Dear OPERA collaboration

I worked at CERN from 1992 to 1997 in the WA98 experiment as well as in the R&D 28 project. Although I haven't done much HEP since, I eagerly studied your interesting publication on the neutrino velocity.

You want to continue the investigation of "possible still unknown systematic effects that could explain the observed anomaly" and, to summarise my concern, I found that the Coriolis force acting on the neutrino during its journey from the source to the detector has an effect that should not be neglected in the analysis. The neutrino covers an additional distance in the order of 10cm because its path is curved rather than straight. Or, observed from an inertial system, the detector moves faster than the source due to its lower latitude on Earth.

I will do the calculation of this effect now using approximations where it is possible. In a later analysis, all calculations can easily be modified to get more accurate results.

You assume the neutrinos to travel from Geneva to Grand Sasso following a straight line. This would be true if the Earth was an inertial system, and it goes without saying that this is not the case since the Earth rotates around its axis. The flight paths of the neutrinos as seen from Earth are curved due to the Coriolis force

$$\vec{F}_C = 2 \cdot m \cdot (\vec{v} \times \vec{\omega})$$

leading to a neutrino acceleration of

$$\vec{a}_C = 2 \cdot (\vec{v} \times \vec{\omega}) .$$

In the OPERA experiment; the neutrinos have an approximate velocity of

$$\vec{v} = 0 \cdot \hat{v}_r + \frac{c}{\sqrt{2}} \cdot \hat{v}_\varphi + \frac{c}{\sqrt{2}} \cdot \hat{v}_\vartheta ,$$

Where c is the speed of light, and where $\hat{v}_r$ (upward), $\hat{v}_\varphi$ (east) and $\hat{v}_\vartheta$ (south) are the unit vectors in the polar coordinate system. This velocity points exactly into a southeast direction, and its magnitude is

$$v = \sqrt{0^2 + \left(\frac{c}{\sqrt{2}}\right)^2 + \left(\frac{c}{\sqrt{2}}\right)^2} = c \ .$$

Of course, the heading of the neutrinos is not exactly southeast, but this is a very good approximation (as I checked on Google Earth).

The angular velocity of the Earth in polar coordinates depends on the latitude λ of the location on Earth and is well known to be given by the expression

$$\vec{\omega} = \omega \cdot (\sin(90° - \lambda) \cdot \hat{v}_r + 0 \cdot \hat{v}_\varphi + \cos(90° - \lambda) \cdot \hat{v}_\vartheta).$$

where $\omega = \frac{2 \cdot \pi}{T}$ with T=24h.

Therefore it is

$$\begin{aligned}
\vec{a}_C &= 2 \cdot (\vec{v} \times \vec{\omega}) \\
&= 2 \cdot \left[\frac{c}{\sqrt{2}} \cdot \hat{v}_\varphi + \frac{c}{\sqrt{2}} \cdot \hat{v}_\vartheta \right] \times [\omega \cdot (\sin(90° - \lambda) \cdot \hat{v}_r + \cos(90° - \lambda) \cdot \hat{v}_\vartheta)] = \\
&= 2 \cdot \left[\frac{c}{\sqrt{2}} \cdot \hat{v}_\varphi + \frac{c}{\sqrt{2}} \cdot \hat{v}_\vartheta \right] \times [\omega \cdot (\sin(90° - \lambda) \cdot \hat{v}_r + \cos(90° - \lambda) \cdot \hat{v}_\vartheta)] \\
&= \sqrt{2} \cdot c \cdot \omega [\hat{v}_\varphi + \hat{v}_\vartheta] \times [\sin(90° - \lambda) \cdot \hat{v}_r + \cos(90° - \lambda) \cdot \hat{v}_\vartheta]
\end{aligned}$$

The neutrino's latitude is ca. 42° at Grand Sasso and ca. 46° at Geneva. Using for simplicity an "average value" of 45°, it follows that

$$\begin{aligned}
\vec{a}_C &= \sqrt{2} \cdot c \cdot \omega \cdot [\hat{v}_\varphi + \hat{v}_\vartheta] \times [\sin(90° - \lambda) \cdot \hat{v}_r + \cos(90° - \lambda) \cdot \hat{v}_\vartheta] \\
&= \sqrt{2} \cdot c \cdot \omega \cdot [\hat{v}_\varphi + \hat{v}_\vartheta] \times [\sin(45°) \cdot \hat{v}_r + \cos(45°) \cdot \hat{v}_\vartheta] \\
&= \sqrt{2} \cdot c \cdot \omega \cdot [\hat{v}_\varphi + \hat{v}_\vartheta] \times \left[\frac{1}{\sqrt{2}} \cdot \hat{v}_r + \frac{1}{\sqrt{2}} \cdot \hat{v}_\vartheta \right] \\
&= c \cdot \omega \cdot [\hat{v}_\varphi + \hat{v}_\vartheta] \times [\hat{v}_r + \hat{v}_\vartheta] \\
&= c \cdot \omega \cdot [\hat{v}_\varphi + \hat{v}_\vartheta] \times [\hat{v}_r + \hat{v}_\vartheta] \\
&= c \cdot \omega \cdot (\hat{v}_\varphi \times \hat{v}_r + \hat{v}_\vartheta \times \hat{v}_r + \hat{v}_\varphi \times \hat{v}_\vartheta + \hat{v}_\vartheta \times \hat{v}_\vartheta) \\
&= c \cdot \omega \cdot (\hat{v}_\vartheta - \hat{v}_\varphi - \hat{v}_r + \vec{o}) \\
&= c \cdot \omega \cdot (\hat{v}_\vartheta - \hat{v}_\varphi - \hat{v}_r)
\end{aligned}$$

