The International Monetary Transmission Mechanism: A Model of Real Exchange Rate Adjustment Under Pricing-To-Market[1]

Caroline Betts
University of Southern California

Michael B. Devereux
University of British Columbia

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Abstract

This paper presents a number of stylized facts regarding the international monetary transmission mechanism, and develops a two-country dynamic general equilibrium model which can account for these facts. We argue that the model can make progress in understanding the following stylized facts for G-7 countries (i) a positive monetary shock in a country causes a persistent real exchange rate depreciation ii) there is a large liquidity effect of such a monetary shock for interest rate differentials iii) in response to a monetary shock, cross-country comovements of output levels are positive and large and iv) there is a "J-curve" effect for a country's trade balance following a monetary shock in that country. The model we develop has two key features; firms set prices in advance, and some firms can 'price-to-market' by segmenting home and foreign markets in their price-setting decisions. The presence of pricing-to-market causes transitory deviations from the law of one price and from purchasing power parity. In this model, the effects of unanticipated monetary shocks are strikingly similar to those we observe in the G-7 data. We show that the international transmission of monetary policy is quite different in an environment of pricing-to-market than in the case where there is a unified world price of traded goods.

JEL Classification F3, F4.

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Section 1 Introduction

One of the central issues in open economy macroeconomics is the international monetary transmission mechanism. How do monetary shocks affect output, the exchange rate, interest rates, and the trade balance in an environment of open international capital markets, where monetary policy has international repercussions? This paper documents a set of empirical findings concerning the international effects of monetary policy, and develops an intertemporal general equilibrium model whose properties can be compared to the empirical findings. Two important features of the model are the presence of nominal price rigidity, and the ability of some firms to price- discriminate between national markets, a phenomenon we call 'pricing-to-market' . With these two features, we show that the model's properties are strikingly consistent with the empirical findings regarding the international monetary transmission mechanism.

For the purposes of this paper, the international monetary transmission mechanism is treated as the set of responses to exogenous money shocks of a small number of international macro variables derived from a vector autoregression. Exogenous shocks to money are defined in a way that has been used in previous literature in closed economy macroeconomics, as orthogonalized innovations to a narrow and policy determined measure of money. Our definition of the monetary transmission mechanism is closely related to work by Eichenbaum and Evans (1995), and to a lesser extent, Clarida and Gali (1995)[1]. We extend the findings of these papers however by deriving the effects of monetary policy shocks on the responses of a wider set of variables.

When looking at G7 countries, we show that in response to an exogenous US monetary policy shock, the international monetary transmission mechanism has the following four features. First, there is a persistent US (CPI based) real exchange rate depreciation. Second, there is an immediate fall in US nominal interest rates relative to foreign interest rates. This is part of the liquidity effect of monetary policy shocks that has been studied by Christiano and Eichenbaum (1994) and others. Third, there is a slight increase in US output relative to foreign output. Finally, the US bilateral trade

[1] See also Grilli and Roubini (1995), Kim and Roubini (1996), and Kumar and Prasad (1996).

balance with the foreign country first deteriorates, but later improves. This feature is popularly known as the 'J-curve' effect suggesting that the impact of a real depreciation on the trade balance may initially be negative, and becomes positive only over time. Here, we show that after a money shock, the US trade balance displays a J-curve response.

We go on to construct a two-country intertemporal general equilibrium monetary model. The simulated responses of the model can be compared to the empirical responses from the VARS. The model has two main features. First, nominal prices are sticky. We introduce price stickiness through a model of staggered price setting. The second feature is that at least for some firms in the model, markets are segmented by country, and prices are set in currency of local sale. Since these firms can separate their domestic and foreign markets, we call these 'pricing-to-market' (PTM) firms. The presence of PTM allows the law of one price to be violated for at least a subset of traded goods, in response to shocks to monetary policy. In fact, all real exchange rate variability in the model comes from the existence of PTM. This is consistent with the findings of Engel (1996), who documents that for OECD countries most real exchange rate variability arises from cross-country differences in traded goods prices.

We show that a calibrated version of our model displays theoretical responses to a money shock that are very similar to the empirical features of the international monetary transmission mechanism. In the presence of price stickiness, and PTM, a monetary shock generates a real exchange rate depreciation, and a fall in the nominal interest rate differential between countries (a liquidity effect). In addition, the money shock causes very similar positive responses of domestic and foreign output, so there is only a slight increase in the differential between domestic and foreign output. Finally, our model shows that a money shock generates a distinct J-curve. That is, the immediate response of the trade balance is negative, but over time, the response turns positive.

From this we conclude that a model encompassing sticky prices and PTM displays features that are consistent with the main features of the empirical international monetary transmission mechanism.

The consistency of the model predictions with those of the empirical international monetary transmission mechanism are dependent on the presence of PTM. Nominal rigidities alone, when markets are not segmented and the law of one price holds, cannot account for any of the features of monetary transmission that we identify in the data. However, when prices are set and sticky in local currency, a monetary expansion generates a nominal exchange rate depreciation which translates into a real exchange rate depreciation, due to deviations from the law of one price. In addition, by suitable calibration of our model, we find that the nominal exchange rate overshoots so that the home nominal interest rate falls relative to the foreign interest rate. Because the presence of PTM implies that the real exchange rate depreciation is *not* fully reflected in a change in the relative price of home goods to foreign goods that consumers face, the 'expenditure-switching' effect of a depreciation is dampened. Positive money shocks therefore have a similar impact on the demand for all goods. This explains why output comovements across countries are positive. Finally, because PTM drives a wedge between home and foreign aggregate price levels after a money shock, it leads real interest rates to differ across countries. The home real interest rate falls while the foreign real interest rate rises. Consumption and investment in the home economy rise on impact, while they fall in the foreign economy. The home country therefore experiences an immediate trade balance deterioration. However, as the home country capital stock rises over time this trade deficit is gradually eliminated, and eventually the home country trade balance goes into surplus.

We also characterize the impact of international monetary transmission in our PTM model for the moments and correlations of relevant macroeconomic aggregates. We suggest that this model of international transmission may help to account for some puzzles of international business cycle data noted by Backus, Kehoe and Kydland (1995). In particular, we show that in the presence of price stickiness and PTM, a) cross country correlations of output levels are higher than cross country correlations of consumption, and b) the variability of the real exchange rate and the terms of trade are higher than the variability of output. While both a) and b) are features which characterize international data, they are in general at odds with the properties of theoretical intertemporal general equilibrium models of the international economy.

Our paper builds on some previous papers in the literature. An early paper by Svennson and Van Wijnbergen (1987) explored the role of monopolistic competition and price stickiness in an international endowment economy with a cash-in-advance constraint. Stockman and Ohanian (1993) look at the exchange rate effects of price stickiness for a subset of goods. Our model is a direct extension of the two country model of Obstfeld and Rogoff (1995), except allowing for PTM. Other recent papers by Chari, McGratten and Kehoe (1996) and Kollman (1996) examine the effects of monetary shocks on real exchange rates in similar frameworks to the one used here. These differ from the present paper in a number of ways. They compare the quantitative properties of their model to the unconditional moments of the data, rather than matching up the responses of the model to those of the empirical monetary transmission mechanism. They also do not focus on the trade balance responses to a monetary shock. Their models do not allow for a partial degree of pricing to market, and so do not subsume PPP as a special case or allow for calibration to data on the observed degree of pricing to market. Here, empirical estimates from the price pass-through literature can be used to pin down the appropriate extent of PTM in our calibrated model.

The rest of the paper is organized as follows. Section 2 documents the empirical features of the international monetary transmission mechanism. Section 3 constructs the theoretical model. Section 4 sets out the equilibrium of the model. Section 5 discusses calibration. Section 6 describes the empirical results of the model. Finally, section 7 concludes.

Section 2 Empirical Evidence

We first develop a set of empirical measures of the international monetary transmission mechanism. We identify the conditional responses in the data of some key international macroeconomic variables to unanticipated, exogenous shocks to the quantity of money. In particular, we focus on the impact of a measure of US monetary policy shocks for the relative output, relative nominal interest rate, the bilateral real exchange rate, and the bilateral trade balance of the US with respect to her G-7 trading partners; Japan, Germany, the United Kingdom, Italy, Canada and France.

Methodology

We follow Eichenbaum and Evans (1995) by identifying exogenous, unanticipated US monetary policy shocks with orthogonalized innovations to a narrow, policy determined measure of money; the ratio of non-borrowed to total reserves of the Federal Reserve. Such innovations can be derived by transforming the residuals from an unconstrained vector autoregression, which includes both the measure of monetary policy as well as a set of endogenous variables that are jointly determined with movements in the quantity of (narrow) money. As suggested by Sims (1980), we can also use the vector autoregression to measure the responses of the endogenous variables to a typical orthogonalized innovation in the quantity of narrow money by tracing out the associated moving average representation for these variables. This representation directly measures the conditional, contemporaneous and dynamic correlations of the endogenous variables of interest - here, relative outputs, the nominal interest rate differential, the bilateral real exchange rate and the bilateral trade balance/output ratio - with orthogonalized monetary innovations.

More formally, we first estimate an unconstrained vector autoregression (VAR) of the form,

$$X_t = A(L)X_{t-1} + V_t \qquad (2-1)$$

where X_t is the vector of demeaned and detrended international macroeconomic variables of interest, including non-borrowed reserves, $A(L)$ is a matrix lag operator, and V_t is an error vector which satisfies $V_t \sim IID(0, \Sigma_V)$. We assure that the elements of V_t are serially uncorrelated by selecting a lag length in $A(L)$ according to standard statistical criteria. We then invert the estimated vector autoregressions to produce reduced form Wold (moving average) representations for the vector of endogenous variables,

$$X_t = G(L)V_t \qquad (2-2)$$

where $G(L)$ is a matrix lag operator with lag length determined by truncation, such as to exclude statistically insignificant parameters at distant lags, and $G(0) = I$.

The reduced form Wold representation is characterized by a correlated innovation vector, so that the measured impact for the endogenous variables of a shock to non-borrowed reserves, for example, cannot be separately identified from the impact of shocks to the other endogenous variables in X_t. However, we can always reparameterize the Wold form and identify a "structural" representation for the endogenous variables in which innovations to each variable are orthogonal. In other words, we can correctly think of each element of V_t as representing a linear combination of variable specific innovations that can be separately identified if we are willing to impose parameter restrictions that allow this. In particular, we can identify a parameterization of (2-2)

$$X_t = \Gamma(L)W_t \tag{2-3}$$

where $W_t \sim IID(0, \Sigma_W)$, Σ_W is a diagonal matrix, the ith row of $\Gamma(L)$ summarizes the response the ith endogenous variable in X_t to each orthogonal innovation in W_t, and $\Gamma(0)$ is lower triangular. Obviously, the diagonal form of Σ_W reflects the orthogonality of the elements of W_t. The triangular form of the impact matrix $\Gamma(0)$ for the shocks W_t means that the selected ordering of variables in X_t corresponds to a ranking of the unanticipated innovations X_t in terms of their contemporaneous impact for the other endogenous variables - a ranking of their temporal "exogeneity". In our VAR's, M_t - the quantity of non-borrowed Fed. reserves - is always the first element of X_t, which reflects the "model" of unanticipated monetary policy that we have in mind.

The prior ordering of non-borrowed/total reserves corresponds to an assumption that orthogonal, unanticipated innovations in these reserves (W_1) may have an immediate impact for any endogenous macroeconomic variable. However, non-borrowed/total reserves respond in a predictable way to other macroeconomic shocks with a lag only (of one month, in our data). If the Fed controls non-borrowed reserves, as we assume, then the specification of the VAR implies that all monetary policy actions are determined independently of current shocks to the economy, although the quantity of base money is allowed to respond endogenously and in a predictable way to past states[2].

[2] We could easily allow for alternative specifications of monetary policy, in which the authorities respond contemporaneously to innovations in output, prices and interest rates for example, by placing such other variables ahead of money in the Wold ordering as is done by Eichenbaum and Evans (1995). In fact, the subset of our empirical results which overlap with those studied by Eichenbaum and Evans - the responses of real and nominal exchange rates and of nominal interest differentials to money shocks - are essentially identical to theirs, despite the difference in ordering of variables in the VAR.

Identification of a triangular Wold representation is achieved by using the covariance condition

$$\Gamma(0)\Sigma_W\Gamma(0)' = \Sigma_V. \qquad (2-3)$$

Since Σ_V is symmetric and positive definite, we can compute the Choleski decomposition of this matrix. Assuming a normalization of the structural covariance matrix $\Sigma_W = I$, this Choleski decomposition is equivalent to a lower triangular matrix of impact multipliers Γ_0 for each element in X_t of the orthogonal innovations W_t. The entire Wold representation is retrieved from applying $G(0)V_t = \Gamma(0)W_t \Rightarrow W_t = \Gamma(0)^{-1}V_t \Rightarrow \Gamma(L) = G(L)\Gamma(0)$. The entire structural innovation vector and moving average parameter matrix $\Gamma(L)$ are then identified. This methodology is simply an application of techniqes advanced by Sims (1980) for identifying the conditional dynamic responses of a set of endogenous variables to unanticipated macroeconomic shocks.

Data

We estimate six, bilateral country pair vector autoregressions, which we view as crude empirical representations of a dynamic two-country world. Specifically, we consider six US-foreign country pairs as described above, in which Canada, France, Germany, Italy, Japan, and the UK each represent the foreign country respectively. In each case, our vector of endogenous variables is given by $X_t = [M_t, I_t - I_t^*, Y_t/Y_t^*, E_tP_t^*/P_t, TB_t/Y_t]'$, where non-asterixed variables denote the values of a home country - US - variable and asterixed variables denote the values of a foreign country variable. Here, M_t is the ratio of non-borrowed to total reserves of the Federal Reserve, $I_t - I_t^*$ is the short-term market net nominal interest rate differential between the US and a trading partner, Y_t/Y_t^* is relative output that we measure by relative industrial production, E_t is the home currency price of foreign exchange which we measure as the average noon spot exchange rate, and P_t/P_t^* is the relative consumer price index. Thus, $E_tP_t^*/P_t$ is the consumption based real exchange rate of the domestic country. Finally, TB_t/Y_t is US trade surplus/output ratio with respect to a foreign country trading partner.

The raw data are monthly, Hodrick-Prescott filtered and seasonally adjusted series. The data are drawn from the IMF's International Financial Statistics from 1974-1 until

1994-12, except Federal Reserve non-borrowed and total reserves and the bilateral trade balance of the US. The monthly seasonally adjusted data for the Fed's non-borrowed reserves and total reserves are drawn from the Federal Reserve database. In addition, the bilateral trade balance data are from the direction of trade flows statistics of the IMF, and comprise the bilateral US exports minus bilateral US imports. In each bilateral VAR, foreign country reported data on bilateral exports and imports to the US are used to construct the US trade balance. However, the results obtained by using US reported trade balance data are essentially qualitatively identical. In addition, we take logs of M_t, Y_t/Y_t^*, and $E_tP_t^*/P_t$, while $I_t - I_t^*$ and TB_t/Y_t appear in levels. The variables that we include in X_t are intended to represent a parsimonious set of open economy macroeconomic variables the responses of which to monetary policy shocks can summarize the central features of the empirical international monetary transmission mechanism[3].

We estimate the unrestricted VAR's (as in (2-1)), selecting the lag length for each bilateral model according to standard (log likelihood) test criteria. We then invert the VAR's and compute the orthogonal decomposition described by (2-3). To analyze the international transmission of US money shocks, we study the impulse response function of each endogenous variable with respect to a one standard deviation increase in the ratio of US non-borrowed/total reserves. These functions are simply the first column in our estimate of $\Gamma(L)$. We then generate the empirical distribution of the estimated parameter matrix $\Gamma(L)$ by Monte Carlo integration, and use it to compute standard errors for the estimated parameters of the impulse response functions. To do this, we assume that the distributions of the parameters in $A(L)$ and of the variance-covariance matrix Σ_V (the parameters which determine $\Gamma(L)$) are known. In paricular, we assume that $V_t \sim N(0, \Sigma_V)$ and that $S = \sum_{t=1}^{T} V_t V_t' \sim W_n(\Sigma_V, T)$ where W_n denotes the Wishart (chi-squared) distribution. We construct a sequence of 1000 Monte Carlo draws

[3] We have also experimented with VAR specifications in which national outputs, prices and the nominal exchange rate appear separately in the VAR, the results of which are qualitatively the same as those we present in this paper. The results presented here capture all of the important features of the data that are produced by "finer" empirical representations in a more compact form.

for the error matrix Σ_V, use each of them to generate a draw for the vector autoregression parameter matrix $A(L)$ and hence $\Gamma(L)$, and thereby compute the means and standard errors of $\Gamma(L)$ where the mean of the draws is treated as the "true" parameter value.

Results

The results we obtain are as follows. The impulse response functions, which we present below in Figures 1a-1e, show that US money shocks produce an initial real exchange rate depreciation for each of the country pairs, although this depreciation is small for Canada. The real depreciations are persistent for all country pairs - the total impact lasting for about two years in all cases except that of Japan, which lasts for about three years. These real exchange rate depreciations imply half-lives of monetary shocks to the real exchange rate of about 1 year, and so are not nearly as persistent as estimates of "unconditional" real exchange rate shocks which exhibit roughly 4.5 year half-lives. Our results are consistent with the work of Clarida and Gali (1994) which contends that monetary shocks are important for real exchange rate movements over short horizons.

