APPENDIX - FOR ONLINE PUBLICATION

Appendix: What do Treasury Bond Risks Say about Supply and Demand Shocks?

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A Phillips Curve Microfoundations

A.1 Final good

A final consumption good is produced by a representative perfectly competitive firm from a continuum of differentiated goods $Y_{i,t}$:

$$Y_t = \left(Y_{i,t}^{\frac{\epsilon_p - 1}{\epsilon_p}} \right)^{\frac{\epsilon_p}{\epsilon_p - 1}}. \tag{A1}$$

The constant $\epsilon_p > 1$ is the elasticity of substitution across intermediate goods. The resulting demand for the differentiated good i is downward-sloping in its product price $P_{i,t}$:

$$Y_{i,t} = Y_t \left(\frac{P_{i,t}}{P_t} \right)^{-\epsilon_p}. \tag{A2}$$

The aggregate price level is given by

$$P_t = \left(\int_0^1 P_{i,t}^{-(\epsilon_p - 1)} di \right)^{-\frac{1}{\epsilon_p - 1}}. \tag{A3}$$

A.2 Intermediate good producers

Intermediate goods firm i produces according to a Cobb-Douglas production function with constant returns to scale

$$Y_{i,t} = A_t N_{i,t}, \tag{A4}$$

where productivity equals A_t and N_t is the supply of the aggregate labor index. Each firm takes the downward-sloping demand schedule as given (A3) and may therefore choose a different amount of the aggregate labor index. With the the final good equation (A1) aggregate output equals

$$Y_t = A_t N_t \tag{A5}$$

where

$$N_t = \int_0^1 N_{i,t} di. \tag{A6}$$

The aggregate resource constraint is simple because there is no real investment and consumption equals output:

$$C_t = Y_t. \tag{A7}$$

Following Lucas (1988) we assume that productivity depends on past skills gained by all agents, and depends on past market labor, n_{t-1}:

$$a_t = \nu + a_{t-1} + (1-\phi)n_{t-1}, \tag{A8}$$

where $0 \leq \phi \leq 1$ and $\nu > 0$ are constants. The assumption (A8) ensures that potential output increases with past output. The process(A8) can equivalently be interpreted as a simple endogenous capital stock, similarly to Woodford (2003, Chapter 5), if a fixed proportion of market labor each period is used to produce investment goods with a constant-returns-to-scale technology, and the total amount of labor is scaled accordingly.

Intermediate firm i' real profit in period t equals

$$Pr_{i,t} = \frac{P_{i,t}}{P_t}Y_{i,t} - \frac{W_t}{P_t}N_{i,t}, \tag{A9}$$

subject to the production function (A4), demand for differentiated goods (A2), and taking the wage W_t as given.

A.3 Employment agency

There is a continuum of monopolistically competitive households, each of which supplies a differentiated labor service, $L_{h,t}$, to the production sector. A representative employment agency aggregates households' labor hours according to a CES production technology with elasticity of substitution $\epsilon_w > 1$:

$$N_t = \left(\int_0^1 L_{h,t}^{\frac{\epsilon_w - 1}{\epsilon_w}}\right)^{\frac{\epsilon_w}{\epsilon_w - 1}} \tag{A10}$$

The agency produces the aggregate labor index, N_t, taking each household's wage rate, $W_{h,t}$ as given, and then sells it to the production sector at the unit cost W_t. The profit maximization of the employment agency is:

$$max_{L_{h,t}} W_t \left(\int_0^1 L_{h,t}^{\epsilon_w - 1}\epsilon_w\right) - \int_0^1 W_{h,t}L_{h,t}dh, \tag{A11}$$

which yields the following demand schedule for the labor hours of household h:

$$L_{h,t} \;=\; \left(\frac{W_{h,t}}{W_t}\right)^{-\epsilon_w} N_t. \tag{A12}$$

The wage index faced by intermediary producers is then given by

$$W_t \;=\; \left(\int_0^1 W_{h,t}^{1-\epsilon_w}\right)^{\frac{1}{1-\epsilon_w}}. \tag{A13}$$

A.4 Labor-leisure choice

Following the classic model of Greenwood et al. (1988), we assume that total consumption consists of a combination of market consumption and home production, given by:

$$C_{h,t}^{home} = A_t\left(1 - \frac{L_{h,t}^{1+\eta}}{1+\eta}\right) \tag{A14}$$

Home production has decreasing returns to scale as in Campbell and Ludvigson (2001), and the parameter η determines the elasticity of market labor supply. Household h's utility depends on market and home good consumption and the corresponding external habit levels H_t and H_t^{home}:

$$U_{h,t} \;=\; \frac{\left((C_{h,t} - H_t) + \left(C_{h,t}^{home} - H_t^{home}\right)\right)^{1-\gamma} - 1}{1-\gamma} \tag{A15}$$

We assume that home good habits are shaped by the aggregate consumption of home goods, so $H_t^{home} = C_t^{home}$ and in equilibrium home goods drop out of the utility function because all households end up choosing the same labor supply in equilibrium. Home production nonetheless matters for the wage-setting first-order condition, which depends on the marginal change in utility from choosing an off-equilibrium path labor supply. External market habit is described by the surplus consumption dynamics in the main paper.

A.5 Price- and wage-setting

We consider the simplified case with flexible product prices but sticky wages. This is in line with Christiano et al. (1999) who find that sticky wages are much more important for aggregate inflation dynamics than sticky wages. It is also in line with Favilukis and Lin (2016) who find that wage-setting frictions are important to capture pro-cyclical firm profits and that a claim to firm profits behaves similarly to a claim to consumption in an asset pricing sense. Wage-setting frictions take the form of Rotemberg (1982). Specifically, we assume that wage-setters face a quadratic cost if they raise wages faster than past inflation. The indexing to past inflation is analogous to the indexing assumption in Smets and Wouters (2007) and Christiano et al. (2005). The cost of re-setting wages for household h in terms of aggregate output equals

$$Cost^h = \frac{\gamma_w}{2}\left(\frac{W_{h,t}}{W_{h,t-1}}/\frac{W_{t-1}}{W_{t-2}} - 1\right)^2 Y_t. \tag{A16}$$

We assume that wage-setting costs get rebated to households lump-sum, i.e. aggregate consumption is unaffected.

A.6 Profit first-order condition

Because product prices are flexible, intermediate firm i's profit becomes

$$Pr_{i,t} = Y_t\left(\left(\frac{P_{i,t}}{P_t}\right)^{-(\epsilon_p-1)} - \frac{W_t}{P_t A_t}\left(\frac{P_{i,t}}{P_t}\right)^{-\epsilon_p}\right). \tag{A17}$$

Taking the first-order condition with respect to the relative price $\frac{P_{i,t}}{P_t}$ gives

$$\frac{P_{i,t}}{P_t} = \frac{\epsilon_p}{\epsilon_p - 1}\frac{W_t}{P_t A_t}. \tag{A18}$$

Because in equilibrium all firms end up choosing the same price, we have that the real wage equals

$$\frac{W_t}{P_t} = \frac{\epsilon_p - 1}{\epsilon_p}A_t. \tag{A19}$$

This means that due to partially monopolistic competition the real wage is compressed by a constant fraction relative to productivity and equilibrium profits of intermediary i are exactly proportional to aggregate output:

$$Pr_{i,t} = \frac{1}{\epsilon_p}Y_t. \tag{A20}$$

This is good because a consumption claim is the same as a claim to firm profits.

