# Starter Activity...

Adam is twice as old as Barry.
Charlie is 3 years younger than Barry.
The sum of all their ages is 53
How old is Barry?

# An Introduction to Singapore 'Bar Models'

## Course Lead

White Rose Maths

@WhiteRoseMaths

# Content

- What are ‘bar models’?
- Models that use a single bar
- Models that use two or more bars
- Problem solving

# What are bar models?

# What are bar models?

Adam, Barry and Charlie are brothers.

Adam is twice as old as Barry.

Charlie is 3 years younger than Barry.

The sum of all their ages is 53

How old is Barry?

$$9 + 7 = ?$$

Bar models can be used as a problem solving tool.

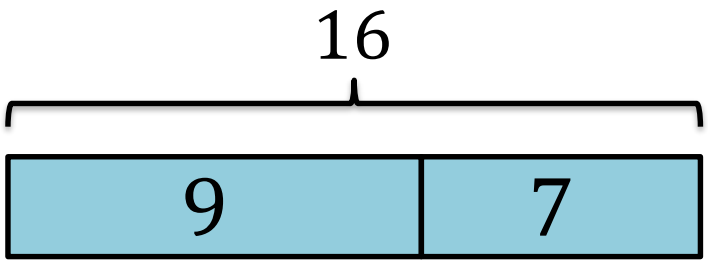
Bar models can be used to illustrate basic concepts.

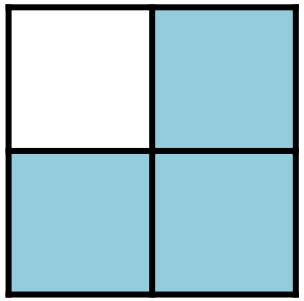
*'In as early as the 4th grade, algebra story problems begin to appear.'*

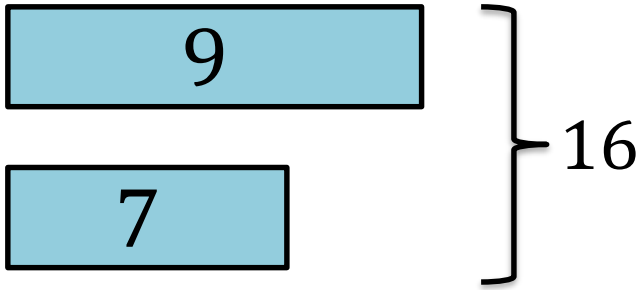
*'These strip diagrams make it possible for children who have not studied algebra to attempt remarkably complex problems.'*

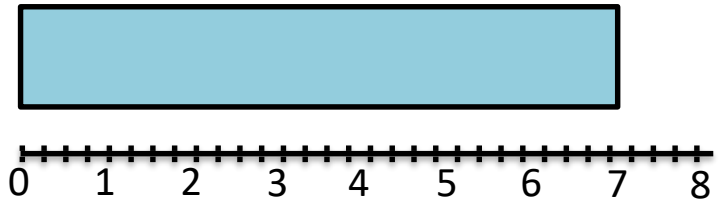
Beckmann, S. (2004)

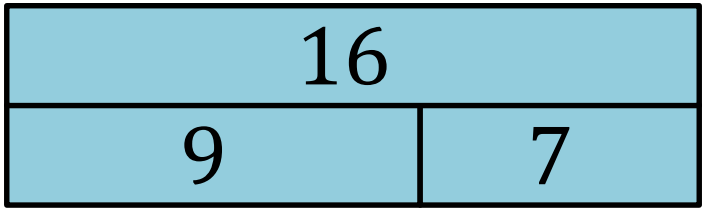
# What are bar models?





*'Although bar models will not always help children carry out required calculations, they are clearly designed to help children decide which operations to use. Instead of relying on superficial and unreliable clues like key words, the simple visual diagrams help children understand why the appropriate operations make sense.'*

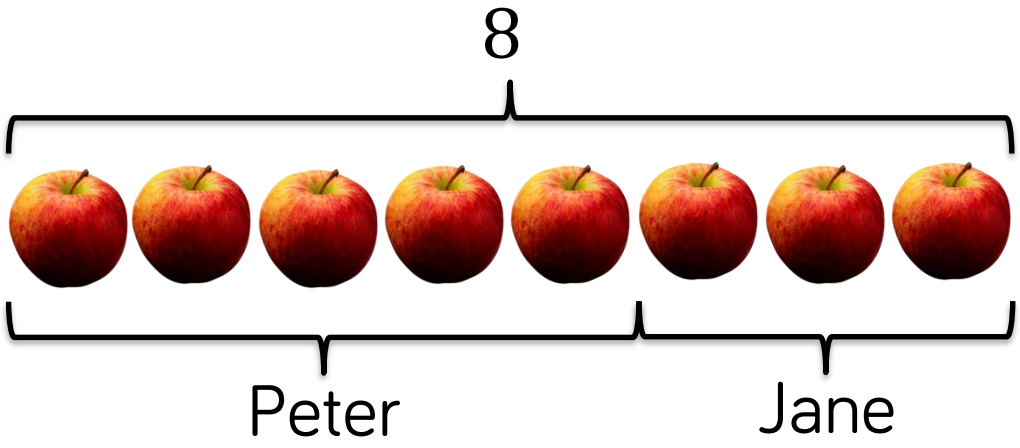
Beckmann, S. (2004)

# Models with a single bar ('Part-whole models')

# Addition

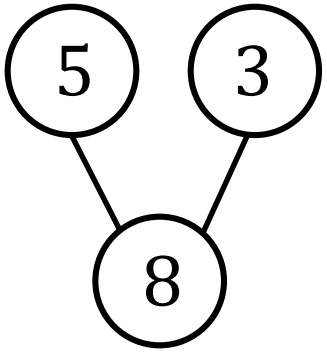
Peter has 5 apples and Jane has 3 apples.
How many apples do they have altogether?

## Model - aggregation




## Calculations

$5 + 3 = 8$

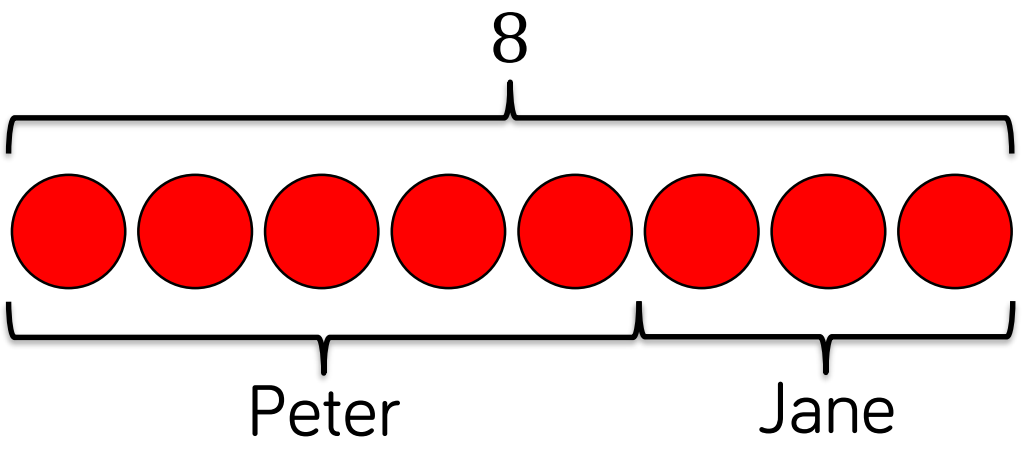



In this model, we are adding two parts together (aggregation).

# Addition

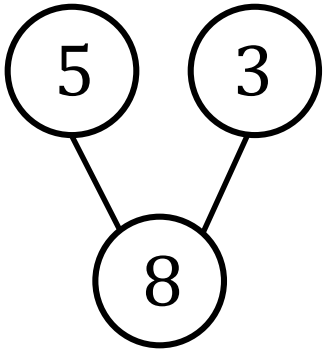
Peter has 5 apples and Jane has 3 apples.
How many apples do they have altogether?

## Model




## Calculations

$5 + 3 = 8$




Students practise by arranging counters on a mini whiteboard.

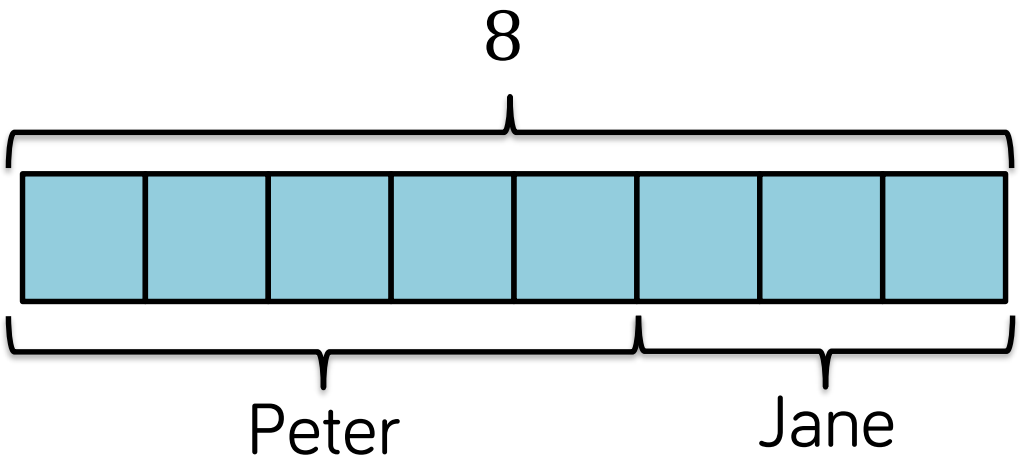
*'If we do not use concrete manipulations, then we can not understand mathematics. If we only use concrete manipulations, then we are not doing mathematics.'*

Gu, J. (2015)

# Addition

Peter has 5 apples and Jane has 3 apples.
How many apples do they have altogether?

