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What Makes Aggregate Fluctuations in Canada Different?

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Abstract

This paper attempts to learn about the differences in aggregate fluctuations in Canada and the United States. Our method for studying this issue is to estimate empirical models in which theory provides the identifying restrictions. We find that aggregate fluctuations in Canada differ from those in the U.S. along three dimensions. First, in Canada, aggregate fluctuations are more sensitive to foreign shocks. Second, nominal shocks do not seem to be as important in Canada as these shocks are for the U.S.. Lastly, Canada appears not only to be more sensitive to foreign shocks, ~~Canada is~~ but also to b more sensitive to regime shifts which originate abroad.

I. Introduction

The purpose of this paper is to learn about the differences, if any, in aggregate fluctuations of Canada and the United States. At the start, we are faced with ~~to~~ two competing views of aggregate fluctuations in Canada and the U.S.. On the one hand theory predicts that the different structures of the two economies, and especially their relative vulnerabilities to external disturbances, should induce different aggregate behaviour. However, anecdotal evidence and previous work suggests that aggregate fluctuations in Canada and the U.S. are more alike than theory predicts.[1] To attempt to reconcile theory and observation, we estimate empirical models in which simple theoretical models provide the identifying restrictions. This allows us to study the dynamic responses of important macroeconomic variables across the two economies and perhaps to gain a ~~bit~~ more knowledge about the similarities and differences in the Canadian and U.S. economies.

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We employ structural vector autoregressions to identify the different sources of aggregate fluctuations in Canada and the U.S.. Recent empirical analyses of aggregate fluctuations in both closed and open economy contexts have employed structural decompositions of aggregate fluctuations. These studies, which include Shapiro and Watson (1988), Blanchard and Quah (1989), King, Plosser, Stock, and Watson (1991), Gali (1992), Mellander, Vredin, and Warne (1992), and Ahmed, Ickes, Wang, Yoo (1993), all begin with an explicit economic model of aggregate fluctuations. The economic model permits identification of an econometrically tractable model which permits computation of the responses of endogenous variables to the structural shocks of the economy.

Specifically, we apply the methods of Blanchard and Quah (1989). This involves estimation of a vector moving average that is identifiable with some underlying economic

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[1] For example, Kuszczak and Murray (1987) report evidence that it is the degree rather than the response to shocks of international origin which distinguishes the Canadian and U.S. economies.

model. The parameters and residuals of a reduced form VAR are used together with the identifying restrictions of the long run predictions of the economic model to derive a structural moving average representation. The Blanchard and Quah decomposition requires that the economic model predicts some of the sources of aggregate fluctuations have a permanent impact on some of the variables of the structural VAR. Further, the Blanchard and Quah decomposition requires that the structural disturbances are orthogonal.

A straightforward way to construct a theoretical model which satisfies the requirements of the Blanchard and Quah decomposition is to work with a stochastic growth model. The stochastic growth models we work with possess technology shocks which have permanent affects on aggregate output. To this, we add money growth shocks to the stochastic growth model following the approach of Lucas (1990) and Fuerst (1992). In closed and open economy settings, the identifying restrictions of the resulting monetary business cycle model allow application of the Blanchard and Quah decomposition.

An outline and summary of our paper follows. Section II presents a descriptive analysis of aggregate fluctuations in Canada and the U.S.. On the basis of this analyses, section III outlines a closed economy monetary growth model which yields identifying restrictions for a bivariate system of output growth and inflation. The results of section III suggest that only the behavior of aggregate prices in the two economies differ. However, whether the symmetric behavior of output and the asymmetric behavior in prices is caused by model misspecification or is economically meaningful remains an open issue. To clarify this issue, section IV presents a open economy model which is the result of the empirical analyses of sections II and III. The results of section IV suggest that aggregate fluctuations in Canada differ from those in the U.S. along three dimensions. First, in Canada, aggregate fluctuations appear to be more sensitive to foreign shocks. Second, nominal shocks are not as important in Canada as these shocks are for the U.S.. Lastly, Canada appears not only to be more sensitive to foreign shocks, Canada appears to be more sensitive to regime shifts which originate abroad. Section V contains a conclusion.

II. Descriptive Analysis

One problem we face is the lack of guidance in thinking of theoretical models of aggregate fluctuations across Canada and the U.S.. This suggests performing some descriptive data analysis with Canadian and U.S. output data to yield some stylized facts to help in model specification. Our analyses makes use of unit root tests, common feature tests, cointegration tests, and tests for regime shifts in the presence of cointegration.

A. Unit Root Tests of Output

We begin by studying the univariate trend properties of Canadian and U.S. output with three unit root tests. The unit root statistics we compute include the augmented Dickey-Fuller (ADF) t-ratio and the Phillips (1987) and Phillips and Perron (1988) $Z(t)$ and $Z(b)$ statistics; see the technical appendix for details.

The top panel of table 1 contains the results of the ADF, $Z(t)$, and $Z(b)$ tests.[2] The sample period is 1960:1 through 1991:4. All of the test statistics, are larger than asymptotic five percent critical values of the ADF, $Z(t)$, and $Z(b)$ tests. Although strict interpretation of the unit root tests provide evidence that Canadian and U.S. output possess a unit root, recent work indicates that these tests have low power

[2] For Canada, the output series is real GDP (1986 dollars). Real per capita GDP for Canada is constructed using the total population series. U.S. output is real GDP (1987 dollars). The U.S. per capita output is generated using total population which includes members of the armed services overseas. The source of the Canadian data is the Bank of Canada and the CanSim data bank. For the U.S. data, the CITIBASE data bank is the source.

against many alternatives.[3] Hence, these results could have been generated by unit root as well as many other DGPs.

B. A Serial Correlation Common Features Test

Given the persistence we find in the level of per capita output in each country, our next bit of analysis examines the relationship of the growth rates of output in Canada and the U.S.. The test we work with is a test for serial correlation common features (SCCF). If SCCF exists for Canadian and U.S. output, it indicates these series share common cyclical properties. That is SCCF provides a measure of the comovements in Canadian and U.S. output growth.

Engle and Kozicki (1993) develop the notion of common features in multivariate information sets. For example, when the (covariance) stationary time series $x(1,t)$ and $x(2,t)$ share serial correlation common features, there exists a linear combination, $x(1,t) - \delta x(2,t)$, $0 < \delta$, which is white noise. A negative slope coefficient indicates $x(1,t)$ and $x(2,t)$ are out of phase. If $x(1,t)$ and $x(2,t)$ share SCCF, they share common cyclical properties. Common features of this type impose weak restrictions on the bivariate process $\{x(1,t), x(2,t)\}$.

Engle and Kozicki outline an instrumental variable approach to test for SCCF in a bivariate information set. First, a two stage least squares (2SLS) of $x(1,t)$ on an intercept and $x(2,t)$ is computed. The instrument set contains lags $i = 1, \ldots, j$ of $x(1,t-i)$ and $x(2,t-i)$. To test for SCCF, the 2SLS residuals are regressed on the instruments. Under the null of a common cofeature, this yields the LM test statistic $T \times R^2$ which is distributed asymptotically χ^2 with $2 \times j-1$ degrees of freedom. Intuitively, the test considers whether the dependence of $x(1,t)$ on the past can be captured by its

[3] For example, Rudebusch (1992) shows that unit root tests have low power against trend stationary alternatives.

past and the past of x(2,t).[4]

For the sample period 1960:1 through 1991:4, the 2SLS regression of per capita Canadian (U.S.) output growth on per capita U.S. (Canadian) output growth yields a slope coefficient of 1.41 (0.64) with a standard error of 0.45 (0.21). Two lags of each variable is included in the information set. The $T \times R^2$ statistic equals 1.04 (0.99) and with three degrees of freedom the confidence level is 0.79 (0.80).[5] The presence of SCCF in Canadian and U.S. output growth indicates these series share common cyclical properties.

C. Cointegration Tests of Output

Besides finding univariate persistence in Canadian and U.S. output, the presence of SCCF in the growth rates of Canadian and U.S. output suggests testing for a stronger set of restrictions in this bivariate process.[6] In this case, we conduct tests for a common stochastic trend in the Canadian and U.S. output. To test for a cointegrating relation between Canadian and U.S. output, we use a two step procedure. First, the cointegrating relation

$$y(1,t) \quad = \quad a(0) \ + \ a(1)y(2,t) \ + \ v(t)$$

is estimated by least squares. Then the estimates of $\Delta v(t)$ are used either to compute a Dickey-Fuller (DF) regression or to estimate a first order autoregression. The DF regression produces the DF t-ratio. This statistic is computed with 1, 2, and 3 lags of $\Delta v(t)$ as regressors. The latter second stage regression yields $Z(t)$ and $Z(b)$

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4 Engle and Kozicki (1990) point out the connection of this regression cofeature test to tests for instrument validity. Hence, the regression cofeature test and tests for instrument validity share similar properties. It is the interpretation of the tests that differ.

5 Engle and Kozicki (1993) report similar results for quarterly Canadian and U.S. real GNP growth rates.

6 Serial correlation common features impose a weaker set of restrictions on a multivariate information set than does cointegration; see Engle and Kozicki (1993).

statistics; the technical appendix contains details.[7]

The middle panel of table 1 contains the results of the cointegration tests. All of the cointegration tests are unable to reject the null of no cointegration at the five percent level. Hence, we find no evidence that a linear combination of Canadian and U.S. output form a cointegrating relation.

C. Cointegration with Regime Shifts

Another possible explanation of the behaviour of Canadian and U.S. output is that Canadian and U.S output cointegrate but with a regime shift. Gregory and Hansen (1992) study the problem of a cointegrating relation with regime shifts. The null hypothesis is no cointegration versus the alternative of cointegration with a regime shift. To construct ADF, Z(*t*), and Z(*b*) statistics to test for a regime shift, Gregory and Hansen estimate the cointegrating relation

$$y(1,t) = a(0) + d(0)\varphi(t,\tau) + a(1)y(2,t) + d(1)y(2,t)\varphi(t,\tau) + v(t),$$

with the level shift parameter $d(0)$, the regime (slope) shift parameter $d(1)$, and the dummy variable

$$\varphi(t,\tau) = \begin{cases} 0, & \text{if } t \leq [T\tau] \\ 1 & \text{if } t > [T\tau]. \end{cases}$$

The parameter τ is unknown as it defines the change point in the cointegrating relation. In this setup, $\tau \in (0, 1)$ and $[\cdot]$ indicates the integer component of $T\tau$.

The test statistics are computed at each observation in a rolling fashion. At each date τ, DF t-ratio, Z(*t*), and Z(*b*) statistics are computed using the residuals of the first stage regression of the regime shift in the cointegrating relation. The test

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[7] Gregory (1993) presents Monte Carlo evidence that the ADF and the Z(*b*) tests provide the best size and power results among many tests for cointegration. The dynamic linear quadratic model is the DGP which generates the results found by Gregory.

statistic of the structural break test are the minimal values of the ADF, $Z(t)$, and $Z(b)$ statistics on $\tau \in T$. Since the test statistics are not well defined either as $\tau \to 0$ or as $\tau \to 1$, we trim the sample to $\tau \in [0.15, 0.85]$; the technical appendix contains some details.[8]

The results of Gregory and Hansen structural break tests appear in the bottom panel of table 1. None of the test statistics are smaller than their five percent critical values. Structural change does not seem to matter for describing the bivariate relationship of Canadian and U.S. output. Results from these tests confirm the lack of a cointegration relation for Canadian and U.S. output. Since we cannot reject the null of no cointegration against cointegration with a regime shift, we conclude that Canadian and U.S. output do not form a stationary linear combination.

III. A Closed Economy Monetary Business Cycle Model

In this section, we consider a version of the cash in advance constraint model studied by Christiano (1990). This model serves as the basis of our closed economy monetary business cycle model. Since closed economy models are more parsimonious than open economy models with similar structures, we begin with a class of closed economy models to further our understanding of aggregate fluctuations in Canada.