A.7 Wage-setting first-order condition with flexible wages

To derive the wage-setting first-order condition, we first start by understanding what happens if wages are flexible. In this case, the first-order condition equals:

$$
\begin{aligned}
0 &= \frac{d\left(C_{h,t}+C_{h,t}^{home}\right)}{d\left(W_{h,t}/W_t\right)} && \text{(A21)}\\
&= \frac{d}{d\left(W_{h,t}/W_t\right)}\left[\frac{W_{h,t}}{P_t}\left(\frac{W_{h,t}}{W_t}\right)^{-\epsilon_w} N_t - \frac{A_t}{1+\eta}\left(\frac{W_{h,t}}{W_t}\right)^{-\epsilon_w(1+\eta)} N_t^{(1+\eta)}\right], && \text{(A22)}\\
&= \left[(-\epsilon_w+1)\frac{W_t}{P_t}N_t\left(\frac{W_{h,t}}{W_t}\right)^{-\epsilon_w} + \epsilon_w A_t\left(\frac{W_{h,t}}{W_t}\right)^{-\epsilon_w(1+\eta)-1} N_t^{(1+\eta)}\right] && \text{(A23)}
\end{aligned}
$$

Because all wage-setters choose the same flexible-wage wage, we can set $W_{h,t} = W_t$. It then follows that the flexible-wage real wage increases proportionately with productivity and increases with the total amount of labor supplied

$$
\begin{aligned}
\frac{W_t^{flex}}{P_t} &= \frac{\epsilon_w}{\epsilon_w-1}A_t N_t^{\eta}, && \text{(A24)}\\
&= \frac{\epsilon_w}{\epsilon_w-1}A_t^{1-\eta}Y_t^{\eta} && \text{(A25)}
\end{aligned}
$$

A.8 Sticky wage

In the derivation of the wage Phillips curve we use the operator $\tilde{E}_t$ to denote the partially adaptive inflation expectations of wage-setters. With the quadratic wage-setting cost (A14) the first-order condition for wage-setting becomes

$$
\begin{aligned}
0 = {} & \frac{d\left(C_{h,t}+C_{h,t}^{home}\right)}{d\left(\frac{W_{h,t}}{W_t}\right)} - \gamma_w\left(\frac{W_{h,t}}{W_{h,t-1}}\frac{W_{t-2}}{W_{t-1}}-1\right)\frac{W_t}{W_{h,t-1}}\frac{W_{t-2}}{W_{t-1}}Y_t \\
& +\gamma_w\tilde{E}_t M_{h,t+1}\left(\frac{W_{h,t+1}}{W_{h,t}}\frac{W_{t-1}}{W_t}-1\right)\frac{W_{h,t+1}}{W_{h,t}}\frac{W_t}{W_{h,t}}\frac{W_{t-1}}{W_t}Y_{t+1}, \qquad \text{(A26)}
\end{aligned}
$$

Since there is symmetry (i.e. all households face the same problem), we can drop the h index when solving for the aggregate wage.

A.9 Log-linearizing the first-order wage-setting condition

Denoting the flexible wage steady-state output by $\bar{Y}_t$, we have that

$$\bar{Y}_t = A_t \bar{N}, \tag{A27}$$

where the flexible-wage labor supply solves

$$\frac{\epsilon_p - 1}{\epsilon_p} = \frac{\epsilon_w}{\epsilon_w - 1} \bar{N}^{\eta}. \tag{A28}$$

Using lower case for logs and hats to denote deviations from the flexible-wage equilibrium, the log output gap equals

$$x_t \equiv \hat{y}_t = n_t - \bar{n}, \tag{A29}$$

$$= \hat{n}_t. \tag{A30}$$

The steady-state stochastic discount factor equals

$$\bar{M}_{t,t+1} = \beta exp\left(-\gamma g\right). \tag{A31}$$

For convenience we define the constant

$$\beta_g \equiv \beta exp(-(\gamma - 1)g). \tag{A32}$$

Letting $\pi_t^w = \log \frac{W_t}{W_{t-1}}$ denote nominal log wage inflation and taking a first-order approximation around $\pi^w = 0$, expression (A26) simplifies to:

$$\begin{aligned} 0 = {} & (-\epsilon_w + 1)\frac{W_t}{P_t} N_t + \epsilon_w A_t N_t^{(1+\eta)} - \gamma_w \left(\pi_t^w - \pi_{t-1}^w\right) Y_t \\ & + \gamma_w Y_t \tilde{E}_t M_{t+1} \left(\pi_{t+1}^w - \pi_t^w\right) \frac{Y_{t+1}}{Y_t} \end{aligned} \tag{A33}$$

Re-arranging:

$$
\begin{aligned}
\epsilon_w - 1 &= \epsilon_w A_t \frac{P_t}{W_t} N_t^{\eta} - \gamma_w \left(\pi_t^w - \pi_{t-1}^w\right) \frac{P_t}{W_t}\frac{Y_t}{N_t} \\
&\quad + \beta\gamma_w \frac{P_t}{W_t}\frac{Y_t}{N_t} \tilde{E}_t M_{t+1} \left(\pi_{t+1}^w - \pi_t^w\right) \frac{Y_{t+1}}{Y_t}, \qquad \text{(A34)} \\
&= \epsilon_w A_t \frac{P_t}{W_t} N_t^{\eta} - \gamma_w \left(\pi_t^w - \pi_{t-1}^w\right) \frac{P_t}{W_t}\frac{Y_t}{N_t} \\
&\quad + \gamma_w \frac{P_t}{W_t}\frac{Y_t}{N_t} \beta_g \tilde{E}_t \left(\pi_{t+1}^w - \pi_t^w\right), \qquad \text{(A35)}
\end{aligned}
$$

where in the last step we dropped second-order terms in M_{t+1} and output growth interacted with wage inflation. We next substitute the production function into (A35):

$$
(\epsilon_w - 1)\frac{W_t}{A_t P_t} = \epsilon_w N_t^{\eta} - \gamma_w \left(\pi_t^w - \pi_{t-1}^w\right) + \beta^g \gamma_w \tilde{E}_t \left(\pi_{t+1}^w - \pi_t^w\right), \qquad \text{(A36)}
$$

$$
\text{(A37)}
$$

giving the wage Phillips curve

$$
\pi_t^w = \frac{1}{1+\beta_g}\pi_{t-1}^w + \frac{\beta^g}{1+\beta_g}\tilde{E}_t \pi_{t+1}^w + \gamma_w^{-1}\left(\epsilon_w N_t^{\eta} - (\epsilon_w - 1)\frac{W_t}{A_t P_t}\right). \qquad \text{(A38)}
$$

Note that the term in parentheses is the wedge between the real productivity-adjusted wage and workers' productivity-adjusted disutility of labor. Because we have flexible product prices the real productivity-adjusted wage is constant and we can substitute in from (A19):

$$
\pi_t^w = \frac{1}{1+\beta_g}\pi_{t-1}^w + \frac{\beta^g}{1+\beta_g}\tilde{E}_t \pi_{t+1}^w + \gamma_w^{-1}\left(\epsilon_w N_t^{\eta} - (\epsilon_w - 1)\frac{\epsilon_p - 1}{\epsilon_p}\right) \qquad \text{(A39)}
$$

In the flexible-wage equilibrium the term in parentheses is zero, giving the first-order log-linearization

$$
\begin{aligned}
\epsilon_w N_t^{\eta} - (\epsilon_w - 1)\frac{\epsilon_p - 1}{\epsilon_p} &= \epsilon_w \bar{N}^{\eta} exp(\eta \hat{n}_t) - (\epsilon_w - 1)\frac{\epsilon_p - 1}{\epsilon_p}, \qquad \text{(A40)} \\
&\approx \epsilon_w \bar{N}^{\eta} \eta \hat{n}_t, \qquad \text{(A41)} \\
&= \epsilon_w \bar{N}^{\eta} \eta \hat{y}_t \qquad \text{(A42)}
\end{aligned}
$$