## Model




## Calculations

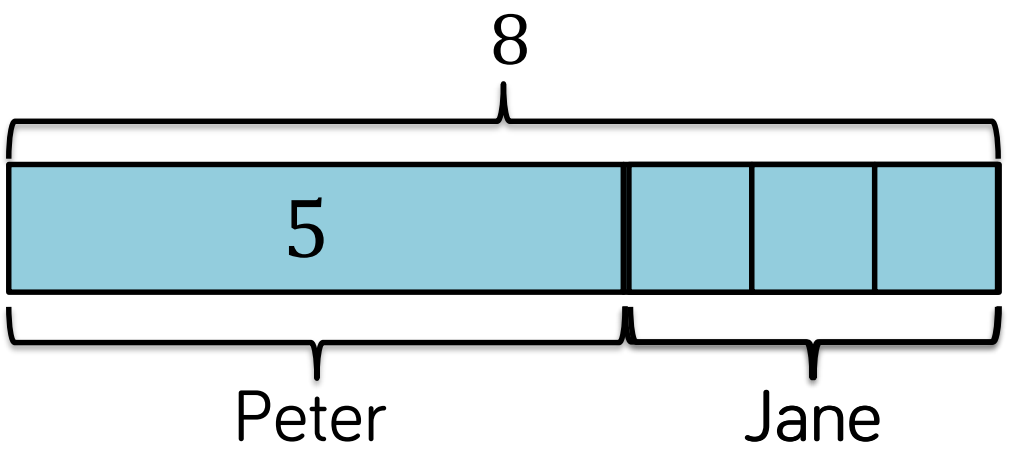
$$5 + 3 = 8$$

This is called a 'discrete bar model', each box represents one whole.

# Addition

Peter has 5 apples and Jane has 3 apples.
How many apples do they have altogether?

## Model

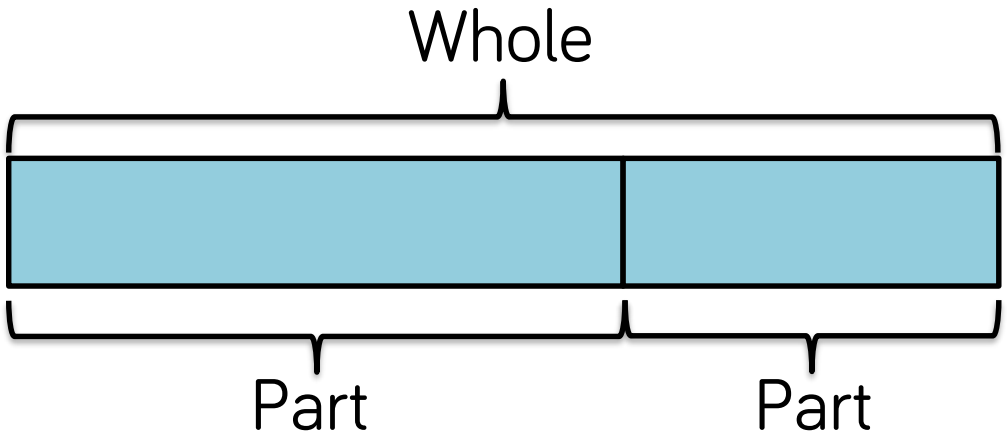



## Calculations

$5 + 3 = 8$

This is a 'continuous model', each rectangle represents a number.

# Terminology



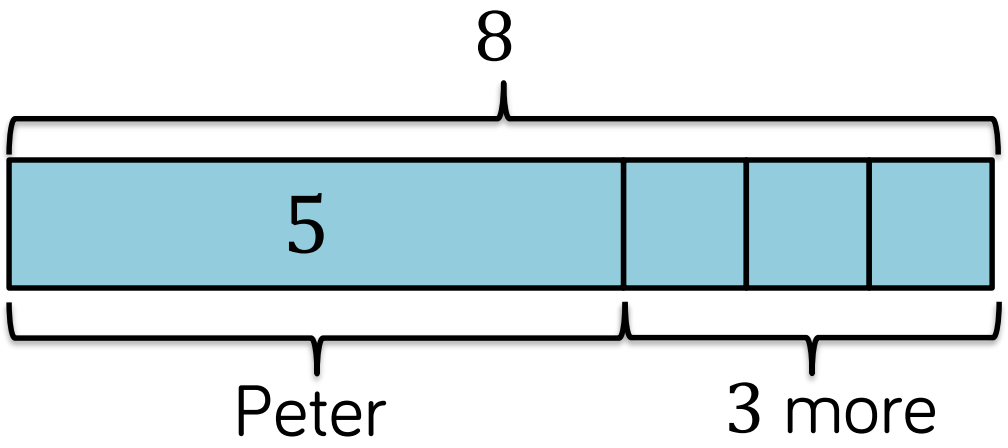

part + part = whole

whole − part = part

# Addition

Peter has 5 apples. He buys 3 more apples.
How many does he have altogether?

## Model - augmentation

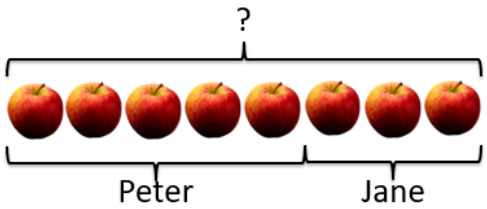



## Calculations

$$5 + 3 = 8$$

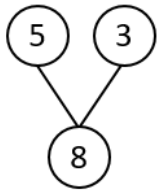
In this example, Peter's amount is increasing – this is augmentation.

## Model - aggregation

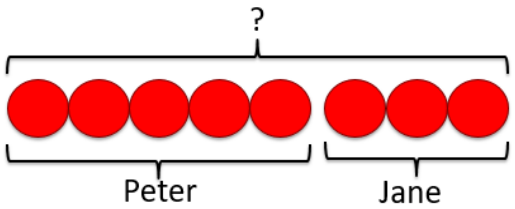



## Calculations

5+3 = 8

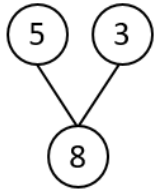



## Model




## Calculations

5 +3 = 8

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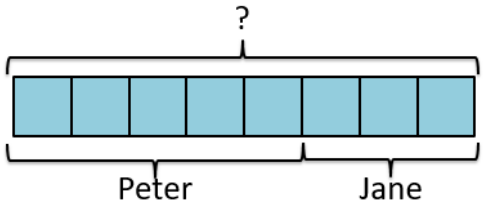
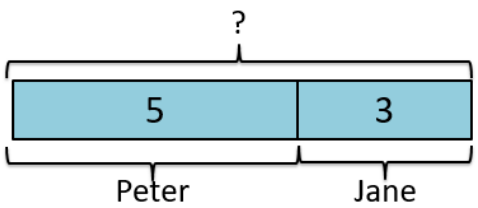



## Calculations

5+3 = 8

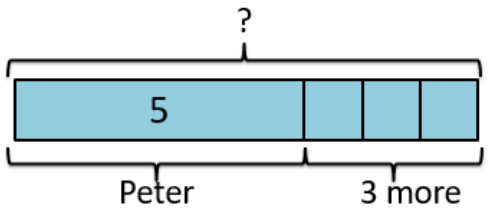
## Model




## Calculations

5 + 3 = 8

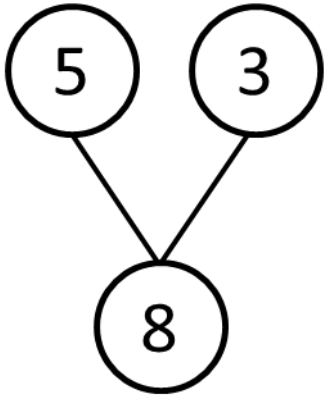
## Model – augmentation




## Calculations

5 + 3= 8

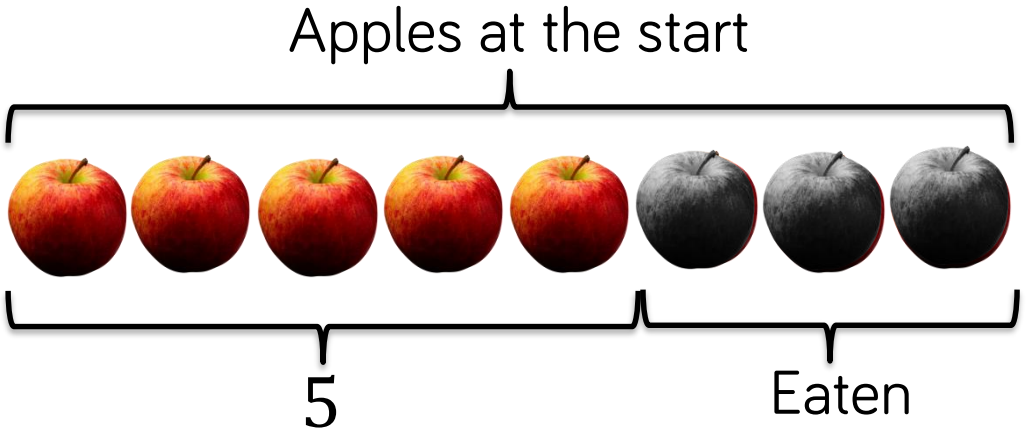
$5 + 3 = ?$



# Subtraction

Jane has 8 apples to begin with. She then eats three apples.
How many apples does she have left?

## Model




## Calculations

$$8 - 3 = 5$$

Jane's amount is decreasing, what structure of subtraction would this be?

# Subtraction

Jane has 8 apples to begin with. She then eats three apples.
How many apples does she have left?

**Discuss and draw:**

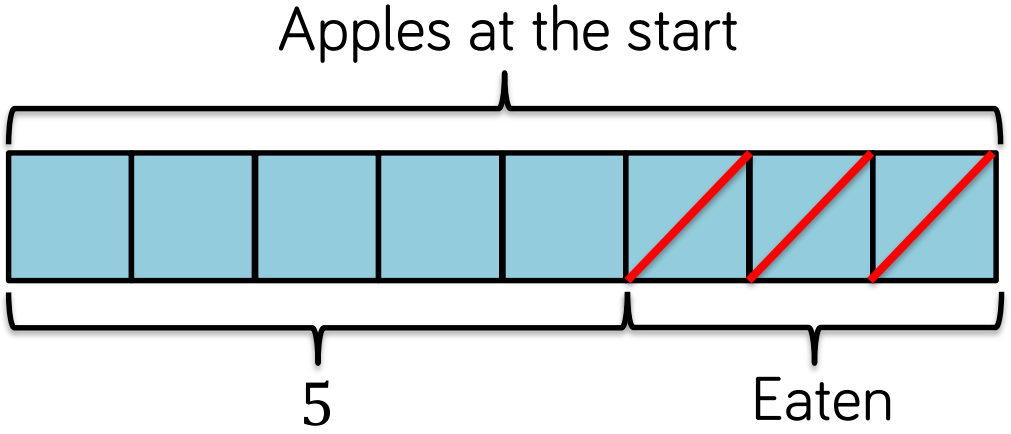
How would you represent this problem with a discrete model?