In addition, we find there is a sharp and significant initial fall in the domestic relative nominal interest rate which is completely reversed within about six months for all country pairs. This is consistent with the liquidity effects of money shocks for relative interest rates obtained by Eichenbaum and Evans (1995). We find that the response of relative outputs Y_t/Y_t^* is small and barely significant for most country pairs. For the cases of Germany, Italy and Jaapan, the output ratio rises slightely but significantly over short horizons, then falls to zero. There are small negative but barely significant responses for the remaining country pairs at short horizons. The somewhat mixed results for impact responses of initial output ratios may reflect Schlagenhauf and Wrase's (1995) result that level output responses to money shocks may be positive or negative. However, what is very clear from our results is that the contemporaneous correlations of outputs across countries conditional on a money shock are very high and positive at all lags.

Finally, we find that US money shocks produce an short-run deterioration of the

US trade balance for all countries, although in cases of Germany and Japan this deterioration occurs after about two months. In all cases, for some initial period, this US trade balance deterioration is significant and in the case of all bilateral country pairs except the US/Canada, it is quite persistent lasting for up to one year in the cases of Germany, Japan and the UK. It is followed period of trade balance improvement for all countries, which is significant (if marginally) for France, Italy and Japan. Eventually, the trade surplus attains balance after a total period of imbalance lasting for several years. Particularly in the cases of France, Germany, Italy and Japan, the bilateral trade balance response to a US money shocks resembles a "J-curve".

Section 3 The Model

In this section we develop a formal model of money-induced real exchange rate volatility. There are two countries; home and foreign. We denote the foreign country variables with an asterisk. Households in each country consume a group of differentiated goods categories of total measure unity. Of these goods a fraction n are produced by the home country, and $1-n$ are produced in the foreign country, where n and $1-n$ also represent the home and foreign country population, respectively [4]. Each commodity category is further comprised of a large number (N) of goods that are imperfect substitutes for one another.

Within each goods category i, each good j is sold exclusively by a price-setting firm. A fraction s of the total measure of categories in each country are goods that can be sold at separate prices across countries, as firms within these categories can price-discriminate across countries[5]. These we refer to as PTM industries. The PTM industries set prices separately for the home and the foreign market, since by assumption only firms can trade these goods. Consumers cannot trade PTM goods directly across

[4] The notation follows Obstfeld and Rogoff (1995), from which the basic structure of the model is taken.

[5] The reason for this double-decomposition of goods structure is to allow a different parameter governing the average markup from that of the substitution elasticity between home and foreign goods. This is discussed further below.

countries. The remaining $1-s$ categories can be freely traded by consumers, so that firms must set a unified price across the two countries.

Let the aggregate state of the economy at any time be denoted z_t, and assume that $z_t \in Z_t$, where Z_t denotes the set of all possible states at time t. Assume that z_t is a Markov process, so that

$$Prob(z_{t+1} \leq z', z_t = z) = G(z', z).$$

For most of the description of the model below, we state only the home country equations. The foreign country equations are derived symmetrically.

Households

Let agents in the home economy have preferences over consumption given by

$$EU = E_0 \sum_{t=0}^{\infty} \beta^t \Big(\frac{C(z_t)^{1-\sigma}}{1-\sigma} + \frac{\gamma}{1-\epsilon}\Big(\frac{M(z_t)}{P(z_t)}\Big)^{1-\epsilon} + \eta \log(1-h(z_t))\Big)$$

where $C(z_t) = (\int_0^1 c(i,z_t)^{\frac{\rho-1}{\rho}} di)^{\frac{\rho}{\rho-1}}$, $(\rho > 1)$, and, for each category i, we have $c(i,z_t) = (\sum j =_0{}^N x(i,j,z_t)^{\frac{\lambda-1}{\lambda}})^{\frac{\lambda}{\lambda-1}}$. $c(i,z_t)$ is the composite consumption of category i good at time t, and $x(i,j,z_t)$ represents that consumption of the individual good j within the category i. There are N goods within each category i. $h(z_t)$ represents total hours worked by the representative domestic household. Households also value real money balances $\frac{M(z_t)}{P(z_t)}$ where $M(z_t)$ are nominal balances and $P(z_t)$ is the home country CPI, defined as

$$P(z_t) =$$

$$\Big[\int_0^n p(i,z_t)^{1-\rho} di + \int_n^{n+(1-n)s} p^*(i,z_t)^{1-\rho} di + \int_{n+(1-n)s}^1 (e(z_t)q^*(i,z_t))^{1-\rho} di\Big]^{\frac{1}{1-\rho}} \quad (3-1)$$

where, in turn

$$p(i,z_t) = \Big[\sum_0^N p(i,j,z_t)^{1-\lambda}\Big]^{\frac{1}{1-\lambda}}.$$

Of the n home categories of goods, all are sold in the home country in home good prices $p(i,z_t)$, while a fraction $1-s$ are sold in the foreign market with prices set in terms of home currency (also at $p(i, z_t)$), while s are sold in the foreign market with prices set in terms of foreign currency $q(i, z_t)$. Of the $1-n$ foreign categories sold in the domestic market, a fraction $1-s$ are priced in foreign currency $q^*(i, z_t)$, while a fraction s are priced in home currency; $p^*(i, z_t)$. Prices denoted p represent home currency prices, while prices denoted q represent foreign currency prices. Thus $p(i, z_t)$ is the home currency price of the home-produced category i good, $p^*(i, z_t)$ is the home currency price of a foreign PTM good i, etc. The exchange rate (domestic unit cost of foreign currency) is given by $e(z_t)$.

In a similar fashion, the foreign CPI is denoted

$$Q(z_t) =$$

$$[\int_0^{(1-n)} q^*(i, z_t)^{1-\rho} di + \int_{1-n}^{1-n+ns} q(i, z_t)^{1-\rho} di + \int_{1-n+ns}^{1} (p(i, z_t)/e(z_t))^{1-\rho} di]^{\frac{1}{1-\rho}} \quad (3-2)$$

where

$$q(i, z_t) = [\sum_0^N q(i, j, z_t)^{1-\lambda}]^{\frac{1}{1-\lambda}}$$

.

Households receive income from wages, $W(z_t)h(z_t)$, returns on their capital holdings $R(z_t) + (1-\delta)P(z_t)$, where $R(z_t)$ denotes the nominal rental return on a unit of capital and δ is the physical rate of depreciation. It is assumed that capital is constructed in exactly the same manner as the composite consumption good, so that it has the same aggregate price index. In addition, households receive profits on their ownership of domestic firms, $\Pi(z_t)$, and returns on state contingent nominal assets. Asset markets are complete, so that there is a nominal asset which has a potentially different payoff for every state of the world. There are also adjustment costs of changing the aggregate domestic capital stock that must be borne by consumers. The adjustment cost function indicates that if consumers wish to accumulate net capital equal to $(K_{t+1} - K_t)$, then they must incur adjustment costs equal to

$$\psi(K_{t+1} - K_t)$$

where $\psi(.) \geq 0, \psi(0) = 0$, $\psi'(.) > 0$, and $\psi''(.) > 0$. The adjustment cost technology requires goods in the same composition as the consumption good and the capital good.

Households also receive transfers $TR(z_t)$ from the central bank. Their budget constraint is

$$P(z_t)C(z_t) + M(z_t) + \int_{z_t \in Z_{t+1}} d(z_{t+1})v(z_{t+1}, z_t) + P(z_t)K_{t+1} + P(z_t)\psi(K_{t+1} - K_t)$$

$$= W(z_t)h(z_t) + \Pi(z_t) + M(z_{t-1}) + TR(z_t) + d(z_t) + R(z_t)K_t + (1-\delta)P(z_t)K_t \quad (3-3)$$

where $M(z_t)$ is the money holding at the end of the period, $d(z_t)$ represents payments in home currency denominated state contingent bonds, and $v(z_{t+1}, z_t)$ is the price of a one dollar payout in state z_{t+1}, given the current state z_t.

Households maximize utility subject to $(3-3)$. The solution to this problem is described as follows. The optimal consumption between each of the differentiated goods is

$$c(i, z_t) = (\frac{f(i, z_t)}{P(z_t)})^{-\rho} C(z_t) \quad (3-4)$$

where $f(i, z_t)$ is equal to either $p(i, z_t)$, $p^*(i, z_t)$, or $e(z_t)q^*(i, z_t)$, depending upon which category good i falls within. Furthermore, it is also the case that

$$x(i, j, z_t) = (\frac{p(i, j, z_t)}{p(i, z_t)})^{-\lambda} C(z_t) \quad (3-5)$$

In addition, the household's optimal money demand schedule can be written as

$$\frac{M(z_t)}{P(z_t)} = (\frac{\gamma C(z_t)}{1 - V(z_t)})^{\frac{1}{\epsilon}} \quad (3-6)$$

where $V(z_t) = \int_{z_{t+1} \in Z_{t+1}} v(z_{t+1}, z_t)$ represents the nominal discount factor (the inverse of one plus the nominal interest rate).

The optimal labour supply decision is characterized by

$$\frac{\eta}{1 - h(z_t)} = \frac{W(z_t)}{P(z_t)C(z_t)} \quad (3-7)$$

The household's choice of contingent nominal assets leads to the condition

$$v(z_{t+1}, z_t)P(z_{t+1})C(z_{t+1}) = \beta P(z_t)C(z_t)dG(z_{t+1}, z_t)$$

Finally, the optimal choice of capital stock leads to the condition

$$1 = \beta \int_{z_{t+1} \in Z_{t+1}} \left(\frac{C(z_t)}{C(z_{t+1})} \frac{(R(z_{t+1}) + P(z_{t+1})(1-\delta) + P_{t+1}\psi'(K_{t+2} - K_{t+1})}{1 + P(z_t)\psi'(K_{t+1} - K_t)} \right) \tag{3-8}$$

The situation of foreign households is entirely analogous.

Since there are security markets for each state of the world, markets are complete. It follows then that all possible gains to risk-sharing are being exploited. It is easy to show the following proposition.

Proposition 1.

In each state of the world $z_t \in Z_t$, for each time period, home and foreign consumption will be related in the following way

$$\frac{C(z_t)^{-\sigma}}{C^*(z_t)^{-\sigma}} = \Lambda \frac{Q(z_t)e(z_t)}{P(z_t)} \tag{3-9}$$

where Λ is a constant.

Proof: see appendix.

What this proposition says is that all deviations from optimal risk-sharing across countries arises due to real exchange rate variability, since

$$\frac{Q(z_t)e(z_t)}{P(z_t)}$$

is just a measure of the real exchange rate. If the real exchange rate is unity (i.e. PPP held), then agents in each country face identical commodity prices, and consumptions are equated, up to a constant Λ which is determined by initial wealth differences. In general however, the real exchange rate will exhibit departures from PPP, as movements in the real exchange rate are produced by the presence of pricing-to-market in national currencies.

Central Banks

Central Banks in each country print domestic currency, transferring the proceeds to domestic agents. The home country central bank budget constraint is

$$TR(z_t) = M(z_t) - M(z_{t-1}) \qquad (3-10).$$

We assume that the Central Banks use the following types of monetary growth rule

$$M(z_{t+1}) = M(z_t)(1 + g_m)\exp(u_{z_t})$$

where g_m is the mean growth rate of money, and u_{z_t} is a mean zero stochastic money shock. Although it is common to look at monetary growth rules in which the growth rate of money is autocorrelated, in our VAR evidence the behaviour of non-borrowed reserves does not display this property. Thus, to remain within our empirical estimates, a rule such as this is more appropriate. We also experiment with a different monetary rule in the presentation of results below however.

Firms

There are two types of firms within each country; firms who may price-to-market, and firms who must set a unified international price. It is assumed that within a category i, all firms have the same pricing policy (i.e. they are all PTM firms or all are not). Within category i, firm j operates a technology

$$y(i,j,z_t) = \theta_t k(i,j,z_t)^{\alpha} l(i,j,z_t)^{1-\alpha} - \upsilon$$

where $y(i,j,z_t)$ is total output of the firm, $l(i,j,z_t)$ is employment, $k(i,j,z_t)$ is capital used, θ_t is a common technology variable, and υ is a fixed cost of production, common to all firms.

In the model, price setting is staggered. Each firm chooses an optimal pre-set price, and faces a constant probability of price readjustment thereafter. This produces a model of gradual price adjustment at the aggregate level as in Calvo (1983) and Yun (1996). By appropriate choice of the exogenous probability of price readjustment, we may choose whatever degree of average price rigidity we wish in the model.

All firms minimize costs in choosing an optimal input bundle. Thus, for firm j in category i, we may define

$$mc(W(z_t), R(z_t))(y+v) = Min\{W_t l_{ij} + R_t k_{ij},\ \theta_t k(i,j,z_t)^{\alpha} l(i,j,z_t)^{1-\alpha} \leq y\}$$

where $mc(W(z_t), R(z_t))$ is the unit marginal cost function, in nominal terms.

Moreover, $mc(W_t, R_t) = mc(z_t)$ must satisfy

$$W(z_t) = mc(z_t)(1-\alpha)\frac{y(i,j,z_t)}{l(i,j,z_t)}$$

$$R(z_t) = mc(z_t)\alpha\frac{y(i,j,z_t)}{k(i,j,z_t)}$$

We may transform these expressions as follows

$$W(z_t) = \frac{p(i,j,z_t)}{\mu(i,j,z_t)}(1-\alpha)\frac{y(i,j,z_t)}{l(i,j,z_t)}$$

$$R(z_t) = \frac{p(i,j,z_t)}{\mu(i,j,z_t)}\alpha\frac{y(i,j,z_t)}{k(i,j,z_t)}.$$

Here the variable $\mu(ij, z_t)$ defines the price-cost markup for a domestic firm selling in the domestic market, i.e.

$$p(i,j,z_t) = \mu(i,j,z_t)mc(z_t) \qquad (3-11).$$

Alternatively, for a PTM firm j' in sector i that sells in the foreign market, we may write

$$e(z_t)q(i,j',z_t) = \mu(i,j',z_t)mc(z_t) \qquad (3-12)$$

Note from $(3-11)$ and $(3-12)$ that if a firm that is selling in two markets chooses the same markup in both markets, the law of one price will hold across countries for this good, because the firms marginal costs are the same for goods sold at home or in the foreign market. In a deterministic economy, with no monetary or exchange rate shocks, firms would in fact desire to set equal markups across markets, because the elasticity of demand is constant and the same in the home and foreign market. But this will hold for all goods. As a result, in a deterministic economy, PPP will hold at all times.

Section 4 Price Setting

All firms within a given sector i change their prices at the same time. Price changes are staggered across sectors though, and, following Calvo (1983) and Yun (1996), a firm within any sector gets to change its price in any period with a constant probability $1-\pi$, irrespective of how long it has had its price fixed for. With a large number of sectors, this implies that $1-\pi$ of the sectors will be altering their price at any given period. A PTM firm j in sector i which has an opportunity to change it price will set prices $\tilde{p}(i,j,z_t)$ and $\tilde{q}(i,j,z_t)$ to maximize state contingent discounted profits, given by

$$\sum_{k=0}^{\infty}\int_{z_{t+k}\in Z_{t+k}} \phi(z_{t+k},z_t)\pi^k\left[p(i,j,z_{t+k})x_d(i,j,z_{t+k}) + e(z_{t+k})q(i,j,z_{t+k})x_d^*(i,j,z_{t+k})\right.$$

$$\left.-mc(z_{t+k})(x_d(i,j,z_{t+k}) + x_d^*(i,j,z_{t+k}))\right]$$

where $\phi(z_{t+k},z_t) = \prod_{i=0}^{k} v(z_{t+i},z_t)$ is the pricing kernel for k period ahead dollars.

Here $x_d(i,j,z_t)$ and $x_d^*(i,j,z_t)$ represent the home and foreign demand functions for the good i,j. From above, we know that each firm faces a constant elasticity of demand given by λ.