In the closed economy model, money is held because households face a cash in advance (CIA) constraint. The consumption good can be purchased with cash carried over from the previous period and current period labour income. However, the typical household has the option of depositing some of its income each period with a financial intermediary (FI). In this model, the monetary aggregate is endogenous.

Besides the deposits of households, the FI receives monetary injections from a

[8] The interested reader should see Gregory and Hansen (1992) for details.

monetary authority each period. The monetary injections take the form of an exogenous stochastic process. Deposits and injections are loaned by the FI to firms.

Firms combine capital and labour with an exogenous technology shock in a constant returns to scale production function to produce the good of the economy. This good serves either as the investment flow into the capital stock of a firm or as the consumption good of households. Since firms own their capital, the only factor firms pay is labour. Firms finance wages with funds they borrow from the FI.

A. Technology

Output, y(t), is generated with the constant returns to scale production function

$$y(t) \quad = \quad k(t)^{\theta}[A(t)n(t)]^{(1-\theta)}, \quad 0 < \theta < 1, \tag{1}$$

where k(t) is capital carried into date t from date t-1 and n(t) is labour input. The cost of a unit of labour to the firm is W(t). The technology shock, A(t), follows a random walk with drift

$$\ln[A(t)] \quad = \quad \gamma t + \ln[z(t)], \quad 0 < \gamma, \tag{2a}$$

where γ is the deterministic trend component of technology growth. The stochastic trend component is

$$\ln[z(t+1)] \quad = \quad \ln[z(t)] + \varepsilon(t+1), \quad \varepsilon(t+1) \sim N(0, \sigma(\varepsilon)^2). \tag{2b}$$

B. Money and Goods Markets

Besides the technology shock, the model includes an exogenous stochastic process for the growth rate of currency. The monetary injection growth shock is a first order autoregression

$$\ln[m(t+1)] \quad = \quad \rho\ln[m(t)] + \nu(t+1), \quad |\rho| < 1, \ \nu(t+1) \sim N(0, \sigma(\nu)^2). \tag{3}$$

Currency growth is defined as

$$\ln[m(t)] \;=\; \ln[M(t+1)] - \ln[M(t)],$$

where M(t) is the stock of currency at the end of date t-1. It is assumed that $\varepsilon(t)$ and $\nu(t)$ are uncorrelated at all leads and lags.

The endogeneity of the monetary aggregate arises from the existence of inside money. The FI must have income each period greater than or equal to the payments for which it has contracted. These payments are the deposits it accepts from households, d(t). The FI's income is the return on loans from firms, l(t). Hence, the balance sheet constraint of the FI is

$$X(t) + d(t) \;\leq\; l(t), \tag{4}$$

where X(t) is the monetary injection during date t, $X(t) = M(t+1) - M(t)$. This means the total money supply of the economy is the sum of the exogenous currency process and the endogenous loan creation process of the FI

$$H(t+1) \;=\; M(t) + X(t) + d(t).$$

Households purchase the consumption good, c(t), at price P(t) with current period labour income and currency brought over from the previous period. Given the CIA constraint,

$$P(t)c(t) + d(t) \;\leq\; M(t) + W(t)n(t), \tag{5}$$

holds with strict equality during each date t, currency will have positive value. However, prices will be affected by the creation of inside money as well as the monetary injection process.

C. The Theoretical DGP

The structure of the closed economy monetary business cycle model yields an econometric specification with an economic interpretation. The model imposes identifying restrictions on its VAR representations. However, the model possess only two sources of underlying uncertainty. Hence, we are limited to studying bivariate VARs. Any higher dimensional VAR would be stochastically singular.

Decision rules of standard real business cycle models make endogenous variables functions of the current state variables. For our monetary business cycle model, the covariance stationary state vector is $S(t) = [\hat{k}(t) \;\; \varepsilon(t) \;\; m(t)]'$, where $\hat{k}(t) = k(t)/A(t\text{-}1)$ is stochastically detrended capital.[9] In turn, stochastically detrended output, $\hat{y}(t) = y(t)/A(t)$, and prices, $\hat{P}(t) = P(t)A(t)/M(t)$, are functions of state variables only. However, the stochastically detrended versions of output and prices are not directly observable. In our environment, observables such as output growth and inflation are predicted to be

$$\Delta \ln[y(t)] = \Delta \ln[\hat{y}(S(t))] + \gamma + \varepsilon(t) \tag{6a}$$

and

$$\Delta \ln[P(t)] = \Delta \ln[\hat{P}(S(t))] - \gamma - \varepsilon(t) + \ln[m(t\text{-}1)]. \tag{6b}$$

Each of these variables is the sum of an over differenced stationary component and a stationary component. In the long run, the over differenced stationary component vanishes. Since the growth rate of stochastically detrended output disappears at frequency zero, technology shocks are the only source of output fluctuations in the long run. Prices are determined by technology shocks and the growth rate of monetary injections in the long run. This provides us sufficient information to identify a bivariate VAR of output and inflation in an economically meaningful way.

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[9] In a Lucas (1990) and Fuerst (1992) style model, the state vector is expended to

$$S(t) = [\hat{k}(t) \;\; \varepsilon(t) \;\; \varepsilon(t\text{-}1) \;\; m(t) \;\; m(t\text{-}1)]'$$

because the household is restricted to make deposit decisions only on the basis of date t-1 information.

Our closed economy monetary business cycle model predicts long run monetary neutrality for output. There are no long run restrictions on inflation. The long run restriction translates into a vertical aggregate supply curve in ΔP-y space. In the long run, movements in the aggregate demand curve translate into fluctuations in inflation. Although many monetary models impose these restrictions on the long run aggregate supply and demand curves, our closed economy monetary model provides a unique interpretation of the structural disturbances.

The formal statement of the identifying restriction of the output-inflation information set implied by (6) is:

The long run level of output is independent of monetary injection growth shocks.

Given this single identifying restriction, the theoretical structural VAR is just identified.[10]

D. Empirical VARs

In order to construct the stylized facts of the output-inflation information set, the sample analog of the theoretical VAR needs to be constructed. The techniques of Blanchard and Quah (1989) provide a straightforward manner in which to do this.[11] For

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[10] The approach to identifying neutrality restriction adopted here differs from that of Shapiro and Watson (1988), King and Watson (1992), and Cogley (1993). Shapiro and Watson work with over identified models in order to uncover the sources of business cycle fluctuations. This requires an instrumental variables estimator. King and Watson's interest is to test various neutrality propositions given integrated variables and prior information about contemporaneous impact and long run multipliers. Problems arise in this environment which requires instrumental variable estimation as well. Cogley estimates an exactly identified model and then imposes on the impulse response functions the over identifying restrictions which are necessary for neutrality. This provides a test of the neutrality propositions.

[11] Recently, Lippi and Reichlin (1993) have developed conditions which signal when the Blanchard and Quah decomposition yields a nonfundmental solution. The Blanchard and Quah decomposition can generate a nonfundamental solution for a number of reasons. For our model, the issue of nonfundamental solutions is an issue of the choice of economic identification of a time series model. This issue is ubiquitous to all structural time series models. Since there are nearly endless variety of identification schemes, the choice of scheme is simply a maintained hypothesis of the model; see Blanchard and Quah

the output growth-inflation system, the unrestricted, finite order, sample VAR yields the VMA(∞)

$$\begin{bmatrix}\Delta \ln[y(t)] \\ \Delta \ln[P(t)]\end{bmatrix} = \Lambda(\mathbf{L})e(t),\ \Lambda(\mathbf{L}) = \sum_{i=0}^{\infty} \Lambda(i)\mathbf{L}^i, \quad \Lambda(0) = \mathbf{I},\ \mathbf{E}\{e(t)e(t)'\} = \Omega.$$

If the theoretical SVMA(∞) process is written

$$\begin{bmatrix}\Delta \ln[y(t)] \\ \Delta \ln[P(t)]\end{bmatrix} = \Phi(\mathbf{L})\mathbf{u}(t),\ \ \Phi(\mathbf{L}) = \sum_{i=0}^{\infty} \Phi(i)\mathbf{L}^i,\ \ \mathbf{E}\{\mathbf{u}(t)\mathbf{u}(t)'\} = \mathbf{I},$$

where $\mathbf{u}(t) = [\varepsilon(t)\ \nu(t)]'$, then, $\mathbf{u}(t) = \Phi(0)^{-1}e(t)$ and $\Phi(j) = \Lambda(j)\Phi(0)$, j = 1, 2, ... Once $\Phi(0)$ is known, the structural disturbances and the structural IRFs can be generated. However, this requires that $\Phi(0)$ be identified.

Identification of the four elements of $\Phi(0)$ requires imposing four conditions on the unrestricted MA(∞). The equation $\mathbf{u}(t) = \Phi(0)^{-1}e(t)$ yields three restrictions because the symmetric matrix $\Omega = \Phi(0)\Phi(0)'$. The fourth condition relies on a restriction on the long run multiplier matrix of the theoretical SVMA(∞)

$$\Phi(1) = \begin{bmatrix}\phi(11) & 0 \\ \phi(21) & \phi(22)\end{bmatrix}$$

and $\Phi(1) = \Lambda(1)\Phi(0)$. That is $\lambda(11)\phi(12,0) + \lambda(12)\phi(22,0) = \phi(12) = 0$. The nonlinear system of four equations uniquely determines the four unknowns of $\Phi(0)$.[12]

(1993).

[12] Blanchard and Quah (1989) provide a proof of this result. The idea is that $\Phi(0)$ is an orthonormal transformation of the unique Choleski factorization of Ω. The long run neutrality restriction acts as an orthogonality condition which uniquely determines $\Phi(0)$. Since $\varepsilon(t)$ and $\nu(t)$ are orthogonal by construction, the Blanchard-Quah decomposition exists under the null.

E. Sample IRFs

The sample IRFs of the output-inflation system appear in figures 1 and 2 for Canada and the U.S., respectively.[13] To compute the structural IRFs, second order VARs are estimated. The sample period is 1960:1 through 1991:4. The response of output to the technology disturbance is the accumulated response over the entire horizon. For example, in figure 1a, the solid line traces the accumulated response of Canadian output to a technology shock over the 41 period horizon.

Figures 1a and 2a contain the responses of output and inflation to the technology disturbance. In each country, output's response to the technology disturbance is a typical capital accumulation path of a stochastic growth model. The only difference across the two economies is all of the response of Canadian output to its technology shock is completed within two years while U.S. output rises over the entire horizon. Since the technology shock is the theoretical permanent component of output, these IRFs represent permanent shifts in the aggregate supply schedules of Canada and the U.S..

The response of inflation to the technology disturbance is similar for Canada and the U.S.. At impact, inflation is lower. Most of the reversion to trend is completed in less than two years. In the long run, inflation returns to its steady state. The only difference between inflation's response across the two countries is that inflation returns to steady state from above in Canada and below in the U.S..

Canada and the U.S. differ in their response to the theoretical monetary injection growth shock as found in figures 1b and 2b. The IRFs of output and inflation to the monetary injection growth shock exhibit a saw tooth pattern in Canada. In the

[13] For Canada, sample y(t) and P(t) are per capita real GDP (1986 dollars) and the implicit GDP price deflator. Real per capita GDP for Canada is constructed using the total population series. Sample y(t) and P(t) are per capita real GDP (1987 dollars) and the implicit GDP price deflator for the U.S.. The U.S. per capita quantities are generated using total population which includes members of the armed services overseas. The source of the Canadian data is the Bank of Canada and the CanSim data bank. For the U.S. data, the CITIBASE data bank is the source.