How would you represent it with a continuous model?

# Subtraction

Jane has 8 apples to begin with. She then eats three apples.
How many apples does she have left?

## Model- discrete



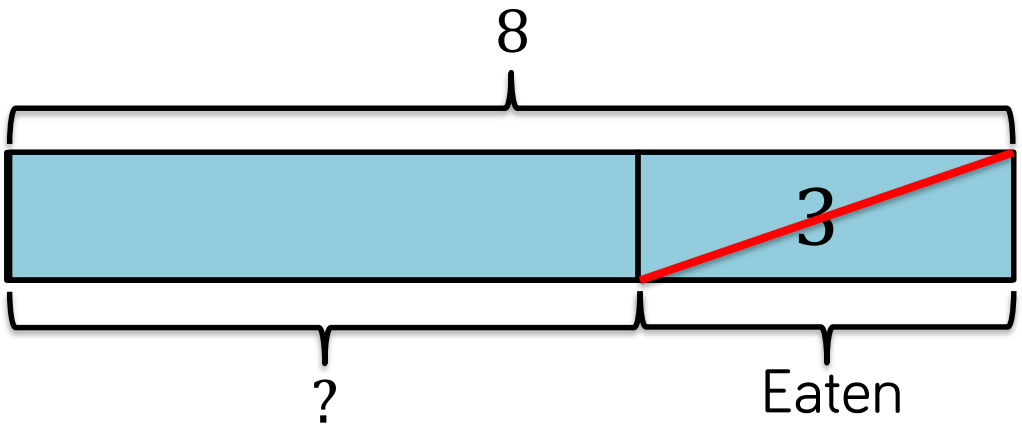

## Calculations

$$8 - 3 = 5$$

# Subtraction

Jane has 8 apples to begin with. She then eats three apples.
How many apples does she have left?

## Model- continuous



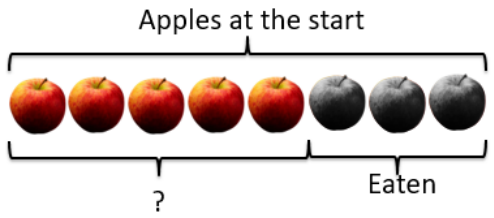

## Calculations

$$8 - 3 = ?$$

# Subtraction

Jane has 8 apples to begin with. She then eats three apples.
How many apples does she have left?

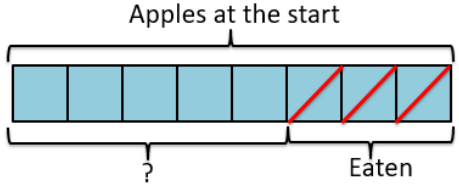
**Model**




**Calculations**

$8 - 3 = 5$

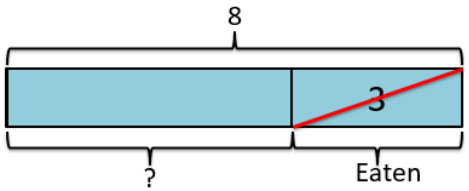
**Model (Discrete)**




**Calculations**

$8 - 3 = \ ?$

**Model (Continuous)**



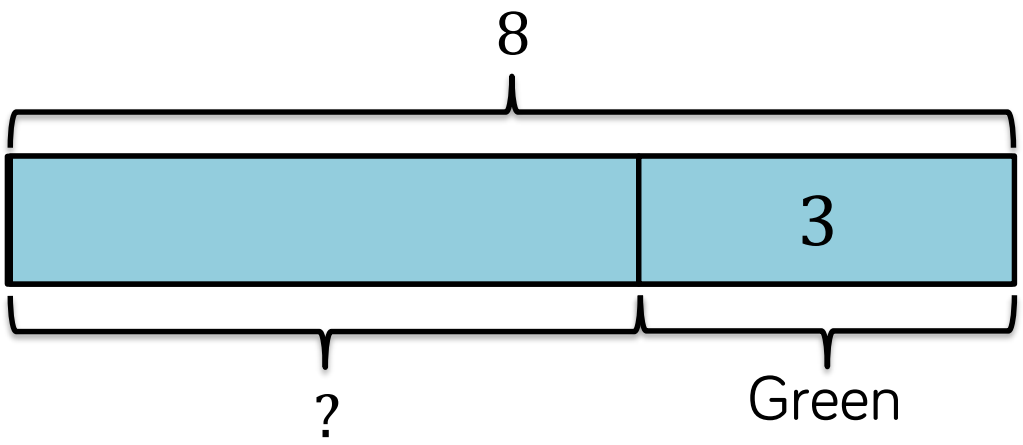

**Calculations**

$8 - 3 = ?$

# Subtraction

Jane has 8 apples. Three are green.
How many apples are red?

## Model- partitioning




## Calculations

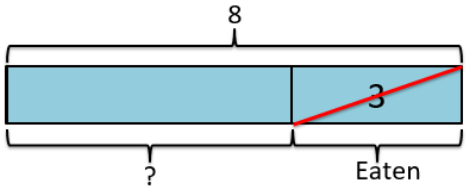
$$8 - 3 = ?$$

In this example, Jane's amount is being split into parts – this is partitioning.

# Subtraction

Jane has 8 apples to begin with. She then eats three apples. How many apples does she have left?

Model (Continuous)

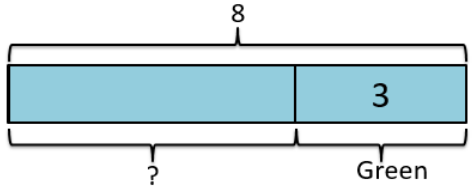



Calculations

$8 - 3 = ?$

Jane has 8 apples. Three are green.
How many apples are red?

Model - Partitioning




Calculations

$8 - 3 = ?$

What's the same?
What's different?

# Other single bars

| | | |
|---|---|---|
| $6 \times 4 = ?$ | $30 \div 5 = ?$ | Find $\frac{3}{5}$ of 30 |

**Discuss and draw:**

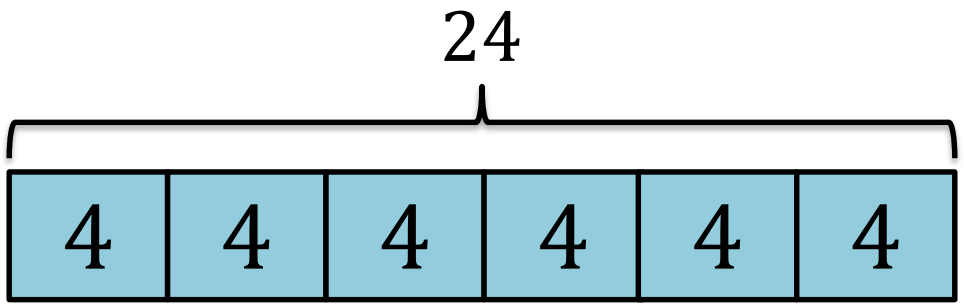
Write a worded question for each calculation above.

Draw a model to represent each problem (any kind of model).

# Multiplication

Muffins come in boxes of 4. Peter buys 6 boxes of muffins.
How many muffins does Peter buy all altogether?

## Model



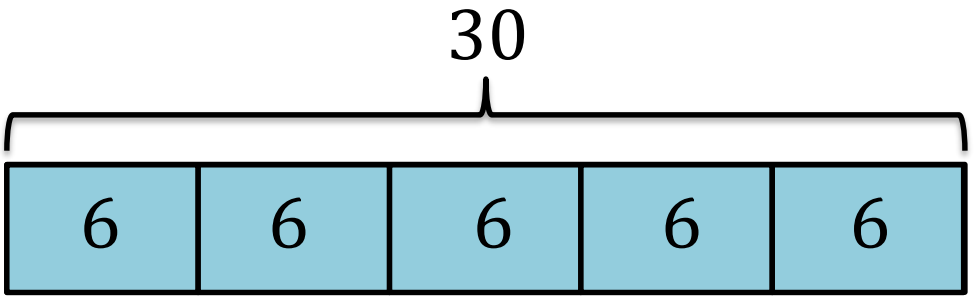

## Calculations

$$6 \times 4 = 24$$

# Division

Jane has 30 cakes. She wants to share them equally between five boxes. How many should go in each box?

## Model - sharing




Number of cakes in each box = 6

## Calculations

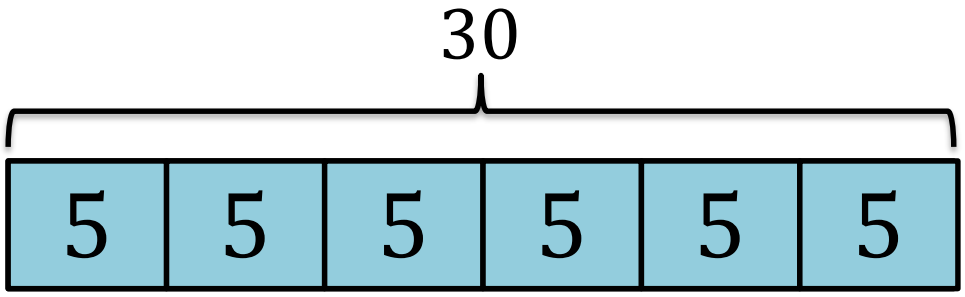
$30 \div 5 = 6$

In this version, we are splitting 30 into 5 equal groups.

# Division

Jane has 30 cakes. She wants to pack them into boxes with 5 cakes in each box. How many boxes will she need?

