Doubling Down on Debt: Limited Liability as a Financial Friction[‡]

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We show that limited liability and pre-existing debt lead to heterogeneous investment distortions, where high leverage firms tend to overinvest while low leverage firms tend to underinvest. In our model, limited liability allows equity holders to change default timing and, thus, "double-sell" cash flows already promised to existing bondholders. We characterize this new financial friction and differentiate it from traditional sources of dilution. With repeated investment opportunities, creditors anticipate equity holders' incentives to change default timing, increasing the cost of funds and reducing investment. Restricting equity payouts exacerbates overinvestment by high leverage firms but mitigates underinvestment by low leverage firms.

Keywords: limited liability, financial friction, investment, debt financing, equity payout

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1 Introduction

Corporations entered the Covid-19 pandemic with historically high levels of leverage, which soared further during the subsequent economic shock and government policies. At the same time, economists and policy makers have been concerned about a persistent broad-based slowdown in worldwide economic growth, as reflected in the discussions about secular stagnation (Summers (2015)), the decline in long-term real interest rates, and the role of highly-leveraged but unproductive zombie firms.[1] We use a simple model linking firm leverage and real investment in a dynamic setting to show that limited liability can simultaneously cause unproductive overinvestment for high leverage firms and a lack of productive investment for low leverage firms. In this framework, a policy of restricting equity payouts tends to move investment towards the first-best for low leverage firms but not for high leverage firms.

Rather than focus on specific institutional features, our framework emphasizes first principles that are likely to hold across countries and endure over time. The central assumption in our model is limited liability, combined with the assumption that direct debt-financed payouts to equity holders are limited. Limited liability in our model is a protection of equity holders' non-firm assets (including human capital) from creditors following a firm's bankruptcy as in Hart and Moore (1994). We make the plausible assumptions that equity holders can optimally choose to finance their investment with debt and equity, and that investment opportunities arrive repeatedly, thereby relaxing key restrictions in Myers (1977)'s seminal study of debt overhang. We think of these assumptions as capturing the arrival of lumpy investment opportunities (Gourio and Kashyap (2007)). We assume that new financing can only be raised when making a lumpy real investment, so real investment and financing are linked as in many models of one-time investment opportunities (e.g. Brunnermeier and Oehmke (2013)). By breaking the assumption that financing can be raised continuously as in the traditional leverage ratcheting literature (Admati et al. (2018a), DeMarzo and He (2021)), we simply capture the mounting evidence of fixed costs in accessing capital markets (e.g. Hennessy and Whited (2007)), and obtain new implications for real investment.

We start with a baseline model with a single investment opportunity and three types of agents. In our model, equity holders operate the firm and are protected

[1] See for example the 2018 OECD study Adalet McGowan et al. (2018).

by limited liability, preexisting debt investors hold existing debt but are otherwise inactive, and new debt investors competitively price new debt issued by the firm. Equity holders simultaneously choose how much to invest, whether to finance this new investment with debt or equity, and how much to directly pay out to themselves. Cash flows are assumed to evolve according to a continuous stochastic process and equity holders can walk away from the firm at any time, in which case debt holders claim the remaining assets. As a result, the optimal default decision is characterized by a threshold on cash flows as in Leland (1994). We assume that equity holders can consume only a fraction of the proceeds from newly issued debt, in line with common debt covenants on dividend and equity payout restrictions (Billett et al. (2007a)).[2]

An incentive to overinvest – defined as investing so much that the firm's expected marginal return on investment is less than the marginal cost – arises due to "double-selling" of promised earlier cash flows. By increasing leverage and bringing bankruptcy forward, investment dilutes existing debt holders' coupon payments, increasing equity holders' return from investment.[3] Since dilution is driven by a change in default timing, different from standard debt dilution in two-period models (Fama and Miller (1972)) it cannot be solved with covenants securing existing debt holders' claims in bankruptcy, such as negative pledge covenant (or other common covenants).

In the case with no future investment opportunities, a policy of allowing debt financed equity payouts is unambiguously efficient because it mitigates inefficient overinvestment. This is because equity payouts provide a more efficient way to increase leverage. Thus, all firms, irrespective of their leverage, limit their inefficient investment by switching to debt-financed equity payouts. In the corner case with unconstrained equity payouts we can split the equity holders' problem into two separate problems: (1) investment and (2) dilution of existing debt holders. Equity holders

[2]Our baseline assumption is that the fraction that can be consumed directly by equity holders is zero.

[3]In contrast to risk-shifting and entrepreneurship models (e.g., Jensen and Meckling (1976) and Vereshchagina and Hopenhayn (2009)), debt in our model can be thought of as collateralized. Because our mechanism operates through the timing of default and in contrast to risk-shifting, the incentive to overinvest in our model applies to the wide range of firms that are still well away from their default threshold, similar to firms with non-investment grade credit ratings and low interest coverage ratios but not currently financially distressed.

choose investment to maximize the net present value of the firm and then choose the level and financing of equity payouts to optimally dilute existing debt holders. Thus, as suggested by Myers (1977), restrictions on equity payouts financed by new debt tend to increase investment. However, in the extreme case of a single investment opportunity such an increase in investment is not desirable in our model because investment is already inefficiently high.

We next build on our baseline model to incorporate repeated investment opportunities, revealing the importance of dynamic considerations. We model repeated investment opportunities with a fixed stochastic arrival rate, where at each investment opportunity the equity holders face an identical problem as in the baseline model. When the investment arrival rate is non-zero, a new channel emerges as debt investors anticipate equity holders' lack of commitment not to dilute their coupons in the future by changing the default timing, raising the cost of debt financing at the time of investment. The resulting incentive to underinvest – defined as foregoing investments whose marginal returns exceed their marginal costs – is particularly relevant for firms with low initial leverage and a high arrival rate of investment opportunities, because these firms have the largest capacity to dilute the coupons of existing debt holders and frequent opportunities to do so. We therefore obtain the prediction that low leverage and high investment opportunity firms tend to underinvest while high leverage firms—just as in the one-shot investment model—tend to overinvest. This contrasts the predictions based on models with collateral constraints in which financially constrained firms that are protected by limited liability always underinvest (e.g. in Buera (2009), Moll (2014), Khan and Thomas (2013), or Buera et al. (2015)).

We investigate the effects of a policy that restricts direct equity payouts from debt issuance in a calibrated version of the repeated investment model. We find that a policy of restricting equity payouts has new heterogeneous implications for the efficiency of investment across the cross-section of firms in our model. By mitigating debt investors' concerns about future dilution, equity payout restrictions lower the cost of debt finance for low leverage firms, leading equity holders to delay bankruptcy at any given level of cash flows. This raises low leverage firms' investment, which however may not be efficient if firms switch from underinvestment to overinvestment. In contrast, for high leverage firms with infrequent investment opportunities, restricting equity payouts exacerbates inefficient overinvestment as in the baseline model with a single investment opportunity.

In the final part of the paper, we discuss the model's empirical implications. We provide suggestive evidence of the model mechanism using firm-level data from Compustat. Using Jordà (2005) projections, we show that following investment high-leverage firms experience relatively lower profitability compared to low leverage firms. The effect is economically meaningful, with a -0.7 percentage point lower return on assets eight quarters after a one standard deviation increase in capital expenditures to assets for high leverage firms compared to low leverage firms.

Literature Review —

Our paper is related to the debt overhang literature (see, for example, Myers (1977), Hennessy (2004), Moyen (2007), and Diamond and He (2014) among many others). While this literature tends to emphasize underinvestment, we show that one key friction – limited liability – can simultaneously lead to over- and under-investment in the cross-section of firms. This difference arises because we permit new investment to be financed with debt and because investment opportunities arrive repeatedly. We also complement the "leverage ratcheting" literature (Admati et al. (2018a), DeMarzo and He (2021)) by focusing on the real investment implications and the effects of restricting equity payouts when equity holders cannot commit against issuing new debt in the future. Our paper is related to Hackbarth and Mauer (2012) who show that debt financing may lead to earlier execution of investment than is prescribed by the first-best solution with this behavior driven by the traditional source of dilution.

The dilution mechanism in our model differs from the standard dilution mechanism in the literature, and in particular is not easily addressed with collateral or debt covenants. In standard two-period models of debt dilution (those in the spirit of Myers (1977)), equity holders issue an excessive amount of debt (particularly just before default) to dilute existing creditors' claims in default. In contrast, in our model, equity holders, by leveraging up and defaulting earlier, only dilute existing debt holders' coupon claims. Finally, the dilution mechanism identified in this paper is missing in recent dynamic models of debt overhang as these models either focus only on equity financing (Hennessy (2004) or Diamond and He (2014)) or assume that recovery value is zero so equity holders cannot benefit from earlier default (DeMarzo and He (2021)). Because dilution happens through the timing of default, our mechanism is reminiscent of the maturity rat race of financial institutions of Brunnermeier and Oehmke (2013), though while their model is about financial institutions with ample access to short-term funding markets, in our all model all corporate debt is long-term.

More broadly our paper is also related to the literature that studies investment distortions arising from different types of credit market frictions within the macroeconomy (see, e.g., Albuquerque and Hopenhayn (2004), Clementi and Hopenhayn (2006), Buera et al. (2011), Khan and Thomas (2013), or Moll (2014))).[4] Relative to these papers, we examine an environment with perfect information and fully competitive debt markets and focus on how—in isolation—limited liability distorts equity holders' decisions.

We also speak to a recent empirical and quantitative literature that investigates investment and corporate leverage at the macroeconomic level (e.g. Atkeson et al. (2017)). Recently, Crouzet and Tourre (2020) investigate how business credit programs can mitigate underinvestment and Acharya and Plantin (2019) argue in a model of agency frictions that equity payouts can lead to underinvestment. We differ from both of these papers in that we emphasize heterogeneous effects across the firm leverage distribution, where overinvestment is possible and equity payout restrictions may be efficient for some firms but not for others.

2 Model

There are three types of agents in the baseline model: equity holders that operate the firm, existing debt holders who hold debt issued in the past, and competitive outside creditors. Equity holders face a one-time investment opportunity at time 0, at which time they can issue new debt and equity and make direct payouts to themselves (i.e., dividends or equity buybacks). Debt is senior to equity but all debt, including newly issued debt, has equal priority (pari passu).[5] All actions are

[4] Our mechanism is also related to debt dilution and debt overhang in the sovereign default literature (see, for example, Arellano and Ramanarayanan (2012), Chatterjee and Eyigungor (2013), Hatchondo et al. (2016), and Aguiar et al. (2019)). The key difference is institutional: sovereigns have no obligation to pay their creditors anything in default; thus, they dilute the existing debt holders by defaulting earlier and re-capturing payments promised to debt holders.

[5] While this might seem to make the traditional source of dilution possible (i.e., selling new pari passu debt to dilute the claims to the firm in default) by construction that will not be the case in our model. See Section 2.4 for more details on how we fully isolate our

perfectly observable and there is complete information. To keep the model analytically tractable and to highlight the underlying intuition, in our baseline model we assume that equity holders have one-time investment opportunity. We extend the model to feature repeated investment in Section 4.

2.1 Firm State, Notation, and Laws of Motion

The state of a firm is summarized by its *cash flows*, Z, and the book value of its *liabilities*, L, defined as the present discounted value (PDV) of all promised cash flows to debt holders.[6] Equity and bond holders discount future payoffs at the same constant rate $r > 0$. In the absence of new investment, $Z(t)$ follows a geometric Brownian motion with risk-neutral drift μ and instantaneous volatility of $\sigma^2 > 0$

$$\mathrm{d}Z(t) = \mu Z(t)\mathrm{d}t + \sigma Z(t)\mathrm{d}\mathbb{W}(t), \quad Z(0) > 0, \tag{1}$$

where $\mathbb{W}(t)$ is a standard Brownian motion. Liabilities, L, may have a one-time jump at time 0 (i.e., at the time of investment if equity holders decide to finance some of the investment with debt) but remain constant for all $t > 0$. Thus, we do not allow equity holders to issue new debt or repurchase existing debt after time 0—and relax this assumption in Section 4.

Real Investment and Payouts to Equity At time 0 equity holders have a one-time investment opportunity that expires immediately if not executed. In particular, at time 0 equity holders can deterministically increase the initial cash flows of the firm from $Z(0)$ to $\hat{Z} \equiv (1+g)Z(0)$, where $g \geq 0$ captures equity holders' investment financed through a combination of new debt and equity. After the initial jump in cash flows, cash flows follow (1). Investment is costly, with the cost function given by $q(g)/Z \equiv \frac{\zeta g^2}{2}$.

At the time of the investment, we also allow equity holders to make direct payouts

forces from traditional sources of dilution.

[6]Due to the absence of fixed costs in this model, cash flows are equivalent to EBITDA profits and proportional to both the assets-in-place and enterprise value. The book value of liabilities L does not take into account the equity holder's option to default. For example, if the firm's liabilities consist of one unit of defaultable consol that promises coupon c every instant of time then $L = \int_0^\infty ce^{-rs}ds$.

to themselves, M. We interpret M as equity buybacks, dividends, or leveraged buyouts financed by issuing new debt. Thus, M captures any payout to equity holders that equity holders can use immediately for consumption. The presence of M allows us to consider proposals to limit share buybacks or dividend payments. We assume that equity holders can only consume $M \in [0, \kappa Z]$, where $\kappa \geq 0$ is the parameter capturing institutional constraints on financing equity buybacks with new debt, with $\kappa = 0$ being our baseline.[7] The assumption that equity holders have at least some ability make equity payouts from financed by issuing debt is supported by empirical evidence in Farre-Mensa et al. (2020).[8]

In our model, equity holders issue debt only at the time of investment. This infrequent adjustment in debt level is consistent with findings of Welch (2004) that much of the variation in market leverage is passive and of DeAngelo and Roll (2015) that departures from stable leverage ratios are often associated with investment.