U.S., these IRFs are smoother. The transitory response of the aggregate supply-demand system to a nominal shock is for output and inflation to rise at impact in each country.[14]

F. Lagrange Multiplier Tests

On the eyeball metric, the IRFs of figures 1 and 2 appear to be drawn from to two disparate DGPs. To quantify this conjecture, a Monte Carlo experiment is conducted in which the DGP of the U.S. bivariate information set is taken as the theoretical model. The first moments of the theoretical DGP are computed by Monte Carlo integration of the second order VAR estimated on U.S. data. To compute the theoretical IRF, the ensemble of artificial replications is averaged

$$\mathbf{F} = N^{-1} \sum_{j=1}^{N} \mathbf{F}(j),$$

where **F** is the mean IRF, N is the number of replications, N = 1000, and **F**(j) is the IRF of replication j.

Formal tests which compare the IRFs are conducted with Lagrange multiplier (LM) tests. The LM test statistics of the IRFs take the quadratic form

$$LM = [\hat{\mathbf{F}} - \mathbf{F}]\hat{\mathbf{V}}^{-1}[\hat{\mathbf{F}} - \mathbf{F}]',$$

where $\hat{\mathbf{F}}$ is the sample IRF from the bivariate Canadian information set and $\hat{\mathbf{V}}$ is the empirical covariance matrix of the theoretical IRF.[15] The empirical covariance matrix of the theoretical IRF is computed as the ensemble average of the outer product of the theoretical IRF

[14] Nason (1993) presents evidence on the ability of Lucas-Fuerst monetary business cycle models to replicate these IRFs for the U.S.. This evidence suggests these models can replicate the IRFs for inflation, but not the IRFs for output.

[15] Similar tests of structural VARs first appear in Cogley and Nason (1993). In that paper, real business cycle models are the source of the theoretical DGP.

$$\hat{V} = N^{-1} \sum_{j=1}^{N} [F(j) - F][F(j) - F]' .$$

Confidence levels of the LM tests can be calculated two different ways. Since the LM statistics are approximately χ^2 with degrees of freedom equal to the number of elements of **F**, the χ^2 distribution can be used for generating confidence levels. However, the small sample bias common to structural VARs poses problems for inference in structural VARs. To avoid this problem, we compute empirical probability values of the LM statistics. The empirical probability value equals the fraction of replications in which the theoretical LM statistics exceeds the sample LM statistic. At replication j, the theoretical LM statistic is constructed as

$$\mathrm{TLM}(j) = [F(j) - F]\hat{V}^{-1}[F(j) - F]' .$$

Hence, the empirical probability values count the number of times the null is found to be true.[16]

The LM tests answer the question of 'how close' two IRFs are to each other after accounting for *sampling uncertainty*.[17] Test results appear in table 2. To read the table, each column contains the LM test statistic of the IRFs of either output, Y, or prices (inflation), P, to the technology, T, or the money growth, M, shock. Since the sample data is quarterly, LM statistics are computed with four, eight, twelve, and sixteen lags. With four lags, there are five elements in **F** because the elements of *C*(0) are included. For example, the response of inflation to the money growth shock at eight lags yields an LM statistic of 23.654 with an empirical probability value of 5.2 percent

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[16] Tim Cogley graciously supplied us with his computer programs for computing the test statistics and empirical probability distributions.

[17] The LM tests have several advantages over other methods for comparing the IRFs. First, under the null, the IRFs have unique interpretations. Hence, rejection of the null provides information for model respecification. Second, under the null, the LM tests have power against the alternative of different DGPs. This is the alternative of interest. Third, unlike the confidence intervals of the IRFs, the LM tests allow for inference to be conducted over any lag length.

and nine degrees of freedom.

The results of table 2 lead to very different conclusions about the disparity of the IRFs than do casual inspection of figures 1 and 2. Once sampling uncertainty is taken into account, it is not possible at the five percent level to reject at any lag length that the responses of output to real and money growth shocks in Canada and the U.S. differ. Much the opposite is true for the response of inflation to real and money growth shocks. The response of inflation to the technology shock is rejected at better than the one percent level at all horizons. Inflation's response to money growth shocks is rejected at the 10 percent level at all horizons.

At this level of detail, these results yield one asymmetry in aggregate fluctuations between Canada and the U.S.. Aggregate price behavior differs in Canada and the U.S.. As for output, it appears to behave in a symmetric manner across the two countries. This backs up the results of the SCCF test. What cannot be inferred from the LM tests without a bit more work are the robustness of these results and the sources of the asymmetries in prices in Canada and the U.S..

IV. An Open Economy Model Monetary Business Cycle Model

Our approach in this section is to specify the real side of an open economy model in accord with our stylized facts. To account for the lack of a cointegrating relation between and the common features in Canadian and U.S. output, we construct a two country model which emphasizes the role of traded and nontraded goods technology shocks. The two sector technology structure allows for SCCF without cointegration in Canadian and U.S. output.

$$\ln[a(j,t+1)] = \ln[a(j,t)] + \eta(j,t+1), \quad \eta(j,t+1) \sim N(0, \sigma^2(\eta,j)). \tag{10}$$

We assume the parameterization of the innovations to the technology shocks is

$$\begin{bmatrix} \varepsilon(t) \\ \eta(h,t) \\ \eta(f,t) \end{bmatrix} = N\left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma^2(\varepsilon) & 0 & 0 \\ & \sigma^2(\eta,h) & 0 \\ & & \sigma^2(\eta,f) \end{bmatrix}\right).$$

B. Resource Constraints

In our two country model, the real side world resource constraint is

$$\sum_{j=1}^{2} \pi(j)[q(j,t) - c(j,q,t) - i(j,t)] \leq 0, \tag{11}$$

where $\pi(j)$, $\pi(h) + \pi(f) = 1$, is the non-negative planner's weight assigned to country j, c(j,q,t) denotes total consumption of traded goods in country j and i(j,t) represents investment in total capital in country j. Within each country, consumption of the nontraded good, c(j,d,t), satisfies

$$c(j,d,t) \leq d(j,t), \quad j = h, f. \tag{12}$$

The total capital stock in country j is the sum of the capital stock in each sector

$$k(j,t) \geq k(j,q,t) + PR(j,t)k(j,d,t). \tag{13}$$

where PR(j,t) is the relative price of the nontraded good to the traded good in country j. This implies the cost of moving capital between the two sectors is the (real) relative price of the nontraded good in units of the traded good in country j. Capital accumulation follows the law of motion

$$k(j,t+1) = [1 - \delta(j)]k(j,t) + i(j,t), \quad 0 < \delta(j) < 1, \tag{14}$$

where $\delta(j)$ is the depreciation rate of country j.

To the real side of the world economy, CIA constraints and monetary injection growth shocks (as in section III) are added for each country.[19] This yields a world economy with a permanent world technology shock, two permanent country specific technology shocks, and two country specific monetary injection growth shocks.

C. A Just Identified Structural VAR

Our goal in this section is to construct an information set of domestic and foreign country variables which yields a just identified structural VAR. As with the construction of the structural VAR of output and inflation, the structure of our open economy business cycle model motivates our identifying restrictions. Although the number of state variables has increased, the manner in which we construct the econometric specification is the same as in section III.

Construction of the structural VAR relies on the structure of our open economy model. The covariance stationary state vector of the model contains the two endogenous state variables and five exogenous state variables

$$S(t) \;=\; [\hat{k}(h,t)\;\hat{k}(f,t)\;\varepsilon(t)\;\eta(h,t)\;\eta(f,t)\;m(h,t)\;m(f,t)]',$$

where $\hat{k}(j,t) \;=\; k(j,t)/[A(t-1)a(j,t-1)]$ is stochastically detrended capital in country j. As in section III, stochastically detrended endogenous variables are functions of $S(t)$. For output, measured total output in each country, y(j,t), is the sum of tradeables and nontradeables output, then, stochastically detrended output becomes $\hat{y}(j,t) \;=\; y(j,t)/[A(t)a(j,t-1)]$.[20] Hence, total output growth in country j follows

[19] The CIA constraints would be modified for the presence of a foreign exchange market. Once again, it is assumed that all technology shocks are uncorrelated with the monetary injection growth shocks at all leads and lags.

[20] Our open economy model permits a balanced growth path. In the long run along this path, total output in a country is determined only by the world and the own country specific shock.

$$\Delta \ln[y(j,t)] = \Delta \ln[\hat{y}(j,\mathcal{S}(t))] + \gamma + \varepsilon(t) + \eta(j,t) \tag{14a}$$

Similarly, the stochastically detrended price level in units of its own currency is $\hat{P}(j,t) = P(j,t)[A(t)a(j,t\text{-}1)]/M(j,t)$. In turn, country j's inflation rate becomes

$$\Delta \ln[P(j,t)] = \Delta \ln[\hat{P}(j,\mathcal{S}(t))] - \gamma - \varepsilon(t) - \eta(j,t) + \ln[m(j,t\text{-}1)], \tag{14b}$$

where M(j,t) and m(j,t) denote the stock of currency and the growth of currency in country j, respectively. These equations and (6a,b) differ because of the presence of the world technology shock in (14a,b).

The information set of the structural VAR is put together using two types of identifying restrictions. First, as the output-inflation information set of section III predicts, long run monetary neutrality for output holds in each country. Second, the open economy model predicts that home country output and prices are unaffected by the foreign country nontraded goods shock in the long run. These two types of identifying restrictions are real side monetary neutrality and foreign country neutrality restrictions. Formally, we state the identifying restrictions as

(i) *The long run level of output in each country is independent of all money growth shocks*, and

(ii) *The long run levels of output and prices in each country are independent of foreign country nontraded good shocks.*

This suggests working with an information set which contains terms of trade variables as well as a dynamic aggregate supply and demand relationship. The information set

$$\Delta \mathbf{X}(t) = \{\Delta \ln y[(h,t)]\ \Delta \ln y[(f,t)]\ \Delta \ln[P(f,t)/\ln P(h,t)]\ \Delta \ln[H(h,t)/P(h,t)]\}'$$

provides a just identified four dimensional, theoretical SVMA(∞)

$$\mathbf{X}(t) = \Phi(\mathbf{L})\mathbf{u}(t), \quad \Phi(\mathbf{L}) = \sum_{i=0}^{\infty} \Phi(i)\mathbf{L}^{i}, \quad \mathrm{E}\{\mathbf{u}(t)\mathbf{u}(t)'\} = \mathbf{I}.$$

We use the growth rate of relative prices between the home and foreign countries

$$\Delta \ln[P(f,t)/P(h,t)] = \Delta \ln[\hat{P}(f,S(t))/\hat{P}(h,S(t))]$$

$$- \eta(f,t) + \eta(h,t) + \ln[m(f,t\text{-}1)] - \mathrm{lm}[m(h,t\text{-}1)], \tag{14c}$$

to capture the terms of trade dynamics between the two countries. Two items about (14c) should be stressed. First, we do not identify separately the relative monetary injection growth shocks of the two countries.[21] To identify these nominal shocks separately, require assumptions about the money supply process and the interactions of monetary policy and nominal exchange rate determination in each country. Second, if the nominal exchange rate is included in (14c), the empirical model would be under identified.[22]

Domestic aggregate demand factors are captured by the presence of home country real balances in $\mathbf{X}(t)$. Domestic country real balances are predicted to follow

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[21] Ahmed, Ickes, Wang, and Yoo (1993) do not distinguish between home and foreign money supply side shocks. As they note, this tack to identifying nominal side shocks rationalizes a plethora of approaches to the money supply process. That is it allows for the money supply process to react to all manner of disturbances. We follow Ahmed, Ickes, Wang, and Yoo as we focus on the responses of real side variables to nominal side shocks. Another way to view this is to understand that the level of sophistication of our theoretical model does not permit the identification of the monetary policy rules of the two countries. To be able to do this, would require us to take make assumptions either explicitly or implicitly about the objective function of the monetary authority.