The first order conditions for the firms pricing decision are

$$\tilde{p}(i,j,z_t) = \frac{\sum_{k=0}^{\infty}\int_{z_{t+k}\in Z_{t+k}} \pi^k\phi(z_{t+k},z_t)\lambda mc(z_{t+k})x_d(i,j,z_{t+k})}{\sum_{k=0}^{\infty}\int_{z_{t+k}\in Z_{t+k}} \phi(z_{t+k},z_t)(\lambda-1)x_d(i,j,z_{t+k})} \qquad (4-1)$$

$$\tilde{q}(i,j,z_t) = \frac{\sum_{k=0}^{\infty}\int_{z_{t+k}\in Z_{t+k}} \pi^k\phi(z_{t+k},z_t)\lambda mc(z_{t+k})x_d^*(i,j,z_{t+k})}{\sum_{k=0}^{\infty}\int_{z_{t+k}\in Z_{t+k}} \phi(z_{t+k},z_t)(\lambda-1)e(z_{t+k})x_d^*(i,j,z_{t+k})} \qquad (4-2)$$

Now we may define the following sub-price indices

$$p(z_t) = (\int_0^n p(i,z_t)^{1-\rho}di)^{\frac{1}{1-\rho}})$$

$$q(z_t) = (\int_0^n q(i,z_t)^{1-\rho}di)^{\frac{1}{1-\rho}}$$

In a symmetric equilibrium all firms within a sector i will charge the same price. In addition, all domestic firms will set the same home currency price $\tilde{p}(z_t)$ when choosing an optimal price, independent of sector, and all domestic PTM firms will set the same foreign currency $\tilde{q}(z_t)$ price, independent of sector. Therefore, we may write these price indices as

$$p(z_t)^{1-\rho} = (1-\pi)\tilde{p}(z_t)^{1-\rho} + \pi p(z_{t-1})^{1-\rho} \qquad (4-3)$$

$$q(z_t)^{1-\rho} = (1-\pi)\tilde{q}(z_t)^{1-\rho} + \pi q(z_{t-1})^{1-\rho} \qquad (4-4)$$

Now using the definition of the markup, we may rewrite $(4-1)$ and $(4-2)$ in a symmetric equilibrium as

$$\tilde{p}(z_t) = \frac{p(t)}{\mu_1(z_t)} \frac{\sum_{k=0}^{\infty} \int_{z_{t+k} \in Z_{t+k}} \pi^k \phi(z_{t+k}, z_t) \lambda \frac{\mu_1(z_t)}{\mu_1(z_{t+k})} x_d(z_{t+k})}{\sum_{k=0}^{\infty} \int_{z_{t+k} \in Z_{t+k}} \phi(z_{t+k}, z_t)(\lambda - 1) x_d(z_{t+k})} \qquad (4-5)$$

$$e(z_t)\tilde{q}(z_t) = \frac{e(z_t)q(t)}{\mu_2(z_t)} \frac{\sum_{k=0}^{\infty} \int_{z_{t+k} \in Z_{t+k}} \pi^k \phi(z_{t+k}, z_t) \lambda \frac{\mu_2(z_{t+k})}{\mu_2(z_t)} x_d(z_{t+k})}{\sum_{k=0}^{\infty} \int_{z_{t+k} \in Z_{t+k}} \phi(z_{t+k}, z_t)(\lambda - 1) \frac{e(z_{t+k})}{e(z_t)} x_d*(z_{t+k})} \qquad (4-6)$$

If the nominal state pricing kernel were constant, these equations would say that prices are set so that the discounted sum of expected markups in both markets is $\frac{\lambda}{\lambda-1}$, which is just the optimal markup for a deterministic economy.

For non-PTM firms the pricing policy is governed by $(4-5)$ alone. The pricing policy for foreign firms is analagous.

Section 5 Equilibrium

Market Clearing Conditions: Factor Markets

It is assumed that within a country there are economy-wide competitive factor markets for labour and capital. The demand for factors comes from home firms. We can divide the output of home firms into three. First, there is the output of home non-PTM firms that sell at a given home currency price to both home and foreign consumers and capital purchasers. Within a sector i, all firms have the same price, so output will be the same for all home firms j within sector i. Denote output of non-PTM sector i as $y_1(i, z_t)$. Second, there is the output of PTM firms who sell to the home market at a given home currency price. Denote the output of PTM sector i sold to home markets as $y_2(i, z_t)$. Finally, there is the output of the home PTM sector that is sold to foreign markets, at a given foreign currency price. This is denoted $y_3(i, z_t)$. It follows that the factor market clearing conditions for labour and capital may be represented as

$$h(z_t) = \int_0^{n(1-s)} mc_1(W, R) y_1(i, z_t) di + \int_{n(1-s)}^{n} mc_1(W, R)(y_2(i, z_t) + y_3(i, z_t)) di \quad (5-1)$$

$$k(z_t) = \int_0^{n(1-s)} mc_2(W, R) y_1(i, z_t) di + \int_{n(1-s)}^{n} mc_2(W, R)(y_2(i, z_t) + y_3(i, z_t)) di \quad (5-2)$$

Market Clearing Conditions: Goods Markets

Goods markets are segmented. Home goods sold domestically may be sold abroad at a different price. Thus, there are three different market clearing conditions for home produced goods. If sector i is not governed by PTM, then the goods market clearing condition for this sector is determined by world demand for good i

$$y_1(i, z_t) = \left(\frac{p(i, z_t)}{P(z_t)}\right)^{-\rho}(C(z_t) + I(z_t) + \psi(.)) + \left(\frac{p(i, t)}{e(z_t)Q(z_t)}\right)^{-\rho}(C(z_t)^* + I(z_t)^* + \psi^*(.)) \quad (5-3)$$

If sector i is characterized by PTM, then there are two market clearing conditions. One is for the home economy, given by

$$y_2(i, z_t) = \left(\frac{p(i, z_t)}{P(z_t)}\right)^{-\rho}(C(z_t) + I(z_t) + \psi(.)) \quad (5-4)$$

while the second is for the foreign economy, given by

$$y_2(i, z_t) = \left(\frac{q(i, z_t)}{Q(z_t)}\right)^{-\rho}(C(z_t)^* + I(z_t)^* + \psi^*(.)) \quad (5-5)$$

The Real Exchange Rate

Movements in the real exchange rate in this economy are due to deviations from the law of one price holding at every moment in time. As we saw above, in an economy with flexible prices, or with $s = 0$, equilibrium markups for all firms will be the same, whether it is for sales in the home market or the foreign market. But with national currency pricing and market segmentation, markups may differ across markets. Differentiating $(3-11)$ and $(3-12)$, and the analogous relationship for the foreign firm, we have that

$$\hat{p}(z_t) - \hat{e}(z_t) - \hat{q}(z_t) = \hat{\mu}_1(z_t) - \hat{\mu}_2(z_t) \qquad (5-6)$$

$$\hat{p}^*(z_t) - \hat{e}(z_t) - \hat{q}^*(z_t) = \hat{\mu}_2^*(z_t) - \hat{\mu}_1^*(z_t) \qquad (5-7)$$

where $\hat{x} = \frac{x(t)-\bar{x}}{\bar{x}}$, and $\bar{x}$ represents the steady state value of x. Using $(5-6)$ and $(5-7)$ in the definition of the the price indices $(3-1)$ and $(3-2)$, we may describe the movement of the real exchange rate as

$$\hat{e}(z_t) + \hat{Q}(z_t) - \hat{P}(z_t) = s(n(\hat{\mu}_2(z_t) - \hat{\mu}_1(z_t) - (1-n)(\hat{\mu}_2^*(z_t) - \hat{\mu}_1^*(z_t))) \qquad (5-8)$$

The intuition behind this equation is clear. The home country experiences a real exchange rate depreciation whenever markups in the foreign market rise relative to those in home markets, whether they be home or foreign firm markups. As we see below, this can be caused by an unanticipated change in the money supply. The change in the real exchange rate is limited by the size of the pricing-to-market sector however. When s is small, the real exchange rate responds by less to changes in country specific markups.

Steady State

We investigate the properties of this model by taking a linear approximation around a non-stochastic steady state. The steady state is independent of the degree of price rigidity. It can be defined by the following equations.

$$1 = \beta(\frac{p\alpha\bar{k}^{\alpha-1}\bar{h}^{1-\alpha}}{\bar{\mu}P} + 1 - \delta) \qquad (5-9)$$

$$1 = \beta(\frac{q^*\alpha\bar{k}^{*(\alpha-1)}\bar{h}^{*(1-\alpha)}}{\bar{\mu}Q} + 1 - \delta) \qquad (5-10)$$

$$\frac{M}{P} = (\frac{\gamma \bar{C}}{(1 - \frac{\beta}{1+g_m})})^{1/\epsilon} \quad (5-11)$$

$$\frac{M^*}{Q} = (\frac{\gamma \bar{C}/\Lambda}{(1 - \frac{\beta}{1+g_m^*})})^{1/\epsilon} \quad (5-12)$$

$$\bar{\theta}\bar{k}^{\alpha}\bar{h}^{1-\alpha} = (p/P)^{-\rho}(\bar{C}(1+1/\Lambda) + \delta(\bar{k}+\bar{k}^*)) \quad (5-13)$$

$$\bar{\theta}\bar{k}^{*\alpha}\bar{h}^{*(1-\alpha)} = (q^*/Q)^{-\rho}(\bar{C}(1+1/\Lambda) + \delta(\bar{k}+\bar{k}^*)) \quad (5-14)$$

$$\frac{\eta}{(1-\bar{h})} = \frac{(1-\alpha)}{\bar{\mu}}\bar{\theta}\bar{k}^{\alpha}\bar{h}^{-\alpha}\frac{p}{P} \quad (5-15)$$

$$\frac{\eta}{(1-\bar{h}^*)} = \frac{(1-\alpha)}{\bar{\mu}}\bar{\theta}^*\bar{k}^{*\alpha}\bar{h}^{*-\alpha}\frac{q^*}{Q} \quad (5-16)$$

$$P = (np^{1-\rho} + (1-n)(eq^*)^{1-\rho})^{\frac{1}{1-\rho}} \quad (5-17)$$

$$Q = (n(p/e)^{1-\rho} + (1-n)q^{*(1-\rho)})^{\frac{1}{1-\rho}} \quad (5-18)$$

The system $(5-9)$-$(5-18)$ gives 10 equations which determine the ten steady state variables $\{\bar{k}, \bar{k}^*, \bar{C}, \bar{h}, \bar{h}^*, \frac{eQ}{P}, \frac{p}{M}, \frac{q^*}{M^*}, \frac{P}{M}, \frac{Q}{M^*}\}$. The steady state markup rate $\bar{\mu}$, is equal to $\lambda/(\lambda-1)$. The variables g_m and g_m^* represent the steady state growth rates of money in the home and foreign country respectively. The variables p/P and q^*/Q are related the the terms of trade. Using $(5-17)$ and $(5-18)$, we see that

$$\frac{P}{p}\frac{q^*}{Q} = \frac{eq^*}{p}$$

which is a measure of the terms of trade between countries (the price of home imports relative to exports).

The system is recursive in the sense that equations $(5-9)-(5-10)$ and $(5-13)-(5-15)$ may be solved for $\bar{k}, \bar{k}^*, \bar{C}, \bar{h}, \bar{h}^*$, and the terms of trade eq^*/p, after substituting from P and Q from $(5-17)$ and $(5-18)$. Then the money market clearing conditions $(5-11)$ and $(5-12)$ can be used to solve for e and either p or q^* separately. Thus there are no long-run real effects of inflation in this model. Inflation reduces welfare of course, through the loss of consumer surplus.

In a completely symmetric steady state where $\Lambda = 1$, and $\theta = \theta^* = 1$, it is easy to see that $k = k^*$, $h = h^*$, and $eq^*/p = 1$.

Solving the Model

The full dynamic system is characterized by 31 equations, represented by $(3-1)$ and $(3-2)$, and $(3-9)$, then $(3-6)$, $(3-7)$, $(3-8)$, $(3-11)$ $(3-12)$, $(4-3)-(4-6)$, and $(5-1)-(5-5)$ for the home economy, with their counterparts for the foreign economy. This system determines the 31 variables $\{C, C^*, k, k^*, h, h^*, W, R, W^*, R^*, p, q, e, p^*, q^*, y_1, y_2, y_3, y_1^*, y_2^*, y_3^*, \mu_1, \mu_2, \mu_1^*, \mu_2^*, P, Q,, \tilde{p}, \tilde{q}, \tilde{p}^*, \tilde{q}^*\}$.

Calibration

The calibrated parameters for the baseline case are reported in Table 1. The parameters required to be chosen are (i) Preference parameters; β, ϵ, η, and σ (ii) Technology parameters α, δ, ρ, λ, υ, and the specification and parameterization of the adjustment cost function, ψ, (iii) Policy parameters; g_m, g_m^*, κ, κ^* (iv) the relative size parameter n, and Λ, and (iv), the pricing-to-market parameter s. In addition there is an additional parameter π governing the frequency of price adjustment.

The rationale for the calibration in Table 1 is as follows. We calibrate so that the unit of time is one quarter. Thus, β is chosen to equal .99, which gives a 4 percent steady state annual real interest rate (we are abstracting from long run growth). Likewise the depreciation rate δ is set at 10 percent per year, so that $\delta = 0.025$. The parameter π is chosen to govern the frequency of price adjustment. This is set so that the average frequency of price adjustment is 4 quarters. This requires $\pi = 0.61$. Results are shown also for alternative values of π.

The value of η is chosen so that the representative agent in both countries chooses to work 30 percent of available time, the standard calibration in RBC models.

The parameters ϵ and σ govern the consumption and interest elasticity of money demand. The consumption elasticity of money demand is equal to $\frac{\sigma}{\epsilon}$. The interest elasticity of money demand is

$$\frac{\beta}{\epsilon(1+g_m)}.$$

As to be expected in a model with price stickiness, these two elasticities are critical for the response of the real and the nominal exchange rate to monetary shocks. Estimates of the consumption elasticity of money demand vary in the literature. Mankiw and

Summers (1986) estimate a consumption elasticity of money demand equal to unity. Other estimates have been reported either higher or lower. Helliwell, Conkerline and Lafrance (1990) report a large number of estimated money demand elasticities for G7 countries that are typically used in macro models. These differ somewhat across countries, but for many countries the income elasticity for narrower definitions of money are below unity. For instance, the reported Fair and Taylor (1983) model uses an estimated elasticity for M1 of .85 for the US and .55 for Japan. [6]

The size of the elasticity of demand matters in much the same way as in Dornbusch (1976). For a high elasticity of demand, the response of the aggregate consumption or income variable to a money shock can be so great that the net result is to generate excess *demand* for money, necessitating a rise in the nominal interest rate. To avoid this, we choose parameters for our baseline case so that $\frac{\sigma}{\epsilon} = 0.85$, consistent with Fair and Taylors estimate. We also report our simulation results for the alternative case of $\frac{\sigma}{\epsilon} = 1$ however. In each case, we then choose σ (or ϵ, equivalently) to determine the interest elasticity of money demand.

Estimates of the interest elasticity of M1 vary from a value of 0.02 reported in Mankiw and Summers (1986) to values around 0.25 reported in the Helliwell et al. (1990). We choose a value of 0.12, approximately half way between these estimates. With money growth rate g_m equal to 6 percent annual, this requires a relatively low intertemporal elasticity of substitution, i.e. $\sigma = 7$. This is still well within the set of reasonable values of σ used by Mehra and Prescott (1985), however. Again, results are given for different interest elasticities of substitution.

The markup parameter λ is set so that markups are equal to those found by Basu

[6] In Betts and Devereux (1996), we showed that, in a model without capital, and risk-sharing through the use of non-contingenet nominal bonds, that a necessary and sufficient condition for the exchange rate to 'overshoot' in response to an unanticipated money shock was that the consumption elasticity of money demand be less than unity. An overshooting exchange rate is a necessary and sufficient condition for a 'liquidity' effect on nominal interest rates (i.e. a fall in the relative domestic to foreign nominal interest rate). Therefore, it would seem to be necessary to calibrate so as to have the consumption elasticity of money demand below unity in order to match the evidence on relative interest rates given in section 2. In fact this is not the case. With complete markets, as assumed here, a unitary consumption elasticity of money demand is consistent with a liquidity effect so long as the elasticity of intertemporal substitution in consumption is low enough.

and Fernald for US data, i.e. in the region of 10 percent. This requires a high elasticity of substitution between goods within a category. On the other hand, the elasticity of substitution between categories is set so that the elasticity of substitution between foreign and domestic goods is equal to 1.5, the number used in Backus, Kehoe, and Kydland(1994). The fixed cost parameter υ is then set to produce average profits of zero, in accordance with evidence of very small pure profits in the US economy. With profits equal to zero, the share of capital in output is α. This is set to 0.36.

The adjustment cost function determines a parameter $\tilde{\psi} = \psi'(.)\bar{K}$, which is the resource cost of adjusting the capital stock. Without adjustment costs, capital would move too quickly across countries, and the variability of investment to output would be counterfactually high. Following Baxter and Crucini (1993), we set this parameter so that the variability of investment relative to output in the simulated model is at reasonable levels. The results are relatively insensitive to alternative values of $\tilde{\psi}$ with the benchmark range.

The pricing-to-market parameter s is the key determinant of the responses of the real exchange rate and macro aggregates to money shocks. There is some independent evidence on this parameter. Knetter (1993) estimates a number which is roughly the analogue of our variable s for US, German and Japanese exporters. The average of his point estimate is 0.614. However, the results of Engel and Rogers (1996), and Engel (1993) suggest that virtually none of the high- frequency variability in nominal exchange rates is transmitted to price levels. This would suggest a value of $s = 1$. We vary this parameter between 0 and 1 to show the effect of PTM on the international monetary transmission mechanism.

We assume equal country size and identical initial wealths , so the relative size parameter n is set equal to .5, and $\Lambda = 1$.