Financing and Limited Liability For tractability we assume that all debt takes the form of defaultable consols, which pay one coupon until the firm defaults and represent a proportional claim to the assets of the firm in bankruptcy.[9] Equity holders can fund the total cost of investment, $q(g)Z$, and the total equity payouts, M, with their own funds (equity financing), by issuing new debt (debt financing), or any linear combination of them. Equity holders are assumed to be deep-pocketed and hence able to finance investment or equity payouts with their own funds if they so choose. We denote the proportion of debt financing by $\psi \in [0, 1]$. If the firm issues only equity (i.e., $\psi = 0$) then the liabilities of the firm, L, have no jump at time 0. If $\psi > 0$, liabilities jump at time 0. In that case, let $\hat{L}$ denote *post-investment liabilities*—capturing the present value of all coupons.

The post-investment liabilities, $\hat{L}$, is implicitly determined by the financing choice, $\{g, \psi, M\}$, and an equilibrium price, $P(\cdot)$, for any newly issued bonds. We defer details

[7]The restriction that $M \geq 0$ is without loss of generality since equity holders would never choose $M < 0$ (which, in the model, corresponds to buying back debt).

[8]We interpret κ as a restriction of equity holders' choices arising from covenants protecting existing debt holders. Alternatively, this restriction can be interpreted as arising from financial regulations that protect existing debt holders.

[9]In Online Appendix A.6 we show that our results continue to hold when equity holders' finance their investment with debt of finite maturity (modeled as in Chatterjee and Eyigungor (2012) or Leland (1998)).

until we can fully specify the problem given the equilibrium actions of equity holders (see Section 2.3).

Equity holders are protected by limited liability. This means that after investment and financing choices have been made, equity holders can choose to default and walk away with nothing at any time, whereupon the firm is taken over by debt holders. Deep-pocketed equity holders are assumed to have sufficient funds (i.e., can inject new equity) to keep the firm as a going concern, if they so choose, when promised debt payments exceed firm cash flows.

Equity Value At any point in time there are two states variables: current cash flows, Z and current liabilities, L. Let $V(Z, L)$ denote the post-investment value of equity (i.e., the value of operating the firm after the investment option was executed) when the current cash flows are Z and current liabilities are L. Similarly, let $V^*(Z, L)$ denote the pre-investment value of equity (i.e., the value of operating the firm to equity holders at the time they make their investment decision) when time 0 cash flows are Z and time 0 liabilities are L.[10]

It will prove useful to rescale the value of equity with cash flows. Thus, we denote the post- and pre-investment equity value relative to cash flows as $v(\cdot) \equiv V(\cdot)/Z$ and $v^*(\cdot) \equiv V^*(\cdot)/Z$, respectively. Similarly, we define current leverage as $\ell \equiv L/Z$ and the equity payouts per unit of Z as $m \equiv M/Z$.

Value in Default Upon default the firm is taken over by the debt holders who continue to operate it. However, default may have real costs in the sense that immediately following default the firm's cash flows decrease from Z to $(1 - \theta)Z$, where $\theta \in [0, 1]$. The parameter θ captures deadweight costs associated with bankruptcy proceedings and debt holders' lower skill in running the firm, but we will use the $\theta = 0$ case as our baseline.

2.2 Equity Holders' Investment and Default Decisions

Equity holders face the following decisions. First, at time 0, they have to choose how much to invest, g, how much to pay out to themselves, M, and how to finance these

[10] We differentiate between pre-investment and post-investment states when discussing the investment decision, in which case we denote the post-investment states by $\hat{L}$ and $Z(1+g)$.

choices, ψ. Having made these choices, at each instant of time they need to decide whether to keep operating the firm or default instead.

Investment Decision Given an initial state (Z, L), equity holders choose the financing mix ψ and real investment g to maximize the sum of the post-investment equity value and direct payouts to equity, net of new equity injected into the firm. The post-investment equity value is given by $V((1+g)Z, \hat{L})$, where $\hat{L}$ are the post-investment liabilities. Equity holders take the equilibrium price $P(\cdot)$ for newly issued bonds as given, and solve

$$V^*(Z,L) = \max_{\substack{g\geq 0\\ \psi\in[0,1]\\ 0\leq M\leq \kappa Z}} \Big\{ \overbrace{V(\underbrace{(1+g)Z}_{=\hat{Z}}, \hat{L})}^{\text{Post-Investment Equity}} - \overbrace{(1-\psi)q(g)Z}^{\text{Equity Financed}} + \overbrace{M}^{\text{Payouts}} \Big\} \tag{2}$$

$$\text{s.t. } \underbrace{P(\hat{L}, Z, L, g, \phi, M)}_{\text{Equilibrium Price}} \underbrace{\left(\hat{L}/r - L/r\right)}_{\text{New Bonds}} = \underbrace{\psi q(g) Z}_{\text{Debt Financed}} + M \tag{3}$$

and subject to the feasibility of the payoffs in default embedded in L and $\hat{L}$. In (3), $(\hat{L} - L)/r$ is the quantity of new bonds issued (given the constant interest rate r). Funds raised from issuing new debt at price $P(\cdot)$ can be used for equity payouts, M, or to finance a portion of investment costs, $\psi q(g)Z$. To understand how existing liabilities distort equity holders' investment choices we consider the following first-best benchmark.

Definition 1 (First-Best Investment)**.** *We define the first-best undistorted investment, g^u, as investment that maximizes the net present value of the firm. That is,*

$$g^u(Z) \equiv \arg\max_g \Big\{ \overbrace{V((1+g)Z, 0)}^{\textit{Post-Investment Equity}} - \overbrace{q(g)Z}^{\textit{Equity Financed}} \Big\} \tag{4}$$

Thus, first-best investment is the level that equity holders would choose if the firm had no preexisting debt and investment had to be fully financed with equity. Since both the payoffs and costs are linear in Z, we can show that g^u is independent of Z throughout our model. The homotheticity that leads to a constant g^u is not essential, but simplifies the analysis.

Default Decision Equity holders optimally choose to default when the equity value, $V(Z,L)$, reaches 0. Note that after investment only the cash flows fluctuate, and the equity holders' *continuation* problem becomes a standard stopping problem (as in Leland (1994) with liabilities L as an additional state), which is given by

$$rV(Z,L) = Z - rL + \mu Z \boldsymbol{\partial}_Z V(Z,L) + \frac{\sigma^2}{2} Z^2 \boldsymbol{\partial}_{ZZ} V(Z,L) \tag{5}$$

$$V(\underline{Z},L) = 0 \tag{6}$$

$$\boldsymbol{\partial}_Z V(\underline{Z},L) = 0, \tag{7}$$

where $\underline{Z}$ is the endogenous default barrier. Here (6) and (7) are the standard value-matching and smooth pasting conditions, respectively. Define

$$\eta \equiv \frac{(\mu - \sigma^2/2) + \sqrt{(\mu - \sigma^2/2)^2 + 2\sigma^2 r}}{\sigma^2} > 0 \tag{8}$$

$$\chi \equiv \left(\frac{(r-\mu)\eta}{\eta+1}\right)^{\eta} > 0 \tag{9}$$

$$s(\ell) \equiv \frac{\chi}{\eta+1}\ell^{\eta}, \tag{10}$$

where $s(\ell)$ captures the value of equity holders' option to default per unit of liabilities (see the discussion below). We now characterize the solution to equity holders' default problem (5)-(7).

Proposition 1 (Post-Investment Equity and Default)**.** *Suppose that the current state of the firms is (Z,L). Then the equity value of the firm is given $V(Z,L) = v(\ell)Z$, where $\ell \equiv L/Z$ is firm's leverage and*

$$v(\ell) = \frac{1}{r-\mu} - \ell(1 - s(\ell)) \tag{11}$$

The endogenous default threshold $\underline{Z}$ satisfies

$$\frac{\underline{Z}(L)}{r-\mu} = \frac{\eta}{1+\eta}L \tag{12}$$

and the liquidation value per unit of liabilities is given by

$$\frac{V((1-\theta)\underline{Z}(L),0)}{L} = \frac{(1-\theta)\eta}{1+\eta} \tag{13}$$

Proof. See Appendix A.1. □

The above proposition characterizes the value of equity and equity holders' optimal default decision. Note that if equity holders were not allowed to default then the value of equity would be given by $\left(\frac{1}{r-\mu} - \ell\right) Z$ as equity holders would have to repay all their liabilities. Thus, $s(\ell)L$ captures the value of equity holders' option to default.

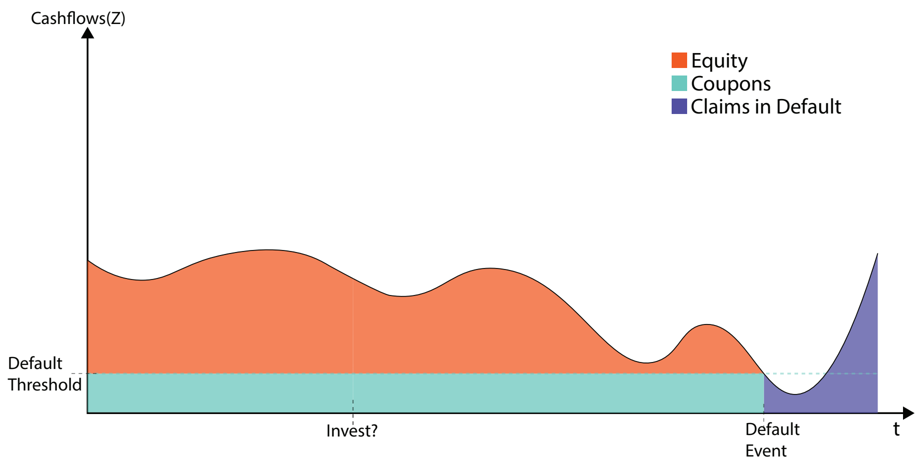

Figure 1: **The effect of limited liability on debt and equity cash flows without investment.** This figure considers the simplified case with no bankruptcy costs ($\theta = 0$).

Before moving on to how debt is priced, it is useful to visualize how firm cash flows are divided between equity and debt holders. Figure 1 depicts a possible path of Z and shows how cash flows are divided among all claimants (for simplicity we set $\theta = 0$). Default occurs at the threshold $\underline{Z}(L)$ optimally chosen by the firm, following Proposition 1. Because we are considering the baseline case without bankruptcy costs ($\theta = 0$), firm's cash flows are not impacted by the default event—instead they simply change claimants. Prior to default, coupons are paid and residual funds are distributed to equity holders. After default, all cash flows are owned by the default claimants. The price of any financial claim is simply the expected present discounted value of cash flows allocated to this claimant, with the expectation taken over all possible realizations of the $Z(t)$ process.

2.3 Pricing Debt Instruments

In this section, we derive how debt is priced, which then determines the equilibrium budget constraint in (3). Debt is priced by outside creditors who are risk-neutral and who anticipate equity holders' optimal default decision (as characterized in Proposition 1). Let T denote the stopping time when cash flows first reach default threshold, $\underline{Z}$, at which point equity holders choose to default.[11]

For tractability, we assume that all debt takes the form of defaultable consols following Leland (1994, 1998). A defaultable consol pays 1 in perpetuity prior to default and receives a share of the bankruptcy value of the firm in default. Conditional on the current state of the firm (Z, L) and equity holders' optimal default decision, the market price of a such bond $P(Z, L)$ equals

$$P(Z,L) \equiv \frac{p(Z,L)}{r} = \underbrace{\mathbb{E}_T\left[\int_0^T e^{-r\tau}\mathrm{d}\tau\right]}_{\substack{\text{PDV of promised coupons} \\ = P^C(Z,L)}} + \underbrace{\mathbb{E}_T\left[e^{-rT}\frac{V((1-\theta)\underline{Z}(L),0)}{rL}\right]}_{\substack{\text{PDV of claims in bankruptcy} \\ = P^B(Z,L)}}, \quad (14)$$

where $p(Z, L)$ denotes the price relative to the risk-free rate.

Equation (14) emphasizes that a defaultable consol consists of pre-bankruptcy component (i.e., the coupon payments prior to default) with market price $P^C(Z, L)$, and a component consisting of claims in bankruptcy (i.e., all debt is equal priority or pari passu) with market price $P^B(Z, L)$. We use $p^C(\cdot, \cdot)$ and $p^B(\cdot, \cdot)$ to denote these prices relative to the risk-free rate, r.

Proposition 2 establishes that leverage ($\ell \equiv L/Z$) is the relevant state for pricing debt, solves for the prices of the defaultable consol and of its pre-bankruptcy and bankruptcy components, and characterizes equity holders' budget constraint, (3).

Proposition 2 (Pricing Debt Instruments)**.** *The relevant state for pricing debt is leverage $\ell \equiv L/Z$. Moreover,*

$$p(\ell) = 1 - (1 + \theta\eta)s(\ell) \quad (15)$$

$$p^C(\ell) = 1 - (1 + \eta)s(\ell) \quad (16)$$

$$p^B(\ell) = (1 - \theta)\eta s(\ell) \quad (17)$$

[11] We assume that $Z_0 > \underline{Z}$.

Given these debt instruments, the budget constraint (3)*, normalized by* Z*, is given by*

$$p(\hat{\ell})\left((1+g)\hat{\ell}-\ell\right)=\psi q(g)+m \tag{18}$$

$$p(\hat{\ell})\geq p^{B}(\hat{\ell}) \tag{19}$$

where $\hat{\ell}\equiv\hat{L}/\hat{Z}$ *is post-investment leverage.*

Proof. See Appendix A.2. □

The budget constraint (18) is standard. The left hand side equals the total value of new debt issued, normalized by Z. The right hand side represents equity holders' need to raise new debt financing, and equals the debt financed portion of investment costs plus equity payouts. The constraint (19) enforces the feasibility of the default payoff embedded in defaultable consols, meaning that equity holders cannot issue so much debt that the firm is strictly above its default threshold immediately after investment. While the constraint (19) never binds when $\kappa=0$, it may bind if direct payments to equity holders are allowed ($\kappa>0$).[12]

In the case without bankruptcy costs (i.e., $\theta=0$) the price of the defaultable consol relative to the risk-free rate simplifies to $p(\ell)=1-s(\ell)$. In this particularly simple case, $s(\ell)$ can be interpreted as the spread relative to the risk-free rate. Recall that $s(\ell)$ also appears in the expression for $v(\ell)$ (the normalized value of equity) with the opposite sign, where it captures the value of equity holders' option to default. Thus, we see that a more valuable default option increases the value of equity at the expense of bond holders.