[22] In our two country model, country j's GDP deflator is a weighted average of traded and nontraded goods, say

$$P(j,t) = P(j,q,t)^{\omega}P(j,d,t)^{1-\omega}, \quad 0 < \omega < 1.$$

Then the ratio of the price of traded goods to P(j,t) is

$$P(j,q,t)/P(j,t) = [P(j,q,t)/P(j,d,t)]^{1-\omega}.$$

Given $P(h,q,t) = e(t)P(f,q,t)$, where e(t) denotes the nominal exchange rate between the countries, the ratio of GDP deflators weighted by the nominal exchange rate measures the relative real exchange rate of the domestic and foreign countries. The identification problem is that e(t) is determined by $\eta(h,t)$, $\eta(f,t)$, $\nu(h,t)$ and $\nu(f,t)$ in the long run.

$$\Delta\ln[H(h,t)/P(h,t)] = \Delta\ln[\hat{H}(h,S(t))/\hat{P}(h,S(t))]$$

$$+ \gamma + \varepsilon(t) + \eta(h,t) - \ln[m(h,t\text{-}1)]. \tag{14d}$$

Nominal balances are equated with the endogenous monetary aggregate as in section III. The structural disturbances vector is $\mathbf{u}(t) = [\eta(h,t)\ \varepsilon(t)\ \eta(f,t)\ \nu(h,t) - \nu(f,t)]'$.

Identification of the 16 elements of $\Phi(0)$ requires imposing 16 conditions on the unrestricted MA(∞) process. The finite order VAR

$$\Xi(\mathbf{L})\Delta\mathbf{X}(t) = \mathbf{e}(t), \quad \Xi(\mathbf{L}) = \sum_{i=0}^{p} \Xi(i)\mathbf{L}^{i}, \quad \Xi(0) = \mathbf{I}, \quad \mathbf{E}\{\mathbf{e}(t)\mathbf{e}(t)'\} = \Omega$$

is the source of the unrestricted MA(∞). The equation $\mathbf{u}(t) = \Phi(0)^{-1}\mathbf{e}(t)$ yields 10 restrictions because the symmetric matrix $\Omega = \Phi(0)\Phi(0)'$. The remaining six restrictions are found in the long neutrality restrictions. Home and foreign output yield four long run neutrality restrictions. The last two restrictions are from (14c) and (14d). Neutrality of the difference of inflation rates to the world shock provides the penultimate restriction. The last restriction is the neutrality of the growth rate of real balances with respect to foreign country, nontraded goods shocks. Hence, the long run multiplier matrix of $\Phi(\mathbf{L})$ becomes

$$\Phi(1) = \begin{bmatrix} \phi(11) & \phi(12) & 0 & 0 \\ 0 & \phi(22) & \phi(23) & 0 \\ \phi(13) & 0 & \phi(33) & \phi(34) \\ \phi(14) & \phi(24) & 0 & \phi(44) \end{bmatrix}.$$

This provides us with a just identified system which uniquely determines $\Phi(0)$.

D. *Empirical Results*

We estimate two versions of our four dimensional structural VAR. In one VAR, Canada plays the role of the domestic country and the U.S. is the foreign country. The other VAR reverses these roles. The sample period is 1960:1 to 1991:4. Output and price series are the same as in section III and the monetary aggregate is the per capita monetary base. First order VARs are estimated in each case.[23]

Before we proceed to the the results of the Blanchard and Quah decomposition, we report multivariate SCCF tests computed with the reduced form VARs. These results appear in table 3. The top panel of table 3 contains the SCCF when Canada is the domestic country and the results for the U.S. information set appear in the bottom panel. Engle and Kozicki (1993) show that Granger causality is a necessary condition for SCCF in a multivariate information set. However, as for the SCCF tests of section II-2, multivariate SCCF imposes weak restrictions on $\Delta\mathbf{X}(t)$. The statistics which we report in table 3 are F-tests with confidence levels in parentheses.

The SCCF tests of table 3 indicate several disparities between the Canadian and U.S. information sets. However, Canadian and U.S. output growth continue to share SCCF. One unique outcome across the information sets is that U.S. output growth share SCCF with real balance growth in each information set. This is not found for Canadian output growth. Something else worth noting is that relative the inflation rates and Canadian output growth possess SCCF across each information set. Finally, growth in U.S. real balances and relative inflation rates share SCCF. Although the SCCF tests impose weak restrictions on each information sets, it appears that nominal (money) demand shares common cyclical properties with U.S. output growth independent of the information set.

[23] The AIC is minimized at lag length one in each VAR. A likelihood ratio test cannot reject the one lag specification when Canada is the domestic country. When the U.S. is the domestic country, the likelihood ratio test rejects the first order VAR in favor of a second order VAR. To maintain symmetry across the two information sets, we estimate first order VARs for each information set.

The Blanchard and Quah (1989) decomposition provides the computational algorithm for our structural VARs. Structural IRFs of the levels of the variables of X(t) appear in figure 3 when Canada is the domestic country and figure 5 when the U.S is the domestic country. Figure 4 contains the IRFs of $\Delta\mathbf{X}(t)$ when Canada is the domestic country. When the U.S. is the domestic country, these figures appear in figure 6. For each information set, the response to the domestic technology shock and the world technology shock appears in the top left and right hand side panels of figures 3 through 6. Responses to the foreign country technology shock and the relative money growth shock appear in the bottom left and right hand side panels of these figures, respectively.

Across either the levels or the growth rate responses to the four structural disturbances, we observe three important asymmetries between the Canadian and U.S. information sets. These differences are

(i) the response of variables in the U.S. information set to the structural disturbances is more persistent than the response of variables in the Canadian information set,

(ii) relative prices respond positively (negatively) in the short and long run in the Canadian (U.S.) information set to the world technology shock,

(iii) at impact Canadian and U.S. real balances rise and in the long run are higher in response to the world shock, however, the response of Canadian real balances is much larger.

Otherwise, the IRFs appear to be similar across the two information sets.

One question the IRFs cannot answer is which shocks are most important. Forecast error variance decompositions (FEVD) are able to answer this question. Results of FEVD of the structural model appear in tables 4a and 4b for the Canadian and U.S. information, respectively. We report standard errors for the FEVD generated with 1000

Monte Carlo replications.

One of the two striking feature of tables 4a and 4b are the differences in the response of domestic output across to the structural shocks across the information sets. In the Canadian information set, the domestic technology shock matters most for domestic output in the short run. The world technology shock matters for Canadian output over short horizons, but to a lesser extent. In the long run, this split is reversed. The split is 2/3 to 1/3 in favor of the world technology in the long run. The other two disturbances have little economic significance for Canadian output throughout the forecast horizon.

For the U.S. information set, domestic output depends most on the world technology shock followed by the relative money growth shock and then the domestic technology shock. By two years, the relative money growth shock has a small but statistically significant impact on U.S. output in this information set. However, the world and U.S. technology shocks continue to have an important effect on U.S. output. Unlike the Canadian information set, the split between these two disturbances is about 50-50 in the long run for the U.S. information set.

The other striking feature is the response of real balances to the structural shocks across the two information sets. Canadian real balances respond only to two of the structural shocks at all forecasting horizons. Relative money growth shocks have the most forecasting power for Canadian real balances from impact throughout the horizon. The world technology shock exhibits forecasting power for Canadian real balances as well. However, the Canadian technology shock appears to have no economic or statistical implication for Canadian real balances at any forecasting horizon.

Real balances in the U.S. are driven by the relative money shock and the U.S technology shock. The Canadian technology has forecasting power for U.S. real balances, however, this information disappears in less than one year. Relative money growth shocks are most important for forecasting real balances at all horizons.

The response of foreign output and relative prices in each information set is symmetric. That is U.S. output has similar responses to the structural shocks in the Canadian information set as does Canadian output in the U.S. information set. The world technology shock is the only shock which matters for foreign output at all forecasting horizons. Likewise, the foreign technology shock is the only shock which matters for relative prices at all forecasting horizons.

Although across the information set foreign output responds symmetrically, the responses of Canadian and U.S. output to the structural shocks is conditional on the information set. That is the results found in figures 3 through 6 and tables 4a and 4b may not be robust. The responses of Canadian and U.S. output to the structural disturbances are conditional on which country fills the role of the domestic country.

To investigate this problem further, we compute tests for structural breaks in the 20 parameters of the reduced form VARs we estimate. We use the tests for structural breaks with unknown change point of Andrews (1993). In particular, the Sup Wald test Andrews constructs is computed over the trimmed sample of 1970:2 through 1979:3. This trimmed sample is chosen because it contains the shift in exchange rate regime of early 1973 as well as the first change in the oil market regime of 1973-1974.[24] In this way, we allow for anticipation of regime change and for delayed response to regime change.

The null hypothesis of the structural break test with unknown change point is equality of the parameters across the first and second subsamples. The results of these tests yield a value of 86.4 for the Sup Wald statistic at 1976:2 with 20 degrees of freedom on the Canadian information set. For the U.S. information set, the Sup Wald

[24] The trimmed sample is equivalent to π = [0.35, 0.65]. The Sup Wald test we use is found in equation 4.1 of Andrews (1993). The Wald test is computed at each observation of the trimmed sample. Over the trimmed sample, the relevant test statistic is the maximal Wald statistic. This statistic has a nonstandard distribution. Andrews provides critical values.

statistic equals 43.5 at 1978:2. Andrews reports critical values of 45.3 at the one percent level and 39.2 at the five percent level. At the midpoint of 1975:1 (change point known), the Wald statistics are 67.3 for the Canadian information set and 28.1 for the U.S. information set. In this case, the statistics are distributed χ^2 with 20 degrees of freedom. The significance levels are 0.0 percent for the Canadian information set and 10.7 percent for the U.S. information set, respectively. These tests point to strong evidence of a structural break in the Canadian information set. For the U.S. information set, the evidence of a structural break is weaker. Compared to the evidence on VAR parameter invariance reported by Kuszczak and Murray (1987), we find significant evidence against VAR parameter invariance across regimes for the Canadian information set. For the U.S, our evidence for parameter invariance is weaker than that reported by Kuszczak and Murray (1987) and Ahmed, Ickes, Wang, Yoo (1993), but is not conclusive in rejecting the null.

V. Conclusion

This paper is an attempt to understand the similarities and differences in aggregate fluctuations in Canada and the U.S.. Our method for studying this issue is to estimate empirical models where the identifying restrictions are provided by simple theoretical models. Most broadly, we find that aggregate fluctuations in Canada differ from those in the U.S. along three dimensions. First, in Canada, aggregate fluctuations are more sensitive to foreign shocks. Second, nominal shocks do not seem to be important for the aggregate Canadian economy as these shocks are for the U.S.. Lastly, Canada appears not only to be more sensitive to foreign shocks, but also Canada is more sensitive to regime shifts which originate abroad.

The fact that the Canadian economy responds more to external shocks than the U.S.

economy does should surprise no one. Hence, this asymmetry between Canada and the U.S. fits within the accepted view of aggregate fluctuations in Canada. Moreover, as other researchers have found the U.S. is affected to a greater extent by external shocks than a traditional (almost) closed economy analysis suggests. For example, in the long run, U.S. output depends almost equally on domestic and world technology shocks. // Para Where our results might surprise some is that we find little evidence that nominal shocks matter for Canadian output. Under our decomposition, nominal shocks matter in an economically meaningful way for U.S. output at horizons of two years and less.

Another somewhat unexpected ~~response~~ result is that Canadian real balances appear to be unaffected by the Canadian technology shock. The forecast error variance decomposition we compute indicates that only the relative money shock and the world technology shock matter for Canadian real balances at all forecast horizons. For U.S. real balances, the U.S. technology shock replaces the world technology shock, in this decomposition.

Taken literally, the results generated by the open economy structural VAR suggest that Canada is an open economy in which only domestic and external technology shocks matter for output. Conditional on the information set we study, the real and nominal sides of the Canadian economy are independent. Nominal shocks appear only to drive real balances. ~~Put another way, our decomposition finds that nominal shocks drive movements in real balances~~ but no other aggregate variable. Our decomposition of the U.S. information set reveals that nominal shocks matter for output at short horizons.