## Model - grouping




Number of boxes needed $= 6$

## Calculations

$30 \div 5 = 6$

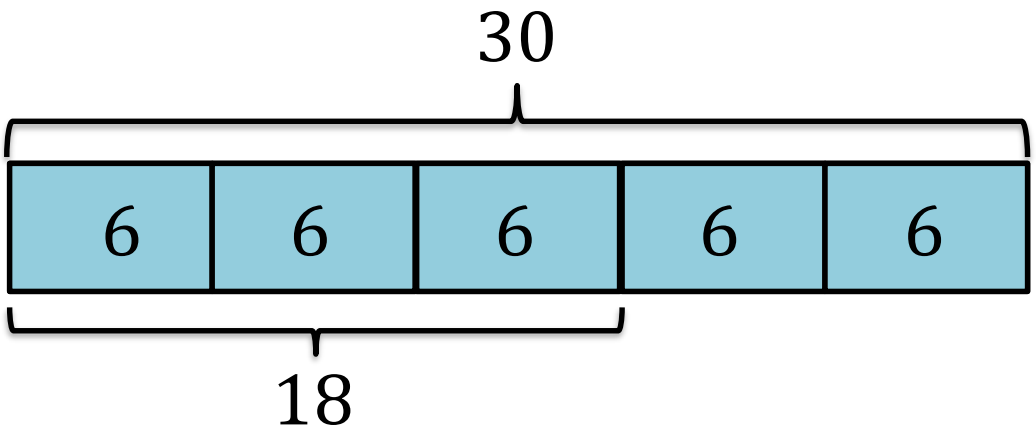
In this version, we are counting how many fives go into thirty.

# Fraction of an amount

Peter starts with $30$ sweets.

He eats $\frac{3}{5}$ of them. How many sweets does he eat?

## Model




## Calculations

$30 \div 5 = 6$

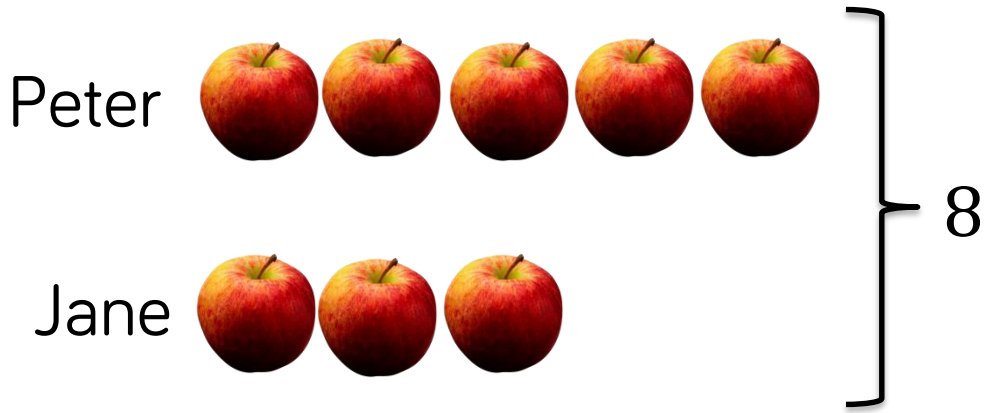
$3 \times 6 = 18$

# Models with more than one bar ('Comparison model')

# Addition

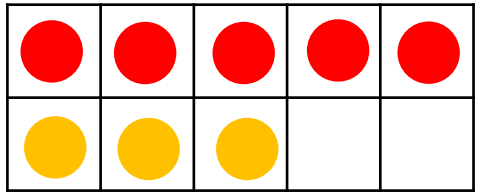
Peter has 5 apples and Jane has 3 apples.
How many apples do they have altogether?

## Model




## Calculations

$$5 + 3 = 8$$



In this model, it is clear who has the most apples.

# Addition

Peter has 5 apples and Jane has 3 apples.
How many apples do they have altogether?

**Discuss and draw:**

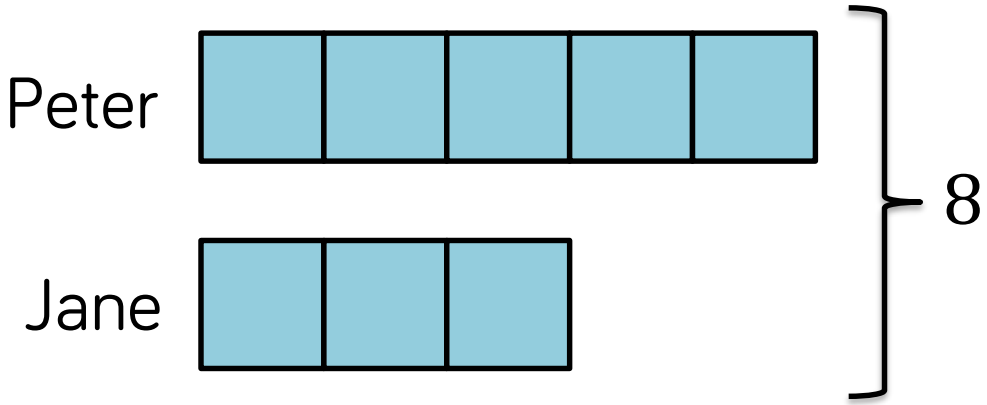
How would you represent this problem with a discrete comparison model?

How would you represent it with a continuous comparison model?

# Addition

Peter has 5 apples and Jane has 3 apples.
How many apples do they have altogether?

## Model - discrete



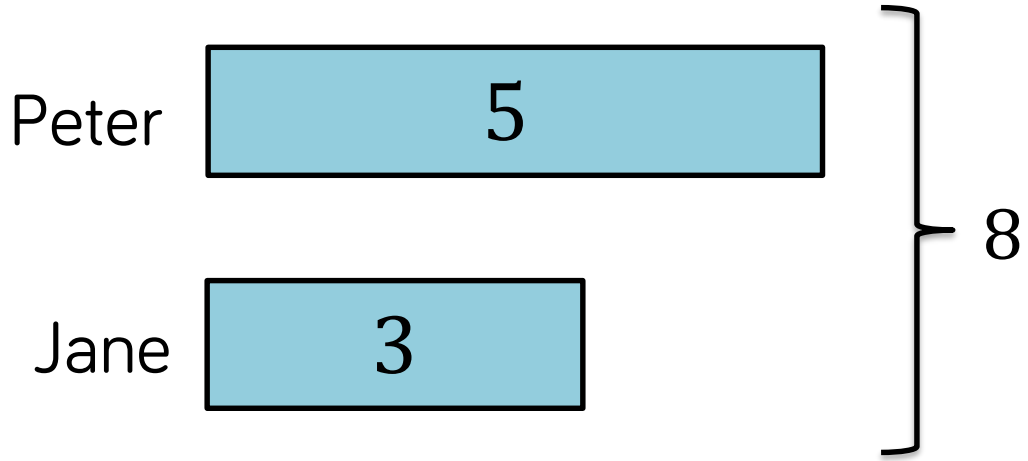

## Calculations

$$5 + 3 = 8$$

# Addition

Peter has 5 apples and Jane has 3 apples.
How many apples do they have altogether?

## Model - continuous




## Calculations

$$5 + 3 = 8$$

# Subtraction

$$10 - 3 = ?$$

**Discuss and draw:**

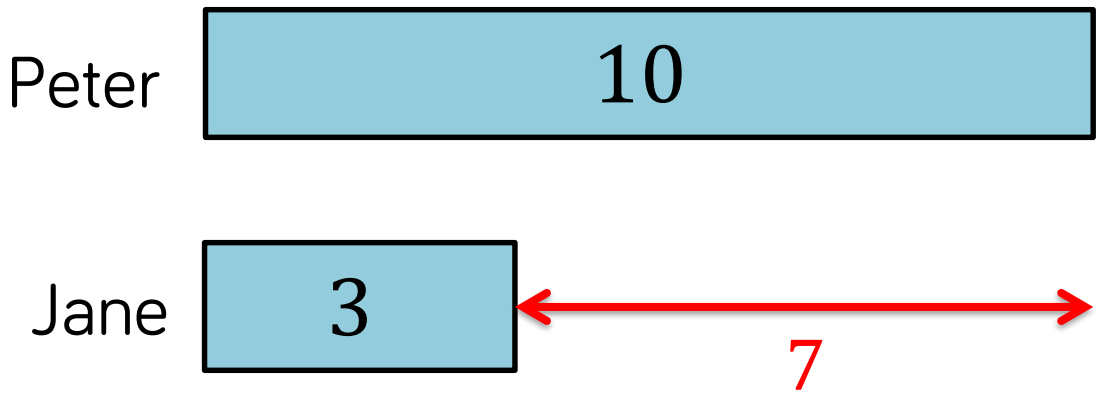
Can you think of a worded subtraction question which would suit a comparison model more than a part-whole model?

How would you represent it with a continuous comparison model?

# Subtraction

Peter has 10 pencils and Jane has 3 pencils.
How many more pencils does Peter have than Jane?

## Model - difference




## Calculations

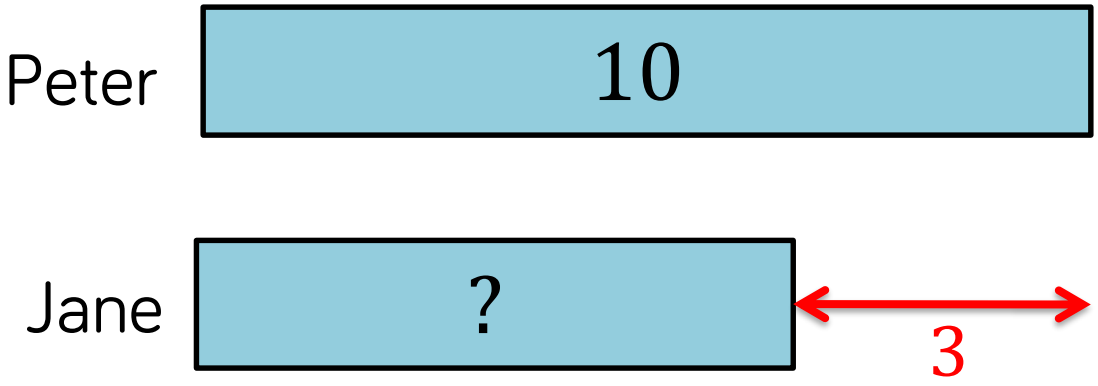
$10 - 3 = 7$

In this question we are 'finding the difference'.