Bankruptcy claims Proposition 2 prices an asset which bundles coupon payments with a claim to a fraction of the liquidation value of the firm upon default (i.e., assuming the proportional distribution of claims is usually referred to as pari passu). A concern might be that since there is no seniority of existing claims on the firm in bankruptcy relative to new claims, the firm is able to capitalize on that by issuing so

[12]This constraint is similar in spirit to a constraint that limits the ability of the firm to sell assets in distress but before bankruptcy (as is the case in common bankruptcy laws). It captures the observation that existing debt holders would be able to (at least partially) block issuance of large amount of debt just before default.

much debt (possibly an infinite quantity) to dilute existing debt holders' bankruptcy claims (see, for example, the discussion in Section 2.D of DeMarzo and He (2021)).

This, however, in not possible in our model. First of all, constraint (19) restricts equity holders to issue debt up to a finite limit implied by the default threshold. Second, for all ℓ that satisfy constraint (19), the value of bankruptcy claims, $p^B(\ell)$ is actually increasing in ℓ. The intuition is that while new debt does increase the number of claimants in bankruptcy, it also induces firms to default with higher Z. This increases the liquidation value of the firm (as the liquidation value is proportional to L in (13)) and offsets the effect of an increase in the number of claimants in bankruptcy. In addition, since the firm now default on average earlier, the value of existing debt holders' bankruptcy claims actually increases.

2.4 Collateralized Debt as an Equivalent Formulation

We can equivalently assume that the existing debt is collateralized to the firm's pre-investment assets, without any changes to the equilibrium. Since the value of the firm in bankruptcy is known with certainty, we can interpret that value as collateral that equity holders' can use to collateralize debt. We then impose the constraint on equity holders' so that any collateral pledged to existing debt holders cannot be pledged to new debt holders (or more generally, that equity holders' cannot pledge the same collateral to multiple debt holders). Let us denote by $p^S(\cdot)$ the value of existing collateralized debt relative to risk-free interest rate, r. Similarly, let $\hat{p}^S(\cdot)$ denote the new collateralized debt sold to finance investment.

As we show in Appendix A.2, in equilibrium, we have $p^S(\hat{\ell}) = \hat{p}^S(\hat{\ell}) = p(\hat{\ell})$. That is, the equilibrium price of the existing and new collateralized debt is identical, and both are the same as the price of the baseline asset. Moreover, we show that the (19) is implied by the pledgeability constraint that ensures that equity holders' cannot pledge the same collateral to multiple debt holders. Thus, we conclude that equity holders will make the same real investment decisions when debt is collateralized—highlighting a key distinction between our model and financial frictions related to limited enforcement around collateral in bankruptcy such as in Buera et al. (2011) and Moll (2014).

A related concern might be that since existing and new debt holders have equal priority claims on the firm in bankruptcy, the firm is able to capitalize on that by issuing so much debt (possibly an infinite quantity) to dilute existing debt holders' bankruptcy claims (see, for example, the discussion in Section 2.D of DeMarzo and He (2021)). This, however, in not possible in our model. First of all, constraint (19) restricts equity holders to issue debt up to a finite limit implied by the default threshold. Second, for all ℓ that satisfy constraint (19), the value of bankruptcy claims, $p^B(\ell)$ is actually increasing in ℓ. The intuition is that while new debt does increase the number of claimants in bankruptcy, it also comes along with new real investment and induces firms to default with higher Z. This increases the liquidation value of the firm (as the liquidation value is proportional to L in (13)) and offsets the effect of an increase in the number of claimants in bankruptcy. In addition, since the firm now default on average earlier, the value of existing debt holders' bankruptcy claims actually increases.

2.5 Equity Holders' Investment Problem and the First-Best

We now restate equity holders' investment problem, normalizing by current cash flows and substituting in the budget constraint associated with defaultable consols (18). Equity holders of a firm with pre-investment leverage ℓ choose $(g, m, \psi, \hat{\ell})$ such that

$$v^*(\ell) = \max_{\substack{g,\hat{\ell}\geq 0 \\ \psi\in[0,1] \\ 0\leq m\leq\kappa}} \Big\{ \overbrace{(1+g)v(\hat{\ell})}^{\text{Post-Investment Equity}} - \overbrace{(1-\psi)q(g)}^{\text{Equity Financed}} + \overbrace{m}^{\text{Payouts}} \Big\} \tag{20}$$

$$\text{s.t.} \quad \underbrace{p(\hat{\ell})}_{\text{Bond Price}} \underbrace{((1+g)\hat{\ell}-\ell)}_{\text{New Bonds}} = \underbrace{\psi q(g)}_{\text{Debt Financed}} + \underbrace{m}_{\text{Payouts}} \tag{21}$$

$$p(\hat{\ell}) \geq p^B(\hat{\ell}) \tag{22}$$

The first-best investment from Definition 1 solves

$$g^u \equiv \arg\max_g \Big\{ \overbrace{(1+g)v(0)}^{\text{Post-Investment Equity}} - \overbrace{q(g)}^{\text{Equity Financed}} \Big\} \tag{23}$$

The equity holders' objective function (20) is a normalization of (2). Similarly, the first-best investment in (23) is just the normalization of (4). As noted in Section 2.3, the constraint (22) enforces the feasibility of the default payoff embedded in defaultable consols, meaning that equity holders cannot issue so much debt that the firm is strictly above its default threshold immediately after investment.

3 Analysis of Investment Decision

In this section, we analyze equity holders' investment and financing decisions in our baseline model with a single investment opportunity. We first characterize the equity holders' problem relative to the first-best. This comparison allows us to identify the sources of inefficiencies in equity holders' investment decisions.

3.1 Sources of Investment Distortions

We define the function $H(\hat{\ell}) \equiv \theta\eta s(\hat{\ell})$ as the deadweight cost of default per unit of leverage, which is non-zero if a share of the firm is dissipated in default (i.e., if $\theta > 0$). We then characterize the investment problem as follows.

Proposition 3. *Equity holders' investment problem can be written as*

$$v^*(\ell) = \max_{\substack{g,\hat{\ell}\geq 0 \\ \psi\in[0,1] \\ 0\leq m\leq\kappa}} \left\{ \frac{1+g}{r-\mu} - q(g) - p(\hat{\ell})\ell - (1+g)H(\hat{\ell})\hat{\ell} \right\} \tag{24}$$

$$s.t.\, p(\hat{\ell})((1+g)\hat{\ell} - \ell) = \psi q(g) + m \tag{25}$$

$$p(\hat{\ell}) \geq p^B(\hat{\ell}) \tag{26}$$

The first-best investment, g^u, is the unique solution to

$$0 = \frac{1}{r-\mu} - q'(g^u) \tag{27}$$

If the investment cost is quadratic and equal to $q(g) = \zeta g^2/2$ then the optimal investment is given by $g^u = \frac{1}{\zeta(r-\mu)}$.

Proof. See Appendix A.3 for details. □

The reformulated objective function (24) shows that the post-investment value of equity equals the expected PDV of cash flows generated by the firm net of (i) the cost of investment ($q(g)$), (ii) the PDV of cash flows promised to the existing debt holders ($p(\hat{\ell})\ell$), and (iii) cash flows lost in default ($(1+g)H(\hat{\ell})\hat{\ell}$), all normalized by Z.[13] Because new debt is fairly priced equity holders bear the full cost of the investment and any change in the expected deadweight cost of default. For the same reason equity payouts m do not appear directly in (24). However, ψ and m affect the post-investment value of equity indirectly through $\hat{\ell}$.

The above characterization of the equity holders' problem emphasizes the sources of inefficient investment. Compared to (23), we see that equity holders face two distortions. The first distortion is due to existing debt, as captured by $p(\hat{\ell})\ell$. Since $p(\hat{\ell})$ is a decreasing function of post-investment leverage, $\hat{\ell}$, equity holders have an incentive to increase leverage. This is the classic conflict between equity and debt holders pointed out by Myers (1977)—though in contrast they assume investment is entirely financed with equity. The second distortion is due to bankruptcy costs and is captured by $(1+g)H(\hat{\ell})\hat{\ell}$. Since $H(\cdot)$ is an increasing function, the presence of bankruptcy costs discourages equity holders from taking on additional leverage.

3.2 Investment Relative to First-Best

We focus first on the case without bankruptcy costs (i.e., $\theta = 0$), so the last term in (24) vanishes, and preexisting leverage is the only source of investment distortions.[14] If in addition preexisting debt is zero (i.e., $\ell = 0$), it follows from the reformulated investment problem in Proposition 3 that equity holders' optimal investment satisfies

$$\frac{1}{r-\mu} - q'(g) = 0, \tag{28}$$

that is investment equals the first-best ($g = g^u$). Equity holders are also indifferent between equity and debt financing and whether to make equity payouts (i.e., they are indifferent over any feasible choices of ψ and m). In other words, when $\ell = 0$ the Modigliani-Miller theorem holds, and the firm value is independent of financing.

Investment deviates from this simple benchmark when the firm has preexisting debt ($\ell > 0$). Without bankruptcy costs, it is immediate from (24) that equity

[13] The reformulated objective function can be obtained by substituting the budget constraint into the expression for the post-investment value of equity (the RHS of (20)).

[14] We investigate how the presence of bankruptcy costs affects investment in Section 3.5.

holders' optimal investment satisfies the following first-order condition (FOC)

$$\frac{1}{r-\mu} - q'(g) - p'(\hat{\ell})\frac{\partial \hat{\ell}}{\partial g}\ell = 0 \tag{29}$$

Equation (29) is the key equation of our model.[15] Compared to (28), which determines equity holders' investment choice when $\ell = 0$, the FOC now includes the additional term $-p'(\hat{\ell})\frac{\partial \hat{\ell}}{\partial g}\ell$. This new term captures the marginal change in the value of existing debt due to the change in the firm's distance to default. When this term is positive at $g = g^u$ equity holders have an incentive to invest beyond the level that would maximize the total value of the firm, while the opposite is true when the term is negative. Since the value of existing debt is decreasing in leverage (i.e., $p'(\hat{\ell}) < 0$) it follows that the sign of this distortion depends on the sign of $\frac{\partial \hat{\ell}}{\partial g}$. If at optimal investment we have $\frac{\partial \hat{\ell}}{\partial g} > 0$ then equity holders *overinvest* relative to first-best. If at the optimal investment we have $\frac{\partial \hat{\ell}}{\partial g} < 0$ then equity holders *underinvest*.

3.3 Dilution Mechanism and Inefficient Investment

We now present our first main result that preexisting debt encourages overinvestment. We first characterize equity holders' choices without equity payouts financed by debt (i.e., $\kappa = 0$).[16]

Proposition 4. *Given $\kappa = 0$, denote g^* as equity holders' optimal investment*

1. *If constrained to use equity financing, equity holders underinvest ($g^* < g^u$)*

2. *If allowed to choose financing optimally, equity holders: (a) finance all their investment with debt; (b) overinvest ($g^* > g^u$)*

Proof. See Online Appendix A.2 □

The first part of Proposition 4 nests the classic underinvestment result of the debt overhang literature (Myers (1977)). Thus, our model makes precise that equity financing is a condition required for this classic result. Figure 2 visualizes the intuition.

[15]Note that once $\{g, \psi, m\}$ are chosen, $\hat{\ell}$ is determined by the budget condition (21). Thus, we can treat $\hat{\ell}$ as a function of $\{g, \psi, m\}$.

[16]As we show in Online Appendix A.6 this result (as well as other results reported in this section) continue to hold when equity holders finance their debt of finite maturity as in Leland (1998).

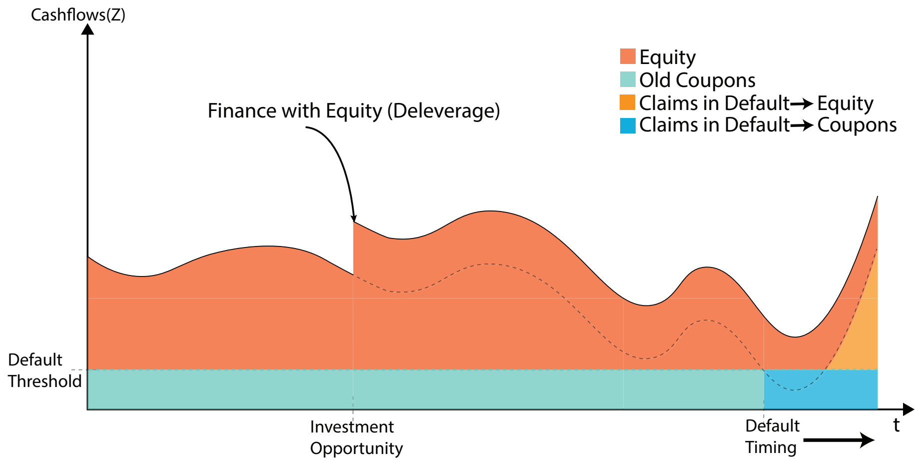

Figure 2: **Equity-financed investment, due to deleveraging, decreases the option value of default.** This figure shows debt and equity cash flows with an equity-financed investment opportunity. We show the simplified case with no bankruptcy costs ($\theta = 0$).