One problem to which we point, but about which we do nothing, is that there is evidence that regime shifts caused by foreign events have a major impact on the aggregate Canadian economy. Once again, this should catch no one off guard. Along with the results from our open economy structural VARs, the evidence on regime shifts suggests that our empirical model for Canadian is not robust. The evidence for the U.S. is a bit more mixed.

Where else is this discussed in the [illegible]

Although our results may appear to lack robustness, we do draw some lessons from our work. However, we pose our lessons as questions for future work for researchers and headaches for the policy analyst. First, how robust is the asymmetry of output and nominal shock across the Canada and the U.S. to alterations in the information of the open economy model? Second, is the evidence of the independence of Canadian output or nominal shocks an example of holding a mirror up to the monetary policy maker? Underlying this questions is an attempt for the policy maker to ask herself if the empirical results of this paper reveal her 'footprints in the sand'. That is have we uncovered, in some way, the policy actions of the monetary policy maker. Third, what are the underlying sources of regime shifts in Canada, is there some way to optimally estimate the date of break, and to develop an economically interesting model which explains the regime shift? Finally, given the evidence we present for at least one recent regime shift in the aggregate Canadian economy, when does the policy maker know she is on top of one? And even if the policy maker knows when she is being buffeted by a regime shift, how does she know its sources? In this case, we think discussions between researchers, policy analysts, and policy makers are welfare improving events.

Obviously, we have no answers, yet, for these questions. Nonetheless, we hope our paper stimulates others to reconsider what makes aggregate fluctuations in Canada different.

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Table 1. **Unit Root and Cointegration Tests of Output**

Unit Root Tests

	ADF(1)	ADF(2)	ADF(3)	Z(*t*)	Z(*b*)
Canadian Y	-0.273	-0.435	-0.880	-1.193	-0.966
U.S. Y	-2.273	-2.571	-2.504	-1.482	-8.512

Cointegration Tests

Dependent Variable	ADF(1)	ADF(2)	ADF(3)	Z(*t*)	Z(*b*)
Canadian Y	-2.493	-2.238	-2.328	-1.550	-4.479
U.S. Y	-2.742	-2.427	-2.549	-1.792	-5.760

Gregory and Hansen Structural Break Tests

Dependent Variable	ADF(1)	ADF(2)	ADF(3)	Z(*t*)	Z(*b*)
Canadian Y	-3.384	-3.626	-3.762	-2.262	-24.513
U.S. Y	-3.038	-3.369	-3.392	-2.136	-21.119

Note: Perron (1988) reports asymptotic critical values for the ADF and Z(*t*) unit root tests of -3.41 and -3.12 at the 5 and 10 percent levels, respectively. The same values for the Z(*b*) unit root test equal -21.8 and -18.3. The ADF and Z(*t*) cointegration tests' asymptotic critical values are -3.78 and -3.50 at the 5 and 10 percent levels as MacKinnon (1991) reports. For the Z(*b*) cointegration test, the values are from Phillips and Ouliaris (1990) and equal -27.09 and 23.19 at the 5 and 10 percent levels. Gregory and Hansen (1992) report asymptotic critical values for the ADF and Z(*t*) structural break tests of -4.95 and -4.68 at the 5 and 10 percent levels. The same values for the Z(*b*) structural break test equal -47.04 and -41.85.

Table 2. Lagrange Multiplier Tests of Canadian Output-Inflation IRFs

No. of Lags	YT	PT	YM	PM
4	11.065	132.854	11.669	18.266
	(0.050)	(0.000)	(0.040)	(0.003)
	[0.074]	[0.000]	[0.059]	[0.022]
8	17.165	173.519	14.302	23.654
	(0.046)	(0.000)	(0.112)	(0.005)
	[0.108]	[0.001]	[0.119]	[0.052]
12	18.252	212.772	15.133	25.652
	(0.148)	(0.000)	(0.299)	(0.019)
	[0.133]	[0.004]	[0.152]	[0.084]
16	21.030	237.504	16.571	35.519
	(0.225)	(0.000)	(0.484)	(0.005)
	[0.146]	[0.007]	[0.170]	[0.075]

Note: The first number in each stack is the LM test statistic. The null hypothesis is that the Canadian data is generated by the DGP of the U.S. economy. The second and third numbers in each stack are the asymptotic Chi-square probability value and the probability value from the empirical distribution of the LM statistic, respectively. The test has degrees of freedom equal to one plus the lag length of the IRF. For example, with four lags for the IRFs, there are five degrees of freedom. The probability value from the empirical distribution of the LM statistic equals the fraction of artificial samples in which the sample LM statistic is larger than the theoretical LM statistic.

Table 3. Multivariate Serial Correlation Common Features Tests

Dependent Variable:	Canadian ΔY	U.S. ΔY	ΔRP	Canadian $\Delta M/P$
Independent Variable				
Canadian ΔY	0.124	0.171	8.546	0.626
	(0.726)	(0.680)	(0.004)	(0.430)
U.S. ΔY	13.566	5.433	2.499	0.010
	(0.000)	(0.021)	(0.116)	(0.919)
ΔRP	1.984	1.560	7.880	0.003
	(0.161)	(0.214)	(0.006)	(0.956)
Canadian $\Delta M/P$	4.026	11.840	0.083	7.438
	(0.047)	(0.001)	(0.774)	(0.070)

Dependent Variable:	U.S. ΔY	Canadian ΔY	ΔRP	U.S. $\Delta M/P$
Independent Variable				
U.S. ΔY	3.324	13.583	1.315	0.844
	(0.071)	(0.000)	(0.254)	(0.360)
Canadian ΔY	1.808	0.538	7.755	0.163
	(0.181)	(0.465)	(0.006)	(0.687)
ΔRP	0.017	0.645	10.455	11.748
	(0.896)	(0.423)	(0.002)	(0.001)
Canadian $\Delta M/P$	14.610	0.302	3.294	70.889
	(0.000)	(0.584)	(0.072)	(0.000)

Note: The multivariate common features test is the Granger causality test. The null hypothesis of no causality is equivalent to no common features. The alternative of (at least one way) causality is equivalent to at least one common serial correlation feature. The F-test statistic is computed after estimating a second order VAR. Significance levels are in parenthesis.

Table 4b. Forecast Error Variance Decomposition

Quarters	U.S. Tech.	World Tech.	CDN Tech	Relative Money
	Percentage of Variance of U.S. Output Explained by:			
1	20.6 (11.1)	42.6 (12.1)	0.0 (2.3)	36.8 (14.3)
2	23.2 (11.9)	51.6 (10.9)	0.5 (1.0)	24.7 (8.0)
4	29.4 (12.8)	56.8 (12.3)	0.6 (0.4)	13.2 (2.4)
8	37.5 (13.5)	56.2 (13.4)	0.3 (0.3)	5.9 (0.5)
12	41.3 (13.6)	54.8 (13.5)	0.2 (0.3)	3.7 (0.6)
16	43.2 (13.7)	53.9 (13.6)	0.1 (0.3)	2.7 (0.7)
32	46.0 (13.7)	52.7 (13.6)	0.1 (0.3)	1.3 (0.7)

Quarters	U.S. Tech.	World Tech.	CDN Tech	Relative Money
	Percentage of Variance of Canadian Output Explained by:			
1	17.5 (8.4)	81.7 (15.2)	0.5 (11.2)	0.3 (3.7)
2	9.5 (4.3)	87.7 (10.6)	0.2 (6.9)	2.5 (4.1)
4	4.6 (1.5)	92.9 (7.6)	0.2 (7.1)	2.2 (1.3)
8	2.1 (0.2)	96.5 (7.6)	0.2 (7.5)	1.2 (0.3)
12	1.4 (0.2)	97.7 (7.7)	0.1 (7.5)	0.7 (0.4)
16	1.0 (0.2)	98.3 (7.8)	0.1 (7.5)	0.6 (0.4)
32	0.5 (0.2)	99.2 (7.8)	0.1 (7.5)	0.3 (0.4)

Quarters	U.S. Tech.	World Tech.	CDN Tech	Relative Money
	Percentage of Variance of Relative Prices Explained by:			
1	0.1 (10.2)	4.8 (6.0)	94.9 (17.2)	0.2 (7.8)
2	1.7 (12.8)	2.8 (2.5)	95.0 (15.9)	0.4 (8.4)
4	4.0 (15.8)	1.4 (0.5)	91.9 (17.0)	2.8 (11.6)
8	5.9 (17.3)	0.6 (0.2)	87.7 (18.3)	5.8 (13.0)
12	6.6 (17.6)	0.4 (0.2)	86.0 (18.7)	7.1 (13.2)
16	6.9 (17.6)	0.3 (0.2)	85.2 (18.8)	7.7 (13.2)
32	7.3 (17.6)	0.1 (0.2)	84.1 (18.8)	8.5 (13.2)

Quarters	U.S. Tech.	World Tech.	CDN Tech	Relative Money
	Percentage of Variance of U.S. Real Balances Explained by:			
1	31.1 (16.2)	7.2 (11.0)	16.8 (8.2)	44.9 (15.6)
2	31.5 (14.5)	7.0 (10.4)	9.1 (3.4)	52.4 (14.8)
4	33.4 (14.3)	5.1 (8.7)	3.6 (0.6)	57.9 (15.0)
8	34.0 (14.4)	3.1 (7.6)	1.2 (0.3)	61.6 (15.2)
12	34.1 (14.4)	2.4 (7.4)	0.7 (0.3)	62.8 (15.2)
16	34.1 (14.4)	2.0 (7.4)	0.5 (0.3)	63.3 (15.2)
32	34.1 (14.4)	1.6 (7.4)	0.2 (0.3)	64.0 (15.2)

Note: Monte Carlo standard errors appear in parentheses.

