrium. This means time very probably moves forwards. This is one of two mechanisms that creates the apparent arrow of time. (The other is a kind of ratcheting due to edge effects that I suspect does not apply outside of ABMs.) I also noted that ΔSX > 0 for the transition VX, except when X∈limit cycle in which case ΔSX ≈ 0. This means that SX must rise over time, and so a rise of SX is predictable.

Then, in my view, rising entropy does not create the arrow of time, so much as it is created by the same asymmetric probability field that creates the arrow of time. The apparently autonomous work done by energy, the apparent rising of entropy, and the apparent arrow of time are all side effects of the probability fields doing what they are wont to do, which is probably the most probable thing.

In other words, I think that the asymmetric probabilities and the gradients in these probability fields are the motive forces behind changes in dynamic systems.

## Transformations of entropy

Science has only identified two ‘forms’ of entropy, that I am aware of: thermodynamic entropy and informational entropy. Mathematicians have generalized these, I am now discovering, to Kolmogorov-Sinai entropy and Tsallis entropy, both extensions of the idea of thermodynamic entropy. At Ref F Yakovenko’s co-authors argue for the equivalence of thermodynamic and informational entropy, with suitable conversion factors proposed. At Ref H, on the other hand, Yakovenko goes further and implies that entropy of a different kind may be active in purely economic data, implying the existence of a fundamental concept of entropy apart from its instantiation in thermodynamic and informational systems. I suspect that they are not the same, but merely co-dependent expressions of a deeper phenomenon. I am beginning to suspect that the mathematical work done on Ergodic Systems, and the study of Kolmogorov-Sinai entropy might be the deepest definition of entropy, that transcends thermodynamics and information theory.

That being said, I am not aware of any theory of transformations of entropy, other than the Ref transformation from energy to entropy. This seems like yet another EA triad proposed in an attempt to add another epicycle to the conservation of energy accounting system.

I would hypothesize that, once entropy is more deeply understood, such conversions of types of entropy, or, at least comparisons or inclusions of them, will become more common.

## Localization of entropy

While entropy is not extensive, a kind of localized computation of thermodynamic entropy makes sense to me. Within a subset of a partition (a subsystem of the system) thermodynamic entropy calculations may change from moment to moment. Consider a dynamic thermodynamic system. If the system, or a subsystem, is not closed, then such calculation of localized thermodynamic entropy might fall over time. The second law of thermodynamics is not abrogated as long as there is some larger closed and enclosing super-system, of which this system is a part, and for which thermodynamic entropy stays constant or rises.

This argument only makes sense if I allow for some sort of localization of entropy, similar to the localization of potential energy in the configuration of a sub-system.

## Grades of entropy (and energy?)

At Ref I, I define the “entropic index” as Ie = S/Smax, where Ie ∈ [0, 1]. My definition is with respect to economic entropy, or ABM entropy. If we extend that definition to apply to thermodynamic entropy as well, then we can go a step further. At Ref J, I define the grade of energy to be Ge = ( 1 – Ie ). where Ge ∈ [0, 1]. When the entropy of a thermodynamic system is “produced” (i.e. it rises) then the entropic index rises and the grade of the contained energy falls. It is degraded. If such a connection exists between entropic index (in a thermodynamic system) and grade of energy (in that same system), then we see there is an absolute and direct connection between entropy production and energy consumption or degradation. Then the entropic index is the grade of entropy, and indirectly a measure of the grade of the conserved extensive characteristic on which it is based.

# Summary

In summary, we use metaphors for energy and entropy that are based on the similarity to physical substances. These metaphors include concepts of localization, or flow from locality to locality, of quality and grade, of transformation from type to type. Such metaphors capture many aspects of energy and entropy, but misrepresent others (such as the intensive nature of entropy, or the lack of localization of potential energy). These metaphors are:

* eminently attractive because they translate very abstract ideas (sets of accounting identities with partitioning rules, averaging formulae and conversion factors) into concrete terms that we understand from everyday experience (flows of liquids);
* extremely powerful because they have advanced our understanding of dynamic systems a very long way in the past 200 years; and
* nevertheless, somewhat misleading in the details, because they may be masking things that we do not currently understand, and need to understand.