Investment financed with equity leads to deleveraging, leading equity holders to pay coupons for longer. As a result, a portion of the cash flows from the new investment is allocated to existing debt holders in form of coupon payments (the portion of the "Claims in Default $\rightarrow$ Coupons" area above the dotted line). The benefit from new investment is hence partly captured by existing debt holders, implying that equity holders' benefit from investment is lower than the social benefit. In terms of the key equation (29), deleveraging implies that $-p'(\hat{\ell})\frac{\partial \hat{\ell}}{\partial g}\ell < 0$, and hence a reduction of equity holder's incentive to invest. This classic argument has been used to explain the historically low investment in the aftermath of the Great Recession in Europe (see, for example, Kalemli-Ozcan et al. (2018)).[17]

By contrast, the second part of Proposition 4 shows that equity holders may want to overinvest if they can choose their financing method freely, provided that the firm has preexisting debt (i.e., $\ell > 0$). Note that investment is deterministic and the volatility of firm's cash flows is assumed to remain the same before and after investment, and thus the mechanism here is distinct from the potential incentive to invest in risky projects with negative present value, so-called risk-shifting (Jensen and Meckling (1976)).

[17] Jungherr and Schott (2021) show how high leverage may lead to slow recoveries from recessions.

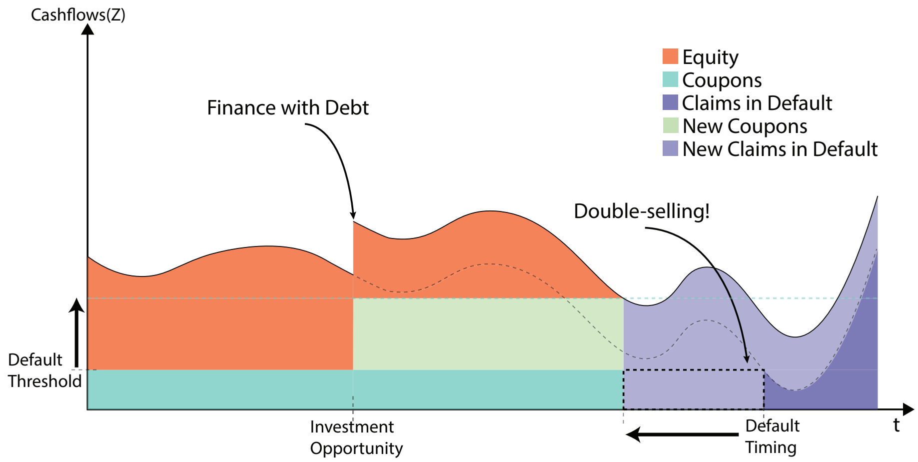

Figure 3: **Debt-financed investment, due to increased leverage, dilutes existing debt holders by double-selling some of their promised coupon payments.** This figure shows debt and equity cash flows with a debt financed investment opportunity. We show the simplified case with no bankruptcy costs ($\theta = 0$).

Why do equity holders overinvest? This is driven by two related but somewhat different forces: (1) an incentive to dilute existing debtholders (that motivates low- and medium-leverage firms' choices), and (2) the incentive to limit the gain from new investment to existing debtholders (that motivates high-leverage firms' choices, for whom costs of dilution exceed its benefits). As we will explain in further detail below, in both cases the incentives are on the expected length of time paying coupons to previous debt holders, and are independent of the traditional source of dilution (i.e., issuing new equal priority claims to the firm in default) that we deliberately shut down to isolate our new mechanism.

We first explain how equity holders can dilute existing debt holders in the model. Figure 3 illustrates that a sufficiently large debt-financed investment leads to higher leverage and earlier default, transforming a portion of the coupon payments that has been promised to existing debt holders (the rectangular area with dashed edges) to claims in default, which have to be shared with new debt holders. Thus, by issuing new debt and increasing leverage, equity holders can sell again claims to some of the cash flows that were previously promised to existing debt holders. It follows that the marginal benefit to equity holders of investing exceeds the social benefit, and hence equity holders overinvest relative to the first-best. In terms of (29), this additional benefit of financing investment with debt is captured by $-p'(\hat{\ell})\frac{\partial \hat{\ell}}{\partial g}\ell > 0$.

It is worth pointing out that only the coupon payments promised to existing debt holders are double-sold by equity holders, not the existing debt holders' bankruptcy claims. In particular, the value of existing debt holders' bankruptcy claims at the time of default is unaffected by the increase in leverage.[18] Furthermore, since after investment default happens on average earlier, the PDV of existing bankruptcy claims actually goes up. Thus, the dilution mechanism in this paper affects negatively only the value of existing debt holders' coupon claims. As such this mechanism differs from the standard dilution mechanism emphasized in the literature, where equity holders issue an excessive amount of debt (particularly just before default) to dilute existing creditors' claims in default (Fama and Miller (1972)).

The dilution mechanism in our model operates through changes in the default timing. By investing above the efficient level the equity holders take on excessive leverage and thus commit to defaulting earlier and at higher cash flows. Since our mechanism is about changes in the timing in default it cannot be captured by simple two-period models of investment in the spirit of Myers (1977).

The above mechanism drives overinvestment by firms with low and medium leverage. If the initial leverage is already high, an additional mechanism comes into play. If initial leverage is sufficiently high equity holders choose to deleverage (i.e., choose $\hat{\ell} < \ell$). This is because at high levels of initial leverage, increasing leverage further requires very large inefficient investment. The cost of such inefficient investment are borne by the equity holders and exceed the benefits of dilution.[19] However, even if equity holders deleverage they still overinvest. This is because, on the margin, increasing investment above the efficient level and financing it with debt allows the equity holders to limit the amount of cash flows generated by new investment that would be

[18] To be precise, *before* the investment takes place, the value of existing debt holders' bankruptcy claims at the time of default is $\frac{\eta}{1+\eta}L$, the value of the firm in default (see (13)). *After* the investment takes place the value of existing debt holders' bankruptcy claims at the time of default is given by $V((1-\theta)\underline{Z}(\hat{L}), 0) \times (L/\hat{L})$, which is also equal to $\frac{\eta}{1+\eta}L$ (since the post-investment firm's value in default ($\frac{\eta}{1+\eta}\hat{L}$) is divided proportionally between new and old debt holders).

[19] To be more precise, define $\bar{g}$ as the level of investment such that $\hat{\ell} \geq \ell$ if and only if $g \geq \bar{g}$. Then $\bar{g}$ is an increasing function of ℓ (see Lemma 12 in the Online Appendix). For high ℓ, choosing $g > \bar{g}$ (i.e., increasing leverage) is associated with investment cost, $q(g)$, which exceeds potential benefits from dilution.

captured by existing debt holders. This observation further underscores the difference between our mechanism and earlier work on risk-shifting and debt dilution.[20]

The overinvestment incentive identified in Proposition 4 stands in a stark contrast to popular models of financial frictions featuring collateral constraints. In the latter, limited liability coupled with additional sources of market incompleteness (such as private information in Clementi et al. (2010) or the ability to abscond funds in Buera et al. (2011)) leads to underinvest by constrained firms (i.e., firms with low assets relative to debt or borrowing needs). In contrast, we show that limited liability by itself provides an incentive to overinvest. In the model with one shot investment this leads to overinvestment while in the presence of repeated investment opportunities it results in heterogeneous investment distortions that depend on firms' leverage (see Section 4).[21]

3.4 Equity Payouts

Having seen that the availability of debt financing can lead to overinvestment in the presence of preexisting debt, we now turn to analyzing how debt financed payouts to equity holders, such as dividends and equity buybacks, affect real investment (i.e., $\kappa > 0$). We define $\bar{m}(\psi, g)$ as the highest equity payout that satisfies the budget constraint (21) given investment, g, and financing choice, ψ. Thus, given g, ψ, equity holders' choice of m has to satisfy $m < \min\{\kappa, \bar{m}(\psi, g)\}$. As shown in Online Appendix A.3 (Equation (A.7)), $\bar{m}(\psi, g) = \frac{1+g}{r-\mu} - \frac{\eta}{1+\eta}\ell - \psi q(g)$.

[20] Note that deleveraging occurs not because firms buy back debt, but rather because following investment cash flows increase more than debt (as issuing more debt becomes too costly for high-leverage firms). Thus, just as in the recent literature on leverage ratcheting (Admati et al. (2018b) and Leland and Hackbarth (2019)), in our model equity holders never have an incentive to buy back debt.

[21] We view our mechanism as complementary to the underinvestment implied by collateral constraints models. The additional sources of market incompleteness emphasized in that literature most likely are of first-order importance for small and young firms or firms with standardized capital. In contrast, the inefficiency emphasized in our paper, while in principle affecting all firms that are protected by limited liability, is probably most relevant for well-established firms with outstanding debt. Indeed, in Section 5 we show that behavior of firms in Compustat is consistent with predictions of our model.

Proposition 5. *Denote g^*, m^*, ψ^* as the equity holders' optimal choices of investment, payouts, and financing, respectively. There exists $\underline{\kappa} \in \mathbb{R}_+$ such that*

1. *If $\kappa < \underline{\kappa}$ then equity holders: (a) overinvest ($g^* > g^u$); (b) finance investment and equity payouts with debt ($\psi^* = 1$); (c) make payouts to themselves up to the constraint ($m^* = \kappa$); (d) continue operating firm*

2. *If $\kappa \geq \underline{\kappa}$ then equity holders: (a) invest the first-best amount($g^* = g^u$); (b) finance investment and equity payouts at least partially with debt ($\psi^* \in [\max\{\underline{\psi}_\kappa, 0\}, 1]$, where $\underline{\psi}_\kappa > 0$ is the unique solution to $\kappa = \bar{m}(g^*, \underline{\psi}_\kappa)$); (c) make maximal feasible payouts to themselves ($m^* = \bar{m}(g^*, \psi^*)$); (d) are indifferent between defaulting and continuing to operate the firm*

The threshold $\underline{\kappa}$ is decreasing in ℓ and r, and increasing in σ.

Proof. See Online Appendix A.3. □

Proposition 5 extends our overinvestment result to the case in which equity payouts financed with debt are permitted at the time of investment. As long as equity holders face sufficiently tight restrictions on equity payouts financed by debt ($\kappa < \underline{\kappa}$), we find that they continue to overinvest. Different from the case with $\kappa = 0$, equity holders accompany investment with direct equity payouts, further increasing post-investment leverage.

By contrast, when the constraint on direct equity payouts is lax ($\kappa \geq \underline{\kappa}$), equity holders invest the first-best amount. In this case, equity holders have a more efficient way of increasing leverage than inefficient investment. Even high-leverage firms are now able to leverage up cheaply. Thus, when $\kappa \geq \underline{\kappa}$, equity holders' problem is decoupled into two separate problems: (1) an investment problem and (2) a dilution of existing debt holders problem. Equity holders choose g to maximize the net present value of the firm and m to maximize the transfer from existing debt holders to themselves. The latter implies choosing the highest feasible m so that the firm defaults right after investment. Thus, when $\kappa \geq \underline{\kappa}$, equity holders essentially sell the firm to the new debt holders.[22]

[22]This suggests that in our setup, equity holders have an incentive to "collude" with new creditors in order to dilute the existing debt holders. A similar mechanism has been emphasized recently by Aguiar et al. (2019) within the context of sovereign default.

Proposition 5 shows that restrictions on equity payouts can increase investment and reduce the probability of bankruptcy, in line with the intuition in Myers (1977). Different from Myers (1977), however, we find that such restrictions might not be desirable. We reach different conclusion since in our model investment is debt financed and tends to be inefficiently high.

The conclusion that equity payout restrictions lead to inefficiently high investment needs to be tempered by the observation that this one-shot investment problem does not account for potential future debt dilutions. We will see in Section 4 that dynamic considerations lead to more nuanced conclusions. However, the overinvestment incentive highlighted in the one-shot model tends to remain particularly for firms with relatively high leverage.

One may wonder if covenants could be used to eliminate the inefficiencies characterized in Propositions 4 and 5. However, as has been first pointed by Myers (1977), it is difficult to write and enforce debt contracts which ensure that equity holders invest first-best amount. For example, unlike in models driven by dilution of collateral claims, covenants that prohibit equity holders from issuing senior debt in bankruptcy have no effect in our environment—as discussed in Section 2.4—and could lead to severe underinvestment if they limit the ability of the firm to collateralize its current assets. The same applies to covenants that limit firms' leverage or negative pledge covenants. Finally, in practice few covenants prohibit firms from issuing new debt with equal priority to the existing debt.[23]

3.5 Bankruptcy Costs

In this section, we analyze how the results in the baseline model are affected by the presence of bankruptcy costs. We find that realistic bankruptcy costs moderate but do not eliminate incentives for overinvestment. Throughout the bankruptcy cost analysis, we assume that there are no equity payouts (i.e., $\kappa = 0$) for simplicity. In the presence of default costs (i.e., $\theta \in (0, 1]$), debt financing is associated with the following trade-off. On the one hand, as before, issuing new debt allows equity holders to resell some of existing debt holders' claims, which we have seen encourages overinvestment. On the other hand, issuing new debt increases the deadweight cost

[23]For example, Billett et al. (2007b) find that less than 1.4% of bond issues have covenants restricting the future issue of senior or pari passu debt.

of default and hence the cost of debt financing, encouraging underinvestment.[24]

Proposition 6. *Suppose that $\theta > 0$. If equity holders are*

1. *constrained to use equity financing, they underinvest ($g^* < g^u$)*
2. *allowed to choose financing optimally then for each ℓ there exists $\underline{\theta}(\ell) > 0$ such that for all $\theta \in [0, \underline{\theta}(\ell)]$ they overinvest ($g^* > g^u$)*

The first part of Proposition 6 shows that the result that equity holders underinvest with equity financing extends to $\theta > 0$. The second part shows that equity holders continue to overinvest with optimal financing even in the presence of bankruptcy costs, as long as those costs are not too large. This is because for $\theta < \underline{\theta}(\ell)$ the marginal benefit associated with issuing additional debt dominates the marginal increase in deadweight cost.

Figure 4 depicts the optimal investment (relative to the first-best investment) for empirically relevant values of θ.[25] We see that these implications are robust to empirically plausible bankruptcy costs. Overinvestment initially increases with leverage, as equity holders can dilute more claims. However, as initial leverage increases so does the cost of increasing leverage (see Section 3.3). Thus, for high levels of initial leverage, the extent of equity holders' overinvestment is decreasing in ℓ.