Figure 1a: Response to Technology Shock, Canada

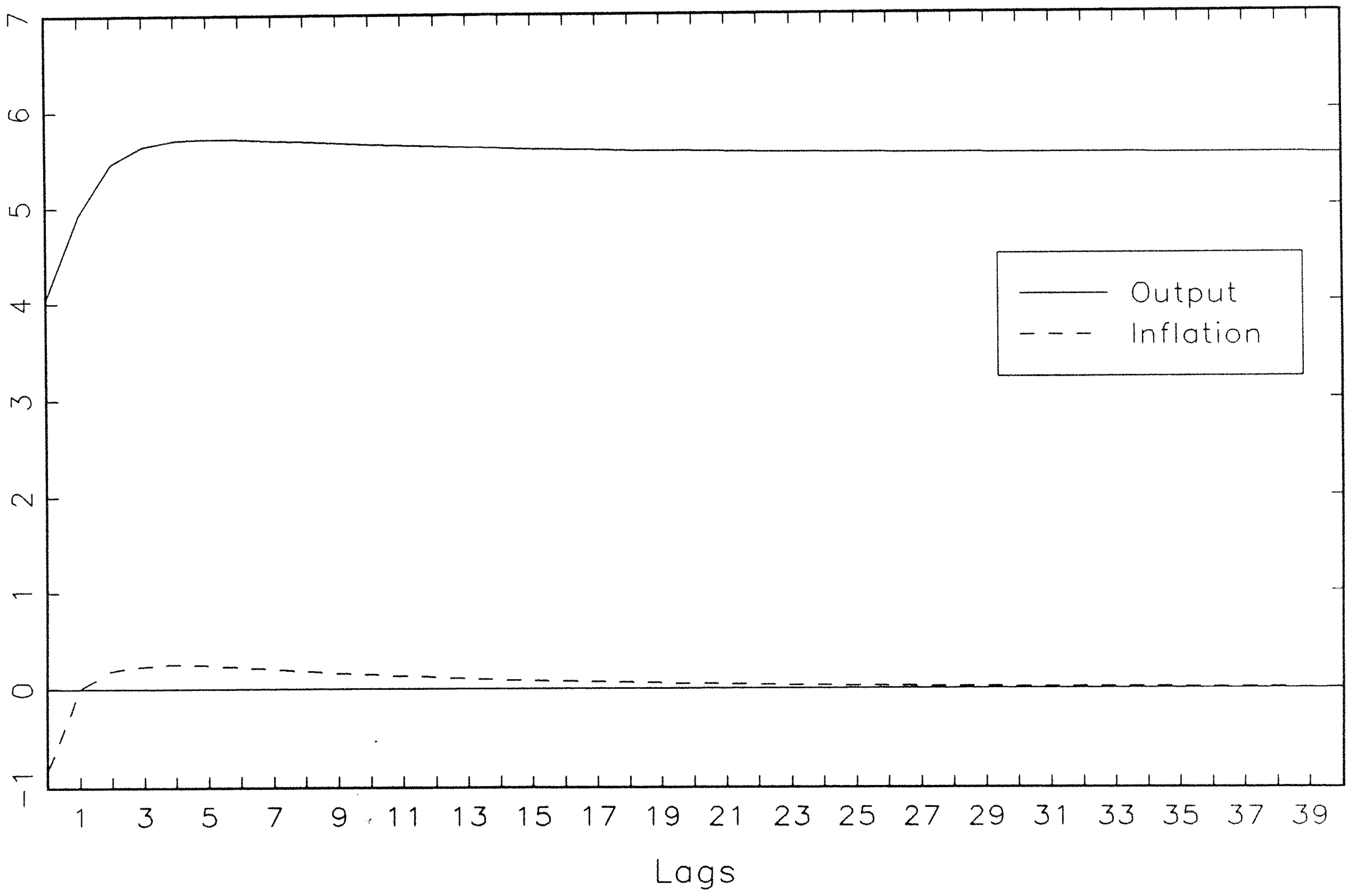


Figure 1b: Response to Money Shock, Canada

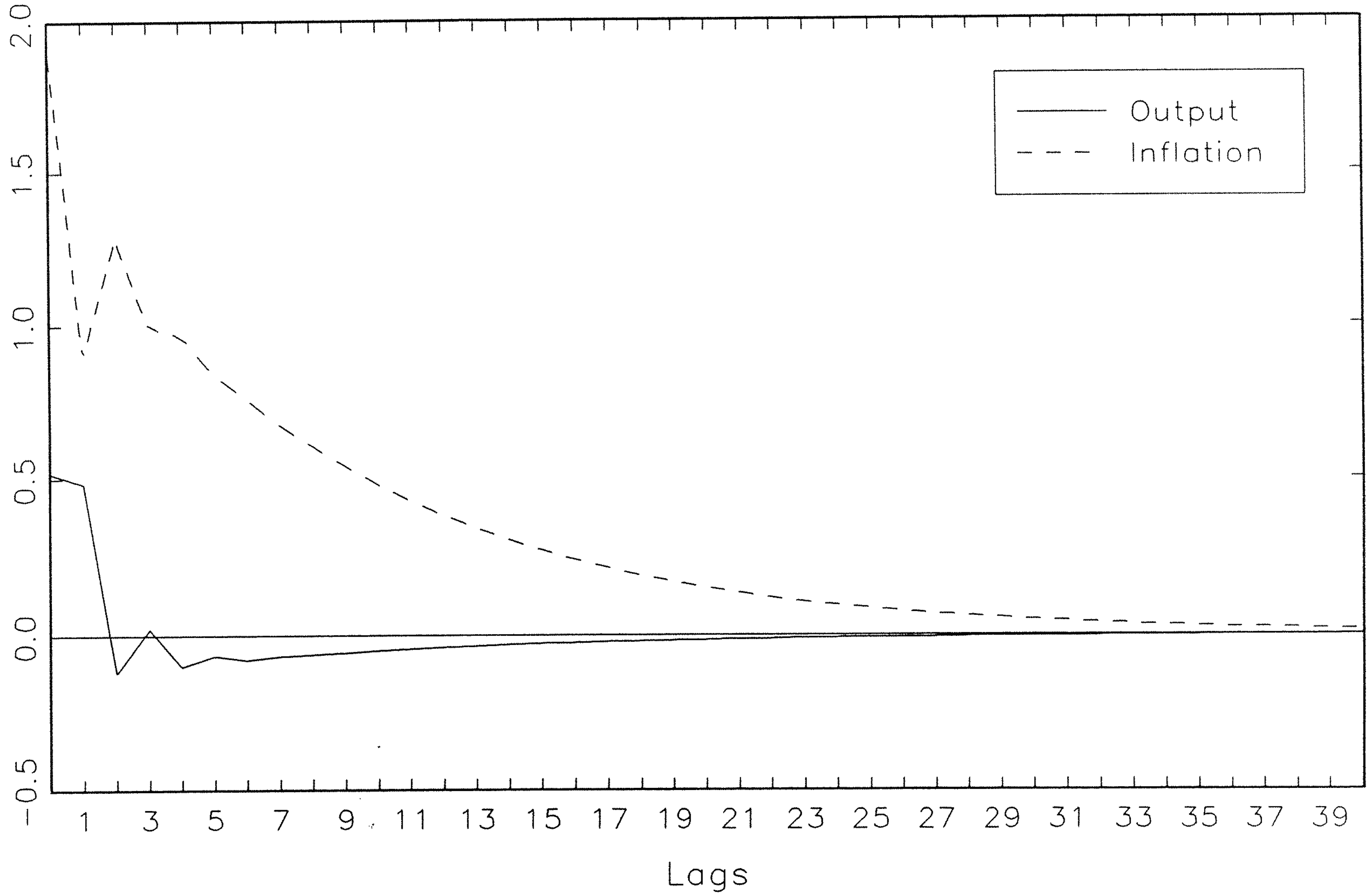


Figure 2a: Response to Technology Shock, U.S.

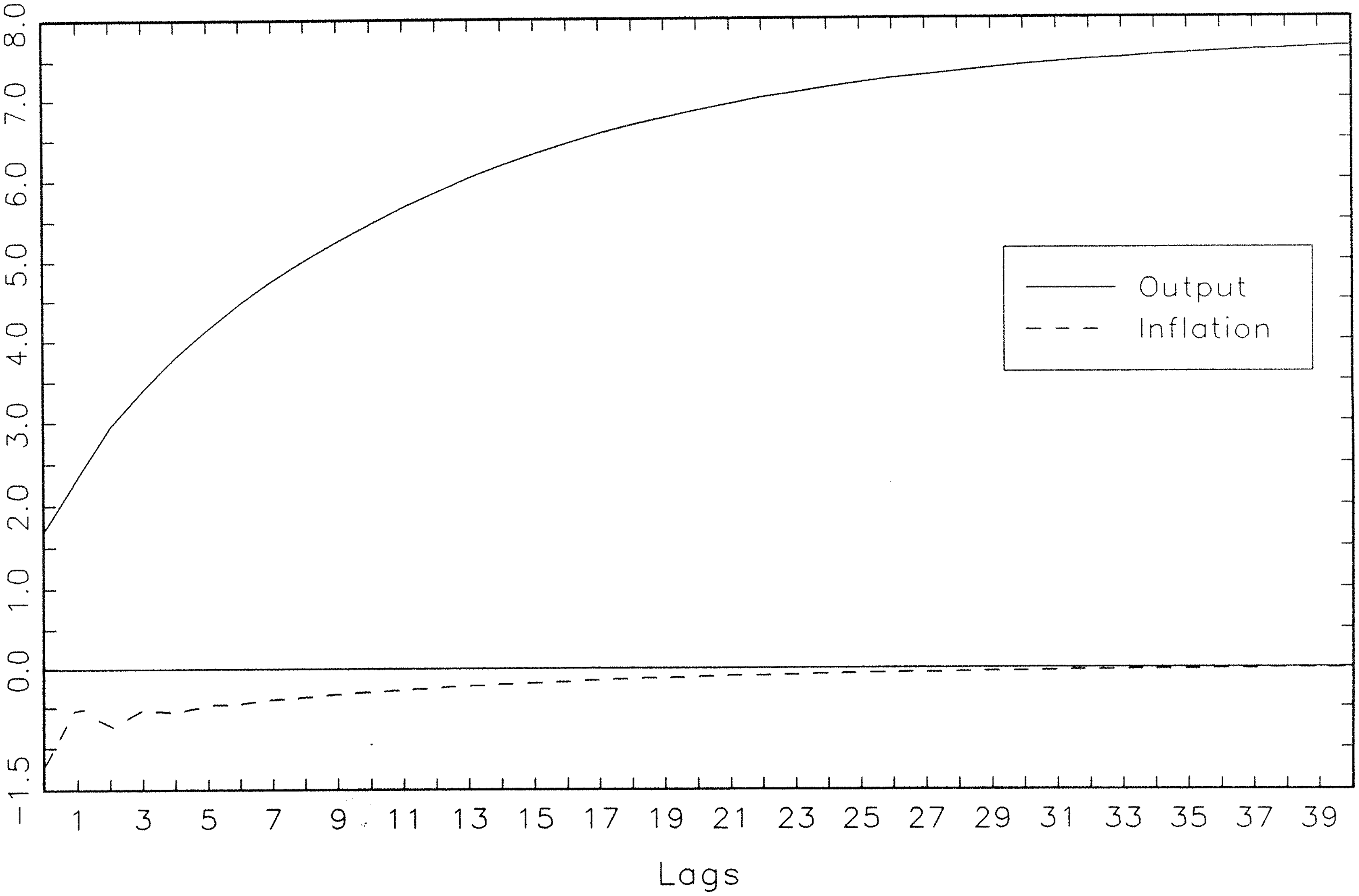


Figure 2b: Response to Money Shock, U.S.