# Subtraction

Peter has $10$ pencils. Jane has $3$ pencils less than Peter.
How many pencils does Jane have?

## Model - difference

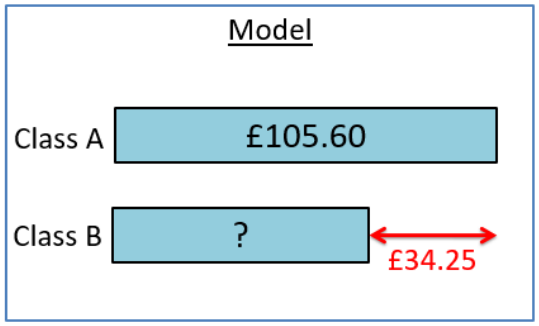


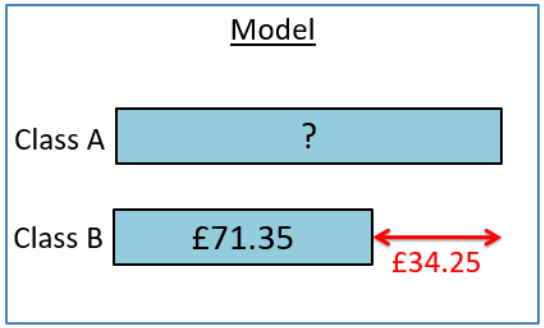

## Calculations

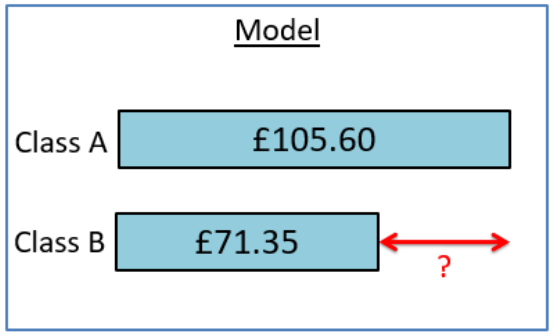
$$10 - 3 = \ ?$$

# Discuss:

Which models match the worded problems?



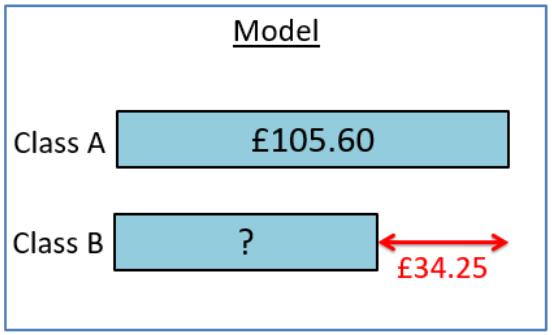




Two classes are raising money for charity. Class A raise £105.60. Class B raise £71.35
How much more money did Class A raise?

Two classes are raising money for charity. Class A raise £105.60. Class B raise £34.25 less than Class A.
How much did Class B raise?

# Discuss:

Which models match the worded problems?

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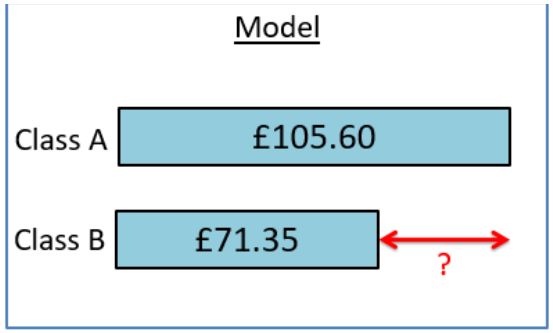


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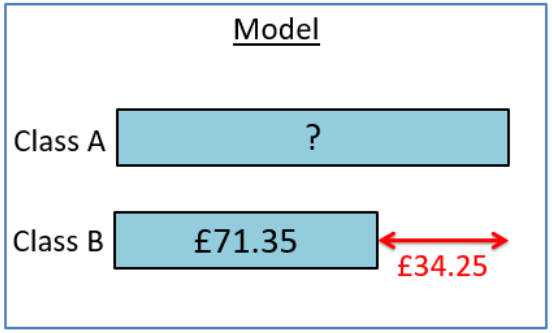
Class A raise £105.60. Class B raise £34.25 less than Class A.
How much did Class B raise?




?

# Other comparison models

Peter and Jane share £40 in the ratio of $3 : 5$
How much money does each person get?

Solve... $3a + 5 = 17$

**Discuss and draw:**

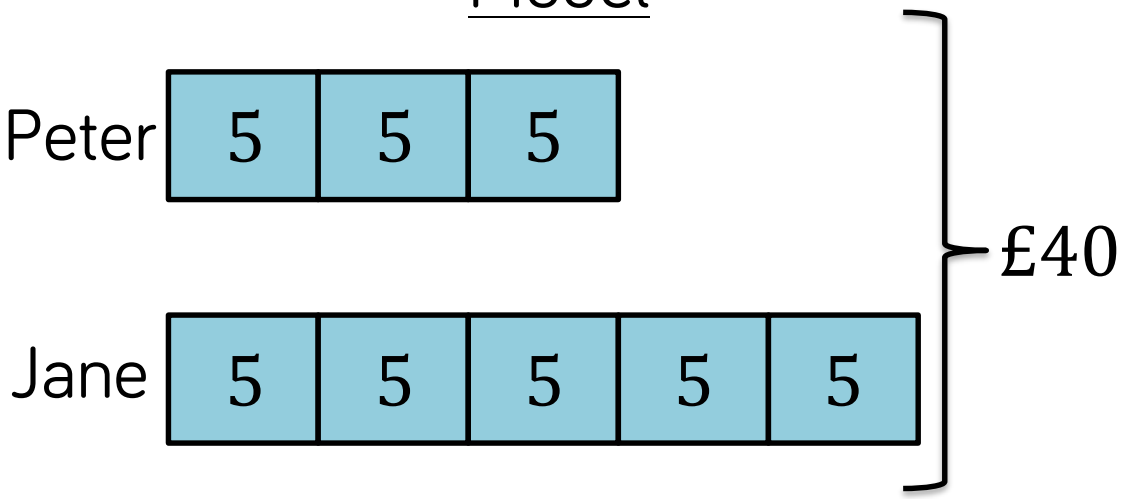
Draw a comparison model to represent each problem.

Manipulate your models to get the answers.

# Ratio

Peter and Jane share £40 in the ratio of 3 : 5
How much money does each person get?

## Model




## Calculations

$40 \div 8 = 5$

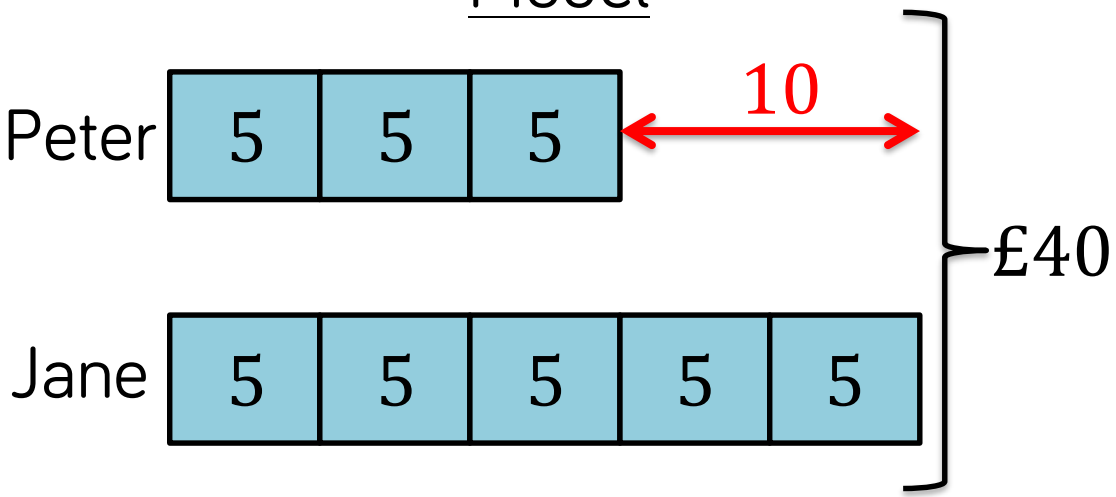
Peter: $3 \times 5 = £15$

Jane: $5 \times 5 = £25$

# Ratio

Peter and Jane share £40 in the ratio of 3 : 5
How much more money does Jane have than Peter?

## Model




## Calculations

$40 \div 8 = 5$

Peter: $3 \times 5 = £15$

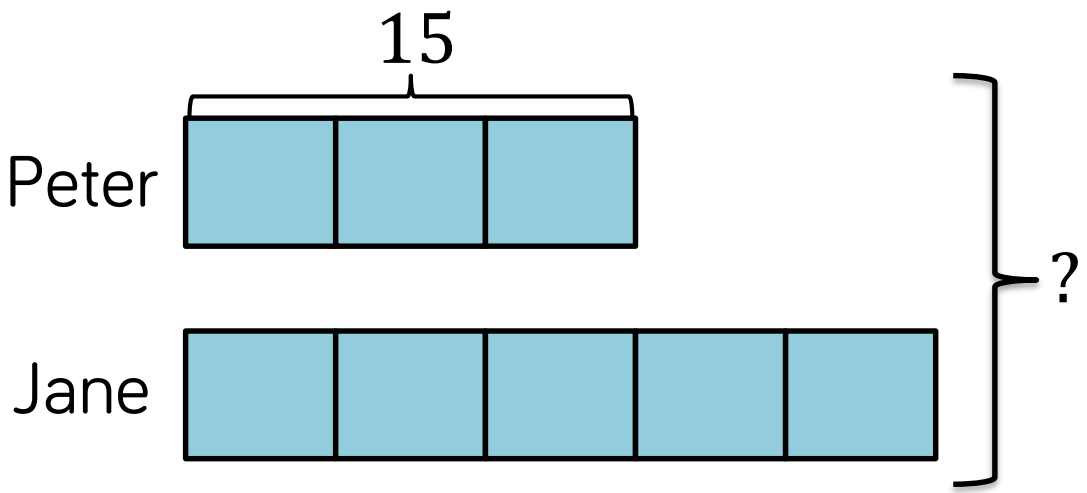
Jane: $5 \times 5 = £25$

$25 - 15 = £10$

# Ratio

Peter and Jane share some money in the ratio of $3 : 5$
Peter gets £15. How much did they share?