[24]When $\theta > 0$ the Modigliani–Miller theorem no longer holds even in the absence of preexisting debt, as in that case equity holders would invest first-best amount but finance it with equity.

[25]These costs are typically estimated to be between 2% and 20% (see, e.g., Bris et al. (2006)).

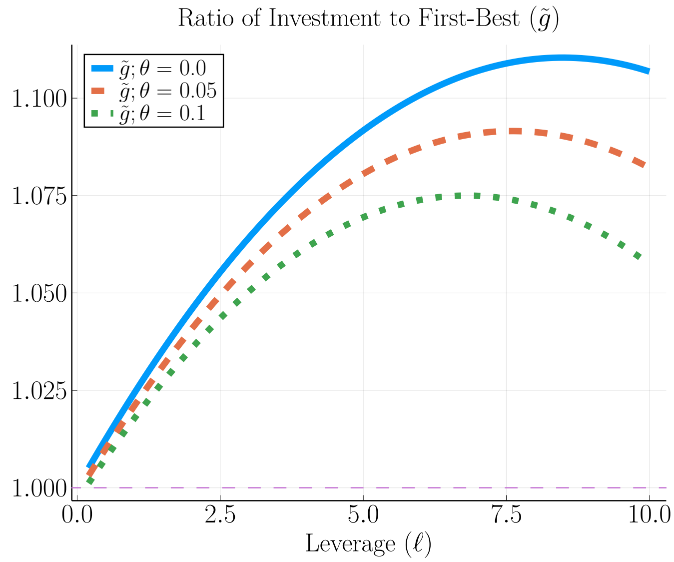

Figure 4: **Investment relative to first-best ($\tilde{g}$) against preexisting leverage (ℓ) for the baseline model with a single investment opportunity.** Each line corresponds to a different level of deadweight bankruptcy costs (θ). The case $\theta = 0$ corresponds to no bankruptcy costs. Model parameters are discussed in Online Appendix B.3 for more details.

4 Repeated Investments

So far, we have analyzed a model with a single investment opportunity. In that setting, we have shown that equity holders have an incentive to finance their investment with debt and overinvest. We have argued that this behavior is driven by incentive to dilute preexisting debt holders coupons (among low leverage firms) and to limit the gain from new investment to existing debt holders (among high-leverage firms). In this section, we build on this model and allow for repeated investment opportunities. We show that repeated investment opportunities have non-trivial consequences as buyers of new debt price in the likelihood of future dilution, thereby increasing the cost of debt financing. We find that dynamic considerations can lead to underinvestment especially among low leverage firms with frequent investment opportunities, and that direct equity payouts (i.e., equity buybacks and dividends) financed out of debt further exacerbate underinvestment among these firms because they make debt financing more expensive.

4.1 Model with Repeated Investment

We consider the same setup as described in Section 2, but assume that investment opportunities arrive at a constant Poisson rate, λ. As above, the state of the firm at any given point in time is $\{Z, L\}$. Upon arrival of an investment opportunity, equity holders have the choice to increase current cash flows from Z to $Z(1+g)$ at cost $Zq(g)$. It follows that cash flows follow a jump diffusion

$$\mathrm{d}Z(t) = \mu Z(t)\mathrm{d}t + \sigma Z(t)\mathrm{d}\mathbb{W}(t) + g(Z(t^-), L(t^-))Z(t^-)d\mathbb{N}(t), \quad Z(0) > 0, \tag{30}$$

where $\mathbb{N}(t)$ is a Poisson process with intensity $\lambda \geq 0$ and $g(Z(t^-), L(t^-))$ is equity holders' investment at time t conditional on the state of the firm $\{Z, L\}$ and the arrival of an investment opportunity. Note that when $\lambda = 0$ we are back to the baseline model with a single investment opportunity.

Investment can be financed by issuing defaultable consols via competitive debt markets (as described in Section 2.3, but with prices reflecting the dynamic decisions of the firm) or equity. This implies that, in contrast to the model of Section 2, liabilities are no longer constant. Rather, $L(t)$ is now a pure jump process with $\mathrm{d}L(t) = (\hat{L}(t) - L(t^-))\mathrm{d}\mathbb{N}(t)$, where $\hat{L}(t)$—as defined in Section 2.1—denotes the value of liabilities immediately after investment implied by equity holders' investment and financing decisions.

4.2 Optimal Investment Problem

Conditional on the arrival of an investment opportunity, the firm solves the natural analogue of the one-shot problem, except that both the $v(\cdot)$ and $p(\cdot)$ functions account for the possible arrival of future investment opportunities. The following proposition describes the equity and debt holders' problems with repeated investment

Proposition 7 (Repeated Investment)**.** *A solution consists of a value of equity $v(\ell)$, price $p(\ell)$, policies $\{g(\ell), m(\ell), \psi(\ell), \hat{\ell}(\ell)\}$, and default threshold, $\bar{\ell}$, such that*

1. *Given $v(\ell)$ and $p(\ell)$:(a) the policies $\{g(\ell), m(\ell), \psi(\ell), \hat{\ell}(\ell)\}$ solve the firm's problem in* (20)*; (b) $v(\ell)$ satisfies the differential variational equation (DVI)*

$$0 = \min\{(r-\mu)v(\ell) + \mu\ell v'(\ell) - \frac{\sigma^2}{2}\ell^2 v''(\ell) - \lambda\left(v(\hat{\ell}(\ell)) - v(\ell)\right) - (1 - r\ell), v(\ell)\} \tag{31}$$

2. *The default threshold $\bar{\ell}$ is optimal and is determined by the indifference in* (31)

3. *Given $v(\ell)$ and the equity holders' policies, the price $p(\ell)$ solves*

$$rp(\ell) = r + (\sigma^2 - \mu)\ell p'(\ell) + \frac{\sigma^2}{2}\ell^2 p''(\ell) + \lambda\left(p(\hat{\ell}(\ell)) - p(\ell)\right) \tag{32}$$

$$p(\bar{\ell}) = \frac{(1-\theta)v(0)}{\bar{\ell}} \tag{33}$$

Furthermore, the first-best investment choice as defined in Definition 1 is

$$g^u = \frac{1}{\zeta(r-\mu)\left(\frac{1}{2}\left(\sqrt{1-\frac{2\lambda}{\zeta(r-\mu)^2}}-1\right)+1\right)} \tag{34}$$

Proof. See Appendix B. □

Unlike the one-shot case, we do not have closed-form solutions when $\lambda > 0$. Thus, we need to solve the model numerically using upwind finite difference methods. To do so, we add artificial reflecting barriers to the stochastic process, $v'(\ell_{\min}) = 0, v'(\ell_{\max}) = 0$, and $p'(\ell_{\min}) = 0$. The absorbing boundary condition for $p(\cdot)$ comes from (33) (i.e., the liquidation value of the firm at the time of default).

4.3 Analysis

Figure 5 depicts investment relative to first-best ($\tilde{g} \equiv g/g^u$) and post-investment leverage relative to its pre-investment level ($\hat{\ell}/\ell$) plotted against leverage when $\lambda = 0.2$ (left panel) and $\lambda = 0.3$ (right panel), for different values of κ. It shows that the repeated arrival of investment opportunities generates heterogeneous investment distortions, with low leverage firms tending to underinvest and high leverage firms tending to overinvest.[26,27] This heterogeneous effect of limited liability on equity holders' invest-

[26]To discipline parameters, we calibrate to moments from the firm dynamics (Sterk et al. (2021)) and investment spikes (Gourio and Kashyap (2007)) literature. The baseline parameters are $r = 0.0765, \sigma = 0.1534, \mu = -0.0514$, and $\zeta = 50.036$. See Online Appendix B.3 for more details.

[27]Grullon and Michaely (2002) show that payouts to equity holders (dividends plus equity repurchases) are around 50% of earnings. Because not all payouts in practice are financed by new debt issuance, we consider this to be an upper bound on average direct equity payouts (given by $\lambda\kappa$ in our model), leading us to consider values for κ between 0 and 1.

ment decisions is our key finding. Figure 5 also shows that firms may overinvest even if they are deleveraging. This observation further highlights that our mechanism differs from the mechanisms emphasized in earlier work on risk-shifting and debt dilution. We next discuss the main intuition behind equity holders' investment and leverage choices

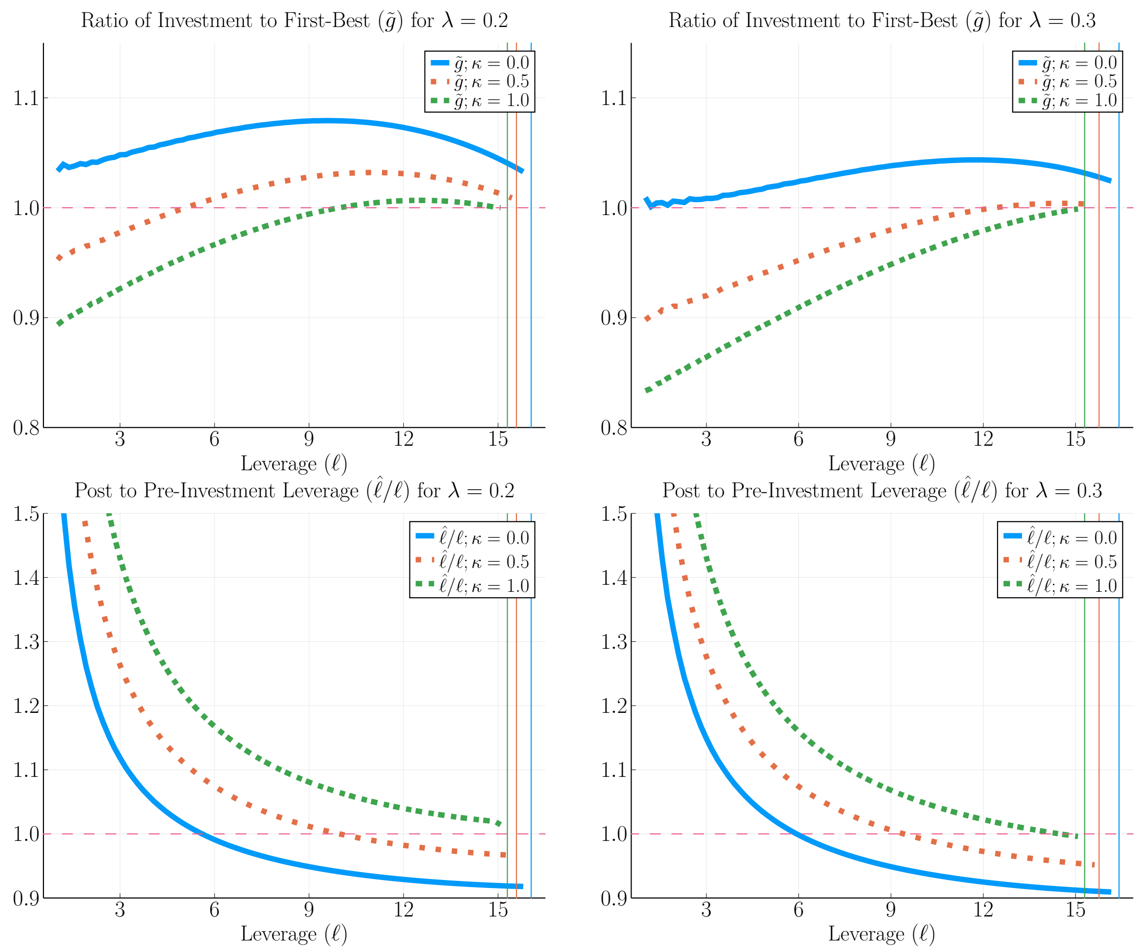

Figure 5: **The effect of equity payout restrictions on the investment-leverage relationship.** The top panel shows investment relative to first-best ($\tilde{g}$) against pre-existing leverage (ℓ), while the bottom panel shows the ratio of ex-post to ex-ante leverage ($\hat{\ell}/\ell$). Each line corresponds to a different value for the constraint on equity payouts κ. The case $\kappa = 0$ is the baseline case of no equity payouts from debt. The left and right panels show this for an arrival rate of new investment opportunities $\lambda = 0.2$ (left) and $\lambda = 0.3$ (right). Bankruptcy costs are set to zero ($\theta = 0$). Vertical lines indicate the default thresholds for each value of κ. For a discussion of model parameter values see the Online Appendix B.3.

Equity holders' choice of investment Consider first the effect of an increase in the arrival rate of investment opportunities, λ, on equity holders' investment decisions for a given level of κ. We see that a higher arrival of investment opportunities decreases equity holders' investment relative to the first-best level for any level of κ. This is driven by two effects. First, a higher λ implies that equity holders have more opportunities to dilute existing debt holders, if such dilution is profitable. Therefore, new creditors expect their claims to be diluted sooner and require to be compensated for that, which increases the cost of debt financing. Second, an increase in λ indirectly increases the cost of inefficient investment.[28] The first of these forces is responsible for the large decrease in investment when κ is high and among low leverage firms when κ is low, while the second effect explains a reduction in investment among high-leverage firms when κ is low.

Consider next the effect of an increase in equity holders' ability to make equity payouts, κ, for a given level of λ. We see that an increase in κ decreases the extent of equity holders' overinvestment. Moreover, this decrease is disproportionally larger for firms with low leverage inducing these firms to underinvest. This is because an increase in κ decreases the cost of dilution as equity holders can now increase their leverage by increasing their equity payouts instead of engaging in inefficient investment. This effect is the strongest when ℓ is relatively low since low leverage firms have the largest capacity to increase leverage (and, hence, dilute debt holders). Since new creditors anticipate this behavior, the cost of debt increases sharply for low leverage firms', leading them to underinvest. As κ increases, this mechanism becomes relevant also for firms with higher levels of leverage, and when $\kappa = 1$ most firms underinvest.