I strongly suspect that energy and entropy do not exist physically, but only as accounting principles. We see effects. We posit a flow of something, or a production of something. We give it a name that is satisfying and suggests that something exists. But nothing (no physical thing) exists other than the effects we observe. To be as blunt as possible, neither energy nor entropy exists, except as practical accounting tools in the minds of scientists and engineers. They are shades of Ptolemy’s epicycles, having practical use, but no real meaning. For this reason, if I really want to understand the sustainability of economic systems, I think I must always keep that in mind.

# Material Added 150101

In searching for an understanding of total energy in a three-body system, such as is found in Atwood’s Machine, I came across the website for the University at Buffalo, and the notes for the 4th year course at Ref P.

It has an equation for the total gravitational energy that looks like this:

|  |  |
| --- | --- |
|  | Equ 12 |

This reminds me of two things I had forgotten about the accounting system used for energy.

**First:** The negative sign on the potential energy is due to the peculiar decision to arbitrarily define zero potential energy as the potential when the two bodies m1 and m2 are an infinite distance r∞ from each other. Consider the two bodies. Suppose I replace with r∞ when they are infinitely distant, and r when they are not, then the change in potential energy P as they move from r to r∞:

|  |  |
| --- | --- |
|  | Equ 13 |

Gravity is an attractive force, and the move from r to r∞ is against gravity, so the potential energy is negative. I have worked these concepts into the above document as part of the revisions.

But, still it troubles me that it is negative in equation 12.

**Second**: Kinetic energy is defined in terms of motion with respect to the inertial frame of reference, the centre of mass, of the system. So, kinetic energy of any mass is not absolute without reference to the system of which it is a part, but is only meaningful in reference to the system in which it exists and moves. Hmmm! And. Hmmm, again!

Soooo!! This means that we cannot precisely compute the total kinetic energy in any system without reference to the super-system of which it is a part. I suppose, for example, that on a second-by-second basis we can consider the centre of mass (COM) of the Earth to provide us with an inertial frame of reference, allowing us to compute the total energy in, say, a lab experiment. But, we rotate every 24 hours, and so, on any period of time longer than a few seconds, there may be terms due to the acceleration resulting therefrom that throw such calculations into error.

More to the point, kinetic energy is only meaningful when computed in reference to the COM of the universe. Where is that?? So, both potential energy and kinetic energy are somewhat localized in masses and sub-systems, but not totally so. Any assumption to the contrary is ignoring a teeny tiny effect. Absolute laws, such as the conservation of energy law, can only be proven in the super-super-systems in which clusters of galaxies play. There is always some effect of distant galaxies that add to the total energy of any sub-system we contemplate, both as potential energy and as kinetic energy. There are wheels within wheels!

Yet, it seems that kinetic energy is much less arbitrary in magnitude than potential energy. What does equation 12 look like when converted to ΔE? Suppose there is some small almost negligible change in the velocities and positions of the masses. ΔE = Eafter – Ebefore. Denote the vi of the ‘before’ state with ui, and the ri of the ‘before’ state with qi.

|  |  |
| --- | --- |
|  | Equ 14 |

|  |  |
| --- | --- |
|  | Equ 15 |

I can collect like terms in the kinetic part. I can replace the references to the COM in the potential part with vectors from mass i to mass j. I.e. the denominators become rij and qij.

|  |  |
| --- | --- |
|  | Equ 16 |

Finding some common denominators for pairs of terms within the sums for potential energy:

|  |  |
| --- | --- |
|  | Equ 17 |

For each (i, j) pair of terms, within the second sum, I can pull out the common factors:

|  |  |
| --- | --- |
|  | Equ 17 |

The part for potential energy reflects the fact that gravity is a conservative force, and the potential energy is entirely dependent on position, and not on path. In path, position is reduced to scalar distances. The kinetic part is dependent on speeds.

Equation 17 is the ‘book-keepers’ version of the law of conservation of energy in an idealized gravitational system, ignoring tidal forces, tectonics, and internal reverberations and waves. This idea has been incorporated into the above note as part of the revision process.

I still don’t really understand the significance of the negative sign in front of the G. for example, when I compare equations 06 and 17. I could remove the negative sign in equation 17 by simply reversing the order of subtraction inside the sum, but, to what end? I would still not understand it.