We therefore obtain the standard log-linearized wage Phillips curve

$$
\pi_t^w = \frac{1}{1+\beta_g}\pi_{t-1}^w + \frac{\beta^g}{1+\beta_g}\tilde{E}_t \pi_{t+1}^w + \kappa \hat{y}_t, \qquad \text{(A43)}
$$

where the constant κ equals

$$\kappa = \gamma_w^{-1} \epsilon_w \bar{N}^{\eta} \eta \tag{A44}$$

Substituting in the adaptive inflation expectations assumption

$$\tilde{E}_t \pi^w_{t+1} = (1-\zeta) E_t \pi^w_{t+1} + \zeta \pi^w_{t-1}, \tag{A45}$$

gives the wage Phillips curve

$$\pi^w_t = \rho^{\pi} \pi^w_{t-1} + f^{\pi} E_t \pi^w_{t+1} + \kappa \hat{y}_t, \tag{A46}$$

where

$$\rho^{\pi} = \frac{1}{1+\beta_g} + \zeta - \frac{1}{1+\beta_g} \zeta, \tag{A47}$$

$$f^{\pi} = 1 - \rho^{\pi}. \tag{A48}$$

Phillips curve shocks to (A46), $v_{\pi,t}$, arise from making the degree of monopolistic wage-setting frictions ϵ_w or the marginal cost of providing labor outside the home η time-varying.

A.10 Price inflation

Product prices equal

$$P_t = \frac{\epsilon_p}{\epsilon_p - 1} \frac{W_t}{A_t}, \tag{A49}$$

so log price inflation equals (up to a constant)

$$\pi^p_t = \pi^w_t - \Delta a_t, \tag{A50}$$

$$= \pi^w_t - (1-\phi) \hat{y}_{t-1}, \tag{A51}$$

where the log deviation of real GDP from potential is the output gap, i.e. $x_t = \hat{y}_t$.

B Solution

B.1 Solving for macroeconomic dynamics

The full macroeconomic dynamics are determined by the Euler equation, the wage Phillips curve (A46) and the monetary policy rule, as well as the short-rate Fisher equation $r_t = i_t - E_t\pi^p_{t+1}$, the relationship between price and wage inflation (A51). The Euler equation is given by

$$x_t = f^x E_t x_{t+1} + \rho^x x_{t-1} - \psi\left(i_t - E_t\pi^p_{t+1}\right) + v_{x,t}, \tag{A52}$$

where

$$\rho^x = \frac{\theta_2}{\phi - \theta_1}, \tag{A53}$$

$$f^x = \frac{1}{\phi - \theta_1}, \tag{A54}$$

$$\psi = \frac{1}{\gamma(\phi - \theta_1)}, \tag{A55}$$

$$\theta_2 = \phi - 1 - \theta_1. \tag{A56}$$

The wage Phillips curve is given by

$$\pi^w_t = \rho^\pi \pi^w_{t-1} + f^\pi E_t \pi^w_{t+1} + \kappa x_t + v_{\pi,t}, \tag{A57}$$

The monetary policy rule is given by

$$i_t = \rho^i i_{t-1} + (1-\rho^i)\left(\gamma^x x_t + \gamma^\pi \pi^p_t\right) + v_{i,t}, \tag{A58}$$

where $v_{x,t} = \frac{1}{\gamma(\phi-\theta_1)}\xi_t$ denotes the demand shock; $v_{\pi,t}$ is the supply shock; and $v_{i,t}$ is the monetary policy shock.

We want to find a solution of the form

$$Y_t = BY_{t-1} + \Sigma v_t, \tag{A59}$$

where the matrix B is $[3 \times 3]$, the matrix Σ is $[3 \times 3]$, and we work with the state vector

$$Y_t = [x_t, \pi^w_t, i_t]', \tag{A60}$$

and the shock vector

$$v_t = [v_{x,t}, v_{\pi,t}, v_{i,t}]'. \tag{A61}$$

Using the relationship (A51), we can write the macroeconomic dynamics in terms of the state vector Y_t:

$$\begin{aligned}
Y_{1,t} &= f^x E_t Y_{1,t+1} + \rho^x Y_{1,t-1} - \psi\left(Y_{3,t} - E_t Y_{2,t+1} + (1-\phi)Y_{1,t}\right) + v_{x,t}, && \text{(A62)}\\
Y_{2,t} &= f^\pi E_t Y_{2,t+1} + \rho^\pi Y_{2,t-1} + \kappa Y_{1,t} + v_{\pi,t}, && \text{(A63)}\\
Y_{3,t} &= \rho^i Y_{3,t-1} + (1-\rho^i)\left(\gamma^x Y_{1,t} + \gamma^\pi Y_{2,t} - \gamma^\pi(1-\phi)Y_{1,t-1}\right) + v_{i,t}. && \text{(A64)}
\end{aligned}$$

We can write this in matrix form:

$$0 = F E_t Y_{t+1} + G Y_t + H Y_{t-1} + M v_t,$$

where the matrices F, G and H are given by

$$\begin{aligned}
F &= \begin{bmatrix} \frac{f^x}{1+\psi(1-\phi)} & \frac{\psi}{1+\psi(1-\phi)} & 0 \\ 0 & f^\pi & 0 \\ 0 & 0 & 0 \end{bmatrix},\\
G &= \begin{bmatrix} -1 & 0 & -\frac{\psi}{1+\psi(1-\phi)} \\ \kappa & -1 & 0 \\ (1-\rho^i)\gamma^x & (1-\rho^i)\gamma^\pi & -1 \end{bmatrix},\\
H &= \begin{bmatrix} \frac{\rho^x}{1+\psi(1-\phi)} & 0 & 0 \\ 0 & \rho^\pi & 0 \\ -(1-\rho^i)(1-\phi)\gamma^\pi & 0 & \rho^i \end{bmatrix}.
\end{aligned}$$

The matrix M is $[3 \times 3]$ and equals:

$$M = \begin{bmatrix} \frac{1}{1+\psi(1-\phi)} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{A65}$$

Following Uhlig (1999), we solve for the generalized eigenvectors and eigenvalues of the matrix Ξ with respect to the matrix Δ, where

$$\Xi = \begin{bmatrix} -G & -H \\ I_3 & 0_3 \end{bmatrix}, \tag{A66}$$

$$\Delta = \begin{bmatrix} F & 0_3 \\ 0_3 & I_3 \end{bmatrix} \tag{A67}$$

To obtain a solution, we then pick three generalized eigenvalues $\lambda_1, \lambda_2, \lambda_3$ with generalized eigenvectors $[\lambda z_1', z_1']'$, $[\lambda_2 z_2', z_2]'$, and $[\lambda_3 z_3', z_3']'$. We denote the diagonal matrix of these eigenvalues by $\Lambda = diag\,(\lambda_1, \lambda_2, \lambda_3)$, and the matrix of the lower $[3 \times 1]$ portion of the eigenvectors by $\Omega = [z_1, z_2, z_3]$. The corresponding solutions for B and Σ are then given by:

$$B = \Omega\Lambda\Omega^{-1}, \tag{A68}$$

$$\Sigma = -\left[FB + G\right]^{-1} M. \tag{A69}$$

For both our calibrations, there exist exactly three generalized eigenvalues with absolute value less than one, and we pick the non-explosive solution corresponding to these three eigenvalues.

B.2 Rotated state vector

Our state space for solving for asset prices is five-dimensional: It consists of $\tilde{Z}_t$, which a scaled version of Y_t, the surplus consumption ratio relative to steady-state $\hat{s}_t$, and the lagged output gap x_{t-1}.