Finally, note that for the highest levels of leverage, Figure 5 shows that investment converges to the first-best as κ increases, similarly to the one-shot model. This is because for high enough ℓ and κ equity holders are able to issue so much debt that they optimally choose to default immediately after investment. This implies that debt holders immediately take over the firm and equity holders have no opportunities to dilute new debt holders' claims. Thus, as long as there are no deadweight bankruptcy costs (or these costs are small), equity holders' and new debt holders' incentives are

[28] To see this note that the last part of Proposition 7 implies that the first-best unconstrained investment is an increasing function of λ. Since the investment cost is strictly convex and is borne by equity holders, overinvestment is more costly when λ is high.

again aligned, just as in the model with one-shot investment.

Equity holders' choice of leverage Next, we consider equity holders' leverage policy given λ and κ. We see that low leverage firms increase their leverage more aggressively than high-leverage firms. This is because low leverage firms find it easier to increase their leverage: their initial leverage is low, so even a small debt issuance increases their leverage above its current level. On the other hand, the same amount of debt issuance for high leverage firms leads to a much more modest change in their post-investment leverage. Since increasing leverage is costly (because at least part of it is driven by inefficient investment whose costs are fully borne by equity holders), firms with high leverage adjust their leverage less aggressively than low leverage.

Somewhat more surprisingly, we see that when equity payout constraints are strict (i.e., when κ is low), firms with high leverage choose to deleverage. This is driven by the fact that when inefficient investment is the main way to increase leverage the investment needed to increase leverage is an increasing function of ℓ.[29] Therefore, as ℓ increases, inefficient investment needed to increase leverage becomes exceedingly costly and, hence, firms with high ℓ choose to deleverage. However, note that this deleveraging is still associated with overinvestment. This is because, on the margin, increasing investment above its efficient level and financing it with debt allows the equity holders to limit the value of cash flows generated by new investment that are captured by existing debt holders.

Finally, we see that an increase in κ increases the post-investment leverage of all firms. An increase in λ has a more subtle effect, but it tends to decrease the leverage of high leverage firms. This decrease is driven by the fact that the price of debt for high leverage firms actually increases when λ increases since these firms deleverage when they invest, and investment is now more frequent. As such, when λ increases these firms can finance their choices with less debt.

Comparison with the model with one-shot investment Two general observations arise when comparing the above results with the results derived in the model with one-shot investment. First, the tendency of equity holders' to overinvest extends to the model with repeated investment opportunities, and the extent of overinvestment decreases as the arrival rate of new investment opportunities increases. Second,

[29]See the discussion following Proposition 4 and Lemma 12 in the Online Appendix.

in contrast to the model with one-shot investment, allowing equity payouts financed with debt (i.e., $\kappa > 0$) need not improve the efficiency of firms' investment and may even induce equity holders to switch from overinvestment to underinvestment. Intuitively, when κ is high debt holders' concerns about the future dilution of their claims are exacerbated when equity holders can make direct payouts to themselves financed with debt more frequently.

Relation to existing literature It is useful to contrast our findings with predictions based on other models of financial frictions and with existing literature on debt overhang. Models of financial frictions that feature collateral constraints also assume that firms are protected by limited liability. However, these models also feature additional sources of market incompleteness such as private information (e.g., Bernanke et al. (1999) or Clementi and Hopenhayn (2006)) or ability to abscond funds (e.g., Buera et al. (2011) or Moll (2014)). In those models, the ability to issue equity payouts would have no effect on constrained firms' investment choices since, in those models, it is typically optimal to delay dividends (see, for example, Clementi and Hopenhayn (2006)). In addition, a higher arrival of investment opportunities would either have no effect (if collateral constraints are modeled as in Buera et al. (2011) or Moll (2014)) or would lead to an increase in investment relative to first-best due to an implied increase in firms' future profitability (if collateral constraints are modeled as in Clementi and Hopenhayn (2006)). In contrast, we find that equity payouts tend to decrease firms' investment relative to first-best, resulting in underinvestment by low leverage firms but limiting overinvestment by high-leverage firms. Furthermore, in our model, a higher arrival rate unambiguously decreases investment by all firms.

Our results are also related to the literature on debt overhang in dynamic models based on Leland (1994). This literature emphasized that in the presence of existing debt firms' underinvest, as predicted by Myers (1977) (see, for example, Hennessy (2004) or Diamond and He (2014)). These papers assume that equity holders finance their investment with equity. In contrast, we allow equity holders to choose their financing optimally. In parallel work, DeMarzo and He (2021) consider a similar model to ours and find that equity holders underinvest even though they can finance their investment with debt. The reason why our predictions differ lies in our modeling choices. DeMarzo and He (2021) assume that equity holders can continuously adjust their leverage and a bankruptcy results in complete loss of the firm's value (i.e., there is zero recovery value). While the assumption of no recovery value in default is convenient for modeling purposes, it implies that the "double-selling" mechanism emphasized in our paper is missing in their setting. Indeed, in our model if bankruptcy losses are very high, equity holders would also find it optimal to always underinvest in our model. However, for empirically plausible values of bankruptcy costs, overinvestment prevails (see Section 3.5).

Covenants In the model, for simplicity, we assumed that existing debt is not protected by covenants. However, there is a large existing literature that documents that covenants are commonly used to protect existing debtholders (see Smith Jr and Warner (1979), Billett et al. (2007b), Chava et al. (2010), Reisel (2014)).[30] Below, we discuss how commonly used covenants would affect our results. Our main conclusion is that, depending on their type, covenants are unlikely to resolve the issues and even further support our modeling choices.

Restrictions on payouts: We think of restrictions on payouts being captured by our model parameter κ. From Figure 5 we see that as these restrictions become tighter, they tend to mitigate underinvestment for low leverage firms but tend to exacerbate overinvestment for the remaining firms. While it is theoretically possible to devise payout restrictions that would restore first-best investment, such covenants would have to be state-contingent. It is therefore unlikely that optimality could be restored through such covenants in practice.

Secured debt restrictions: Secured debt restrictions (often referred to as negative pledge covenants) prohibit firms from issuing secured debt, unless all pre-existing debt also obtains a proportional claim to the same collateral. This covenant is typically used by unsecured lenders to protect themselves from dilution, though it may be difficult to enforce in practice (see Donaldson et al. (2019)). In our benchmark model, all debt already has equal priority claims in bankruptcy, thus this type of covenant does not have any bite.

Restrictions on leverage: Restrictions on leverage are relatively common covenants particularly for non-investment grade firms (see Billett et al. (2007b)). However, note that in our model investment distortions occur even at low to medium leverage levels,

[30] For theoretical analysis of covenants see Smith Jr and Warner (1979), Donaldson et al. (2019, 2020), and references therein.

i.e. even for firms that are unlikely to violate leverage restrictions. At the same time, the highest leverage firms in our model tend to even decrease leverage while overinvesting. It is therefore unlikely that covenants that restrict leverage can correct equity holders' investment incentives in our model.

Senior debt restrictions: Senior debt restrictions prohibit the firms from issuing senior debt. These types of covenants are empirically extremely rare affecting only 0.2% of firms in Billett et al. (2007b) post-2000 sample. In our model, this would imply that all new debt would have to be junior compared to the existing debt. As we show in the Appendix, this covenant would resolve the issue of overinvestment but at the cost of firms underinvesting for all levels of leverage.

Rating and net worth triggers: Some debt issues contain a put option allowing debt holders to sell the bond back to the company if its credit rating or net worth fall below a certain level. These provisions are rare in practice (less than 1.4% of firms in Billett et al. (2007b) post-2000 sample). Further, because our mechanism acts through default timing, it tends to be strongest for firms that are not immediately approaching bankruptcy and are hence less likely to violate any such provisions.

4.4 Dynamic Leverage, Investment, and Bankruptcy

Figure 6 uses a simple calibration of our model to illustrate that the incentives to over- or underinvest can be quantitatively meaningful and respond heterogeneously to a policy of restricting equity payouts. This figure compares the paths of leverage and real investment across equity payout restrictions. This figure shows simulated paths (averaged over 1000 paths) for two firms, one with high leverage and a second one with lower leverage. The left panels assume that equity payouts are prohibited ($\kappa = 0$), while the right panels permit equity buybacks ($\kappa = 0.5$).[31] The top panels show simulated leverage and the bottom panels simulated real investment. We consider two types of firms, depending on their initial leverage ratios. Low leverage firms (blue) are calibrated to have an initial interest coverage ratio of 4, consistent with Palomino et al. (2019) reporting an average interest coverage ratio of around 4 for the

[31] Grullon and Michaely (2002) estimate an equal-weighted ratio of total of equity payouts (including dividends, repurchases, etc.) to earnings of around 0.5 in 2000. We take this as an upper bound for equity payouts in our model because in practice not all dividends/equity buybacks are financed through new debt issuance.

period 1970-2017. High leverage firms (red) are calibrated to have an initial interest coverage ratio of 1.5, representing a typical below investment grade firm.[32]

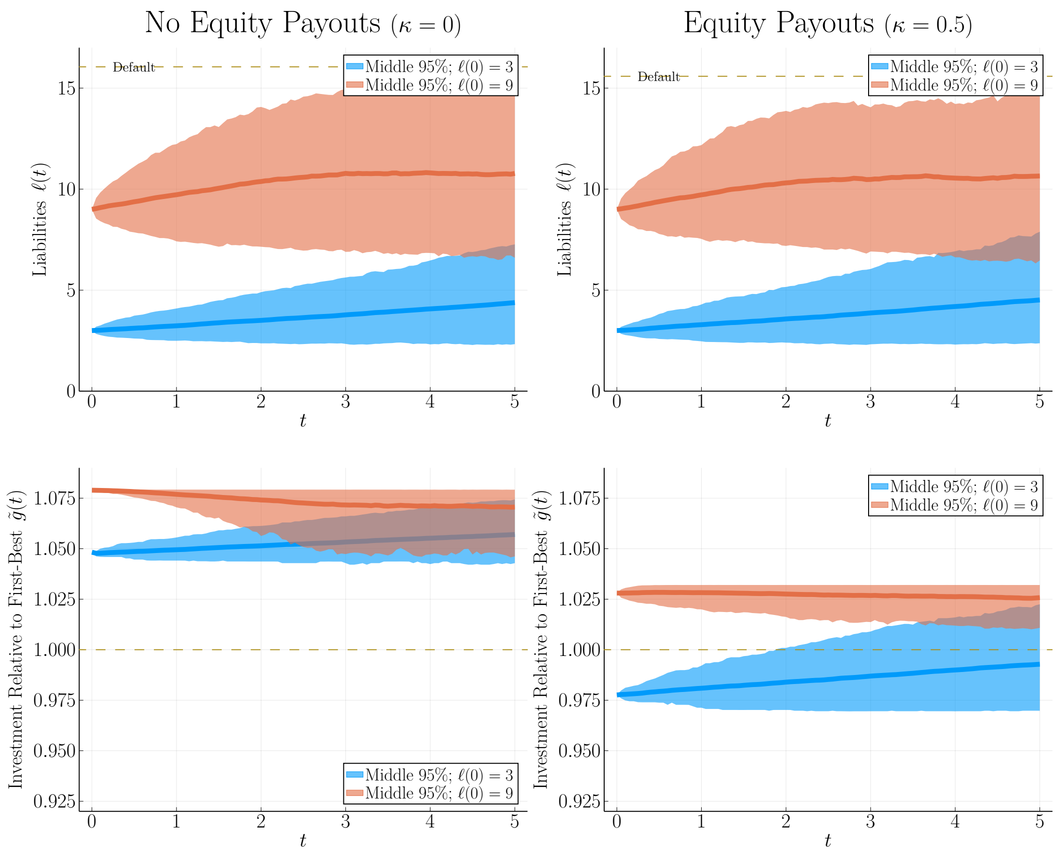

Figure 6: **Simulation of an ensemble of 1000 paths of leverage ℓ (top), investment relative to first-best $\tilde{g}$ (bottom) with and without equity payouts. blue=low initial leverage, red=high initial leverage.** Each panel shows the ensemble starting from $\ell(0) \in \{3, 9\}$ corresponding to interest coverage ratios of 4 and 1.5 respectively. The left panels use $\kappa = 0$ (no equity payouts) and the right panels use $\kappa = 0.5$ (constrained equity payouts from new debt). The investment arrival rate is set to $\lambda = 0.2$. The central line is the mean, the red shaded area shows the 2.5th and 97.5th percentiles. Moments are based on non-defaulted firms.

Comparing the two bottom panels shows how restricting equity payouts affects real investment for high and low leverage firms. In the left panel, where equity payouts from debt issuance are restricted to zero, investment is higher for both types of

[32] Palomino et al. (2019) found that 30% of creditors had an interest coverage ratio of 2 or less, and about 10% of borrowers had an interest coverage ratio of 1 or less.

firms. However, restricting equity payouts has heterogeneous effects, with the difference between the left and right panel being larger for firms with low initial leverage. These quantitatively heterogeneous effects also have heterogeneous efficiency implications. Restricting equity payouts moves the high leverage firm's investment up and exacerbates its tendency to overinvest. By contrast, restricting equity payouts from new debt leads the low leverage firm to switch from underinvestment to overinvestment. Our model therefore tells a cautionary tale about policies restricting equity payouts, as it predicts that the effects on real investment will be smaller and more likely to be inefficient if the initial leverage ratio is high.

The top panels show that permitting equity payouts leads to increases in leverage especially for firms with low initial leverage, as firms optimally take on more debt in order to finance direct payouts to equity holders. In addition, the default threshold is lower in the top right panel than the top left panel. The default threshold is lower when equity payouts are permitted due to dynamic considerations, as buyers of new debt price in future anticipated dilution, and thereby reduce equity holders' incentive to keep the firm as a going concern at any level of cash flows. While the changes in leverage and default thresholds from permitting equity payouts appear visually small, bankruptcy is a tail event and therefore affected by these changes. Within these simulations, the cumulative five-year bankruptcy rate for firms with initially high leverage increases when equity payouts are permitted (from 28.5% for the case of $\kappa = 0$ to 32.0% when $\kappa = 0.5$). The increase in bankruptcy rates arises through a combination of the incentive to lever up and the lower optimal leverage at which equity holders walk away from the firm, as captured by the lower default threshold.