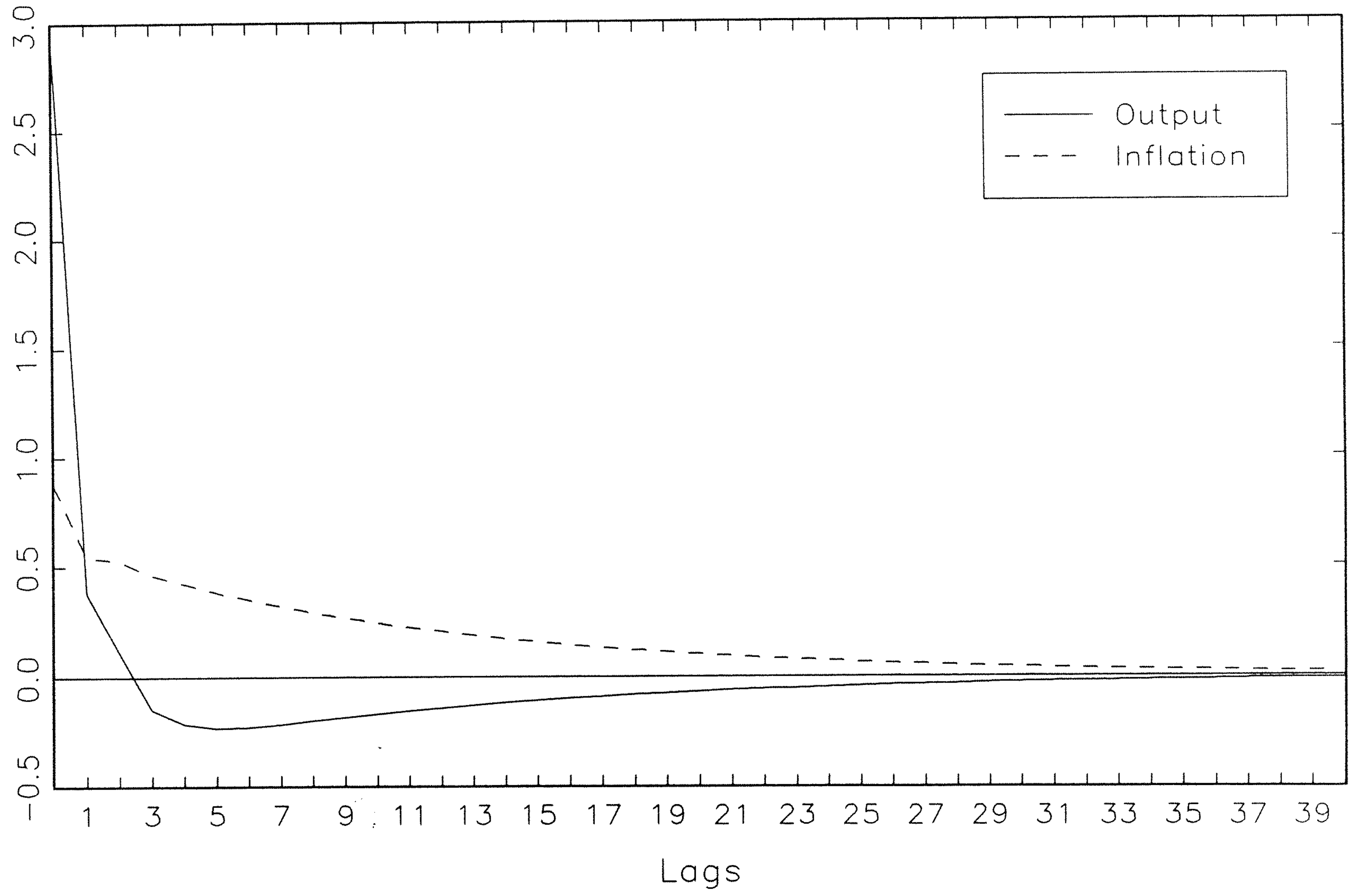


Figure 3: Impulse Responses of Levels, Canada

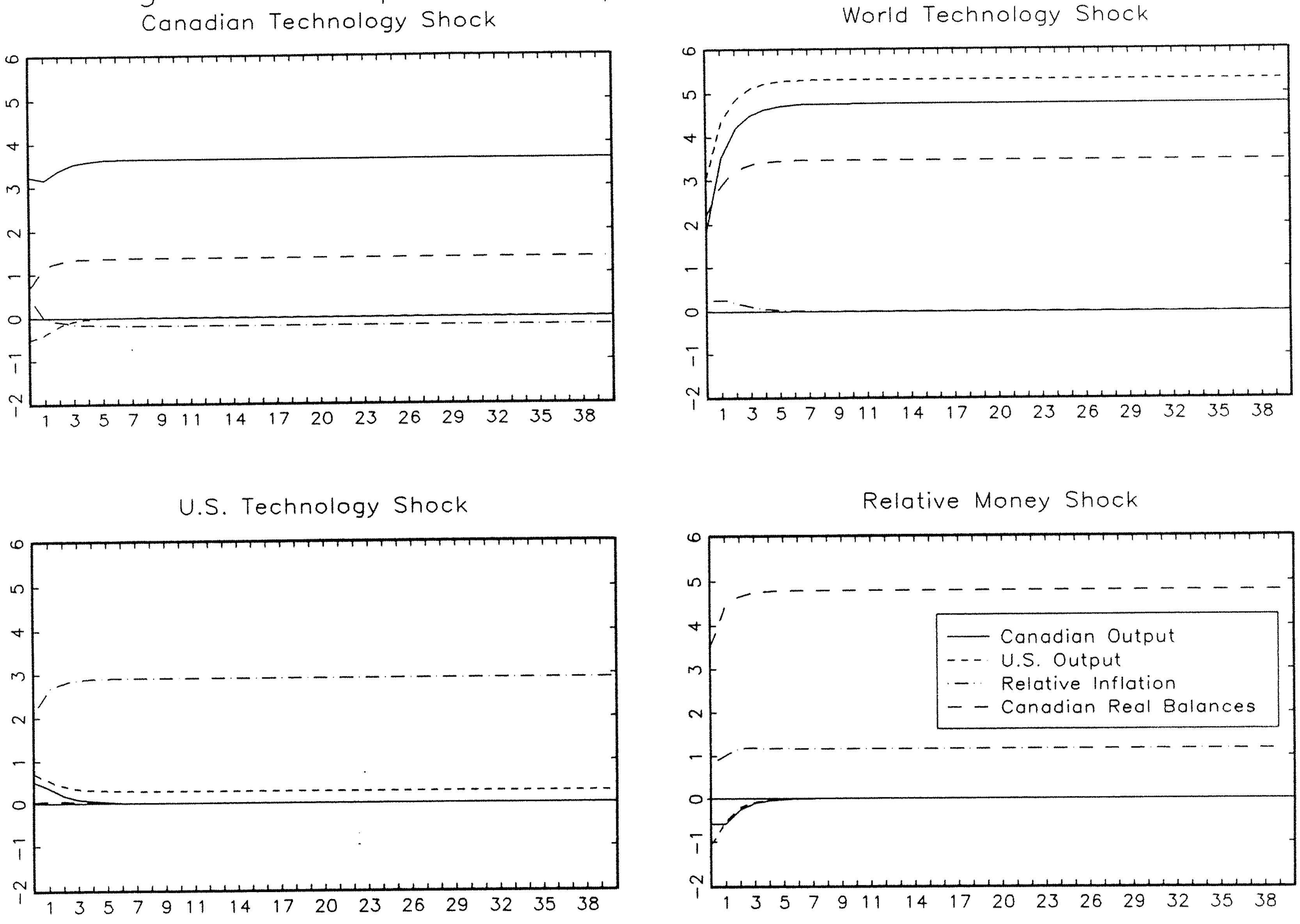


Figure 4: Impulse Responses of Growth Rates, Canada

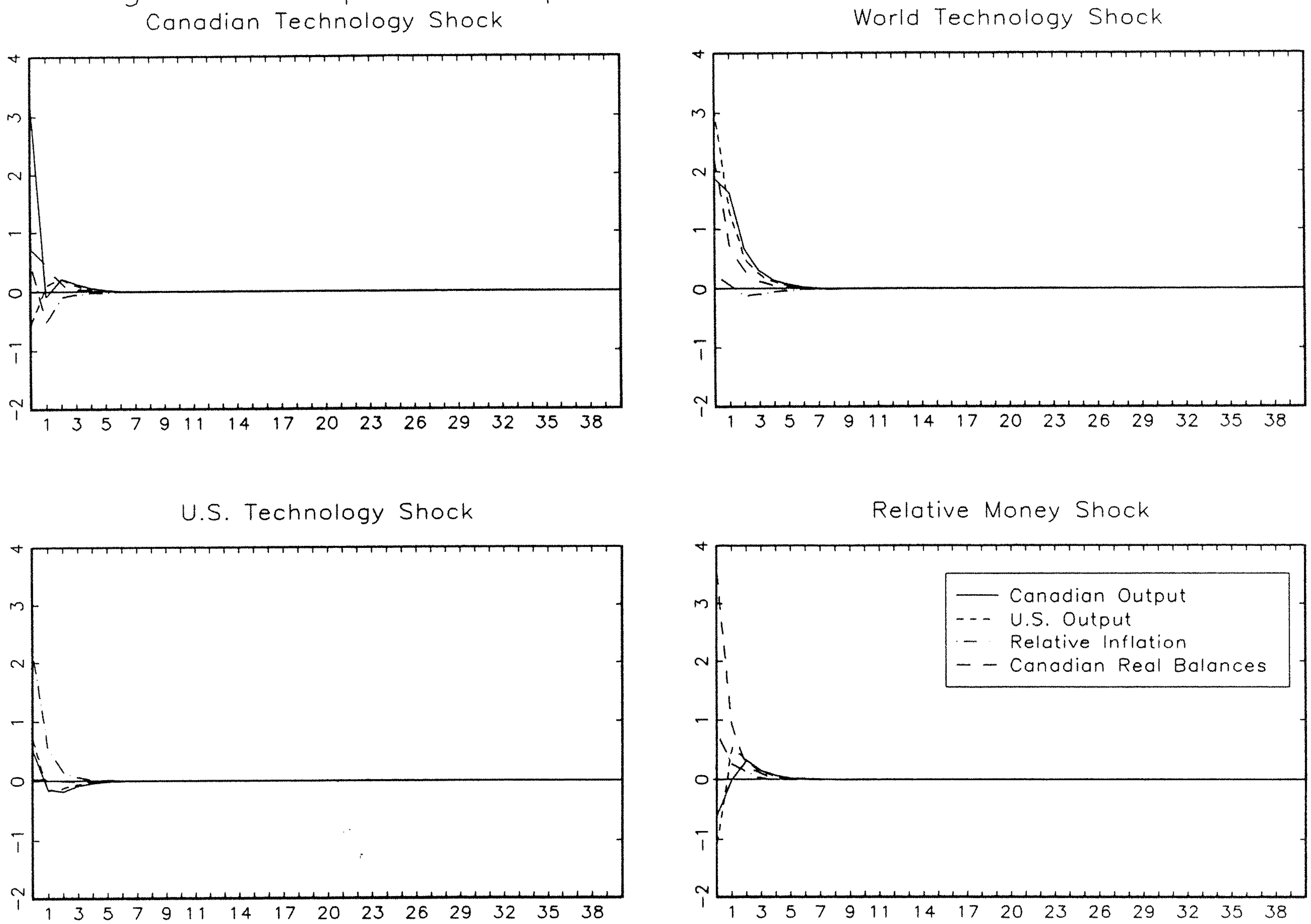


Figure 5: Impulse Responses of Levels, U.S.