Our monetary policy rule implies that a shock to the money supply causes a one-time increase in the level of the money stock. In our VAR estimates for the response of non-borrowed reserves to a non-borrowed reserves shock, this is approximately the correct assumption. That is, the subsequent movements in non-borrowed reserves does not suggest autocorrelation in growth rates. Nevertheless, the unconditional evidence on broader monetary aggregates such as M1 clearly suggests that autocorrelation in growth rates. As a check on the model, therefore, we report results also for an alternative monetary policy rule given by

$$\frac{M(z_{t+1})}{M(z_t)} = (\frac{M(z_t)}{M(z_{t-1}})^{\kappa} \exp(\tilde{g}_m + u_{z_t})$$

where $1 \geq \kappa \geq 0$, $g_m = \tilde{g}_m/(1-\kappa)$ is the mean growth rate of money, and u_{z_t} is a stochastic money shock. In this case, following Christiano and Eichenbaum (1991), we set $\kappa = .3$.

Section 6 Quantitative Evaluation of the Model

We explore the model's responses to a home country money shock. The key question is whether this model can replicate the main features of the international monetary transmission mechanism as defined above.

Impulse Responses.

First, we explore the effect of an unanticipated, permanent increase in the home country money supply of one percent. Figure 2 and 3 illustrates the impulse responses for the real exchange rate, the output differential between the two countries, the nominal interest rate differential, and the home country trade balance. Figure 2a represents the case of $s = 0$, so that PPP holds at all times. Figure 3a represents the case of full PTM, or $s = 1$.

In Figure 2a, the real exchange rate is unaffected by the monetary shock (the nominal exchange rate depreciates by the permanent amount i.e. 1, in the first period), since even with sticky nominal prices, the home price index P rises and the foreign price index Q falls, leaving eQ/P unchanged. In the bottom left panel we see that, since the nominal exchange rate depreciates immediately to its steady state level, there is no affect at all on the interest rate differential, since uncovered interest rate parity (UIRP) has to hold in our model.

In the top right panel of the Figure we see that the output differential between the two countries rises sharply on impact . This is because the home country output rises, but foreign output falls. The nominal depreciation generates a terms of trade deterioration for the hŏme economy, which leads aggregate world expenditure to substitute towards home country output.

However, because with PPP there is effectively perfect risk-sharing, home and foreign consumption responses are identical. Consumption rates in both countries both rise immediately (this is shown in Figure 2b). This implies that the trade balance improves in the home country, as shown in the bottom right panel of Figure 2a.

Figure 3a shows the impulse response for the case of $s = 1$. In this case, the unanticipated money shock leads to an immediate real depreciation, after which the real exchange rate gradually returns to its PPP equilibrium level. The output response in both countries is now positive. With $s = 1$, the exchange rate depreciation does not pass through immediately into import goods prices, thus reducing the 'expenditure switching' of world demand towards the home country product. On impact, output levels rise by approximately the same magnitude in both the home and foreign countries. Therafter, the output differential rises only slightly through the adustment period.

However, as shown in Figure 3b, consumption responses are now driven apart. By the risk-sharing rule $(3 - 9)$, the real depreciation drives a wedge between home and foreign consumption, so that the consumption differential must rise. In levels, home consumption rises, while foreign consumption falls slightly. With sluggish prices, and a lower nominal interest rate (as explained below), real interest rates fall in the home economy, while they rise in the foreign economy. This raises domestic consumption and reduces foreign consumption. Figure 3b shows also that home country investment rises, while foreign investment falls.

The bottom left panel of Figure 3a shows that the nominal interest rate differential falls, giving a liquidity effect of the home monetary expansion. The reason for this is that the nominal exchange rate now 'overshoots', rising by more in the short run than in the long run. Thus by UIRP, the home nominal interest rate falls relative to the foreign nominal rate..[7]

[7] Note that some of the estimates from section 2 suggest that the nominal exchange rate rises only slowly to its maximal value following a non-borrowed reserves shock. In conjunction with a fall in the

This bottom right panel of Figure 3a shows that the response of the trade balance is the opposite of the $s = 0$ case. The trade balance now immediately goes into deficit, and returns to surplus only gradually. There is in effect a 'J' curve phenomenon.

The intuition behind the J-curve in the model is easy to see. Output rises by the same amount in both countries, but absorption rises in the home economy, and falls in the foreign economy. As a result, the initial effect of the money shock is to generate a deterioration in the home economy trade balance. This is gradually eliminated however, and the home country goes into surplus as the initial rise in home investment leads to a gradual increase in home relative output, while the convergence to PPP generates a convergence in consumption levels across countries.

Comparing Figure 3a with Figure 1 in section 2, we are led to the conclusion that the theoretical responses of the real exchange rate, relative output, interest rate differentials, and the trade balance to money shocks in an environment of PTM are very similar to the empirical evidence on the international monetary transmission mechanism. In both the data and the model, a money shock generates a persistent real depreciation, a very small rise in relative output levels, a fall in home relative interest rates, and a J-curve response of the trade balance. Moreover, with the exception of the exchange rate response, discussed above, the scale of the responses is very similar to those of the model.

But comparing Figure 2 and Figure 1, on the other hand, establishes clearly that the PPP based model is inconsistent with the major features of the international monetary transmission mechanism. While it predicts that the real exchange rate should be unchanged, that relative output levels should jump sharply, that the interest rate differential should be unchanged, and that the trade balance should improve, the data suggest that a monetary innovation is followed by a real exchange rate depreciation, a

relative home nominal interest rate, this implies that conditional on a money shock UIRP is violated. This point was noted clearly in Eichenbaum and Evans (1995). Our model requires UIRP to hold, so by necessity, the presence of a liquidity effect requires an exchange rate overshooting. Therefore, the departures of exchange rate movements in the model from those in the data are really part of the well known 'forward premium anomoly'. Gourinchis and Tornell (1996) develop an intriquing exchange rate model based on learning effects to potentially resolve this dilemma.

small rise in relative output, a fall in the interest rate differential, and a trade balance deterioration.

In comparing Figure 2b and Figure 3b, it is also noteworthy that the presence of PTM introduces a considerable degree of endogenous price stickiness into the model. We mean by this that the domestic price level rises by less on impact in an environment of PTM, and takes about 1 to 2 quarters longer to adjust to the shock than in the PPP environment. Intuitively, with PPP, the expenditure switching response of world demand adds to the pressure on the domestic firms to raise their prices (and reduces the pressure on foreign firms - as shown in Figure 2b, foreign prices fall in this case). But with PTM, aggregate demand rises for both countries goods, and prices rise in both countries. But the absence of an expenditure switching towards domestic goods implies prices rise on impact by less, in the home country.

Note that the estimates of the conditional (on a money shock) persistence of the real exchange rate in response to a money shock are considerably less than the unconditional persistence of the real exchange rate. A relatively small persistence in the real exchange rate in the model therefore is not inconsistent with the well-known high persistence in real exchange rate data, since we are attempting only to match conditional responses of the real exchange rate.

Sensitivity Analysis

Figure 4a varies the degree of price stickiness in the model. Figure 4a sets the average price adjustment to take place within one quarter. Figure 4b assumes that the average length of price adjustment is 12 quarters. The qualitative features of the international monetary transmission mechanism are unchanged. A smaller degree of price rigidity eliminates much of the persistence in the real exchange rate and other variables (e.g. see Cooley and Hansen 1995), while a greater degree of price rigidity introduces a more persistent behaviour of all variables .

Figure 5 varies the the consumtion elasiticity of money demand in the model. Figure 5a assumes that $\frac{\sigma}{\epsilon} = 1$, while Figure 5b sets $\frac{\sigma}{\epsilon} = .5$ In each case, σ is adjusted to maintain a constant interest elasticity of money demand. There is very little difference in the responses. For a lower consumption elasticity of money demand, consumption rises by more in the home economy, and the trade balance falls by a greater amount (Figure 5b). The opposite situation occurs as the elasticity rises to 1 (Figure 5a).

Figure 6 increases the elasticity of intertemporal substitution ($\frac{1}{\sigma}$). Here σ is set to 4. This effectively raises the interest elasticity of money demand in the money, holding the consumption elasticity of money demand constant. The main effect is to reduce the scale of the liquidity effect, and increases the initial fall in the trade balance.

Finally, Figure 7 extends the benchmark model to allow for autocorrelation in money growth rates. Here we see that the size of the liquidity effect is reduced. This is intuitive, since with autcorrelation in money growth rates, the magnitude of the Fisherian fundamentals in the nominal interest rate response is enhanced.

Moments

We compute the simulated moments of the model for three different values of s. These are reported in Table 2. In this simulation it is assumed that home and foreign money shocks are independent. For $s = 0$, the cross country correlation of output levels is negative, while the cross country correlation of consumption is unity. As s rises, the cross country correlation of output rises, while the cross country correlation of consumption falls. For $s = .614$, the cross country correlation of output is .55, while the cross country correlation of consumption is .35. As s rises to unity, the cross country correlation of output rises to .79, while the cross country correlation of consumption falls to -.22.

From this we see, in model driven by money shocks alone, the presence of PTM can help explain the 'quantity anomoly' as defined by Backus, Kehoe, and Kydland (1994). The greater is the degree of PTM, the more likely that the output correlations across countries will exceed consumption correlations. Moreover this holds despite the presence of complete financial markets and full exploitation of risk-sharing. The key mechanism driving the results is international market segmentation that allows for national pricing to market. This has the joint effect of reducing the role that relative prices play in the adjustment mechanism, thus increasing the comovement of output, while reducing the comovement of consumption, due to the effect of changes in the real exchange rate on optimal risk-sharing.

Table 2 also shows the the presence of PTM increases both the real exchange rate variability, relative to output, and the terms of trade variability, relative to output. For $s = 0$, the real exchange rate is constant, while the terms of trade variability is about 95 percent of output. For $s = .65$ and $s = 1$, respectively, real exchange rate variability rises to about 1.32 percent of output and 240 percent of output respectively. For $s = .614$, terms of trade variability is 75 percent of output variability, but rises to 235 percent of output variability as s goes to 1. Thus, the model can also partially explain, in the case of money shocks alone, the 'price variability anomoly' as defined by Backus, Kydland and Kehoe (1994). In the data, the terms of trade vary by more than output. In the PPP model, terms of trade variability is less than output. But with a greater and greater share of the market governed by PTM, terms of trade variability rises relative to output. The reason that PTM allows for a greater degree of terms of trade variability, relative to the PPP model, is that, with PTM, the terms of trade response is not restricted by the first order conditions of optimal consumption allocation as in standard models. This allows prices to move without the usual limitations imposed by the elasticity of substitution between home and foreign goods. With PTM, prices responses do not play an allocational role. This fact allows for price responses to be magnified.

Section 7 Conclusions

This paper has documented a set of empirical findings governing the international monetary transmssion mechanism. These findings, some of which have been shown in previous literature, indicate that a US monetary expansion generates a persistent real depreciation, a small positive response in US relative output, a fall in relative US interest rates, and a J-curve response of the trade balance. The paper then develops a theoretical model that is consistent with these empirical findings. The two key ingredients of the model are nominal price stickiness, and segmented international commodity markets. The responses of the model to a money shock are very similar to those produced in the

VAR estimates from the data.

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Table 1: Calibration

a) Preference Parameters	
β	0.99
η	0.50
$\frac{\sigma}{\epsilon}$	0.85
σ	7.00
b) Technology Parameters	
δ	.025
ψ	10.5
α	0.36
ρ	1.50
λ	10.0
c) Policy Parameters	
$g_m = g_m^*$	0.06
d) Relative Size Parameters	
n	0.50
Λ	1.00
e) Pricing parameters	
s	0, .614, 1
π	0.616

Table 2: Moments from Theoretical Model

Variable	s=0	s=0.614	s=1.0
$\frac{\sigma_c}{\sigma_y}$	0.11	0.17	0.21
$\frac{\sigma_I}{\sigma_y}$	2.48	4.10	5.6
$\frac{\sigma_R}{\sigma_y}$	0.00	1.32	2.4
$\frac{\sigma_t}{\sigma_y}$	0.95	0.75	2.35
Corr(y,y^*)	0.01	0.55	0.79
Corr(c,c^*)	1.00	0.35	-0.22