5 Empirical Implications

We now turn to the empirical implications of the model. A central model implication is that high leverage firms tend to engage in unprofitable investment whereas low leverage firms tend to forego otherwise profitable investments. These predictions are in line with the empirical results of Kahle and Stulz (2013), who found that highly levered firms increased their investment relative to low leverage firms during the financial crisis of 2008-2009. However, our model makes predictions for investment relative to the first-best and not necessarily about the absolute level of investment. We therefore use Compustat data 1988-2018 and provide suggestive evidence that investment of high leverage firms leads to relatively lower productivity compared to low leverage firms.

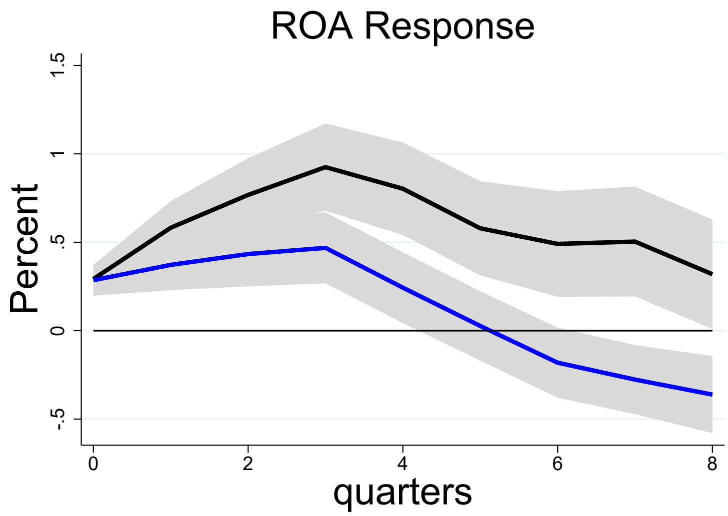

Figure 7: **Empirical impulse responses of firm profitability to investment for high (blue) and low (black) leverage firms in Compustat data 1988-2018.** This figure plots the impulse responses and associated 90% confidence interval bands of return on assets (ROA, in percent) to a one standard deviation increase in *CAPX/Assets* of firm i at time t. We compute Jordà (2005) local projections with firm and time fixed effects: $ROA_{i,t+h} = \alpha_i + \alpha_t + \beta_0 CAPX/Assets_{i,t} \times HighLev_{i,t-1} + \beta_1 CAPX/Assets_{i,t} + \beta_2 HighLev_{i,t-1} + \gamma ROA_{i,t-1} + \varepsilon_{i,t}$. The variable $CAPX/Assets_{i,t}$ is standardized to have a unit standard deviation. The variable $\text{HighLev}_{i,t}$ takes a value of one if $Lev_{i,t}$ is in the top quartile in quarter t, and zero otherwise. The figure shows the coefficients $\beta_0 + \beta_1$ (high leverage) and β_1 (low leverage). Our sample consists of a quarterly panel 1988.Q1-2018.Q1 and excludes the middle 50% of firms by $t-1$ leverage. Standard errors are double-clustered by firm and quarter. For a detailed data description see Online Appendix B.1.

Figure 7 shows impulse responses of firm profitability to a one-standard deviation increase in investment, estimated via local projections (Jordà (2005)). We separately show the responses for firms in the top quartile of pre-investment leverage (blue) and bottom quartile of pre-investment leverage (black), together with 90% confidence intervals. The impulse responses for the low leverage firms are as expected, if these firms face profitable investment opportunities and only a weak incentive to finance with debt. Following a one-standard deviation increase in the CAPX/Assets ratio, low leverage firms' return on assets increases by about one percentage point three quarters after the investment. This magnitude is not only statistically but also economically significant, compared to median return on assets of 6.38% in our sample.

The impulse response for high leverage firms contrasts with the one of low leverage firms, consistent with the model prediction that high leverage firms have an incentive to debt finance their investment and invest more than the first-best. Following a one-standard deviation increase in investment, high-leverage firms' profitability initially increases. However, the profitability response is significantly lower than for low leverage firms at almost all horizons, and even becomes negative by quarter eight. The regressions corresponding to Figure 7 are reported in the Online Appendix, where we also show that the results are robust to including the middle 50% of firms and using a continuous leverage variable rather than a high-leverage dummy.

6 Conclusion

This paper provides a new simple model showing that limited liability can inefficiently distort investment away from low leverage firms and towards highly levered firms. Taken together, in our model high leverage firms have an incentive to overinvest because debt financed investment allows them to (1) dilute current debt holders' coupon claims by increasing leverage and bringing forward bankruptcy; or (2) limit the gains from investment to debt holders from new investment if increasing leverage is too costly. At the same time, our model predicts that low leverage firms with frequent investment opportunities tend to underinvest. This is because these firms have the largest capacity to dilute the coupons of debt holders in the future, which is anticipated by creditors who require high compensation for lending to those firms. These mechanisms are robust if all debt is collateralized and strengthen when equity holders can deplete equity by making equity payouts from new debt issuances.

The analysis in this paper has important policy implications—especially during times with enormous government support for corporations, as in the response to the Covid-19 crisis and the financial crisis of 2008-2009. Our model emphasizes that government programs that increase firms' debt burden may have undesired consequences, as higher leverage induces debt financed overinvestment among highly levered firms. Another lesson from our analysis is that restrictions on equity payouts are no cure-all to excessive leverage and low investment, and the effects of such a policy are heterogenous by initial firm leverage. In our model, equity payout restrictions reduce bankruptcy and raise investment towards the first-best when initial leverage is low. However, they tend to exacerbate inefficient overinvestment when initial firm leverage is high.

To emphasize the role of limited liability, we abstracted from other related mechanism and focused on decisions of a single firm. The model can be extended to allow for information frictions in the quality of collateral (Gorton and Ordoñez (2014)). If collateral quality is cyclical, debt financing would likely be further distorted relative to our benchmark. The channel emphasized in this paper likely has potentially important aggregate implications, both through investment distortions across firms (e.g., Khan and Thomas (2013), Moll (2014), and Buera et al. (2011)), and the cyclicality of the costs of financial distress (e.g. Atkeson et al. (2017)). We believe that investigating these general equilibrium consequences of limited liability will be fruitful.

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Appendix A Proofs for Section 2

A.1 Proof of Proposition 1

Proof of Proposition 1. To find the post-investment value of equity $V(Z, L)$ that solves equity holders' default problem as described by (5)- (7) we use the method of undetermined coefficients with the guess

$$V(Z, L) = \frac{1}{r-\mu}\left(Z + \frac{\omega}{\eta}Z^{-\eta}\right) - L \tag{A.1}$$

Using this guess in (5) and equating undetermined coefficients we arrive at the equation $2(r + \eta\mu) = \eta(1 + \eta)\sigma^2$. We solve this quadratic equation for η and note that the smaller of the two roots is explosive and, hence, it violates the transversality condition. Therefore,

$$\eta = \frac{(\mu - \sigma^2/2) + \sqrt{(\mu - \sigma^2/2)^2 + 2\sigma^2 r}}{\sigma^2} \tag{A.2}$$

Next, we substitute the guess (A.1) into (7) to find that $\omega = \underline{Z}^{\eta+1}$. Then we use (A.1) and the expression for ω in (6) to find that the default threshold is given by

$$\underline{Z} = \frac{(r-\mu)\eta}{\eta+1}L \tag{A.3}$$

(A.3) defines the default threshold and completes derivations of $V(Z, L)$. Since the default threshold depends on equity holders' liabilities L we donate it by $\underline{Z}(L)$.

Next, we show that the value of equity can be expressed as $V(Z, L) = v(\ell)Z$, where $\ell \equiv L/Z$, and derive the expression for $v(\cdot)$. First, we note that

$$\frac{\underline{Z}(L)}{Z} = \frac{(r-\mu)\eta}{\eta+1}\frac{L}{Z} = \frac{(r-\mu)\eta}{\eta+1}\ell, \tag{A.4}$$

Next, we use the expressions for ω and for $\underline{Z}/Z$ found in (A.1) to obtain

$$V(Z, L)/Z = \frac{1}{r-\mu} + \frac{\chi}{\eta+1}\ell^{\eta+1} - \ell, \tag{A.5}$$

where

$$\chi \equiv \left(\frac{(r-\mu)\eta}{\eta+1}\right)^{\eta} \tag{A.6}$$

Finally, we set $v(\ell) = \frac{1}{r-\mu} - \ell(1 - s(\ell))$, were $s(\ell) = \frac{\chi}{\eta+1}\ell^{\eta}$, which implies that $V(Z, L)/Z = v(\ell)$.

To find the liquidation value of the firm we note that equity holders walk away when cash flows Z reach the default threshold $\underline{Z}(L)$. At that time debt holders take over the firm so its liabilities are reset to $L = 0$ but the firm loses fraction $\theta \in [0, 1]$ of its value. It follows that the liquidation value of the firm, from creditors' perspective, is given by

$$V((1-\theta)\underline{Z}(L), 0) = \frac{(1-\theta)\underline{Z}(L)}{r-\mu} \tag{A.7}$$

Thus, the liquidation value per unit of liabilities is given by

$$\frac{V((1-\theta)\underline{Z}(L), 0)}{L} = \frac{(1-\theta)\eta}{\eta+1}, \tag{A.8}$$

where we used the definition of $\underline{Z}(L)$ (see (A.3).

$\square$

A.2 Proof of Proposition 2

Proof of Proposition 2. **(Derivations of debt prices)** Let T be the first-time cash flows, Z, reach the default threshold $\underline{Z}(L)$. Since Z follows a geometric Brownian motion (see (1)) we have

$$\mathbb{E}_T\left[e^{-rT}\right] = \exp\left(\frac{-(\mu - \sigma^2/2) - \sqrt{(\mu - \sigma^2/2)^2 + 2\sigma^2 r}}{\sigma^2}\left(\log Z - \log \underline{Z}(L)\right)\right) \tag{A.9}$$

as shown in Jeanblanc et al. (2009). Using the definition of η and χ (see (A.2) and (A.6), respectively) and the expression obtained in (A.3) we conclude that

$$\mathbb{E}_T\left[e^{-rT}\right] = (Z/\underline{Z}(L))^{-\eta} = \chi\ell^{\eta} \tag{A.10}$$

Using (A.10) we see that

$$P^C(Z,L) \equiv \frac{p^C(Z,L)}{r} = \frac{1}{r}\left[1-\chi\ell^\eta\right] = \frac{1}{r}\left[1-(1+\eta)s(\ell)\right], \tag{A.11}$$

where $s(\ell) = \chi/(1+\eta)\ell^\eta$ and $\ell = L/Z$. Note that the above equation implies that the relevant state variable is ℓ. Hence, we can express the price of the coupon claims of a defaultable consol as

$$P^C(\ell) \equiv \frac{p^C(\ell)}{r} = \frac{1}{r}\left[1-(1+\eta)s(\ell)\right] \tag{A.12}$$

Next, we consider the price of a bankruptcy claim $P^B(Z,L)$. Note that

$$P^B(Z,L) \equiv \frac{p^B(Z,L)}{r} = \frac{V((1-\theta)\underline{Z}(L),0)}{rL}\mathbb{E}_T\left[e^{-rT}\right] \tag{A.13}$$

Using (A.8), (A.9), and the definition of $s(\ell)$ we obtain

$$P^B(Z,L) \equiv \frac{p^B(Z,L)}{r} = \frac{1}{r}(1-\theta)\eta s(\ell), \tag{A.14}$$

We see again that the relevant state variable is ℓ and, thus, we can write the price of claims bankruptcy as $P^B(\ell) \equiv \frac{p^B(\ell)}{r}$.

From the above discussion it follows that the leverage ℓ is the relevant state for pricing defaultable consols so that we can write $P(Z,L) = P(\ell)$ and $p(Z,L) = p(\ell)$. Putting together (A.12) and (A.14) we obtain

$$P(\ell) = \frac{p(\ell)}{r} = \frac{1}{r}\left[1-(1-\theta\eta)s(\ell)\right] \tag{A.15}$$

(The Budget Constraint) Equity holders issue debt to finance their equity payouts, M, and a fraction ψ of the investment cost $Zq(g)$. Let K denote the quantity of new bonds issued by equity holders to finance $\psi Zq(g) + M$. Then, K has to satisfy the following budget constraint

$$P(\hat{\ell})K = \psi Zq(g) + M, \tag{A.16}$$

where $\hat{\ell}$ is the post-investment leverage. Next, we relate K to the change in leverage $\hat{L} - L$. Recall that L is defined as the present discounted value (PDV) of liabilities.

Since each unit of debt promises a payment of a constant coupon of 1 and agents discount these payments at a rate r it follows the PDV of the cash flows promised to new debt holders is given by K/r. Therefore, the post-investment liabilities are given by $\hat{L} = L + \frac{K}{r}$. It follows that

$$K = r(\hat{L} - L) \tag{A.17}$$

Let $\hat{\ell} = \hat{L}/(Z(1+g))$. Substituting the above expression for K into the budget constraint (A.16), dividing both sides of the resulting equation by Z, setting and using the definition of $p(\cdot)$ (see (A.15)) we obtain

$$p(\hat{\ell})\left(\hat{\ell}(1+g) - \ell\right) = \psi q(g) + m, \tag{A.18}$$

which corresponds to (18) in the text. □

(Collateralized Debt) We formally show that (1) the price of debt where default claims are pledged as collateral (as opposed to the proportional claims to the firm in bankruptcy) is identical to the price of debt in our baseline model under suitable pledgeability constraint and (2) under pledgeability constraint, equity holders choices respect constraint (19) that we impose in the benchmark model.