We next describe the definition of $\tilde{Z}_t$. To simplify the numerical implementation of the asset pricing recursions, we require that shocks to the scaled state vector $\tilde{Z}_t$ are independent standard normal and that the first dimension of the scaled state vector is perfectly correlated with consumption innovations. This rotation facilitates the numerical analysis, because it is easier to integrate over independent random variables. Aligning the first dimension of the scaled state vector with output gap innovations (and hence surplus consumption innovations) helps, because it allows us to use a finer grid to integrate numerically over this crucial dimension over which asset prices are most non-linear.

If the scaled state vector equals $\tilde{Z}_t = AY_t$ for some invertible matrix A, the dynamics of

$\tilde{Z}_t$ are given by:

$$\tilde{Z}_t = AY_t, \tag{A70}$$

$$\tilde{Z}_{t+1} = \underbrace{ABA^{-1}}_{\tilde{B}}\tilde{Z}_t + \underbrace{A\Sigma v_{t+1}}_{\epsilon_{t+1}}. \tag{A71}$$

We hence want a matrix, A, such that

$$Var\left(\epsilon_{t+1}\right) = A\Sigma\Sigma_v\Sigma' A', \tag{A72}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{A73}$$

Finding such a matrix A should in general be possible, because the matrix M and therefore $\Sigma\Sigma_v\Sigma'$ generally have rank three. We require that the first dimension of ϵ_{t+1} is perfectly correlated with the consumption shock. We can therefore find the three rows of A using the following steps:

1. Set $A_1 = \frac{e_1}{\sqrt{e_1\Sigma\Sigma_v\Sigma' e_1'}}$.

2. We use the MATLAB function *null* to compute the null space of $A_1\Sigma\Sigma_v\Sigma'$. Let n_2 denote the first vector in $null\left(A_1\Sigma\Sigma_v\Sigma'\right)$. We then define the second row of A as the normalized version of n_2:

$$A_2 = \frac{n_2}{\sqrt{n_2\Sigma\Sigma_v\Sigma' n_2'}}. \tag{A74}$$

3. Let n_3 denote the first vector in $null\left(A_1\Sigma\Sigma_v\Sigma', A_2\Sigma\Sigma_v\Sigma'\right)$. We then define the third row of A as the normalized version of n_3:

$$A_3 = \frac{n_3}{\sqrt{n_3\Sigma\Sigma_v\Sigma' n_3'}}. \tag{A75}$$

It is then straightforward to verify that equation (A73) holds for

$$A = \begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix}. \tag{A76}$$

B.3 Asset pricing recursions

Before deriving the recursions for the numerical asset pricing computations, we derive a convenient form for the dynamics of the log surplus consumption ratio. We use e_i to denote a row vector with 1 in position i and zeros elsewhere. The matrix

$$\Sigma_M = e_1\Sigma \tag{A77}$$

denotes the loading of consumption innovations onto the vector of shocks v_t, where e_1 is a basis vector with a one in the first position and zeros everywhere else. The volatility of consumption surprises equals:

$$\sigma_c^2 = \Sigma_M \Sigma_v \Sigma_M'. \tag{A78}$$

To simplify notation, we define $\hat{s}_t$ as the log deviation of surplus consumption from its steady state. The dynamics of $\hat{s}_t$ are:

$$\hat{s}_t = s_t - \bar{s}, \tag{A79}$$

$$\hat{s}_t = \theta_0 \hat{s}_{t-1} + \theta_1 x_{t-1} + \theta_2 x_{t-2} + \lambda(\hat{s}_{t-1})\varepsilon_{c,t}, \tag{A80}$$

where with an abuse of notation we write:

$$\lambda(\hat{s}_t) = \lambda_0\sqrt{1-2\hat{s}_t} - 1, \hat{s}_t \leq s_{max} - \bar{s}, \tag{A81}$$

$$\lambda(\hat{s}_t) = 0, \hat{s}_t \geq s_{max} - \bar{s}. \tag{A82}$$

The steady-state surplus consumption sensitivity equals:

$$\lambda_0 = \frac{1}{\bar{S}}. \tag{A83}$$

In our calculations of bond prices, we repeatedly substitute out expected log SDF growth, which equals:

$$E_t[m_{t+1}] = \log\beta - \gamma E_t \Delta\hat{s}_{t+1} - \gamma E_t \Delta c_{t+1}, \tag{A84}$$

$$= -r_t + \xi_t - \frac{\gamma}{2}(1-\theta_0)(1-2\hat{s}_t), \tag{A85}$$

$$= -(e_3 - e_2 B + (1-\phi)e_1)Y_t + \xi_t - \frac{\gamma}{2}(1-\theta_0)(1-2\hat{s}_t) \tag{A86}$$

We often combine this with $r_t = \bar{r} + (e_3 - e_2 B)Z_t$ and $\hat{r}_t = (e_3 - e_2 B)Z_t$.

Including the constant, consumption growth is given by:

$$\Delta c_{t+1} = g + x_{t+1} - \phi x_t. \tag{A87}$$

The steady state real short-term interest rate at $x_t = 0$ and $s_t = \bar{s}$ is the same as in Campbell and Cochrane (1999):

$$\bar{r} = \gamma g - \frac{1}{2}\gamma^2\sigma_c^2/\bar{S}^2 - \log(\beta). \tag{A88}$$

The updating rule for the log surplus consumption ratio can then be written in terms of the state variables as:

$$\hat{s}_{t+1} = \theta_0\hat{s}_t + \theta_1 e_1 A^{-1}\tilde{Z}_t + \theta_2 x_{t-1} + \lambda(\hat{s}_t)\varepsilon_{c,t+1}. \tag{A89}$$

B.3.1 Recursion for zero-coupon consumption claims

We now derive the recursion for zero-coupon consumption claims in terms of state variables $\tilde{Z}_t$, $\hat{s}_t$ and x_{t-1}. Let P^c_{nt}/C_t denote the price-dividend ratio of a zero-coupon claim on consumption at time $t+n$. The outline of our strategy here is that we first derive an analytic expression for the price-dividend ratio for P^c_{1t}/C_t. For $n \geq 1$ we guess and verify recursively that there exists a function $F_n(\tilde{Z}_t, \hat{s}_t, x_{t-1})$, such that

$$\frac{P^c_{nt}}{C_t} = F_n\left(\tilde{Z}_t, \hat{s}_t, x_{t-1}\right). \tag{A90}$$

The ex-dividend price-consumption ratio for a claim to all future consumption is then given by

$$\frac{P_t}{C_t} = F\left(\tilde{Z}_t, \hat{s}_t, x_{t-1}\right), \tag{A91}$$

where we define

$$F\left(\tilde{Z}_t, \hat{s}_t, x_{t-1}\right) = \sum_{n=1}^{\infty} F_n\left(\tilde{Z}_t, \hat{s}_t, x_{t-1}\right). \tag{A92}$$

We now derive the recursion of zero-coupon consumption claims in terms of state variables $\tilde{Z}_t$ and $\hat{s}_t$. The one-period zero coupon price-consumption ratio solves:

$$\frac{P^c_{1,t}}{C_t} = E_t\left[\frac{M_{t+1}C_{t+1}}{C_t}\right] \tag{A93}$$

We simplify

$$\begin{aligned}\frac{M_{t+1}C_{t+1}}{C_t} &= \beta\exp(-\gamma E_t\Delta\hat{s}_{t+1}-(\gamma-1)E_t\Delta c_{t+1}\\ &\quad -\gamma(\hat{s}_{t+1}-E_t s_{t+1})-(\gamma-1)(c_{t+1}-E_t c_{t+1})).\end{aligned}$$