Figure 1a – US/Canada: EP*/P Response to US Money Shock

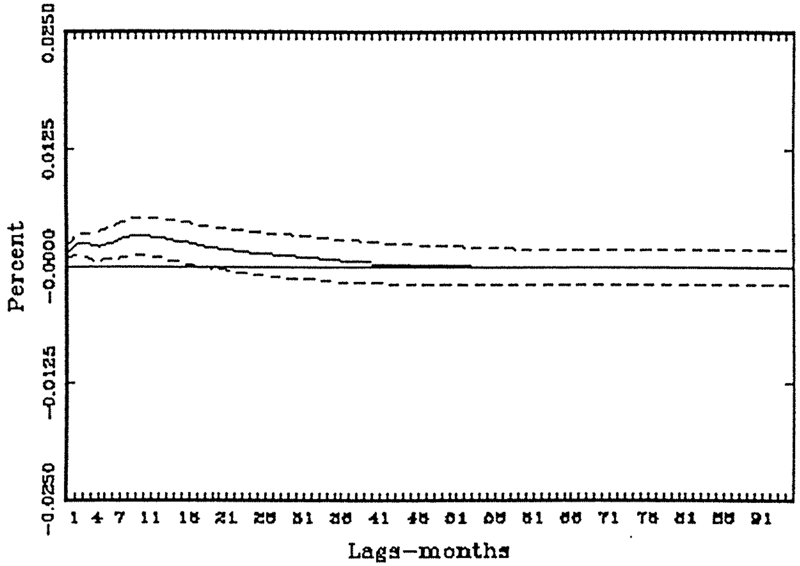


Figure 1a – US/Canada: Y/Y* Response to US Money Shock

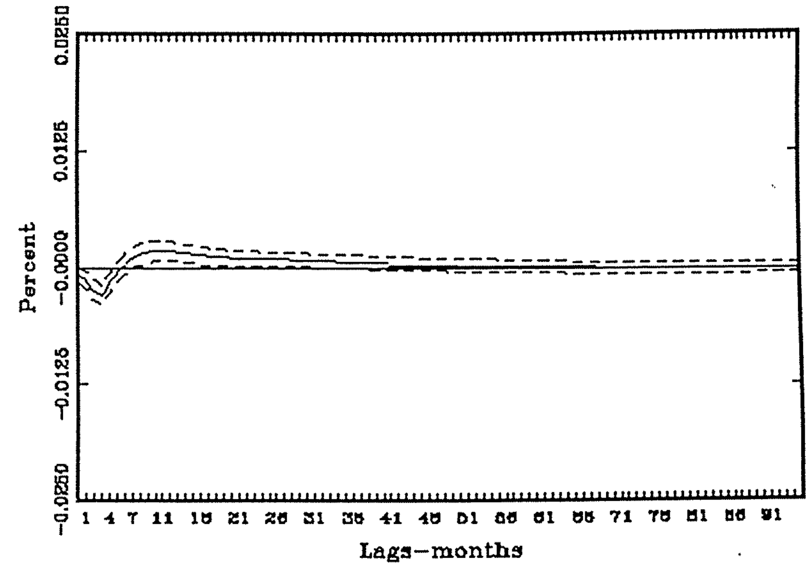


Figure 1a – US/Canada: I–I* Response to US Money Shock

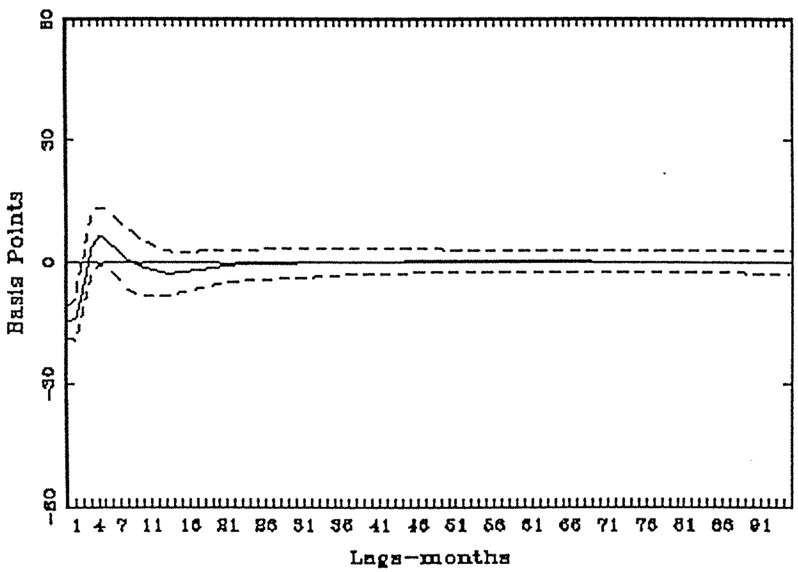


Figure 1a – US/Canada: TB/Y Response to US Money Shock

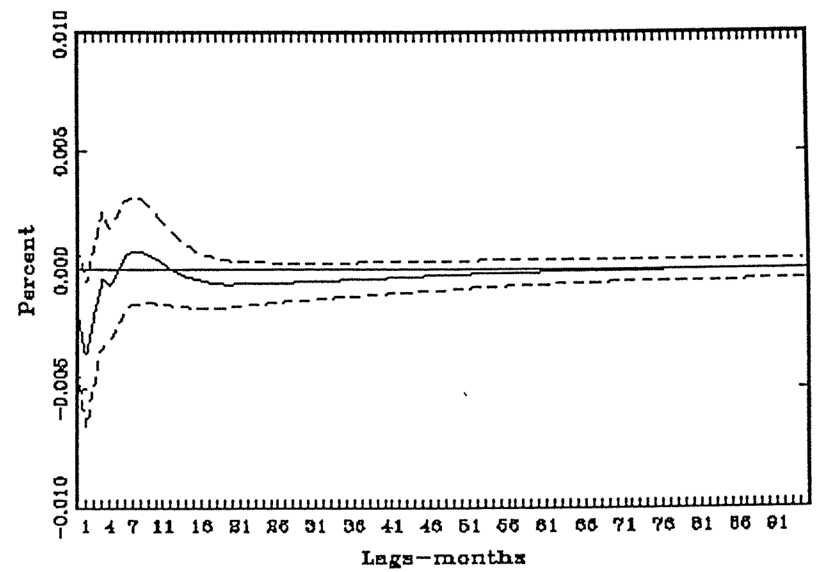


Figure 1b — US/France: EP*/P Response to US Money Shock

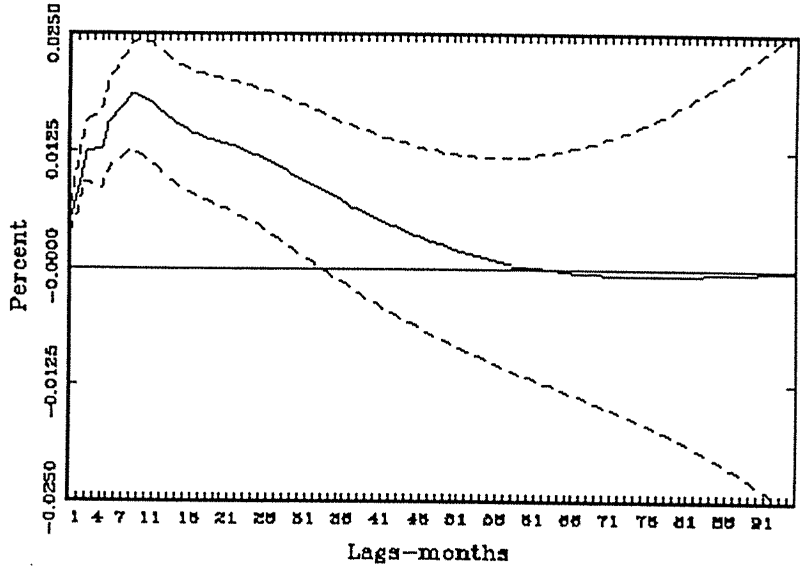


Figure 1b — US/France: Y/Y* Response to US Money Shock

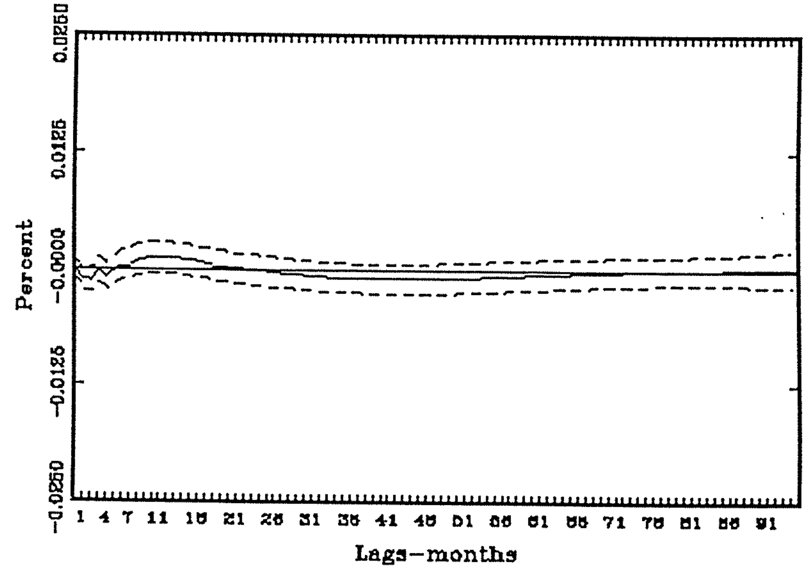


Figure 1b — US/France: I-I* Response to US Money Shock

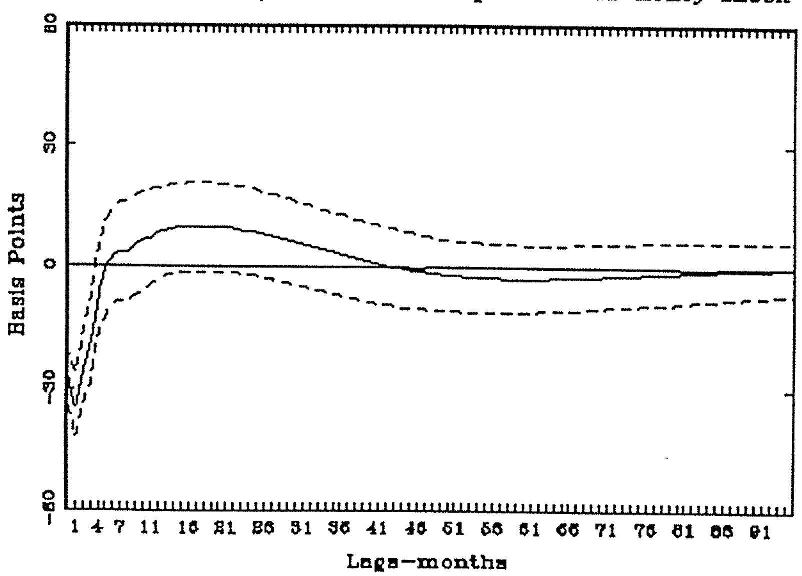


Figure 1b — US/France: TB/Y Response to US Money Shock

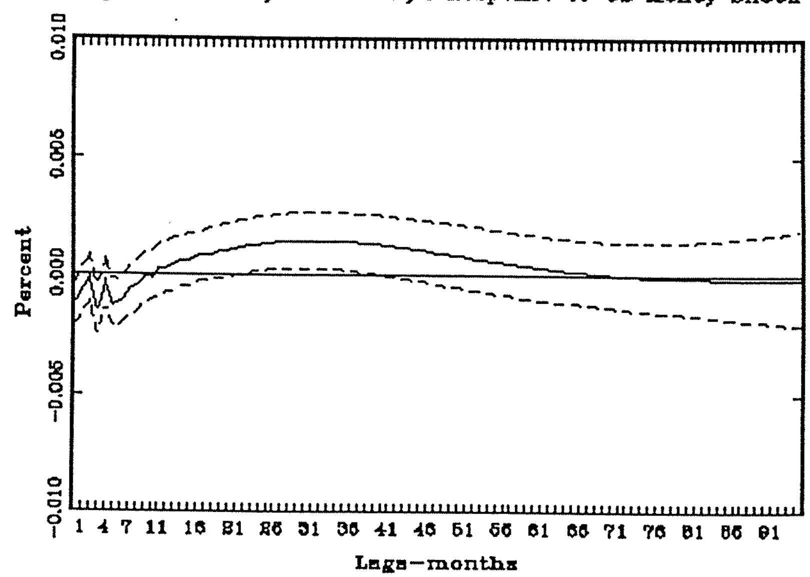


Figure 1c — US/Germany: EP*/P Response to US Money Shock

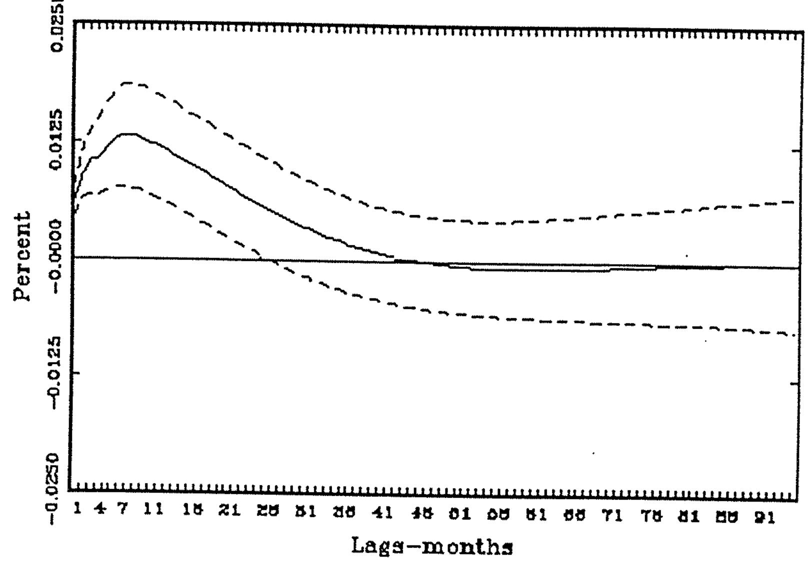


Figure 1c — US/Germany: Y/Y* Response to US Money Shock

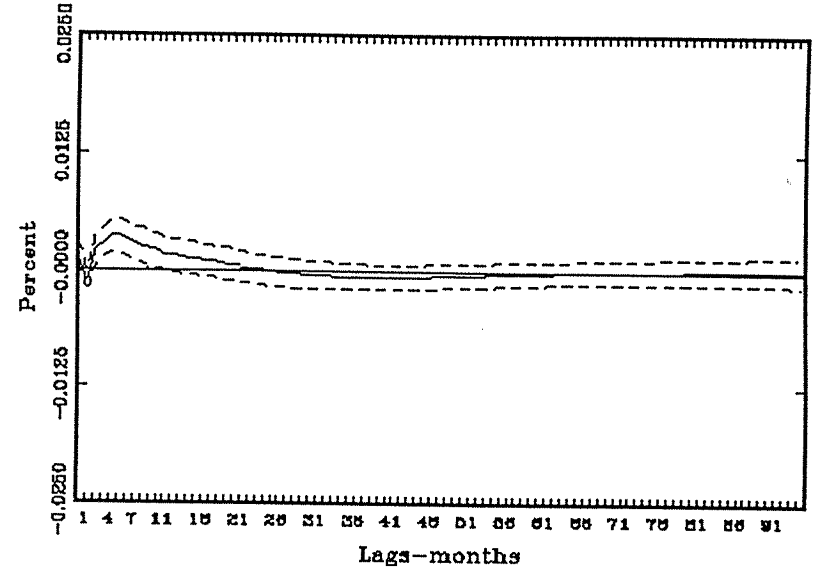


Figure 1c — US/Germany: I-I* Response to US Money Shock

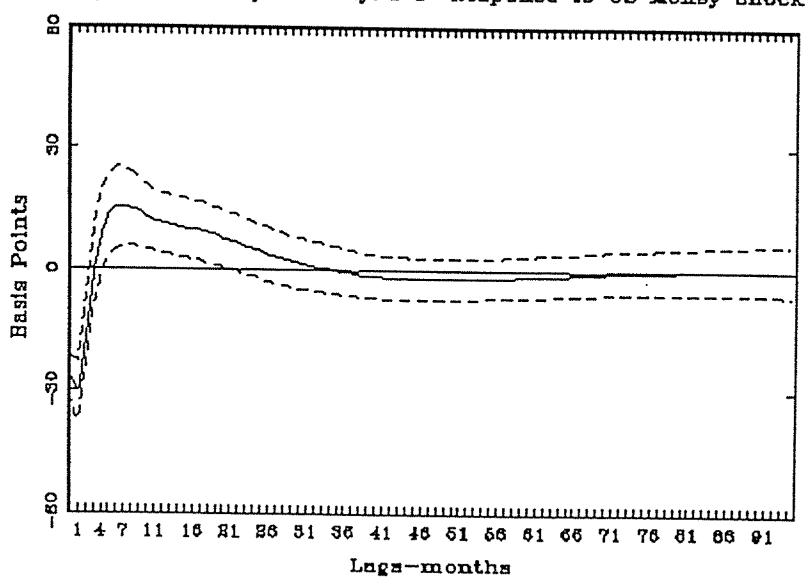


Figure 1c — US/Germany: TB/Y Response to US Money Shock

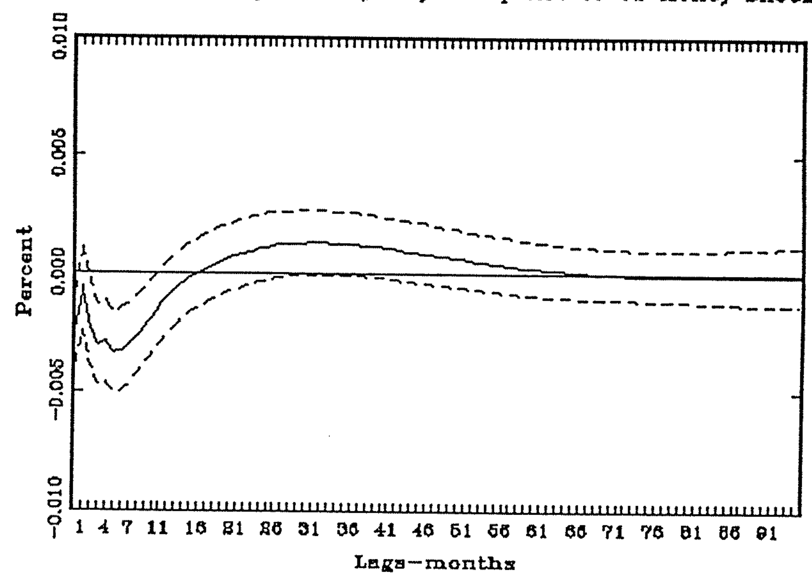


Figure 1d – US/Italy: EP*/P Response to US Money Shock

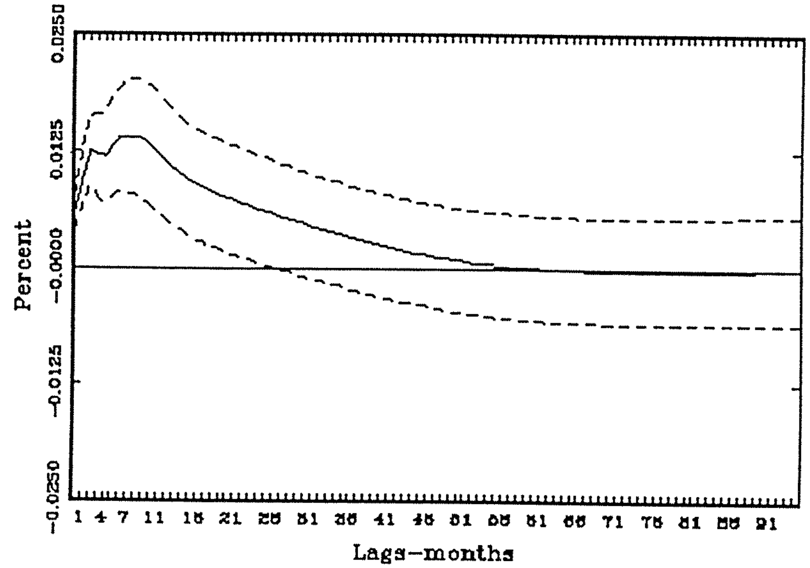


Figure 1d – US/Italy: Y/Y* Response to US Money Shock