## Model




## Calculations

$15 \div 3 = 5$

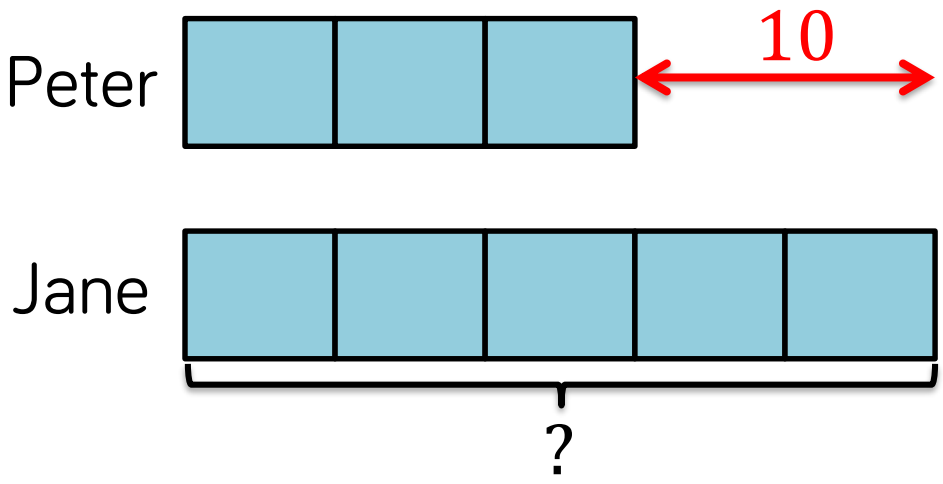
Jane: $5 \times 5 = £25$

$25 + 15 = £40$

# Ratio

Peter and Jane share some money in the ratio of $3 : 5$
Jane gets £10 more than Peter. How much did Jane get?

## Model




## Calculations

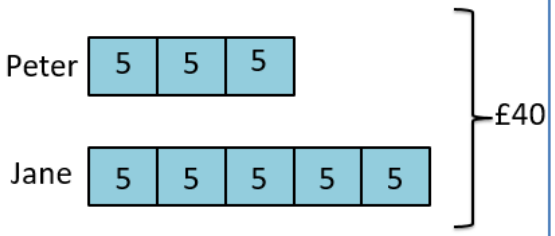
$10 \div 2 = 5$

Jane: $5 \times 5 = £25$

# Ratio

Peter and Jane share £40 in the ratio of 3:5
How much money does each person get?

Model




Calculations

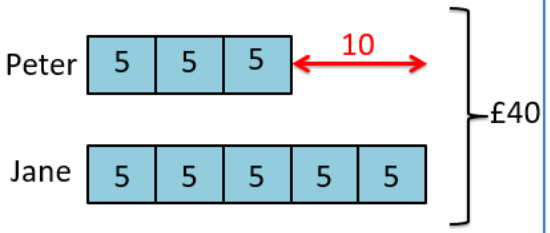
$40 \div 8 = 5$

Peter: $3 \times 5 = £15$

Jane: $5 \times 5 = £25$

Peter and Jane share £40 in the ratio of 3:5
How much more money does Jane have than Peter?

Model




Calculations

$40 \div 8 = 5$

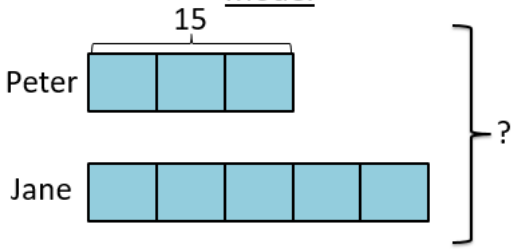
Peter: $3 \times 5 = £15$

Jane: $5 \times 5 = £25$

$25 - 15 = £10$

Peter and Jane share some money in the ratio of 3:5
Peter gets £15. How much did they share?

Model




Calculations

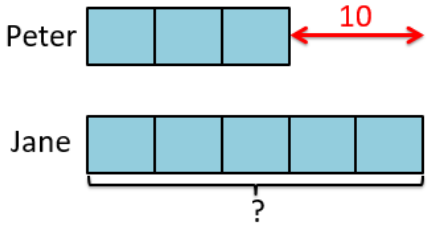
$15 \div 3 = 5$

Jane: $5 \times 5 = £25$

$25 + 15 = £40$

Peter and Jane share some money in the ratio of 3:5
Jane gets £10 more than Peter. How much did Jane get?

Model




Calculations

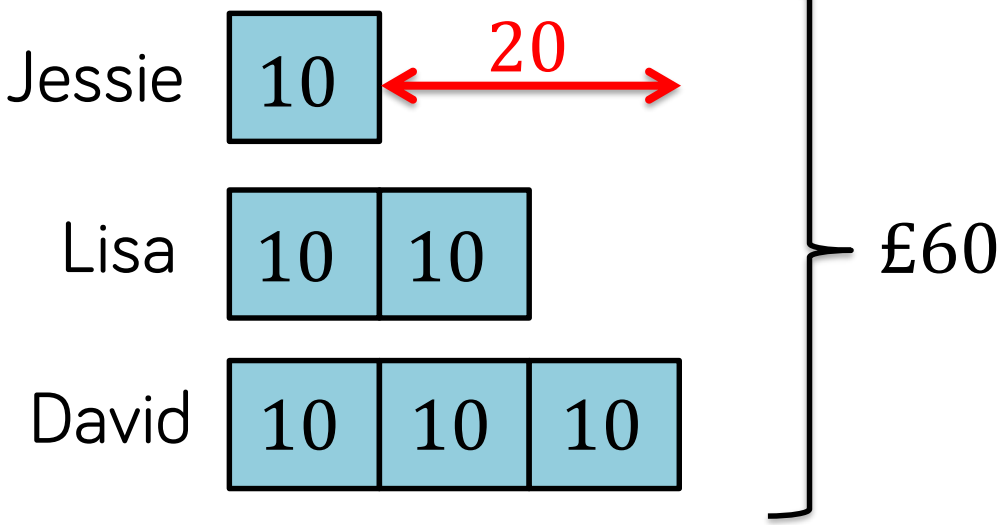
$10 \div 2 = 5$

Jane: $5 \times 5 = £25$

# Ratio

You try... Jessie, Lisa and David share £60 in the ratio of 1 : 2 : 3
How much more money does David get than Jessie?

Model




Calculations

$60 \div 6 = 10$

Jessie: $1 \times 10 = 10$

Lisa: $2 \times 10 = 20$

David: $3 \times 10 = 30$

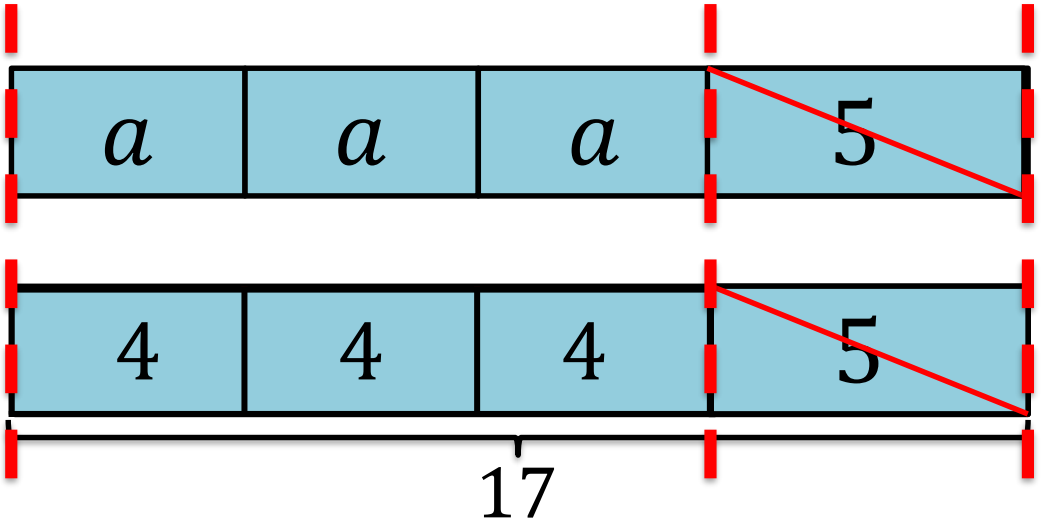
$30 - 10 = £20$

What other questions could you ask? How would this change the bar?

# Solving equations

Solve... $3a + 5 = 17$

## Model




## Calculations

$$3a + 5 = 17$$

$-5$ $-5$

$$3a = 12$$

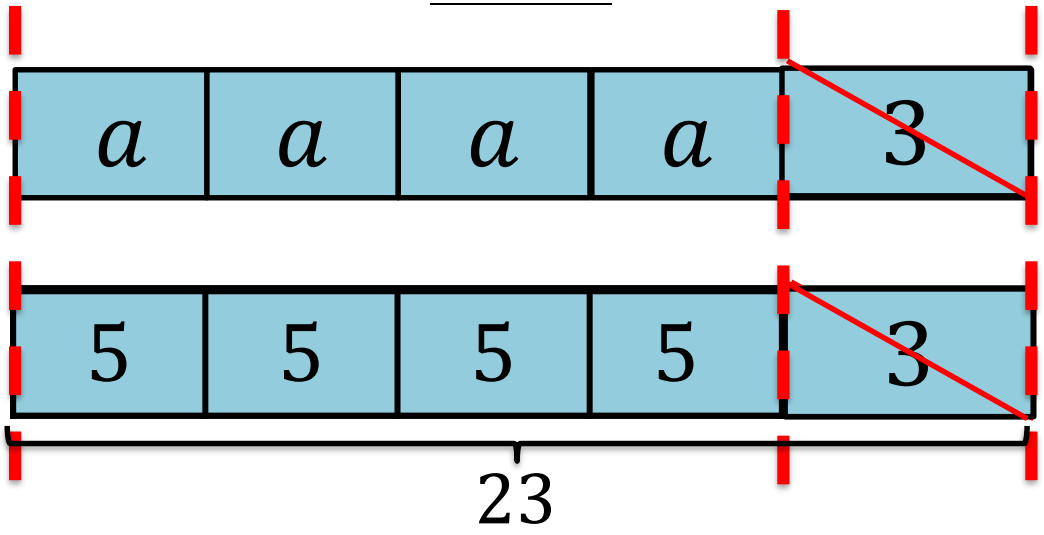
$\div 3$ $\div 3$

$$a = 4$$

# Solving equations

Now you try... $4a + 3 = 23$

## Model




## Calculations

$$4a + 3 = 23$$

$-3$ $-3$

$$4a = 20$$

$\div 4$ $\div 4$

$$a = 5$$

# Solving equations

Sarah has 13 cherries. She shares them equally between 3 bags.
She has one left over.