Using the notation $f_n=\log(F_n)$, this gives the log one-period price-consumption ratio as:

$$\begin{aligned}f_1\left(\tilde{Z}_t,\hat{s}_t,x_{t-1}\right) &= \log\beta-\gamma[(\theta_0-1)\hat{s}_t+\theta_1 x_t+\theta_2 x_{t-1}]-(\gamma-1)[g+E_t x_{t+1}-\phi x_t]\\ &\quad +\frac{1}{2}\left(\gamma\lambda(\hat{s}_t)+(\gamma-1)\right)^2\sigma_c^2, \qquad \text{(A94)}\\ &= \log\beta-(\gamma-1)g-e_1[(\gamma\theta_1-\gamma\phi+\phi)I+(\gamma-1)B]A^{-1}\tilde{Z}_t\\ &\quad -\gamma(\theta_0-1)\hat{s}_t-\gamma\theta_2 x_{t-1}+\frac{1}{2}\left(\gamma\lambda(\hat{s}_t)+(\gamma-1)\right)^2\sigma_c^2 \qquad \text{(A95)}\end{aligned}$$

Next, we solve for f_n, $n\geq 2$ iteratively. Note that:

$$\frac{P^c_{nt}}{C_t}=\mathbb{E}_t\left[\frac{M_{t+1}C_{t+1}}{C_t}\frac{P^c_{n-1,t+1}}{C_{t+1}}\right]=\mathbb{E}_t\left[\frac{M_{t+1}C_{t+1}}{C_t}F_{n-1}\left(\tilde{Z}_{t+1},\hat{s}_{t+1},x_t\right)\right] \qquad \text{(A96)}$$

This gives the following expression for f_n:

$$\begin{aligned}f_n(\tilde{Z}_t,\hat{s}_t,x_{t-1}) &= \log\Bigg[\mathbb{E}_t\Bigg[\exp\Bigg(\log\beta-(\gamma-1)g-e_1[(\gamma\theta_1-\gamma\phi+\phi)I+(\gamma-1)B]A^{-1}\tilde{Z}_t\\ &\quad -\gamma(\theta_0-1)\hat{s}_t-\gamma\theta_2 x_{t-1}-(\gamma(1+\lambda(\hat{s}_t))-1)\sigma_c\epsilon_{1,t+1}\\ &\quad +f_{n-1}(\tilde{Z}_{t+1},\hat{s}_{t+1},x_t)\Bigg)\Bigg]\Bigg]. \qquad \text{(A97)}\end{aligned}$$

Here, $\epsilon_{1,t+1}$ denotes the first dimension of the shock ϵ_{t+1}.

B.3.2 Recursion for zero-coupon bond prices

We use $P^{\$}_{n,t}$ and $P_{n,t}$ to denote the prices of nominal and real n-period zero-coupon bonds. The strategy is to develop analytic expressions for one- and two-period bond prices. We then guess and verify recursively that the prices of real and nominal zero-coupon bonds with maturity $n\geq 2$ can be written in the following form:

$$\begin{aligned}P_{n,t} &= B_n(\tilde{Z}_t,\hat{s}_t,x_{t-1}), \qquad \text{(A98)}\\ P^{\$}_{n,t} &= B^{\$}_n(\tilde{Z}_t,\hat{s}_t,x_{t-1}), \qquad \text{(A99)}\end{aligned}$$

where $B_n(\tilde{Z}_t,\hat{s}_t,x_{t-1})$ and $B^{\$}_n(\tilde{Z}_t,\hat{s}_t,x_{t-1})$ are functions of the state variables. As discussed in the main paper, we assume that the short-term nominal interest rate contains no risk premium, so the one-period log nominal interest rate equals $i_t = r_t + E_t\pi_{t+1}$. Taking account of the constants, one-period bond prices equal:

$$P^{\$}_{1,t} = \exp(-Y_{3,t} - \bar{r}), \tag{A100}$$

$$P_{1,t} = \exp(-Y_{3,t} + \mathbb{E}_t Y_{2,t+1} - (1-\phi)Y_{1,t} - \bar{r}). \tag{A101}$$

We next solve for longer-term bond prices including risk premia. Substituting in (A100) into the bond-pricing recursion gives:

$$P^{\$}_{2,t} = \exp(-\xi_t)\mathbb{E}_t\left[M_{t+1}P^{\$}_{1,t+1}\exp(-Y_{2,t+1} + (1-\phi)Y_{1,t})\right] \tag{A102}$$

$$= \exp(-\xi_t)\mathbb{E}_t\left[M_{t+1}\exp(-Y_{3,t+1} - Y_{2,t+1} + (1-\phi)Y_{1,t} - \bar{r})\right]. \tag{A103}$$

We can now verify that the two-period nominal bond price takes the form (A99):

$$\begin{aligned} B^{\$}_2(\tilde{Z}_t, \hat{s}_t, x_{t-1}) &= \exp\left(E_t\left(m_{t+1} - \xi_t - Y_{3,t+1} - Y_{2,t+1} + (1-\phi)Y_{1,t}\right) - \bar{r}\right) \\ &\times\mathbb{E}_t\left[\exp\left(\left(-\gamma\left(\lambda(\hat{s}_t)+1\right)\Sigma_M - \underbrace{[(e_2+e_3)\Sigma]}_{v_{\$}}\right)v_{t+1}\right)\right]. \end{aligned} \tag{A104}$$

Here, we define the vector $v_{\$}$ to simplify notation. Taking logs, substituting out for $E_t m_{t+1}$, and using the definition for the sensitivity function $\lambda(\hat{s}_t)$, we get:

$$\begin{aligned} b^{\$}_2 &= -e_3[I+B]A^{-1}\tilde{Z}_t + \frac{1}{2}v_{\$}\Sigma_v v_{\$}\prime \\ &+\gamma\left(\lambda(\hat{s}_t)+1\right)\Sigma_M\Sigma_v v'_{\$} - 2\bar{r}. \end{aligned} \tag{A105}$$

The closed-form solution for the two-period real bond price becomes

$$\begin{aligned} P_{2,t} &= \exp\left(E_t\left(m_{t+1} - \xi_t - Y_{3,t+1} - (1-\phi)Y_{1,t+1} + Y_{2,t+2}\right) - \bar{r}\right) \\ &\times\mathbb{E}_t\left[\exp\left((-\gamma(\lambda(\hat{s}_t)+1)\Sigma_M - \underbrace{(e_3 + (1-\phi)e_1 - e_2B)\Sigma)}_{v_r}v_{t+1}\right)\right] \end{aligned} \tag{A106}$$

We define the vector v_r to simplify notation. Taking logs, substituting out for $E_t m_{t+1}$, and using the definition for $\lambda(\hat{s}_t)$ gives:

$$b_2(\tilde{Z}_t, \hat{s}_t, x_{t-1}) = -v_r \left[I + B\right] A^{-1}\tilde{Z}_t + \frac{1}{2}v_r\Sigma_v v_r' + \gamma\left(\lambda(\hat{s}_t) + 1\right)\Sigma_M\Sigma_v v_r' - 2\bar{r}. \tag{A107}$$