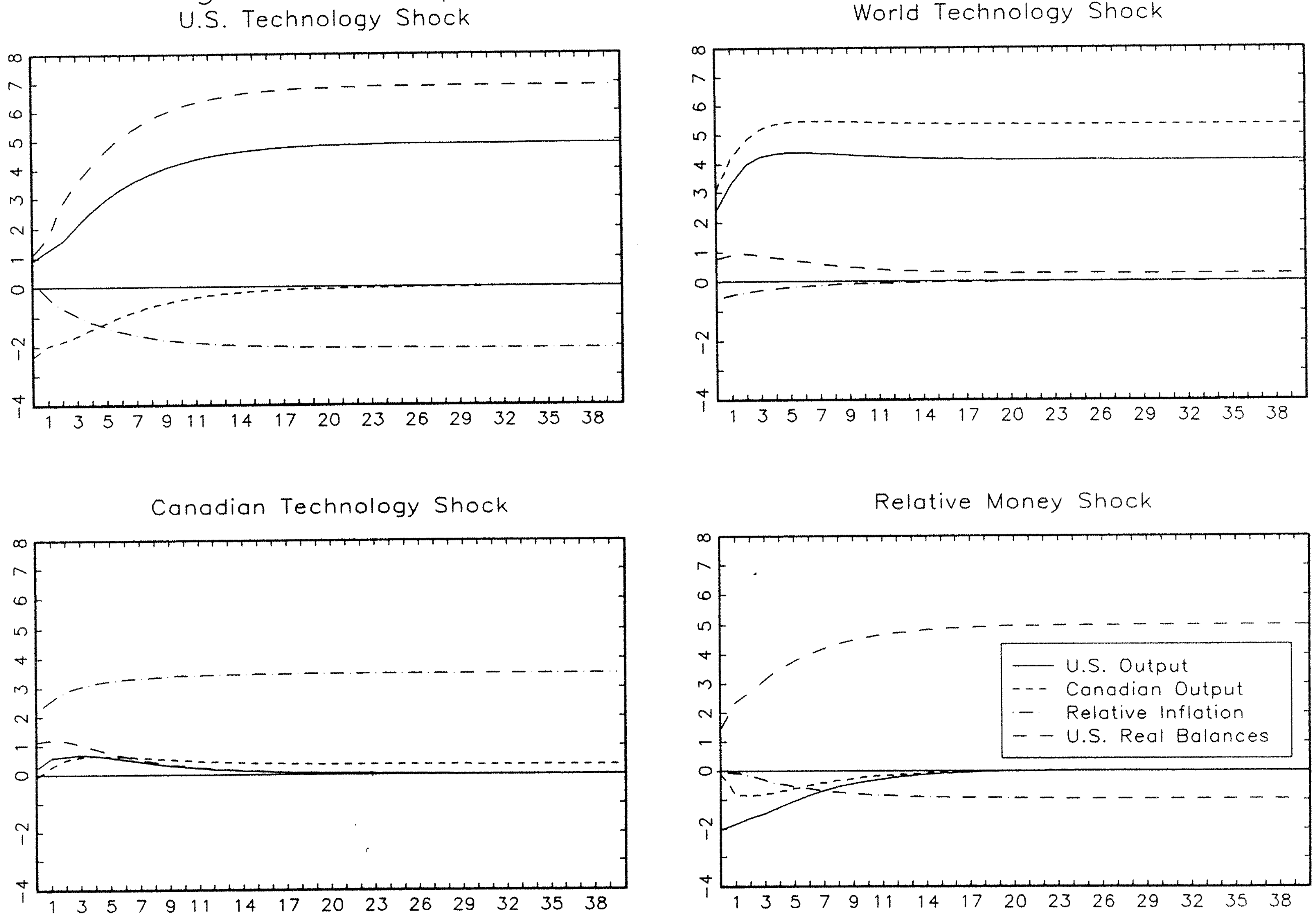
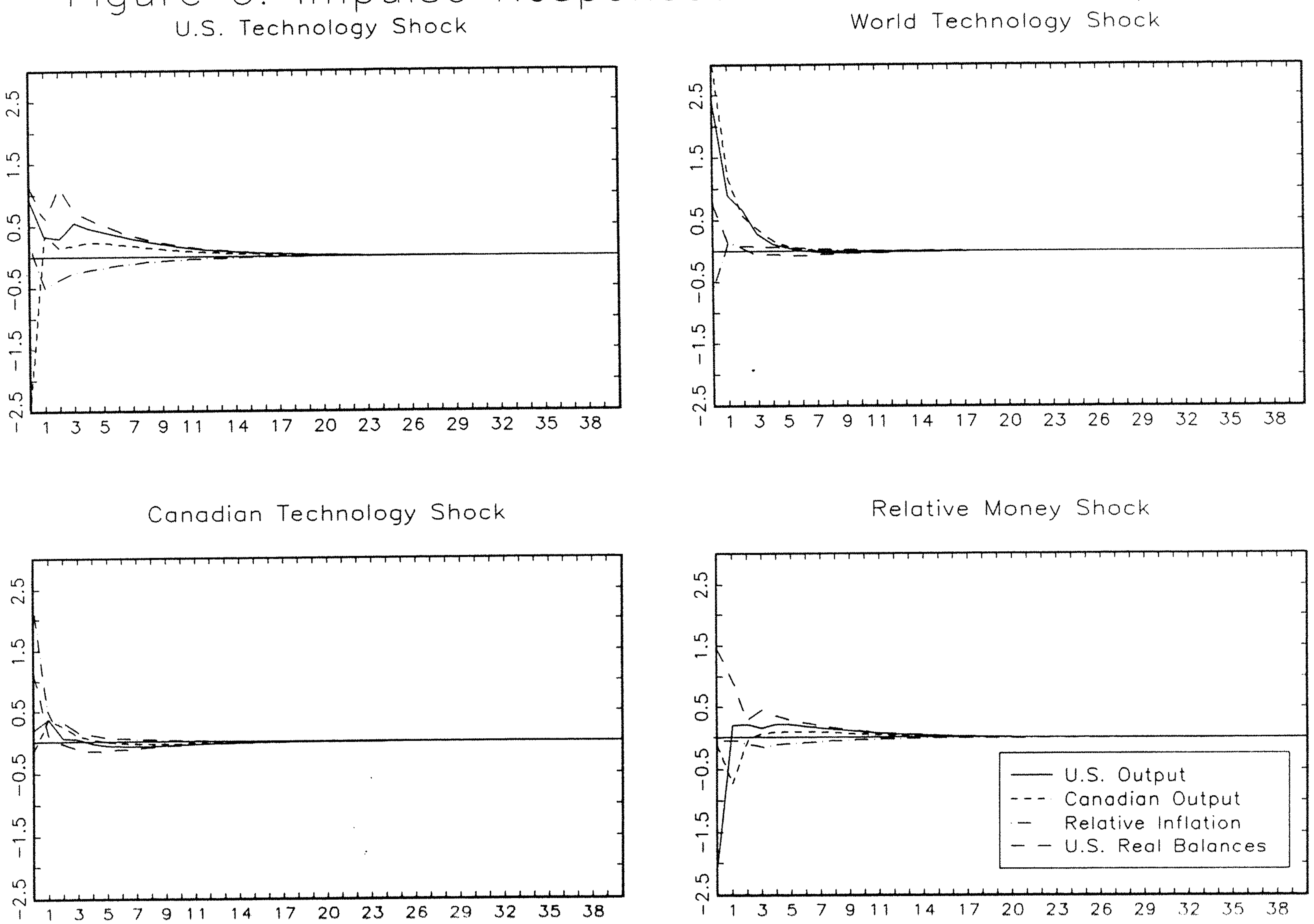


Figure 6: Impulse Responses of Growth Rates, U.S.



Technical Appendix

1. Structural VAR Decomposition

The decomposition of the vector autoregressions (VARs) follows Blanchard and Quah (1989) and draws heavily on computer programs provided by Tim Cogley. Since the monetary business cycle models predict structural VARs that are just identified, the Blanchard and Quah decomposition provides the most computationally efficient method. We outline the procedure for the bivariate output-inflation information set.

The construction of the sample impulse response functions (IRFs) use second order VARs for each system. After estimation of the VAR, reduced form IRFs of length 128 [the sample size] are computed for the integrated variable of the system in response to $\mathbf{e}(t)$. These IRFs are summed for all 128 steps and used as numerical approximation of $\lambda(11)$ and $\lambda(12)$. The estimates of $\sigma(e1)^2$, $\sigma(e2)^2$, and $\sigma(e12)$ from the VAR serves as the sample analog of Ω. This gives the nonlinear system of four equations in four unknowns

$$\phi(11,0)^2 + \phi(12,0)^2 - \sigma(e1)^2 = 0,$$

$$\phi(21,0)^2 + \phi(22,0)^2 - \sigma(e2)^2 = 0,$$

$$\phi(11,0)\phi(21,0) + \phi(12,0)\phi(22,0) - \sigma(e12) = 0,$$

$$\lambda(11)\phi(12,0) + \lambda(12)\phi(22,0) = \phi(12) = 0.$$

This is solved for the elements of $\Phi(0)$ using a Newton-Raphson algorithm; see Press, Flannery, Teukolsky, and Vettering (1986). Given $\Phi(0)$, the sample structural IRFs can be computed.

2. Unit Root Tests

Three unit root tests are applied to Canadian and U.S. output series. The ADF t-ratio test is taken from the regression

$$\Delta x(t) = a + \mu t + [1-b(0)]x(t-1) + \sum_{j=1}^{n} b(j)\Delta x(t-j) + u(t),$$

where n is chosen so that u(t) is white noise under the null of x(t) possessing a unit root. The ADF test statistic is the t-statistic of $[1-b(0)]$ and under the null it equals zero. We compute the ADF t-ratio setting n = 1, 2, and 3.

The other two unit root tests use the regression

$$x(t) = a + \mu t + bx(t-1) + u(t),$$

to construct the

$$Z(t) = [\sigma(u)/\sigma(S)]t(b) - [\sigma^2(u) - \sigma^2(S)][T^3/4\sqrt{3\left|X'X\right|}\sigma(S)],$$

and

$$Z(b) = T[b - 1] - [\sigma^2(u) - \sigma^2(S)][T^6/24\left|X'X\right|]$$

of Phillips (1987) and Phillips and Perron (1988) where $\sigma^2(u) = E\{u^2(t)\}$, $t(b)$ is the t-ratio of b, T is sample size, and $X(t) = [1 \ t \ x(t-1)]$. The long run covariance matrix of u(t), $\sigma^2(S)$, is estimated using the quadratic spectral (QS) kernel discussed in Andrews (1990). To set the lag length of the QS kernel, we employ the Newey and West (1991) automatic bandwidth rule.

3. Cointegration Tests

The cointegration tests we compute parallel the unit roots. The test statistics are the Dickey-Fuller (DF) t-ratio, the $Z(t)$, and the $Z(b)$ of Phillips and Perron. To test for a cointegrating relation between Canadian and U.S. output, we use a two step procedure. First, the cointegrating relation

$$y(1,t) = a(0) + a(1)y(2,t) + v(t)$$

is estimated by least squares. Then either a DF regression

$$\Delta v(t) = [1-b(0)]v(t-1) + \sum_{j=1}^{n} b(j)\Delta v(t-j) + u(t),$$

or the regression

$$v(t) = bv(t-1) + u(t),$$

are estimated by least squares. The DF test statistic is the t-ratio of $[1-b(0)]$. The number of lags of $\Delta v(t)$ is set so that u(t) is white noise under the null. The DF t-ratio is generated with n = 1, 2, and 3. The $Z(t)$ and $Z(b)$ tests statistics are computed as

$$Z(t) = [b - 1][\sum_{t=2}^{T} v^2(t-1)/\sigma^2(S)]^{1/2} - 0.5[\sigma^2(u) - \sigma^2(S)][\sigma^2(S)T^{-2} \sum_{t=2}^{T} v^2(t-1)]^{-1/2},$$

and

$$Z(b) = T[b - 1] - 0.5[\sigma^2(u) - \sigma^2(S)][1/T^{-2} \sum_{t=2}^{T} v^2(t-1)]$$

To compute $\sigma^2(S)$, we use the QS kernel with the Newey-West automatic bandwidth rule.

4. Cointegration Tests in the Presence of Regime Shifts

The null hypothesis is no cointegration versus the alternative of cointegration with a regime shift. To construct ADF, $Z(t)$, and $Z(b)$ statistics to test for a regime shift, estimate the cointegrating relation

$$y(1,t) = a(0) + d(0)\varphi(t,\tau) + a(1)y(2,t) + d(1)y(2,t)\varphi(t,\tau) + v(t),$$

with the level shift parameter $d(0)$, the regime (slope) shift parameter $d(1)$, and the dummy variable

$$\varphi(t,\tau) = \begin{cases} 0, & \text{if } t \leq [T\tau] \\ 1 & \text{if } t > [T\tau]. \end{cases}$$

The parameter τ is unknown as it defines the change point in the cointegrating relation. In this setup, $\tau \in (0, 1)$ and $[\cdot]$ indicates the integer component of $T\tau$.

The Gregory and Hansen tests compute the DF t-ratio, Z(t), and Z(b) statistics at each date τ using the residuals of the first stage regression of the regime shift in the cointegrating relation. It is straightforward to compute the DF t-ratio. However, the Z(t) and Z(b) statistics require a bias correction for the estimate of the first order autoregressive parameter, b, from the regression

$$v(t) = bv(t\text{-}1) + u(t).$$

The bias correction of b removes the dependence of v(t) on u(t)

$$b = \sum_{t=1}^{T-1} [v(t\tau)v(t+1\tau) - \lambda(\tau)] / \sum_{t=1}^{T-1} v^2(t\tau),$$

where $\lambda(\tau)$ is the covariance of $u(t\tau)$. We compute $\lambda(t)$ using the quadratic spectral kernel of Andrews (1990) with the Newey and West (1991) automatic bandwidth rule.