# Final thoughts

I have a final few thoughts on all of this (at least, for now).

* If energy is NOT something that flows, if its magnitude is arbitrary and/or specific to the sub-system in which it is found, what, exactly, is it that a conservative force conserves?
	+ My answer: It conserves an accountable characteristic of the system as the system changes dynamically, managing a continual trade-off between positions and velocities, constraining the system to accept only specific exact trade-offs in positions and velocities. (Not a great answer!)
	+ This sounds like Newton’s laws of motion plus the laws of thermodynamcis. That’s where we started, I believe. It’s a tautological answer.
* What, exactly, happens when energy ‘degrades’?
	+ I could say it becomes more localized, but that’s reverting back to the language of ‘energy as substance’, however true is seems to be.
	+ I could say kinetic energy becomes less coherent (‘all co-situated atoms at same velocity’ degrades to ‘all co-situated atoms at random velocities in both magnitude and direction’). This also seems to be true.
	+ I could say potential energy cycles more quickly to kinetic and back to potential. Planets orbit at a slow pace but tidal forces convert that to heat, associated with vibrating atoms, and speeding atoms that careen off of each other in a potential/kinetic frenzy of conversions. This seems to be true, but with a clear exception.
	+ Consideration of ultra-violet light and infra-red light seems to give the lie to that last idea. UV light and visible light is high grade, able to do a lot of work such as create sugars and starches via photosynthesis, whereas IR light seems unable to do much other that heat a cold object, or fly off into space. UV cycles between electric potential and magnetic potential quickly but IR cycles slowly. Maybe that’s because the cycle is potential to potential, rather than potential to kinetic.
* There are several kinds of duality associated with energy:
	+ Like the wave/particle duality of light, there seems to be a potential/kinetic duality of energy.
	+ Energy has a local/non-local duality. When localized (as EM waves, as kinetic energy of large masses of coherent matter) it can move (must move). When non-localized (as potential energy in a stable non-degrading or slowly degrading state) it stays still. When in incoherent matter cycling quickly between kinetic and potential forms, it is localized.
	+ Again, like the wave/particle duality of light, there seems to be a liquid/gas duality of energy. By this I mean, the language of liquids applies when we talk of energy flows, but the language of expanding gases applies when we talk of heat diffusing or spreading. Perhaps it acquires some of the characteristics of the material in which it localizes.
	+ But, energy embodied in light steps out of all of those metaphors.
* Entropy also has a dual nature, or even multiplicity of natures.
	+ Thermodynamic entropy can appear to be localized, in closed systems or in flows of mass. Or, it can appear to be non-local within a system.
	+ I suspect that, whenever a system contains countable objects, and the reproduction of those objects is constrained in some fashion, a form of entropy will exist, and its tendency will be to rise over time. So, we get thermodynamic entropy, informational entropy, economic entropy (as implied by Yakovenko’s work in econophysics), and ABM entropy (as implied by Yakovenko’s capital exchange models).
* Energy has a tendency, or even necessity, to degrade, a process that never reverses. And entropy has a tendency, or even necessity, to increase.
	+ When the energy associated with a system degrades, the thermodynamic entropy associated with the system rises.
	+ In fact, if the MPP and/or MEPP are true, then energy must degrade (and entropy increase) at the maximum possible rate, and there is some sort of selection process whereby the universe, or parts of it, self-organize to achieve higher rates of degradation/increase. The evidence for this is the mere existence of self-organising systems, such as living organisms, ecosystems, societies, or economies.
	+ Newton’s laws of motion and the laws of conservation of mass and energy are merely the constraints on this need for energy to degrade, and the need for entropy to increase.
	+ What drives energy to degrade, and entropy to rise in tandem?
	+ My answer: probabilities. The universe is slowly moving towards an inevitable state of heat death, moving from a somewhat improbable configuration to a vastly more probable configuration, taking gazillions of steps, but in a clockworks fashion, just one highly probable step after another. The mathematics of counting combinations and permutations drives the universe towards its future. . . . This is the source of the ‘arrow of time’, in every circumstance in which it can be applied.

What a great puzzle?!