For $n \geq 3$, we repeatedly substitute out for $E_t m_{t+1}$ to obtain the following recursion for nominal and real bond prices, respectively:

$$\begin{aligned} B_n^{\$}(\tilde{Z}_t, \hat{s}_t, x_{t-1}) &= \mathbb{E}_t\left[\exp\left(m_{t+1} - \xi_t - Y_{2,t+1} + (1-\phi)Y_{1,t} + b_{n-1}^{\$}(\tilde{Z}_{t+1}, \hat{s}_{t+1}, B^{\$}x_t)\right)\right] \\ &= \mathbb{E}_t\left[\exp\left(-\bar{r} - e_3 A^{-1}\tilde{Z}_t - \frac{\gamma}{2}(1-\theta_0)(1-2\hat{s}_t)\right.\right. \\ &\qquad \left.\left. -\gamma(1+\lambda(\hat{s}_t))\sigma_c\epsilon_{1,t+1} + b_{n-1}^{\$}(\tilde{Z}_{t+1}, \hat{s}_{t+1}, x_t)\right)\right]. \end{aligned} \tag{A108}$$

The value function iteration for real bond prices then becomes

$$\begin{aligned} B_n(\tilde{Z}_t, \hat{s}_t, x_{t-1}) &= \mathbb{E}_t\left[\exp\left(m_{t+1} - \xi_t + b_{n-1}(\tilde{Z}_{t+1}, \hat{s}_{t+1}, x_t)\right)\right] \\ &= \mathbb{E}_t\left[\exp\left(-\bar{r} - (e_3 - e_2 B + (1-\phi)e_1)A^{-1}\tilde{Z}_t - \frac{\gamma}{2}(1-\theta_0)(1-2\hat{s}_t)\right.\right. \\ &\qquad \left.\left. -\gamma(1+\lambda(\hat{s}_t))\sigma_c\epsilon_{1,t+1} + b_{n-1}(\tilde{Z}_{t+1}, \hat{s}_{t+1}, x_t)\right)\right]. \end{aligned} \tag{A109}$$

B.3.3 Computing returns

The log return on the consumption claim equals:

$$r_{t+1}^c = \log\left(\frac{P_{t+1}^c + C_{t+1}}{P_t^c}\right), \tag{A110}$$

$$= \Delta c_{t+1} + \log\left(\frac{1 + \frac{P_{t+1}^c}{C_{t+1}}}{\frac{P_t^c}{C_t}}\right). \tag{A111}$$

Real and nominal log bond yields equal:

$$y_{n,t} = -\frac{1}{n}b_{n,t}, \tag{A112}$$

$$y_{n,t}^{\$} = -\frac{1}{n}b_{n,t}^{\$}. \tag{A113}$$

Real log bond returns equal:

$$r_{n,t+1} = b_{n-1,t+1} - b_{n,t}. \tag{A114}$$

Nominal log bond returns equal:

$$r^{\$}_{n,t+1} = b^{\$}_{n-1,t+1} - b^{\$}_{n,t}. \tag{A115}$$

Real and nominal bond log excess returns then equal:

$$xr_{n,t+1} = r_{n,t+1} - r_t, \tag{A116}$$
$$xr^{\$}_{n,t+1} = r^{\$}_{n,t+1} - i_t. \tag{A117}$$

B.3.4 Levered stock prices and returns

We note that the price of the levered equity claim is δP^c_t, so the price-dividend ratio equals:

$$\frac{P^\delta_t}{D^\delta_t} = \delta \frac{C_t}{D^\delta_t}\frac{P^c_t}{C_t}. \tag{A118}$$

Using the expression

$$D^\delta_{t+1} = P^c_{t+1} + C_{t+1} - (1-\delta)P^c_t \exp(r_t) - \delta P^c_t, \tag{A119}$$

and

$$P^\delta_t = \delta P^c_t \tag{A120}$$

gives the gross return on levered stocks:

$$\left(1 + R^\delta_{t+1}\right) = \frac{D^\delta_{t+1} + P^\delta_{t+1}}{P^\delta_t}, \tag{A121}$$
$$= \frac{1}{\delta}\frac{P^c_{t+1} + C_{t+1} - (1-\delta)P^c_t \exp(r_t)}{P^c_t}, \tag{A122}$$
$$= \frac{1}{\delta}\left(1 + R^c_{t+1}\right) - \frac{1-\delta}{\delta}\exp(r_t). \tag{A123}$$

Log stock excess returns then equal:

$$xr^\delta_{t+1} = r^\delta_{t+1} - r_t. \tag{A124}$$

To mimic firms' dividend smoothing in the data, we report simulated moments for the price of equities dividend by dividends smoothed over the past 64 quarters:

$$P_t^\delta / \left(\frac{1}{64} (D_t^\delta + D_{t-1}^\delta + ... + D_{t-63}^\delta) \right). \tag{A125}$$

B.4 Risk-premium decomposition

We use the superscript ^{rn} for risk-neutral, superscript ^{cf} for cash flow, and ^{rp} for risk premium. Risk-neutral valuations are expected cash flows discounted with the risk-neutral discount factor, given by:

$$M_{t+1}^{rn} = \exp(-(r_t - \xi_t)). \tag{A126}$$

Note that since we are not interested in risk-neutral bond and stock prices, but only a decomposition of returns, multiplying M_{t+1}^{rn} by a constant discount rate does not matter. For any zero-coupon claim it would shift risk-neutral returns merely by a constant and therefore leave our decomposition into risk-neutral and risk-premium components unaffected. For a claim to all future consumption or stock returns, a constant discount rate could theoretically shift the weights between nearer-term consumption claims and longer-term consumption claims, and therefore change risk-neutral returns. However, since consumption growth is stationary we have found that this makes very little different to risk-neutral stock returns in any of our numerical applications.

B.4.1 Risk-neutral zero-coupon bond prices

We use analogous recursions to solve for risk-neutral bond prices. One-period risk-neutral bond prices are given exactly as before by equations (A100) and (A101). For $n > 1$, we guess and verify that the prices of real and nominal risk-neutral zero-coupon bonds with maturity n can be written in the following form

$$P_{n,t}^{rn} = B_n^{rn}(\tilde{Z}_t, \hat{s}_t, x_{t-1}), \tag{A127}$$

$$P_{n,t}^{\$,rn} = B_n^{\$,rn}(\tilde{Z}_t, \hat{s}_t, x_{t-1}). \tag{A128}$$

for some functions $B_n^{rn}(\tilde{Z}_t, \hat{s}_t, x_{t-1})$ and $B_n^{\$,rn}(\tilde{Z}_t, \hat{s}_t, x_{t-1})$.

We derive the two-period risk-neutral nominal bond price analytically:

$$P_{2,t}^{\$,rn} = \exp(-\xi_t)\mathbb{E}_t \left[M_{t+1}^{rn} P_{1,t+1}^{\$,rn} \exp(-Y_{2,t+1} + (1-\phi)Y_{1,t}) \right] \tag{A129}$$

$$= \exp(-r_t)\mathbb{E}_t \left[\exp(-Y_{3,t+1} - Y_{2,t+1} + (1-\phi)Y_{1,t} - \bar{r}) \right]. \tag{A130}$$

We can hence verify that the two-period risk-neutral nominal bond price takes the form (A99)

$$b_2^{\$,rn} \quad = \quad -e_3\left[I+B\right]A^{-1}\tilde{Z}_t+\frac{1}{2}v_{\$}\Sigma_v v_{\$}\prime-2\bar{r} \tag{A131}$$

Here, the vector $v_{\$}$ is identical to the case with risk aversion. Comparing expressions (A131) and (A105) shows that they agree when $\gamma=0$. We similarly solve for 2-period real bond prices in closed form:

$$\begin{aligned} P_{2,t}^{rn} \quad = \quad & \exp\left(-Y_{3,t}+\mathbb{E}_t Y_{2,t+1}-(1-\phi)Y_{1,t}-\bar{r}\right)\times\exp\left(\mathbb{E}_t\left(-Y_{3,t+1}+\mathbb{E}_{t+1}Y_{2,t+2}+(1-\phi)Y_{1,t+1}-\bar{r}\right)\right) \\ & \times\mathbb{E}_t\left[\exp\left(-\underbrace{(e_3+(1-\phi)e_1-e_2B)\Sigma}_{v_r}v_{t+1}\right)\right]. \end{aligned} \tag{A132}$$

The vector v_r is again identical to the case with risk aversion. Taking logs gives:

$$b_2^{rn}(\tilde{Z}_t,\hat{s}_t,x_{t-1}) \quad = \quad -(e_3+(1-\phi)e_1-e_2B)\left[I+B\right]A^{-1}\tilde{Z}_t+\frac{1}{2}v_r\Sigma_v v_r'-2\bar{r}. \tag{A133}$$

We note that the risk-neutral bond prices (A133) and bond prices with risk aversion (A107) are identical when the utility curvature parameter γ equals zero.