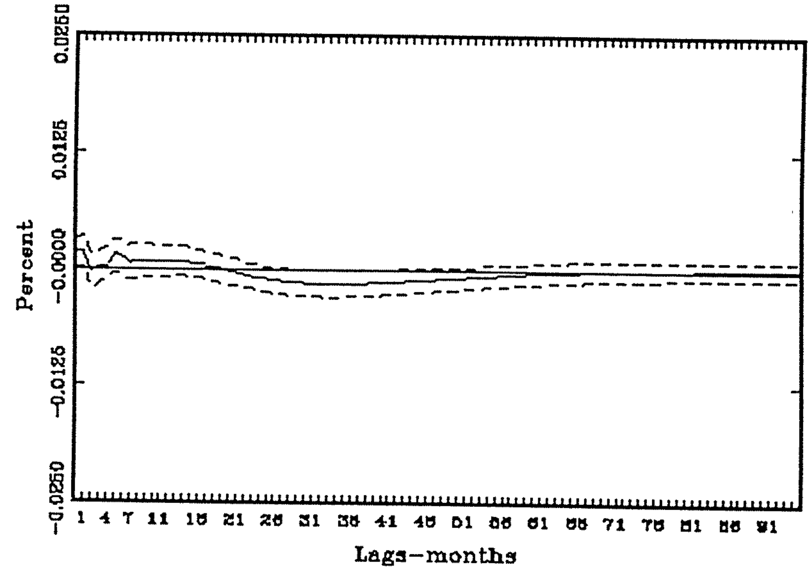


Figure 1d – US/Italy: I–I* Response to US Money Shock

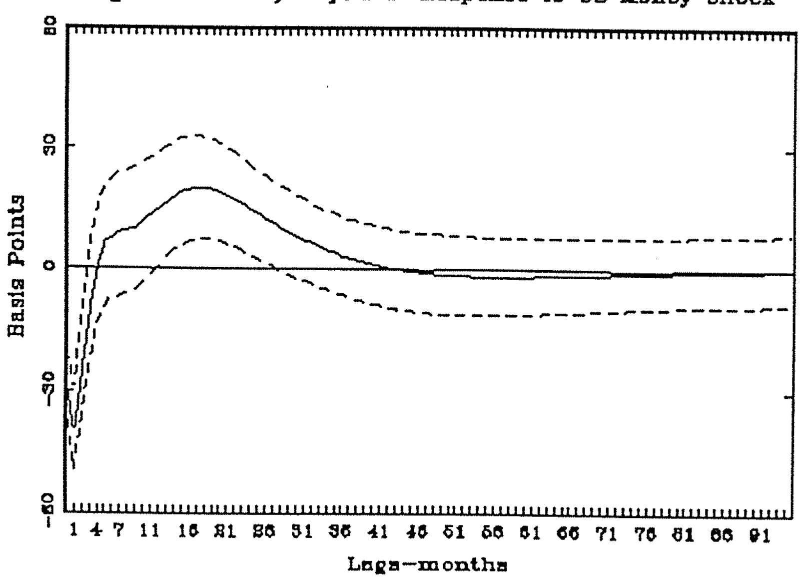


Figure 1d – US/Italy: TB/Y Response to US Money Shock

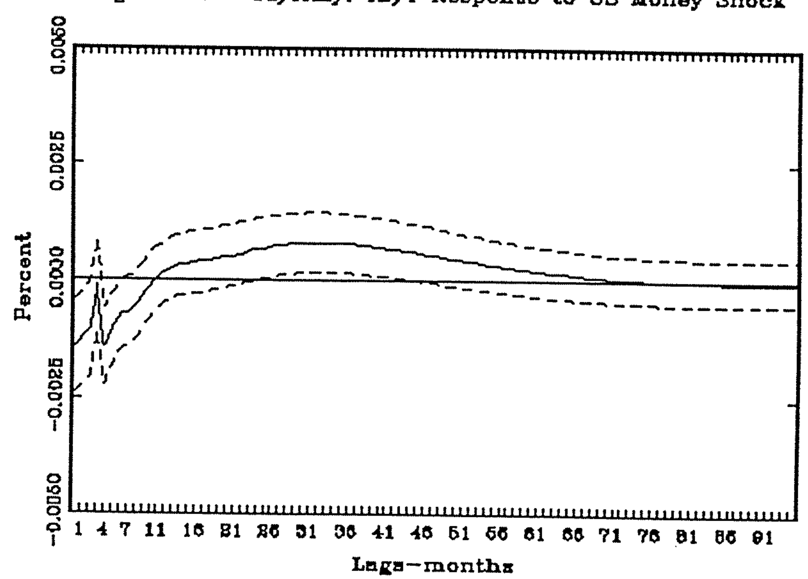


Figure 1e — US/Japan: EP*/P Response to US Money Shock

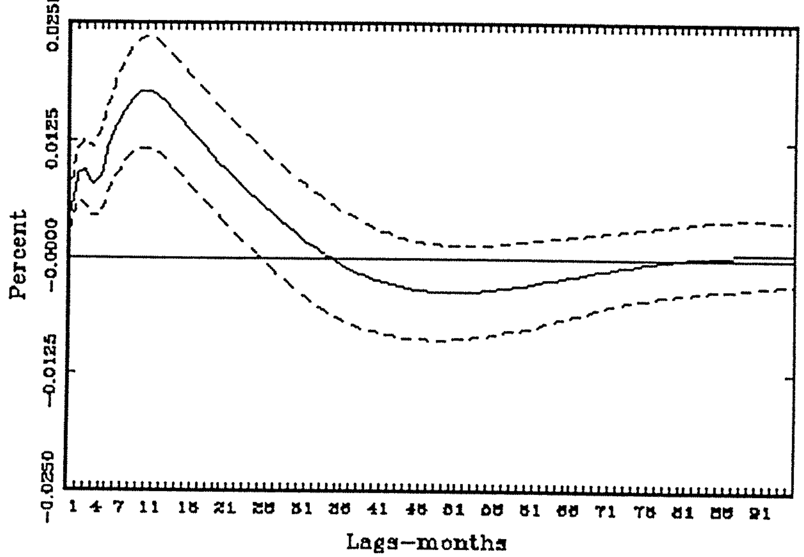


Figure 1e — US/Japan: Y/Y* Response to US Money Shock

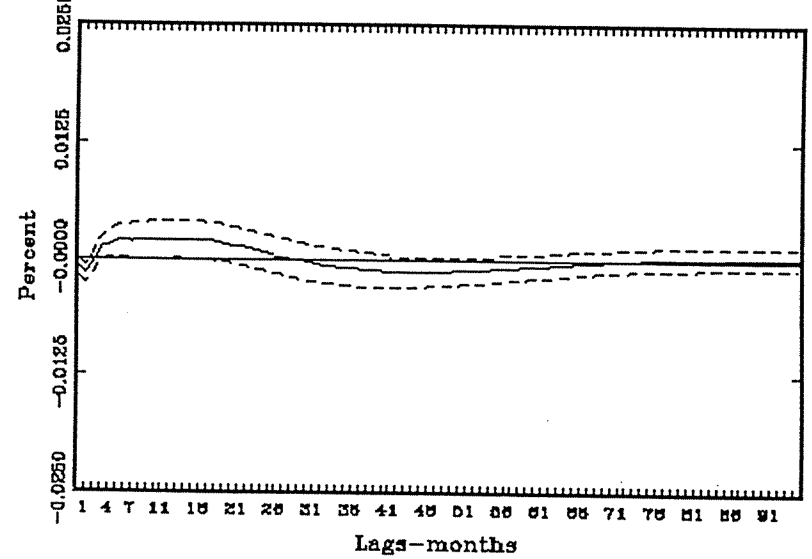


Figure 1e — US/Japan: I—I* Response to US Money Shock

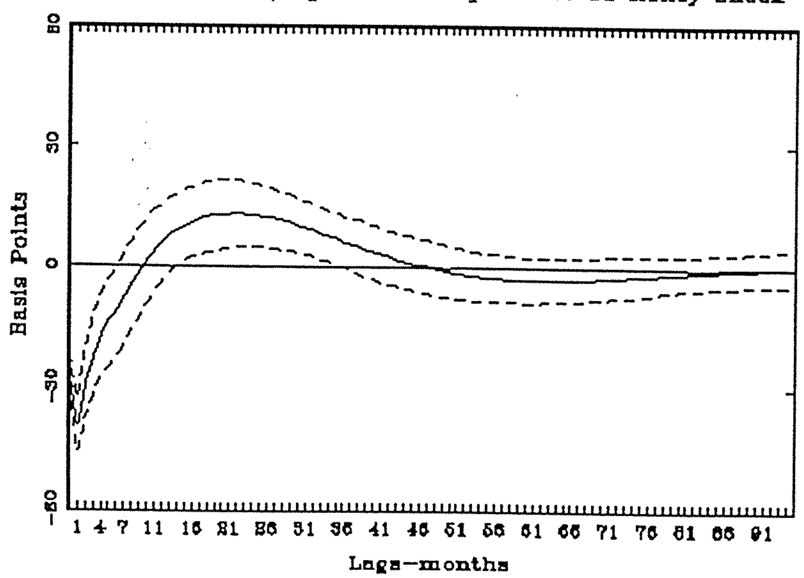


Figure 1e — US/Japan: TB/Y Response to US Money Shock

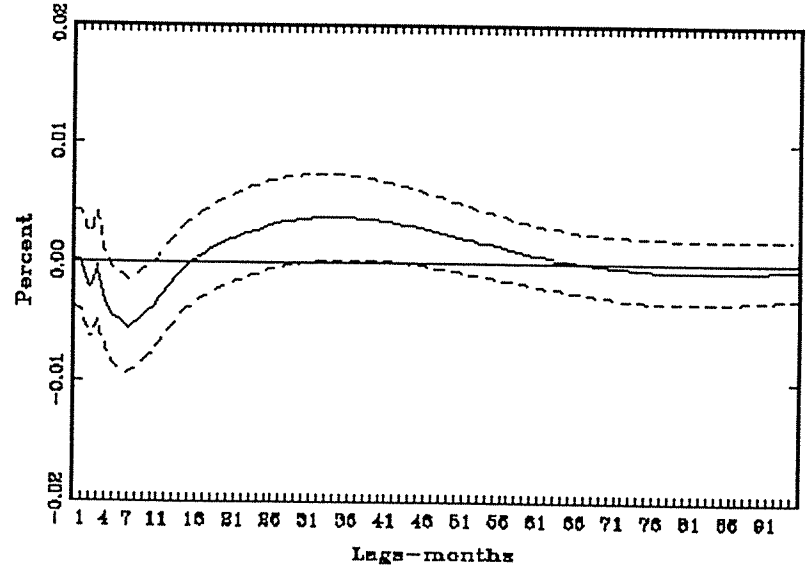


Figure 1f – US/UK: EP*/P Response to US Money Shock

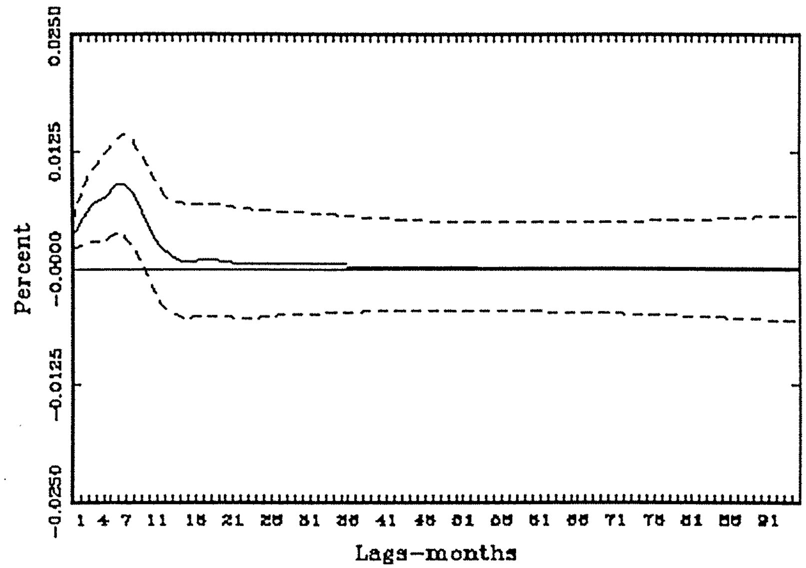


Figure 1f – US/UK: Y/Y* Response to US Money Shock

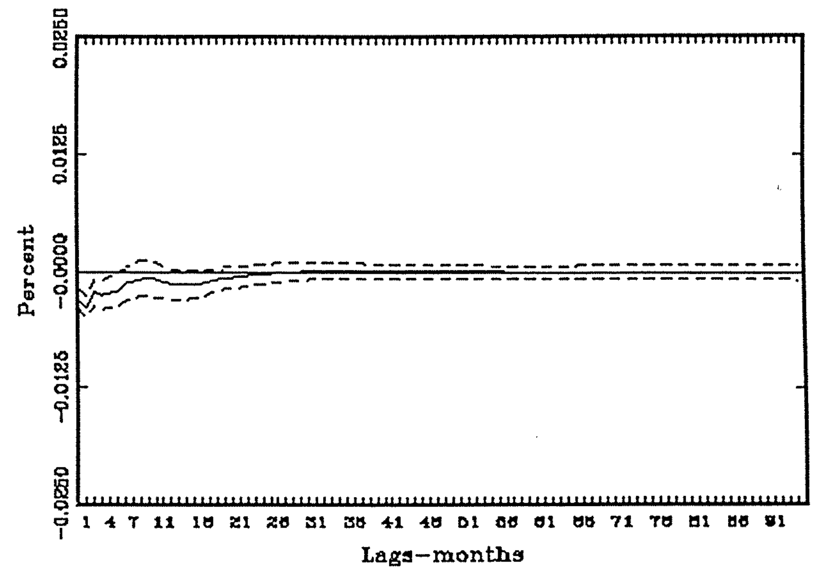


Figure 1f – US/UK: I–I* Response to US Money Shock

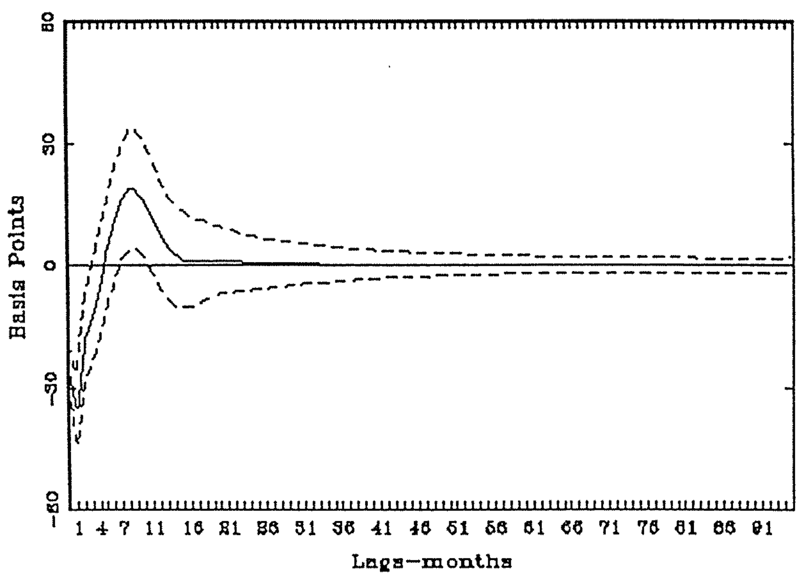


Figure 1f – US/UK: TB/Y Response to US Money Shock

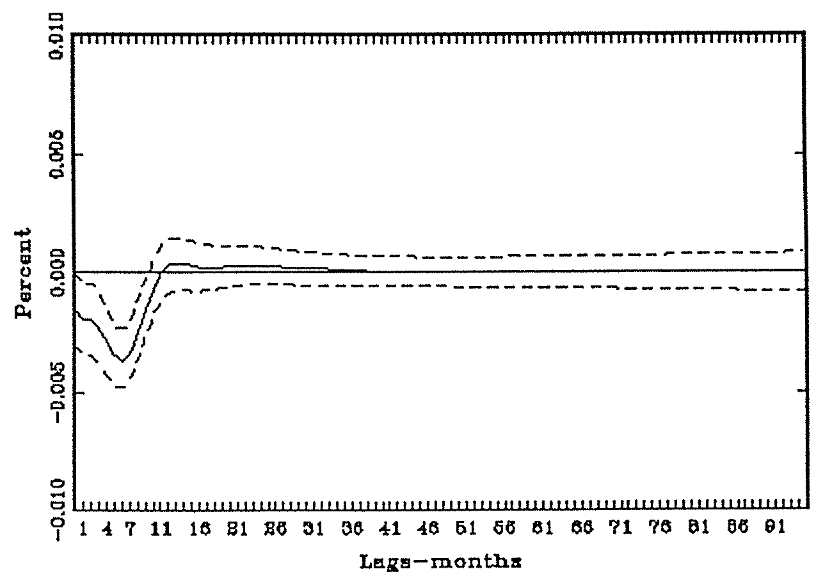


Figure 2A

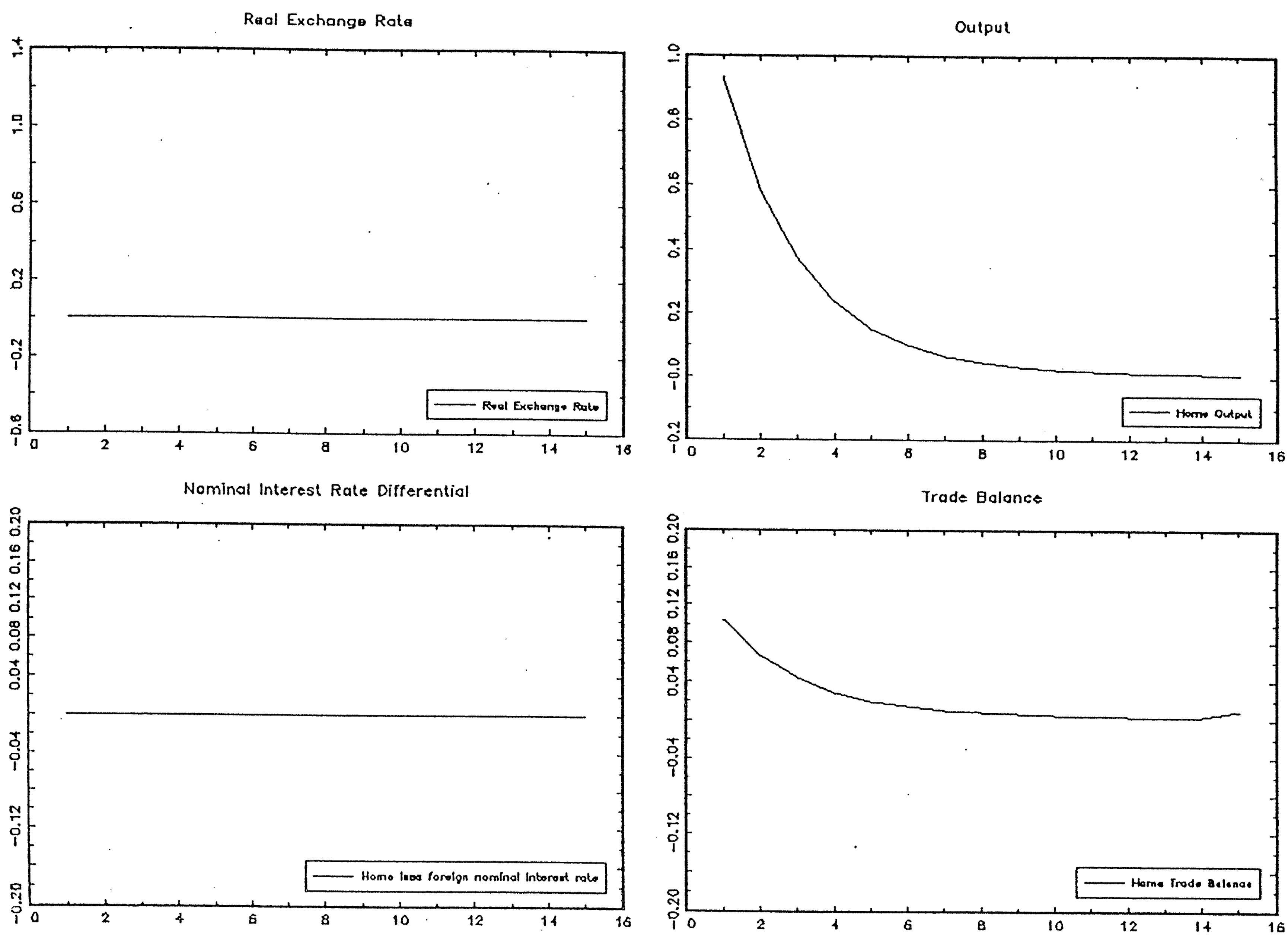

Real Exchange Rate
Real Exchange Rate
1.4
1.0
0.6
0.2
-0.2
-0.6
0
2
4
6
8
10
12
14
16
Output
Home Output
1.0
0.8
0.6
0.4
0.2
-0.0
-0.2
Nominal Interest Rate Differential
Home less foreign nominal interest rate
0.20
0.16
0.12
0.08
0.04
-0.04
-0.12
-0.20
Trade Balance
Home Trade Balance