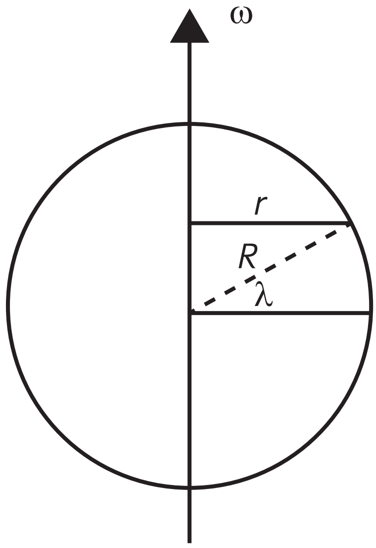
Due to the approximation of an angle of 45°, this vector points southwest and into the Earth, accelerating the neutrino with

$$a_C = \sqrt{3} \cdot c \cdot \omega$$

Pointing southwest, this acceleration vector is exactly perpendicular to the velocity as it could be expected by the relation $\vec{a}_C = 2 \cdot (\vec{v} \times \vec{\omega})$. It therefore doesn't change the magnitude of the velocity, but its direction, and the neutrino doesn't follow a straight line, but it performs (part of) a circular motion. It therefore covers a longer distance between Geneva and Grand Sasso than taken into account in the analysis so far.

To calculate the length of the additional flight path, it is easier to observe the neutrino from an inertial system (located far above Earth). Now, the particle follows a straight line when travelling from the source to the detector, but the source and the detector have different velocities due to the different latitudes of their locations: It is

$$\begin{aligned} v_\lambda &= \omega \cdot r \\ &= \omega \cdot R_e \cdot \cos\lambda \end{aligned}$$



where ω is again the angular speed of the Earth, R_e is the Earth's radius and λ is the latitude of the location.

It is approximately

$$v_{source} = \omega \cdot R_e \cdot \cos 46°$$

and

$$v_{det\,ector} = \omega \cdot R_e \cdot \cos 42°\,,$$

and the speed of the detector relative to the source is

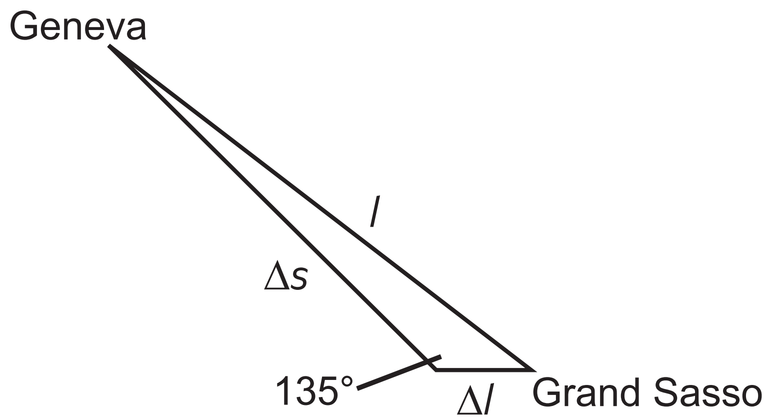
$$\begin{aligned} v_{rel} &= v_{det\,ector} - v_{source} \\ &= \omega \cdot R_e \cdot \cos 42° - \omega \cdot R_e \cdot \cos 46° \\ &= \frac{2 \cdot \pi}{24h} \cdot 6.371 \cdot 10^6 \, m \cdot (0.7431 - 0.6947) \\ &= 22.5 \frac{m}{s} \end{aligned}$$

The neutrino covers the distance of $\Delta s = 7.32 \cdot 10^5\,\mathrm{m}$ between Geneva and Grand Sasso in a time $\Delta t = \frac{\Delta s}{c}$. The distance covered by the detector (as seen from Geneva) during the neutrino's journey to Italy is therefore

$$\begin{aligned}\Delta\ell &= v_{el} \cdot \Delta t \\ &= 22.5\frac{m}{s} \cdot \frac{\Delta s}{c} \\ &= 22.5\frac{m}{s} \cdot \frac{7.32 \cdot 10^5\,m}{c} \\ &= 5.49 \cdot 10^{-2}\,m\end{aligned}$$

Since between the neutrino velocity and the velocity of the detector, there is an angle of 135°, the neutrino in fact covers a distance of

$$\ell = \sqrt{(\Delta s)^2 + (\Delta\ell)^2 - 2 \cdot \Delta s \cdot \Delta\ell \cdot \cos 135°}$$



which corresponds to an additional distance of

$$\begin{aligned}\ell - \Delta s &= \sqrt{(\Delta s)^2 + (\Delta\ell)^2 - 2 \cdot \Delta s \cdot \Delta\ell \cdot \cos 135°} - \Delta s \\ &= \sqrt{(7.32 \cdot 10^5\,m)^2 + (5.49 \cdot 10^{-2}\,m)^2 - 2 \cdot 7.32 \cdot 10^5\,m \cdot 5.49 \cdot 10^{-2}\,m \cdot \cos 135°} - 7.32 \cdot 10^5\,m \\ &= 3.88 \cdot 10^{-2}\,m\end{aligned}$$

This result means that the neutrino, due to the Coriolis force, covers additional 3.88cm. This value may change in the order of ca. 20% in a more accurate analysis which will use more precise values for the heading of the neutrino's velocity and for the distance between source and detector.

This effect is in the order of the accuracy of the GPS determination of the distance between source and detector ($\pm 0.2\,m$), and it therefore seems necessary to me to include this effect in the continued analysis.

I am open for discussion.

Kind regards

Matthias Heidrich

Matthias Heidrich, Prof. Dr. rer. nat.
Huebstrasse 5811
CH-9112 Schachen b. Herisau
www.matthias-heidrich.com
Tel.: +41 (0)71 5344470