To consider collateralized debt we interpret the value of the firm in bankruptcy, $\frac{\eta}{1+\eta}L$, as pledgeable collateral.[33,34] Let C denote the collateral promised to the existing debt holders. We assume that the firm used all of its assets in available in bankruptcy as collateral when it issued existing debt in the past —as would be optimal— so that $C = \frac{\eta}{1+\eta}L$. In order for the equity holder to deliver this value following their investment and financing choices, it has to be the case that

$$\frac{\eta}{1+\eta}\hat{L} \geq \frac{\eta}{1+\eta}L \tag{A.19}$$

[33]This interpretation is possible since the value of the firm in bankruptcy is known with certainty since investment is deterministic and the post-investment paths for $Z(t)$ are continuous.

[34]As in the benchmark model, equity holders' liability are equal to the PDV of promised coupon payments. For that reason, equity holders' liabilities are unchanged if debt is collateralized or carry a proportional claim to the value of the firm in default.

and that only

$$\min\left\{\frac{\hat{Z}}{r-\mu}, \frac{\eta}{1+\eta}\hat{L}\right\} - \frac{\eta}{1+\eta}L \tag{A.20}$$

can be used for collateral for the new debt.[35] We refer to (A.19) and (A.20) as the pledgeability constraint.

Now, consider the price of new collateralized debt, denoted by $\hat{P}^S(\hat{Z}, \hat{L}, \hat{C})$, where $\hat{Z}$ and $\hat{L}$ are post-investment cash flows and liabilities, respectively, and $\hat{C}$ is the value of collateral promised to new debt holders. Then, following the same argument as we used in the proof of Proposition 2, we obtain

$$\hat{P}^S(\hat{Z}, \hat{L}, \hat{C}) = P^C(L, Z) + \frac{\hat{C}}{r(\hat{L}-L)}\chi\left(\frac{\hat{L}}{\hat{Z}}\right)^{\eta}, \tag{A.21}$$

where $P^C(L, Z)$ is defined in (14). Since the value of new debt is increasing in collateral promised and equity holders do not get to keep any collateral unused, it follows that $\hat{C} = \min\left\{\frac{\hat{Z}}{r-\mu}, \frac{\eta}{1+\eta}\hat{L}\right\} - \frac{\eta}{1+\eta}L$. Therefore, if $\frac{\eta}{1+\eta}\hat{L} \leq \frac{\hat{Z}}{r-\mu}$ (as we argue below), then

$$\hat{P}^S(\hat{Z}, \hat{L}, \hat{C}) = P^C(\hat{L}, \hat{Z}) + \frac{1}{r}\frac{\eta}{1+\eta}\chi\left(\frac{\hat{L}}{\hat{Z}}\right)^{\eta} = P^C(\hat{L}, \hat{Z}) + P^B(\hat{L}, \hat{Z}) = P(\hat{L}, \hat{Z}) \tag{A.22}$$

This implies that the price of new debt is exactly the same as in the benchmark model. A similar argument can be used to show that the price of existing collateralized debt, which we denote by $P^S(Z, L, C)$, is equal to $P(Z, L)$.

We now argue that equity holders find it optimal to choose $\hat{L} \leq \frac{\hat{Z}}{r-\mu}$. To see that this is the case, note that choosing $\hat{L} > \frac{\hat{Z}}{r-\mu}$ yields the same payoff as $\hat{L} = \frac{\hat{Z}}{r-\mu}$. This is because in both cases equity holders decide to walk away from the firm and the total value of the new debt issued is the same. From the above discussion, we know

[35] The maximum value that equity holders can possibly promise as collateral is the post-investment value of the firm (if they choose to default immediately), $\frac{\hat{Z}}{r-\mu}$, net of the collateral promised to existing debt holders. This happens if equity holders choose $\frac{\eta}{1+\eta}\hat{L} > \frac{\hat{Z}}{r-\mu}$. Otherwise, if equity holders choose optimally to continue operating the firm, the maximal value of collateral they can promised to new debt holders is $\frac{\eta}{1+\eta}(\hat{L}-L)$.

that if $\hat{L} \leq \frac{\hat{Z}}{r-\mu}$ then equity holders' problem is the same as in the baseline model. Thus, if $\kappa = 0$ or $\kappa < \underline{\kappa}$ (where $\underline{\kappa}$ is defined as in Proposition 5) then equity holders strictly prefer to choose $\hat{L} \leq \frac{\hat{Z}}{r-\mu}$. If $\kappa \geq \underline{\kappa}$ then equity holders are indifferent between any $\hat{L} \geq \frac{\hat{Z}}{r-\mu}$ and so we can assume that they choose $\hat{L} = \frac{\hat{Z}}{r-\mu}$. This completes our argument.

A.3 Proof of Proposition 3

Proof of Proposition 3. To derive (24) consider equity holders' objective function (20) and substitute the budget constraint ((A.18)) to eliminate $\psi q(g) + m$ and obtain

$$(1+g)v(\hat{\ell}) - q(g) + p(\hat{\ell})(\hat{\ell}(1+g) - \ell) \tag{A.23}$$

Using the expression we found for $v(\hat{\ell})$ (Proposition 1), (A.23) can be written as

$$\frac{1+g}{r-\mu} - (1+g)\hat{\ell}\left(1 - s(\hat{\ell})\right) - q(g) + p(\hat{\ell})\left(\hat{\ell}(1+g) - \ell\right) \tag{A.24}$$

Using the observation $p(\hat{\ell}) = 1 - (1+\theta\eta)s(\hat{\ell})$ and simplifying (A.24) we obtain

$$\frac{1+g}{r-\mu} - p(\hat{\ell})\ell - q(g) - \theta\eta s(\hat{\ell})\hat{\ell}(1+g) \tag{A.25}$$

Defining $H(\hat{\ell}) = \theta\eta s(\hat{\ell})$ and using this definition in (A.25) we obtain the equity holders' objective function (24) in Proposition 3.

To obtain (27), note that $v(0) = \frac{1}{r-\mu}$. Thus, (23) implies that g^u is a unique solution to the F.O.C. given by $0 = \frac{1}{r-\mu} - q'(g^u)$.

$\square$

Appendix B Repeated Investment Derivations

This section derives the ODEs for a repeated investment decisions, which introduces a controlled jump-process.

Assume that upon an arrival of an investment opportunity, the state jumps to a deterministic function of the current state, $\hat{\ell}(\ell)$. Define the jump size as $\tilde{g}(\ell) \equiv \hat{\ell}(\ell) - \ell$.

Then the SDE for ℓ is

$$d\ell_t = (\sigma^2 - \mu)\ell_t \mathrm{d}t + \sigma \ell_t d\mathbb{W}_t + \left(\hat{\ell}(\ell) - \ell\right) d\mathbb{N}_t \tag{B.1}$$

where $\mathbb{N}_t$ is a homogeneous Poisson process with arrival rate $\lambda \geq 0$.

Firm's HJBE First, we will derive the HJBE in ℓ-space without the jumps, and add them. Set $V(Z, L) = Zv(L/Z)$ and differentiate w.r.t. Z

$$\boldsymbol{\partial}_Z V(Z, L) = v(L/Z) - \frac{L}{Z}\boldsymbol{\partial}_\ell v(L/Z) = v(\ell) - \ell \boldsymbol{\partial}_\ell v(\ell) \tag{B.2}$$

$$\boldsymbol{\partial}_{ZZ} V(Z, L) = \frac{L^2}{Z^3}\boldsymbol{\partial}_{\ell\ell} v(L/Z) = \frac{1}{Z}\ell^2 \boldsymbol{\partial}_{\ell\ell} v(\ell) \tag{B.3}$$

Use the ODE in (5), divide by Z, and use the above derivatives to obtain

$$(r - \mu)v(\ell) = 1 - r\ell - \mu\ell\boldsymbol{\partial}_\ell v(\ell) + \frac{\sigma^2}{2}\ell^2\boldsymbol{\partial}_{\ell\ell} v(\ell) \tag{B.4}$$

Default Decision The notation denotes $\cdot|_\ell$ as the evaluation of a function at ℓ.

For the firm's boundary conditions, add in artificial reflecting barriers at some $\ell_{\min}$ and $\ell_{\max}$. We will ensure that the equilibrium $\ell_{\min} < \overline{\ell}$ so it is never binding in the solution, the $\ell_{\max}$ will be chosen large enough to not effect the solution.

Then, for the firm, we can write the DVI for their stopping problem as

$$u(c) \equiv 1 - r\ell \tag{B.5}$$

$$\mathcal{L}_v \equiv r - \mu + \mu\ell\boldsymbol{\partial}_\ell - \frac{\sigma^2}{2}\ell^2\boldsymbol{\partial}_{\ell\ell} - \lambda\left(\cdot|_{\ell+\tilde{g}(\ell)} - \cdot|_\ell\right) \tag{B.6}$$

$$0 = \min\{\mathcal{L}_\ell v(\ell) - u(\ell), v(\ell)\} \tag{B.7}$$

$$\boldsymbol{\partial}_\ell v(l_{\min}) = 0 \tag{B.8}$$

$$\boldsymbol{\partial}_\ell v(l_{\max}) = 0 \tag{B.9}$$

We would numerically find a $\bar{\ell}$ which fulfills the indifference point, and then find the value of liquidation per unit of PV of liabilities is

$$v^{\text{liq}} \equiv (1 - \theta)\frac{\lim_{\ell\to 0} v(\ell)}{\bar{\ell}} \tag{B.10}$$

Bond Pricing The price of a bond, $P(Z, L)$ pays 1 unit until default. The ODE in the continuation region without jumps is

$$rP(Z, L) = 1 + \mu Z \boldsymbol{\partial}_Z P(Z, L) + \frac{\sigma^2}{2} Z^2 \boldsymbol{\partial}_{ZZ} P(Z, L) \tag{B.11}$$

Take the defintion $rP(Z, L) \equiv p(L/Z)$ and differentiate with respect to Z

$$r\boldsymbol{\partial}_Z P(Z, L) = -\frac{1}{Z} \ell \boldsymbol{\partial}_\ell p(\ell) \tag{B.12}$$

$$r\boldsymbol{\partial}_{ZZ} P(Z, L) = \frac{1}{Z^2} \left(2\ell \boldsymbol{\partial}_\ell p(\ell) + \ell^2 \boldsymbol{\partial}_{\ell\ell} p(\ell) \right) \tag{B.13}$$

Multiply by r and substitute the derivatives into (B.11)

$$rp(\ell) = r + (\sigma^2 - \mu)\ell \boldsymbol{\partial}_\ell p(\ell) + \frac{\sigma^2}{2} \ell^2 \boldsymbol{\partial}_{\ell\ell} p(\ell) \tag{B.14}$$

In default, the bond is entitled to a share of rL units of the liquidation value $V((1-\theta)\underline{Z}(L), 0)$, hence $P(\overline{\ell}) = \frac{V((1-\theta)\underline{Z}(L),0)}{rL}$. Divide by r and use the definitions of v^{liq} and $p(\cdot) = P(\cdot)/r$ to find that the boundary condition is $p(\overline{\ell}) = v^{\text{liq}}$.

Summarizing, bond pricers take v^{liq} and $\overline{\ell}$ as given, and then solve

$$\mathcal{L}_p \equiv r - (\sigma^2 - \mu)\ell \boldsymbol{\partial}_\ell - \frac{\sigma^2}{2} \ell^2 \boldsymbol{\partial}_{\ell\ell} - \lambda \left(\cdot|_{\ell + \tilde{g}(\ell)} - \cdot|_\ell \right) \tag{B.15}$$

$$\mathcal{L}_p p(\ell) = r \tag{B.16}$$

$$\boldsymbol{\partial}_\ell p(\ell_{\min}) = 0 \tag{B.17}$$

$$p(\overline{\ell}) = v^{\text{liq}} \tag{B.18}$$

where the lower boundary is an artificial reflecting barrier and the upper boundary is the liquidation absorbing barrier.

Investment Finally, the objective function of the firm at every arrival point λ remains to maximize the equity value. Given an equilibrium $p(\ell)$ and $v(\ell)$ functions—consistent with the optimal jump process, the agent solves the problem described by (20)–(22).

First-Best The first-best is derived through a guess-and-verify approach. First, guess that the user would choose a constant g due to the homotheticity of the problem.

With that, the unnormalized Bellman equation (with jumps) is

$$rV(Z) = Z + \mu Z V'(Z) + \frac{\sigma^2}{2} Z^2 V''(Z) + \lambda \max_g \left\{ V((1+g)Z) - V(Z) - \zeta \frac{g^2}{2} \right\} \tag{B.19}$$

Take the first-order condition

$$\zeta g Z = Z V'((1+g)Z) \tag{B.20}$$

Guess the solution to the problem is $V(Z) = AZ$ for an undetermined Z, and subtitute into the (B.19) and solve for A to find,

$$A = \frac{1 - \frac{1}{2}\zeta g^2 \lambda}{-g\lambda - \mu + r} \tag{B.21}$$

Similarly, substitute the guess into (B.20) to find $g = \frac{A}{\zeta}$. Use this expression to eliminate A in (B.21), solve the quadratic for g, and choose the positive root to find,

$$g^u = \frac{1}{\zeta(r-\mu)\left(\frac{1}{2}\left(\sqrt{1 - \frac{2\lambda}{\zeta(r-\mu)^2}} - 1\right) + 1\right)} = \frac{2}{\sqrt{\zeta\left(\zeta(r-\mu)^2 - 2\lambda\right)} + \zeta(r-\mu)} \tag{B.22}$$

In addition, given that $V(Z) = AZ$ and noting that $V(Z, 0)$ we obtain $v(0) = \frac{V(Z,0)}{Z} = A$. Consequently, for a default threshold $\bar{\ell}$, the liquidation value per unit of defaultable console in (B.10) is,

$$v^{\text{liq}} = \frac{1-\theta}{\bar{\ell}} \frac{1 - \frac{1}{2}\zeta (g^u)^2 \lambda}{r - \mu - g^u \lambda} \tag{B.23}$$

When $\lambda = 0$, these all nest the $g^u = \frac{1}{\zeta(r-\mu)}$ case.