**Discuss and draw:**

How would you represent this situation?

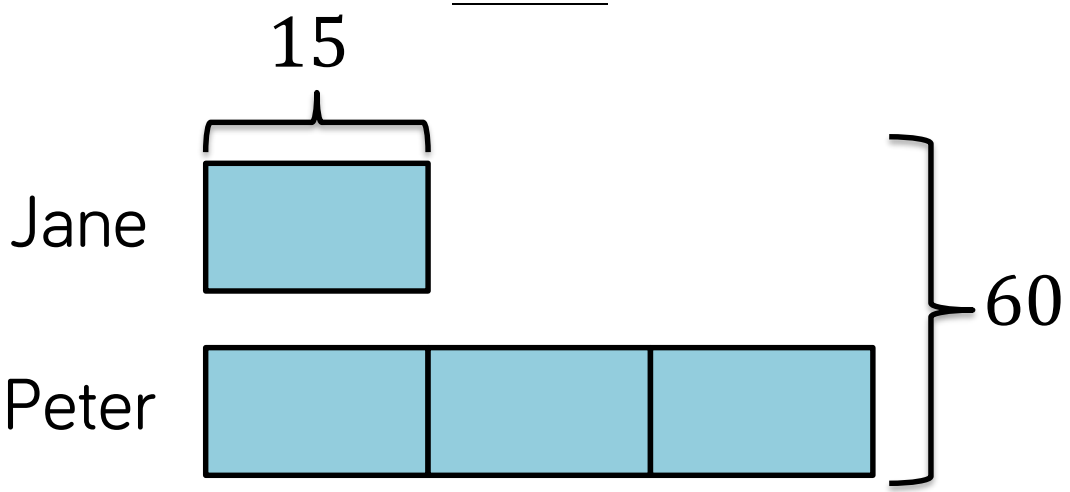
How would you represent it with a continuous comparison model?

# Solving problems

# Problem 1

Peter and Jane have $60$ sweets in total between them.
Peter has three times as many sweets as Jane.
How many sweets does Jane have?

Model




Calculations

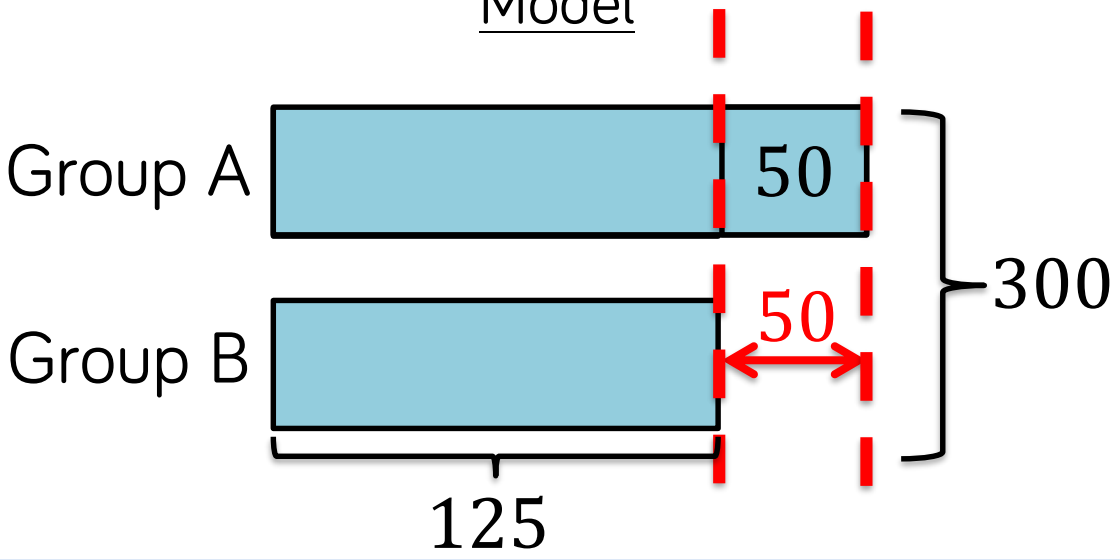
$$60 \div 4 = 15$$

What other questions could you ask? How would this change the bar?

# Problem 2

300 children are divided into two groups. There are 50 more children in the first group than in the second group.
How many children are there in the second group?

Model




Calculations

$300 - 50 = 250$

$250 \div 2 = 125$

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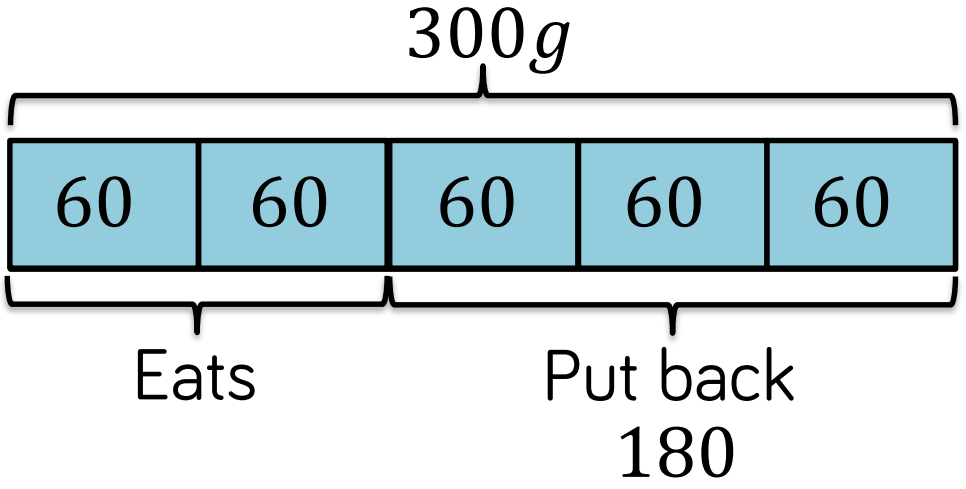
# Problem 3

Matthew has a $300g$ block of cheese. He eats two fifths of the cheese and puts the rest back in the fridge.
How much cheese did Matthew put back in the fridge?

Model




Calculations

$300 \div 5 = 60$

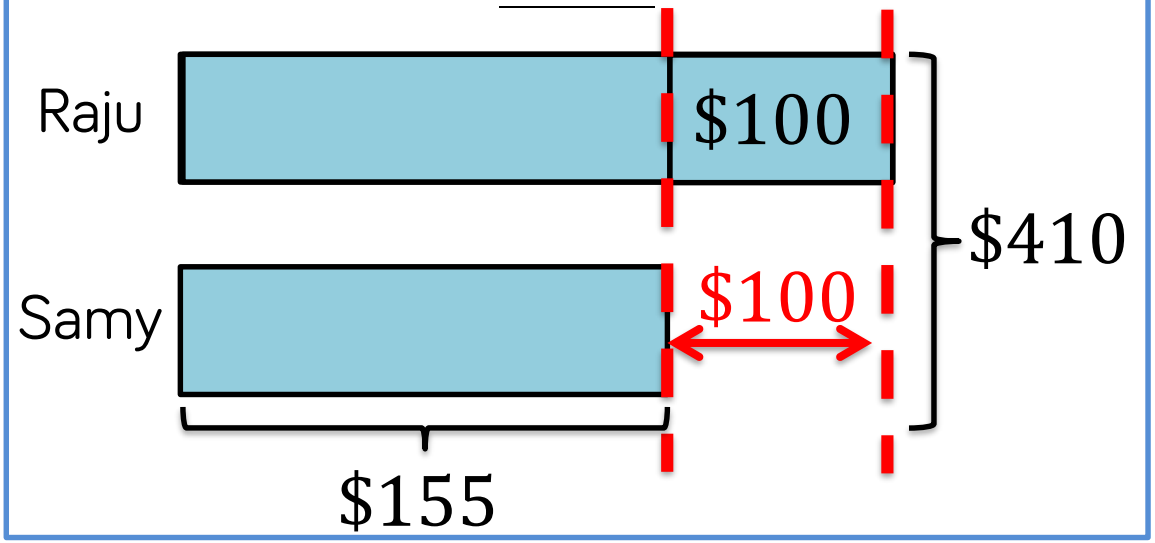
$3 \times 60 = 180$

# Solving problems
# Now you try!

# Problem 4

Raju and Samy shared $410 between them.
Raju received $100 more than Samy.
How much money did Samy receive?