For $n\geq 3$ the n-period risk neutral nominal and real bond prices satisfy the following recursions, respectively:

$$B_n^{\$,rn}(\tilde{Z}_t,\hat{s}_t,x_{t-1}) \quad = \quad \mathbb{E}_t\left[\exp\left(-\bar{r}-e_3A^{-1}\tilde{Z}_t+b_{n-1}^{\$}(\tilde{Z}_{t+1},\hat{s}_{t+1},x_t)\right)\right], \tag{A134}$$

$$B_n^{rn}(\tilde{Z}_t,\hat{s}_t,x_{t-1}) \quad = \quad \mathbb{E}_t\left[\exp\left(-\bar{r}-(e_3+(1-\phi)e_1-e_2B)A^{-1}\tilde{Z}_t+b_{n-1}(\tilde{Z}_{t+1},\hat{s}_{t+1},x_t)\right)\right] \tag{A135}$$

B.4.2 Risk-neutral zero-coupon consumption claims

Next, we derive recursive solutions for the risk-neutral prices of zero-coupon consumption claims. Let $P_{nt}^{c,rn}/C_t$ denote the risk-neutral price-dividend ratio of a zero-coupon claim on consumption at time $t+n$. The risk-neutral price-consumption ratio of a claim to the entire stream of future consumption equals:

$$\frac{P_t^{c,rn}}{C_t} = \sum_{n=1}^{\infty} \frac{P_{nt}^{c,rn}}{C_t}. \tag{A136}$$

For $n \geq 1$, we guess and verify there exists a function $F_n^{rn}(\tilde{Z}_t, \hat{s}_t, x_{t-1})$, such that

$$\frac{P_{nt}^{c,rn}}{C_t} = F_n^{rn}\left(\tilde{Z}_t, \hat{s}_t, x_{t-1}\right). \tag{A137}$$

We start by deriving the analytic expression for F_1^{rn}. The one-period risk-neutral zero-coupon price-consumption ratio solves

$$\begin{aligned}
\frac{P_{1,t}^{c,rn}}{C_t} &= \mathbb{E}_t\left[M_{t+1}^{rn}\frac{C_{t+1}}{C_t}\right] &\text{(A138)}\\
&= \exp(-r_t + u_t)\mathbb{E}_t\left[\frac{C_{t+1}}{C_t}\right] &\text{(A139)}\\
&= \exp(-\gamma\mathbb{E}_t x_{t+1} + \gamma(\phi-\theta_1)x_t - \gamma\theta_2 x_{t-1} - \bar{r})\mathbb{E}_t\left[\frac{C_{t+1}}{C_t}\right] &\text{(A140)}
\end{aligned}$$

Using (A87) to substitute for consumption growth, we can derive the following analytic expression for f_1^{rn}:

$$f_1^{rn}(\tilde{Z}_t, \hat{s}_t, x_{t-1}) = g - \bar{r} - e_1[(\gamma\theta_1 - \gamma\phi + \phi)I - (1-\gamma)B]A^{-1}\tilde{Z}_t - \gamma\theta_2 x_{t-1} + \frac{1}{2}\sigma_c^2. \tag{A141}$$

Next, we solve for f_n, $n \geq 2$ iteratively:

$$\frac{P_{nt}^{c,rn}}{C_t} = \exp\left(-\gamma\mathbb{E}_t Y_{1,t+1} + \gamma(\phi-\theta_1)Y_{1,t} - \gamma\theta_2 x_{t-1} - \bar{r}\right)\mathbb{E}_t\left[\frac{C_{t+1}}{C_t}F_{n-1}^{rn}\left(\tilde{Z}_{t+1}, \hat{s}_{t+1}, x_t\right)\right] \tag{A142}$$

This gives the following expression for f_n^{rn}:

$$\begin{aligned}
f_n^{rn}(\tilde{Z}_t, \hat{s}_t, x_{t-1}) = \log[\mathbb{E}_t[\exp(&-\gamma\mathbb{E}_t Y_{1,t+1} + \gamma(\phi-\theta_1)Y_{1,t} - \gamma\theta_2 x_{t-1} - \bar{r}\\
&+ g - \phi Y_{1,t} + \mathbb{E}_t Y_{1,t+1} + \sigma_c\epsilon_{1,t+1}\\
&+ f_{n-1}^{rn}(\tilde{Z}_{t+1}, \hat{s}_{t+1}, x_t)\Big)\Big]\Big]. &\text{(A143)}
\end{aligned}$$

Finally, we re-write $f_{n,t}^{rn}$ as an expectation involving $f_{n-1,t+1}^{rn}$, the state variables $\tilde{Z}_t$, and period $t+1$ shocks:

$$f_n^{rn}(\tilde{Z}_t, \hat{s}_t, x_{t-1}) = \log\Big[\mathbb{E}_t\Big[\exp\Big(e_1[(1-\gamma)B - (\phi - \gamma(\phi-\theta_1))I]A^{-1}\tilde{Z}_t - \gamma\theta_2 x_{t-1} + g - \bar{r} + \sigma_c\epsilon_{1,t+1} + f_{n-1}^{rn}(\tilde{Z}_{t+1}, \hat{s}_{t+1}, x_t)\Big)\Big]\Big]. \tag{A144}$$

B.5 Risk-neutral returns

We plug risk-neutral price-consumption ratios and bond prices into equations (A111) through (A117). This gives risk-neutral returns on the consumption claim, risk-neutral log excess bond returns, and risk-neutral bond yields. We then substitute risk-neutral returns on the consumption claim into (A123)-(A124) to obtain risk-neutral log excess stock returns.

B.6 Relationship between preference and expected growth shocks

We now show that modeling preference shocks to the Euler equation of holding bonds is isomorphic to expected growth shocks, up to the difference between price and wage inflation, and provided that the income and intertemporal substitution effects for the consumption claim cancel as would be the case if $\gamma = 1$. To show this, assume that ξ_t is proportional to shock to expected productivity growth

$$a_t = \nu + a_{t-1} + (1-\phi)n_{t-1} + \frac{1}{\gamma}\xi_{t-1}. \tag{A145}$$

Then potential output equals (up to constant)

$$a_t = \left((1-\phi)c_{t-1} + \frac{1}{\gamma}\xi_{t-1}\right) + \phi a_{t-1}, \tag{A146}$$

$$= \sum_{i=0}^{\infty}\phi^i\left((1-\phi)c_{t-1-i} + \frac{1}{\gamma}\xi_{t-1-i}\right), \tag{A147}$$

and the link between the output gap and consumption is given by

$$x_t = c_t - a_t, \tag{A148}$$

$$= c_t - \sum_{i=0}^{\infty}\phi^i\left((1-\phi)c_{t-1-i} + \frac{1}{\gamma}\xi_{t-1-i}\right) \tag{A149}$$