Figure 2.B

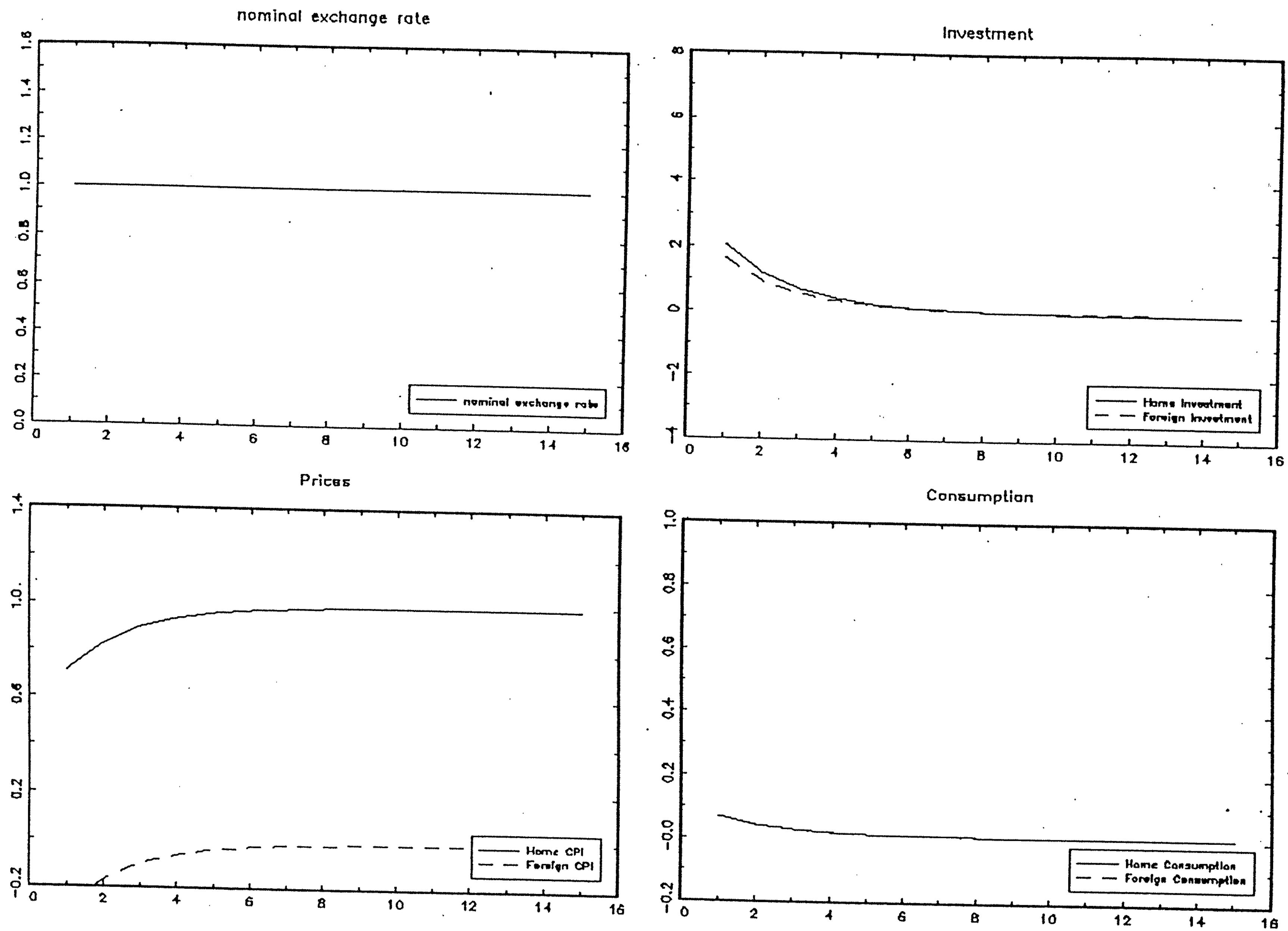
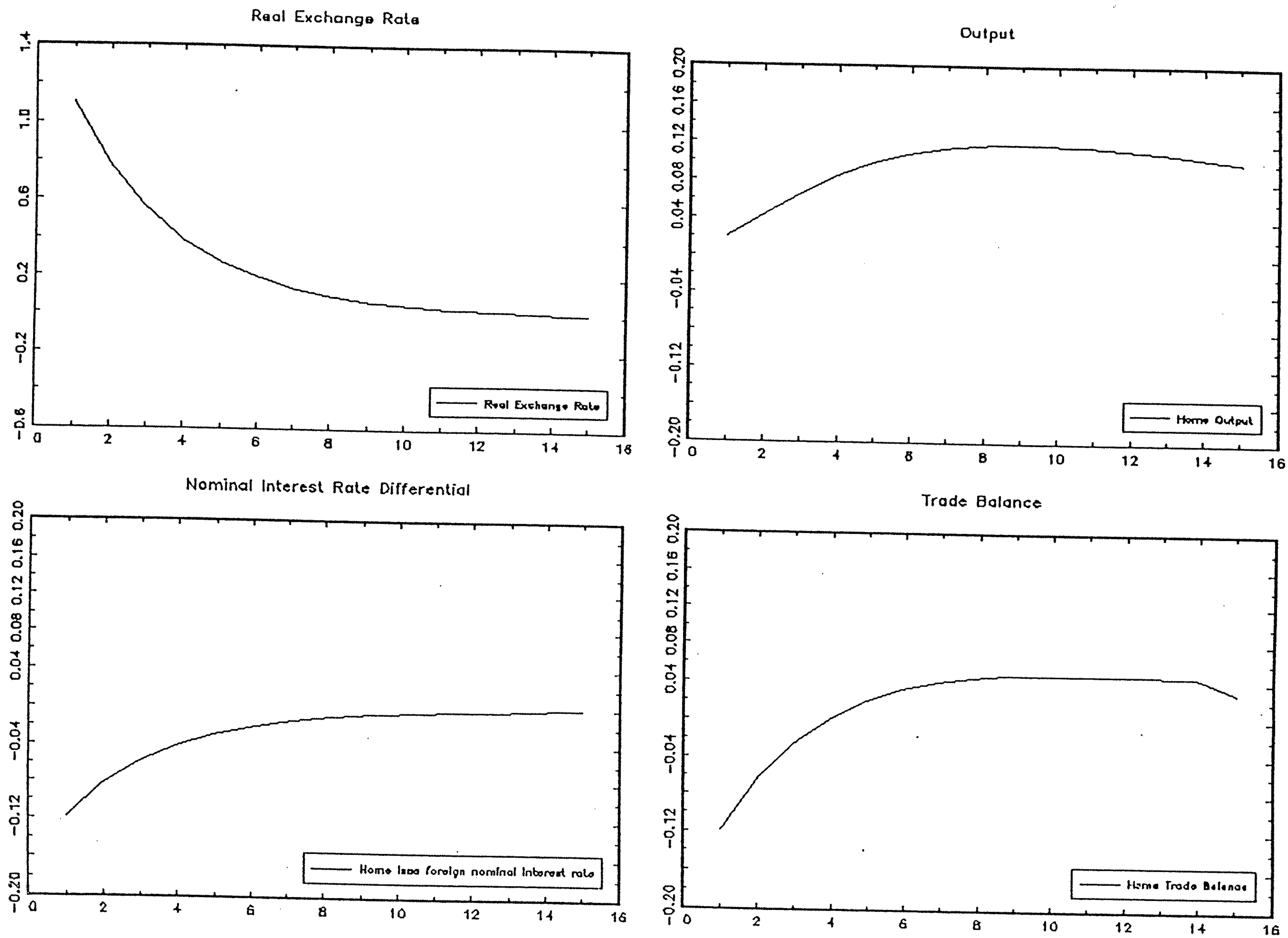
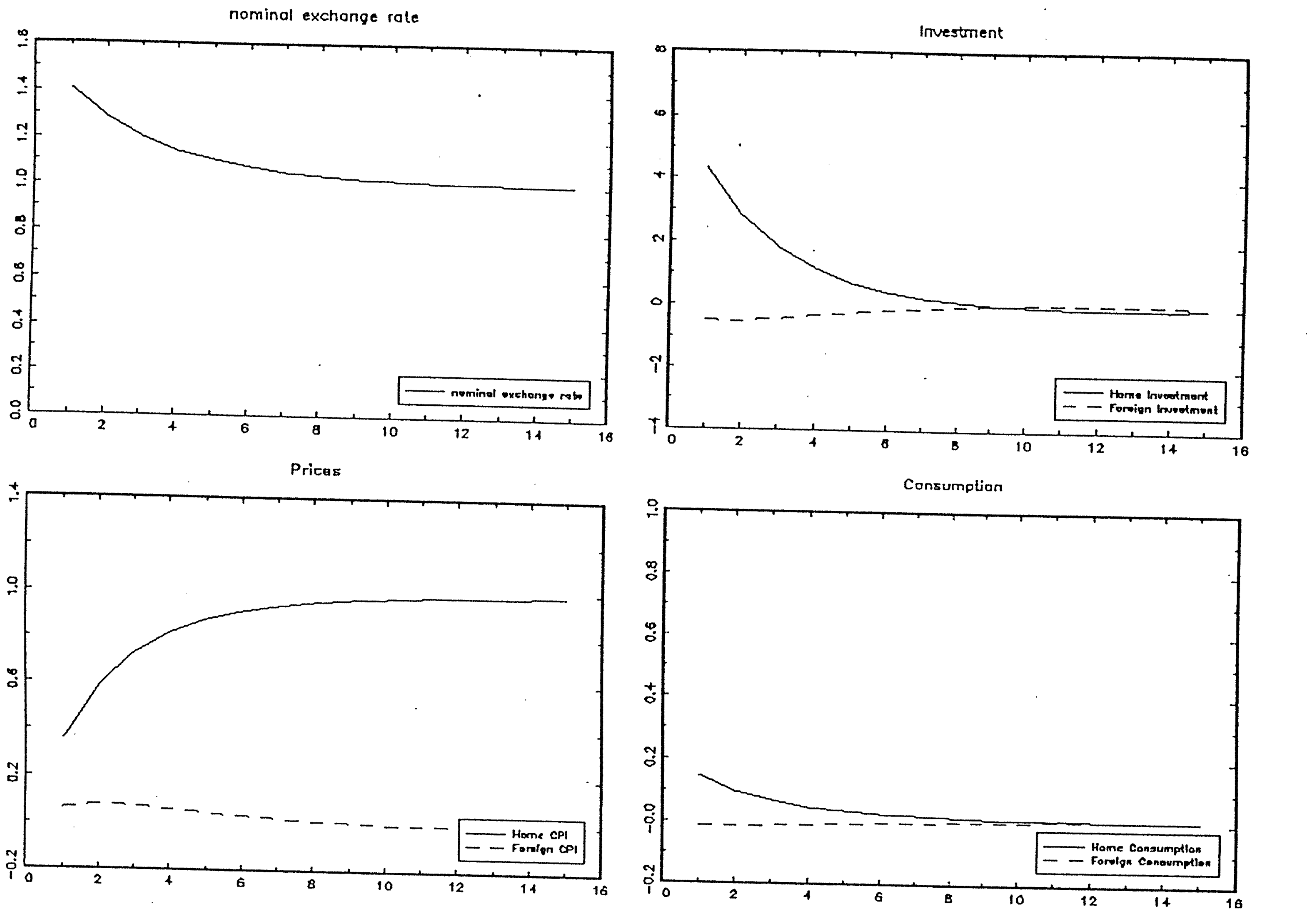

nominal exchange rate
nominal exchange rate
Investment
Home Investment
Foreign Investment
Prices
Home CPI
Foreign CPI
Consumption
Home Consumption
Foreign Consumption

Figure 3A



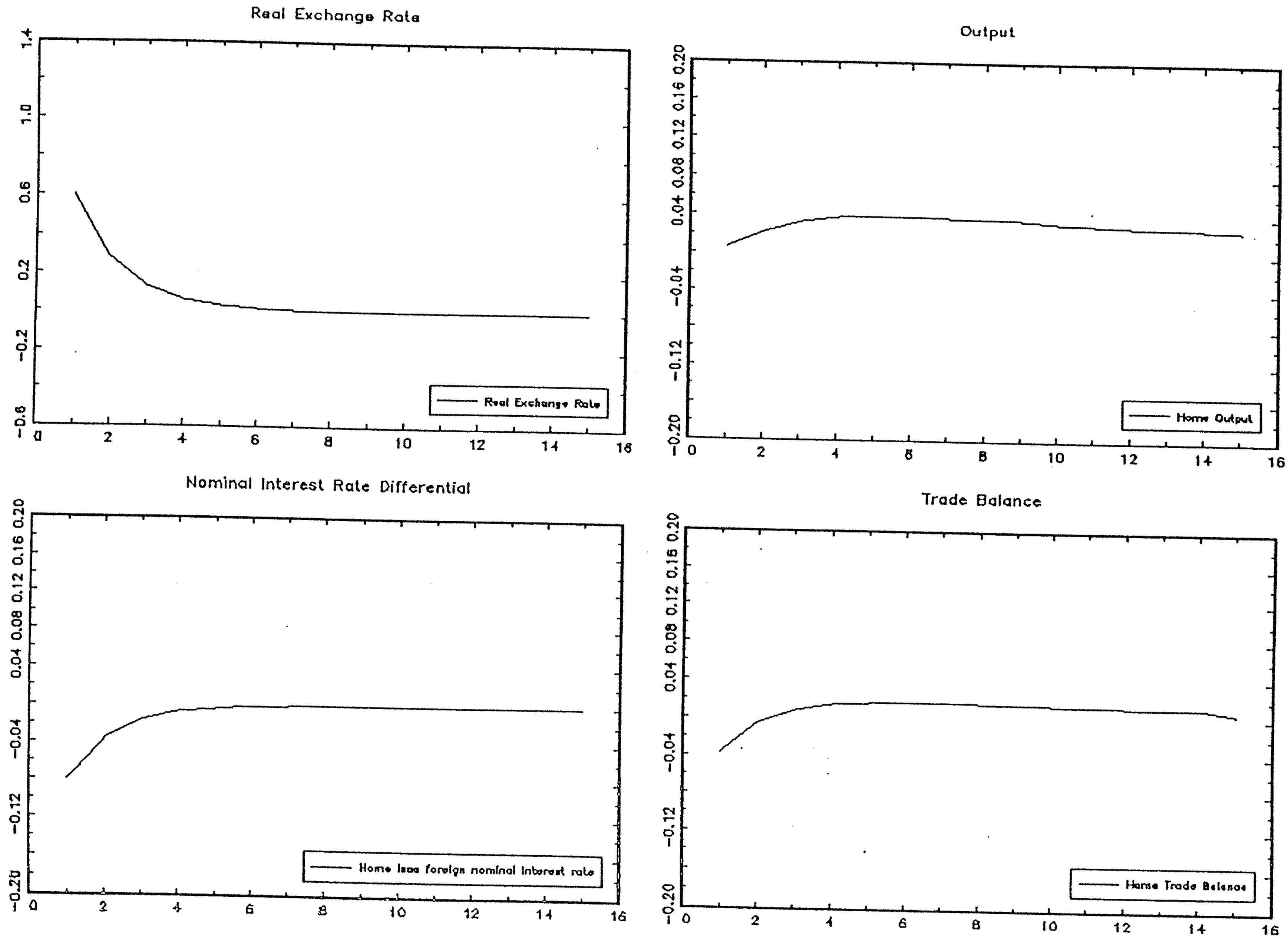
Real Exchange Rate
Real Exchange Rate
Output
Home Output
Nominal Interest Rate Differential
Home less foreign nominal interest rate
Trade Balance
Home Trade Balance