Model




Calculations

$410 - 100 = 310$

$310 \div 2 = 155$

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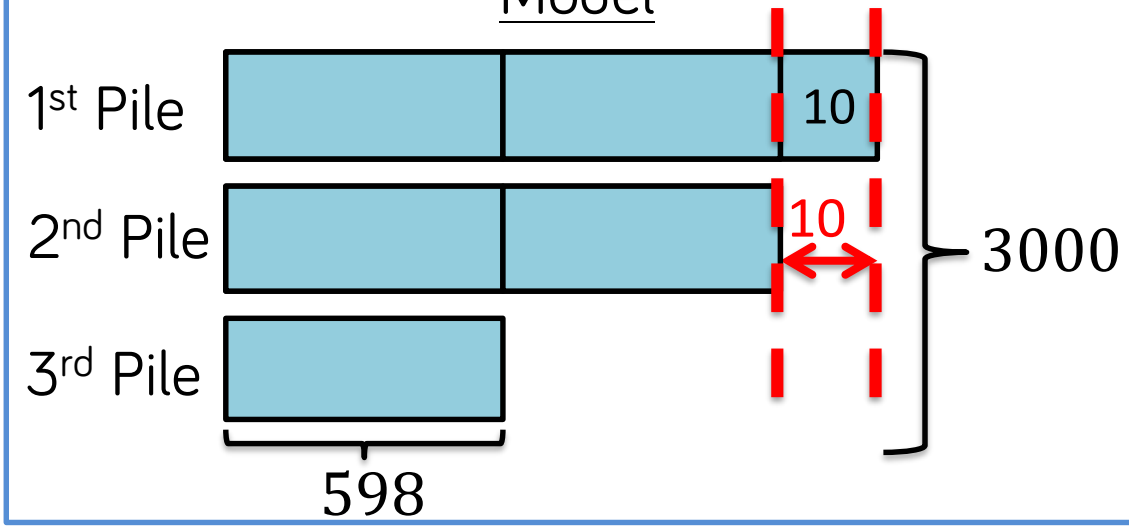
# Problem 5

3000 exercise books are arranged into 3 piles. The first pile has 10 more books than the second pile. The number of books in the second pile is twice the number of books in the third pile.
How many books are in the third pile?

## Model




## Calculations

$3000 - 10 = 2990$

$2990 \div 5 = 598$

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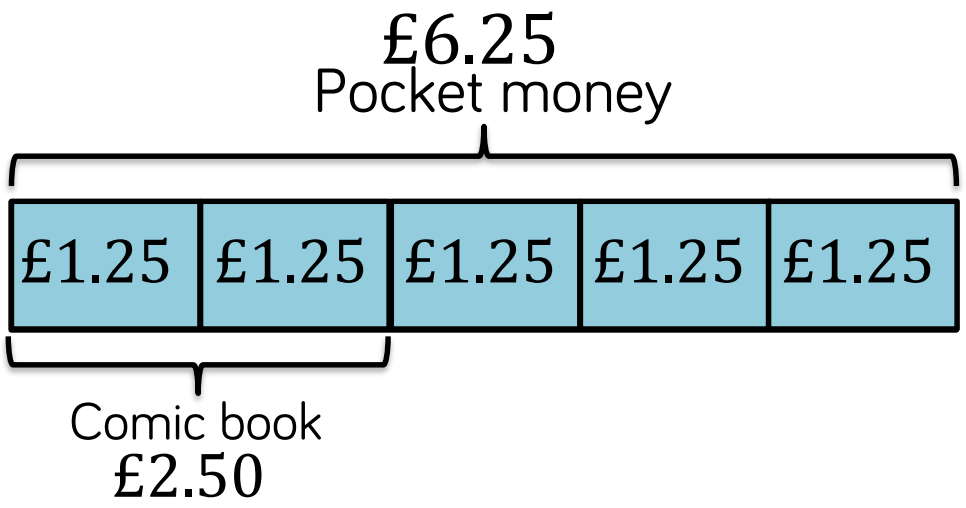
# Problem 6

Jenny spent $\frac{2}{5}$ of her pocket money on a comic book.

The price of the comic book was £2.50

How much pocket money did Jenny get?

## Model




## Calculations

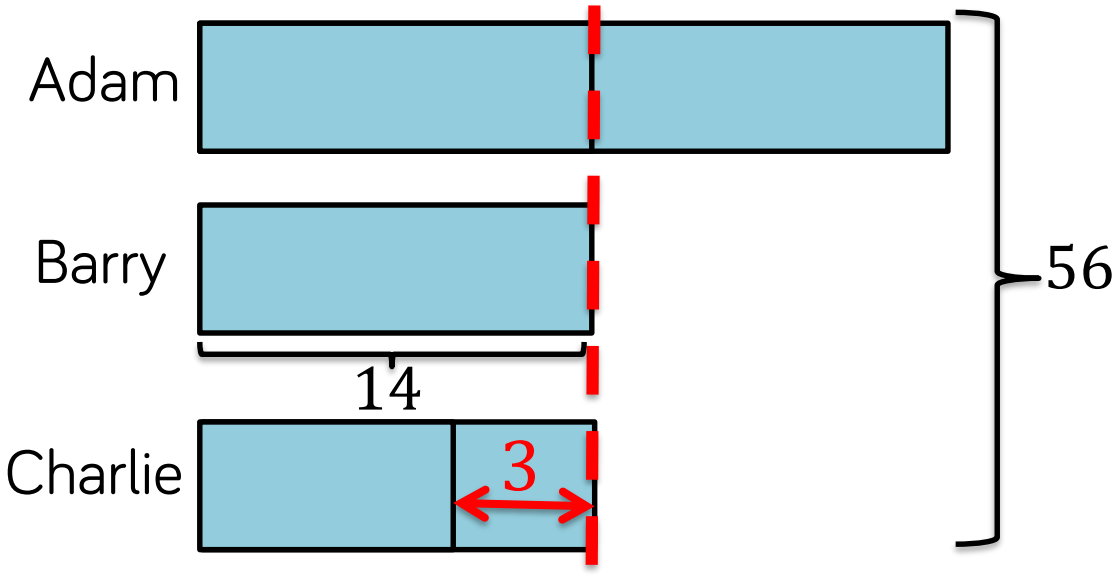
$2.50 \div 2 = 1.25$

$5 \times 1.25 = 6.25$

# Problem 7

Adam is twice as old as Barry. Charlie is 3 years younger than Barry. The sum of all their ages is 53. How old is Barry?

## Model



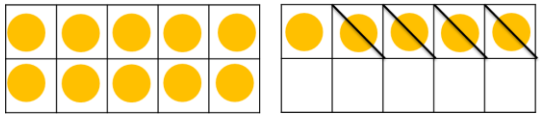

## Calculations

$$53 + 3 = 56$$

$$56 \div 4 = 14$$

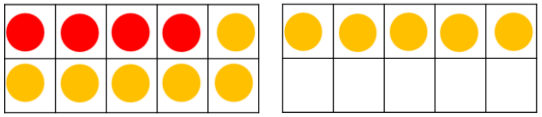
# A consistent picture

$15 - 4 = ?$

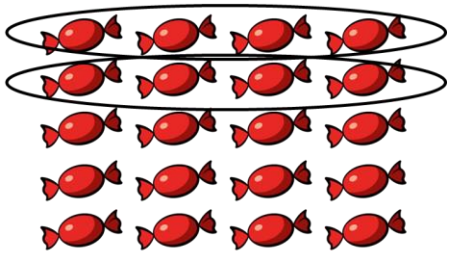


$4 + 11 = ?$

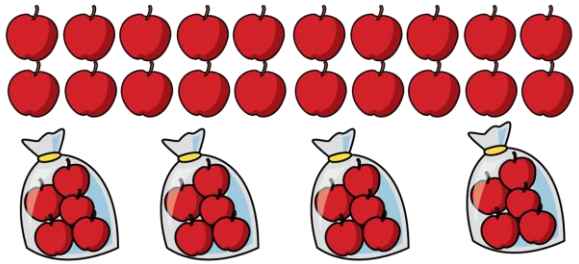
6 5



$\frac{2}{5}$ of $20 = ?$



$20 \div 4 = ?$

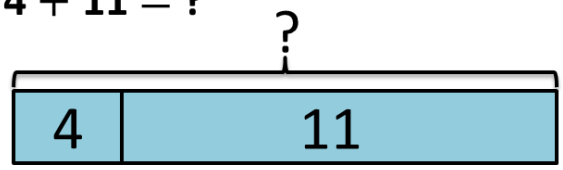


Share 20 in the ratio 2:3

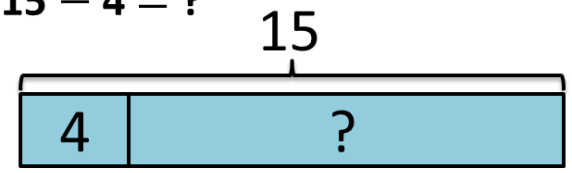
$5 \times 4 = ?$

# A consistent picture

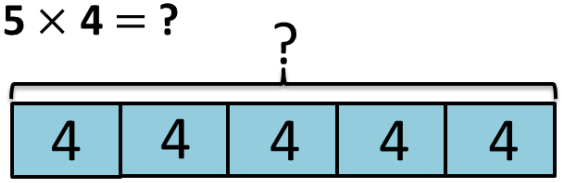
$4 + 11 = ?$



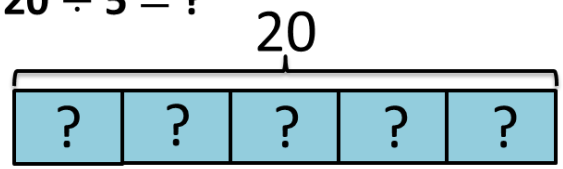

$15 - 4 = ?$



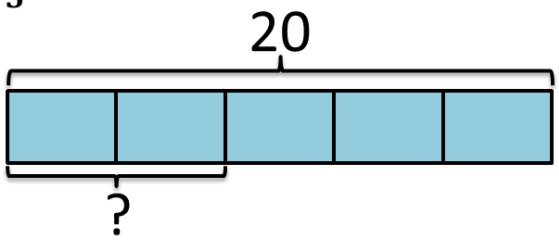

$5 \times 4 = ?$



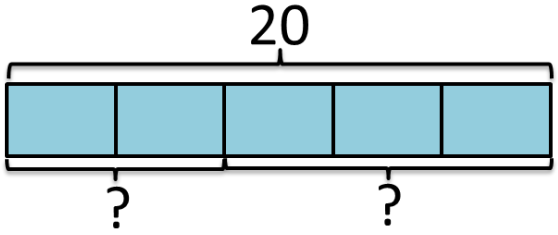

$20 \div 5 = ?$




$\frac{2}{5}$ of 20 = ?




Share 20 in the ratio 2:3



# Any Questions?

# Thank you

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# References


- Beckmann, S. (2004) Solving Algebra and Other Story Problems with Simple Diagrams: a Method Demonstrated in Grade 4 - 6 Texts Used in Singapore, The Mathematics Educator, 14, (1), pp. 42 – 46
- Gu, D. (2015). Analysis of Four Lessons. 8th September 2015, Shanghai Normal University, Shanghai, China.