Consumption growth then equals

$$\Delta c_{t+1} = x_{t+1} - \phi x_t + \frac{1}{\gamma}\xi_t. \tag{A150}$$

The consumption surprise in the SDF and its volatility therefore remain unchanged:

$$c_{t+1} - E_t c_{t+1} = x_{t+1} - E_t x_{t+1}, \tag{A151}$$
$$\sigma_c^2 = e_1 \Sigma \Sigma_u M' e_1'. \tag{A152}$$

The first-order asset pricing condition for the real risk-free rate takes the form

$$r_t = \gamma E_t \Delta c_{t+1} + \gamma \theta_1 x_t + \gamma \theta_2 x_{t-1}, \tag{A153}$$
$$= \gamma E_t x_{t+1} - \gamma \phi x_t + \gamma \theta_1 x_t + \gamma \theta_2 x_{t-1} + \xi_t \tag{A154}$$

We then obtain the identical macroeconomic Euler equation as before, with the expected growth shock ξ_t acting as a demand shock:

$$x_t = f^x E_t x_{t+1} + \rho^x x_{t-1} - \psi r_t + \underbrace{\frac{1}{\gamma(\phi - \theta_1)} \xi_t}_{v_{x,t}} \tag{A155}$$

What happens to the bond pricing equation? The real bond pricing recursion is given by

$$P_{n,t} = E_t \left[M_{t+1} P_{n-1,t+1} \right], \tag{A156}$$
$$= E_t \left[exp\left(-\xi_t - \gamma \left(x_{t+1} - \phi x_t \right) - \gamma \Delta s_{t+1} \right) P_{n-1,t+1} \right]. \tag{A157}$$

The nominal bond pricing recursion similarly becomes

$$P_{n,t}^{\$} = E_t \left[M_{t+1} P_{n-1,t+1} \right], \tag{A158}$$
$$= E_t \left[exp\left(-\xi_t - \gamma \left(x_{t+1} - \phi x_t \right) - \gamma \Delta s_{t+1} \right) P_{n-1,t+1}^{\$} exp(-\pi_{t+1}) \right]. \tag{A159}$$

This shows that the bond pricing recursion is exactly as before, when expressed in terms of the output gap rather than consumption.

The recursion for a zero-coupon consumption claim becomes

$$\frac{P_{n,t}^c}{C_t} = E_t \left[M_{t+1} \frac{C_{t+1}}{C_t} \frac{P_{n-1,t+1}^c}{C_{t+1}} \right], \tag{A160}$$
$$= exp(-\xi_t + \frac{1}{\gamma} \xi_t) E_t \left[exp\left(-\gamma \left(x_{t+1} - \phi x_t \right) - \gamma \Delta s_{t+1} \right) exp\left(x_{t+1} - \phi x_t \right) \frac{P_{n-1,t+1}^c}{C_{t+1}} \right]$$

That is, the pricing equation for the consumption claim is unaffected by ξ_t provided that $\gamma = 1$ because the intertemporal substitution and expected dividend growth effects exactly cancel. However, if we set $\gamma = 1$ the intertemporal substitution and wealth effects exactly cancel and we can equivalently interpret ξ_t as a shock to expected consumption growth or a preference shock for bonds. If we re-solve for consumption growth using the realized process for ξ_t, this shock would represent a perfect foresight shock to expected consumption growth. Alternatively, the shock ξ_t could represent a noisy signal of future consumption growth or even perceived future consumption growth that is not realized, in which case the realized consumption growth process would not incorporate ξ_t separately from its effect on x_t.

The equivalence breaks down when considering price and wage inflation, as the relationship between price- and wage inflation would also need to be modified to

$$\pi_t^p = \pi_t^w - (1-\phi)\hat{n}_{t-1} - \frac{1}{\gamma}\xi_{t-1} \tag{A161}$$

It would be counterfactual and inconsistent with the reduced-form inflation-output gap relationship in the 2001.Q2-2019.Q4 period if a demand shock were to lead to lower rather than higher price inflation. In order to interpret the shock ξ_t as an expected growth shock we would therefore need to complicate the model by introducing sticky prices in addition to sticky wages, to bring price and wage inflation closer.

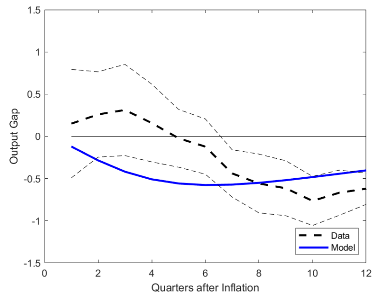
C Additional Model Results

Because our calibration procedure separately calibrates the adaptive inflation expectations parameter ζ holding all other parameters constant, it is important to check that the macroeconomic moments used to calibrate the volatilities of shocks and monetary policy parameters do not suffer unreasonably from changing ζ. Because our 2001.Q2-2019.Q4 calibration uses $\zeta = 0$ this is not a concern here. Figure A1 shows the model-implied impulse responses against the data for the 1979.Q5-2001.Q1 calibration with $\zeta = 0$. They are analogous to Figure 2 in the main paper except that in the main paper we use $\zeta = 0.6$. Comparing against Figure 2 in the main paper shows that most impulse responses are unaffected by changing ζ, and the only difference in the impulse responses with $\zeta = 0.6$ is in the long end of Panel C. The model interest rate response with $\zeta = 0.6$ is much more persistent with $\eta = 0.6$ than with $\zeta = 0$ simply because inflation is more persistent. We are not concerned about this discrepancy because a volatile persistent component in inflation during this period is in line with a long-standing econometrics literature (Stock and Watson (2007)). Further, our empirical measure of inflation combines persistent and short-lived fluctuations in inflation and unit roots are hard to estimate and detect in finite samples.

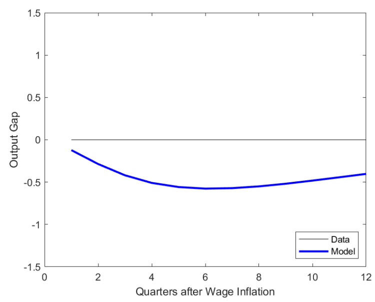
Figure A1: Output Gap, Inflation, and Policy Rate Dynamics Pre-2001 with Rational Inflation Expectations ($\zeta = 0$)

Panel A: Output Gap onto Lagged Price Inflation

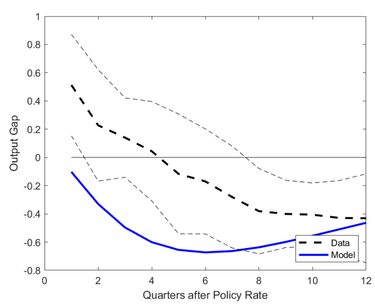
1979.Q4-2001.Q1 2001.Q2-2019.Q4



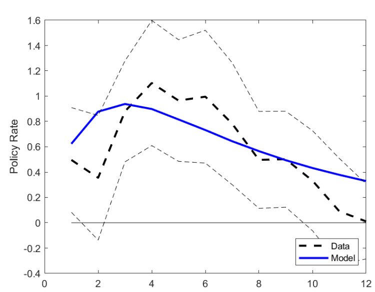
Panel B: Output Gap onto Lagged Wage Inflation



Panel C: Output Gap onto Lagged Policy Rate



Panel D: Policy Rate onto Lagged Price Inflation



This figure shows the model-implied impulse responses against the data for the 1979.Q5-2001.Q1 calibration with $\zeta = 0$. They are analogous to Figure 2 in the main paper except that in the main paper we use $\zeta = 0.6$.

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