Figure 3 B



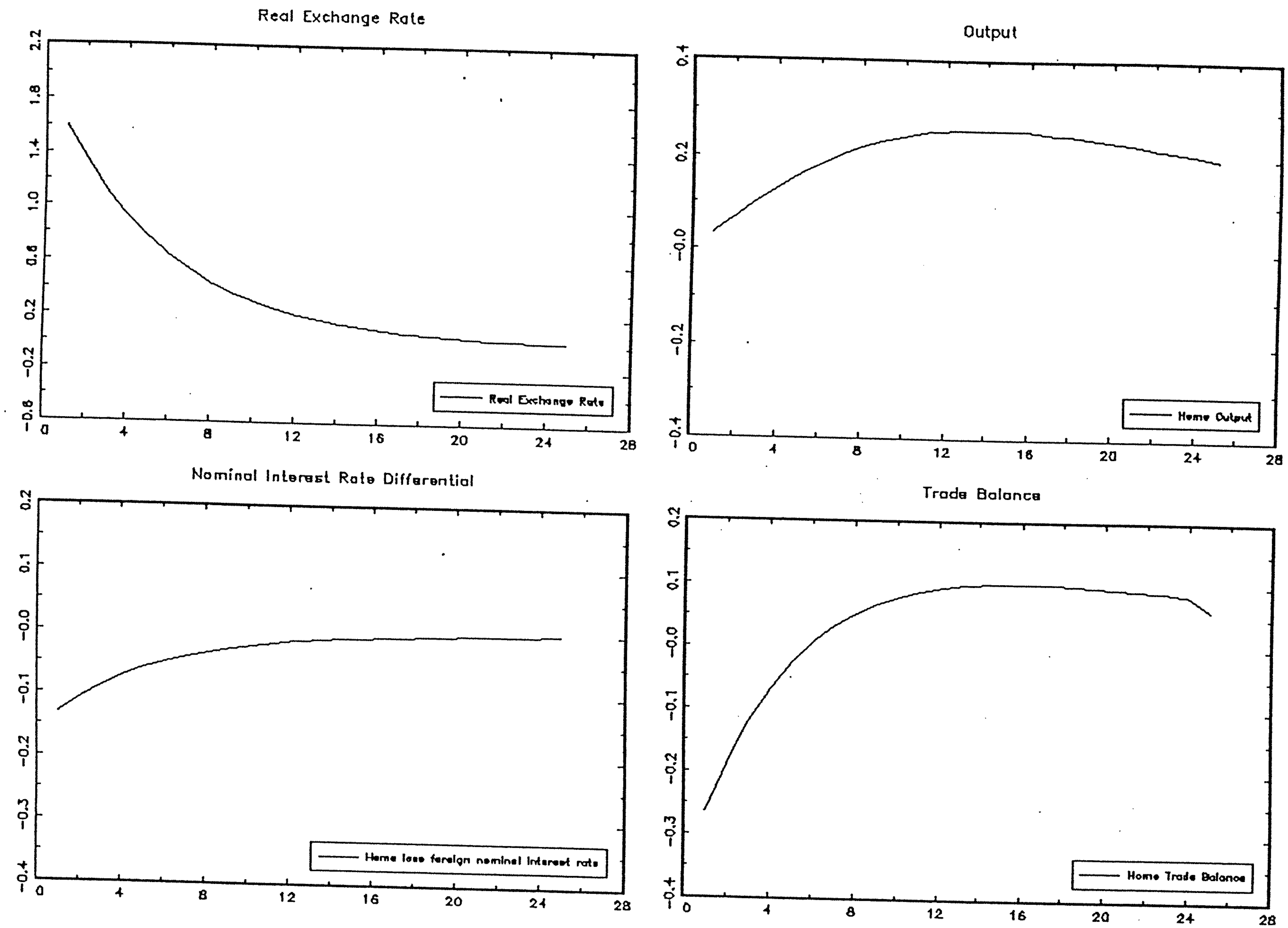
nominal exchange rate
nominal exchange rate
Investment
Home Investment
Foreign Investment
Prices
Home CPI
Foreign CPI
Consumption
Home Consumption
Foreign Consumption

Figure 4 A



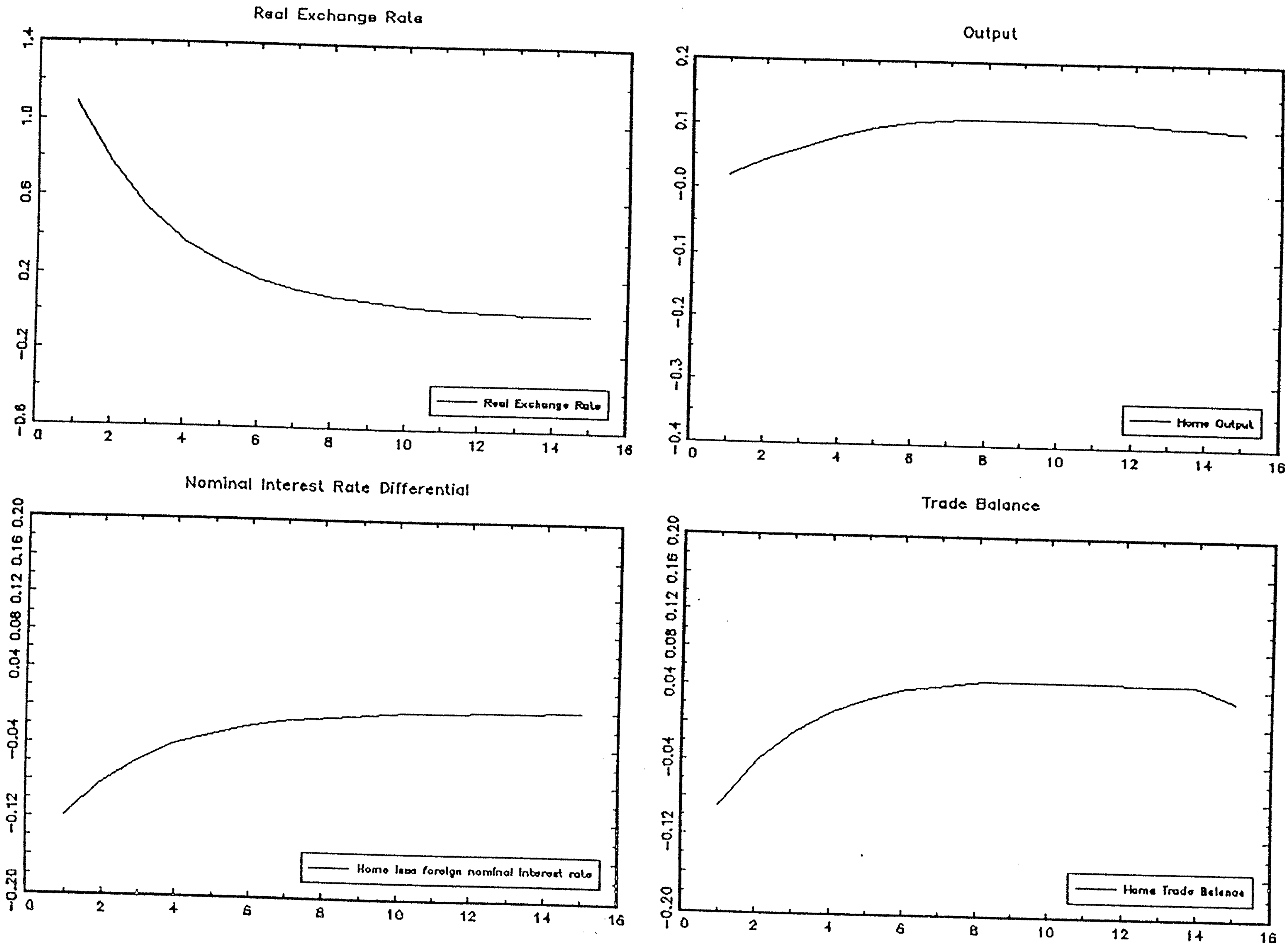
Real Exchange Rate
Real Exchange Rate
Output
Home Output
Nominal Interest Rate Differential
Home less foreign nominal interest rate
Trade Balance
Home Trade Balance

Figure 4 B



Real Exchange Rate
Real Exchange Rate
Output
Home Output
Nominal Interest Rate Differential
Home less foreign nominal interest rate
Trade Balance
Home Trade Balance

Figure 5A



Real Exchange Rate
Real Exchange Rate
Output
Home Output
Nominal Interest Rate Differential
Home less foreign nominal interest rate
Trade Balance
Home Trade Balance

Figure 5B

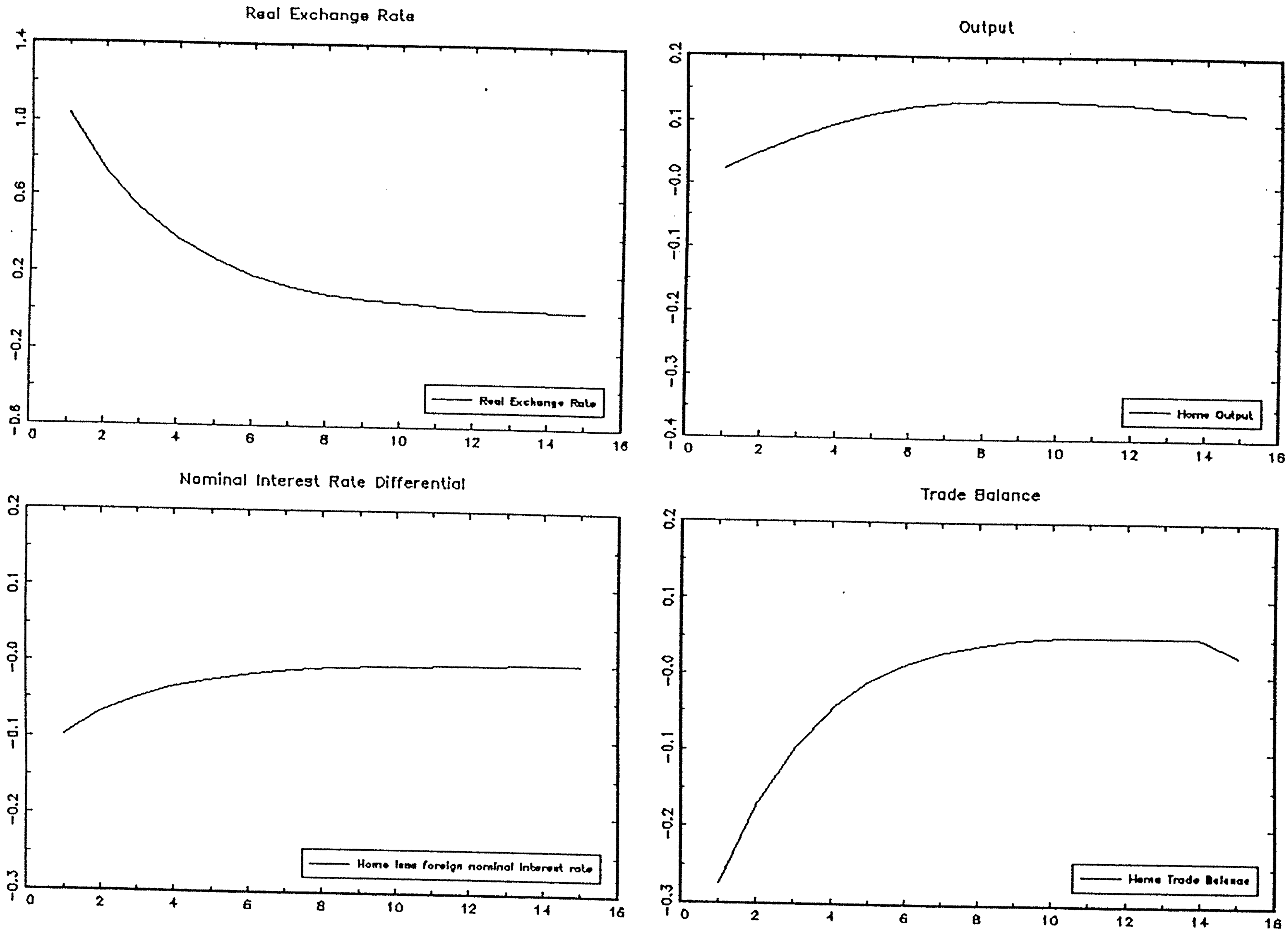
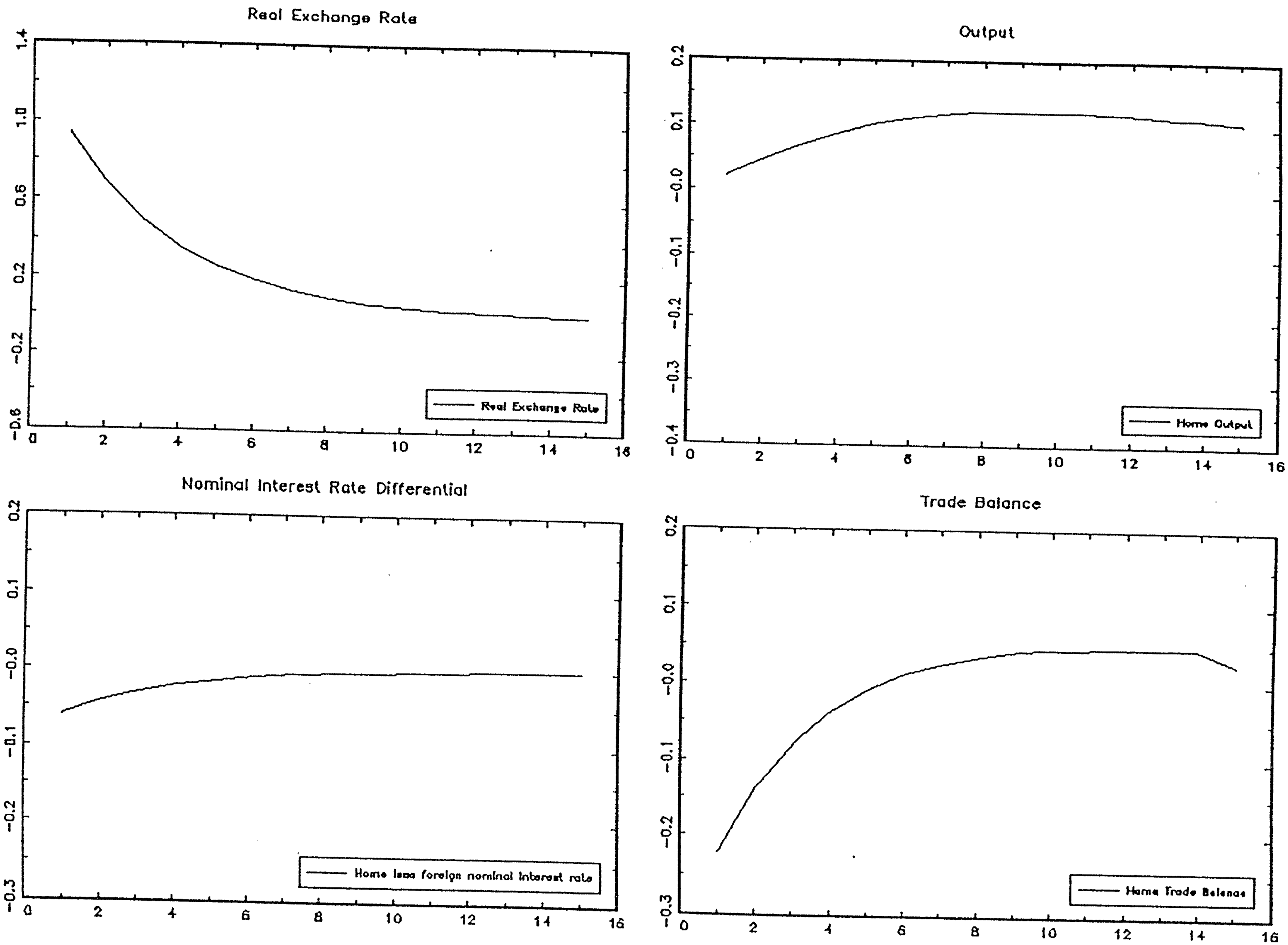

Real Exchange Rate
Real Exchange Rate
1.4
1.0
0.6
0.2
-0.2
-0.6
0
2
4
6
8
10
12
14
16
Output
Home Output
0.2
0.1
-0.0
-0.1
-0.2
-0.3
-0.4
Nominal Interest Rate Differential
Home less foreign nominal interest rate
Trade Balance
Home Trade Balance

Figure 6


Real Exchange Rate
Real Exchange Rate
Output
Home Output
Nominal Interest Rate Differential
Home less foreign nominal interest rate
Trade Balance
Home Trade Balance