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The Acoustics and Historic Development of String Instruments

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Abstract: Plucked and bowed string instruments have existed for many hundreds, even thousands, of years. However, most of our knowledge about the sounds of instruments from before around 1500 relies on the iconography provided by sculptures and paintings. Our understanding of the acoustics of the violin and related instruments like the viola, cello and double bass has advanced significantly over the last twenty or so years, largely through the use of digital technology. This project attempts to build on this progress, and more recent insights, to understand the historic development of stringed instruments from an acoustic perspective. This has involved measuring the acoustical properties of a number of modern reconstructions of older instruments like the lira da braccio and vihuela. Their properties are compared with those of the modern violin family. This is an ongoing investigation, with preliminary results described in this chapter.

1. Introduction

Around one hundred instruments, both newly built and valuable Old Italian violins have been examined by experimental modal analysis. They have been measured fully setup for playing, at various making or assembly stages and as separate parts examined in isolation. In addition, we now have detailed measurements of the sound radiation spectra for well over a hundred classic Italian instruments including violins by, Stradivari, Guarneri del Gesu and members of the Amati family. Data is collected and shared in a community of makers and researchers.

Over the last dozen or so years the present authors have independently and in collaboration played an active role in advancing our understanding of the vibrations of the violin and related instruments (Gough[1-4], Stoppani[5-8]). One of us (Stoppani) is a violin maker, who has developed powerful data acquisition and analysis software for makers to use in their workshops [7]. This is now widely used by both makers and researchers to measure, record and analyse the vibrational and acoustical properties of interesting instruments wherever they can be investigated. The modal analysis measurements illustrated have been made with Stoppani software using a *roving* impact hammer (PCB MODEL 086E80) and fixed accelerometer (Dytran 3225F SERIES). (In their research the authors have also made use of 3 dimensional analysis with triple Polytec Doppler lasers.) The other (Gough) is a physicist/ acoustician/ violinist, who has used COMSOL computational finite element analysis (FEA) to model and thereby understand the vibrations and radiated sound of instruments investigated by Stoppani and others. This integrates the quest to understand how the physical and acoustic properties of both modern and historic instruments are important for the perceived sound and the playability of an instrument.

As part of the present investigation, preliminary measurements and computations have been made on a number of original and copied historic instruments, including the viola da gamba, lira da braccio and vihuela and compared with those of the violin family. The modelling of such instruments, starting from a simple rectangular box to those of the violin's shallow, thin-walled, narrow-waisted, box-like, shell structure, with orthotropic materials, arched top and back plates supported by thin ribs, closely mirrors the historic development of both bowed and plucked stringed instruments in many cultures.

2. Violin Acoustics

Figure 1 shows an overlay of five Stradivari violins measured by the American violin maker Joseph Curtin. All high quality violins, classic Italian and modern, have very similar spectra. The frequency range below 1 kHz is referred to as the signature mode region with sound radiated almost uniformly in all directions. There are three dominant acoustical resonances: the Helmholtz *A0* resonance associated with air bouncing in and out of the *f*-holes, which is driven by the of the *B1-* and *B1+* modes. The latter modes are formed from a coupled combination of the volume-changing, strongly radiating, *breathing* mode of the violin and an otherwise only very weakly radiating *bending* mode of the body shell. In addition there are often weaker contributions from what are known as *CBR* and *C4* modes. Above 1 kHz the acoustic wavelength becomes comparable with the size of the instrument and the radiation becomes increasingly directional, though the directionality is quickly averaged out in a confined performance space. There is then a cluster of relatively strongly radiating resonances around 1 kHz known as the *transition* region and a broader cluster from around 2-3 kHz, originally referred to as the *BH* Bridge Hill feature, but now recognised to involve the properties of the island area between the *f*-holes as well [4].

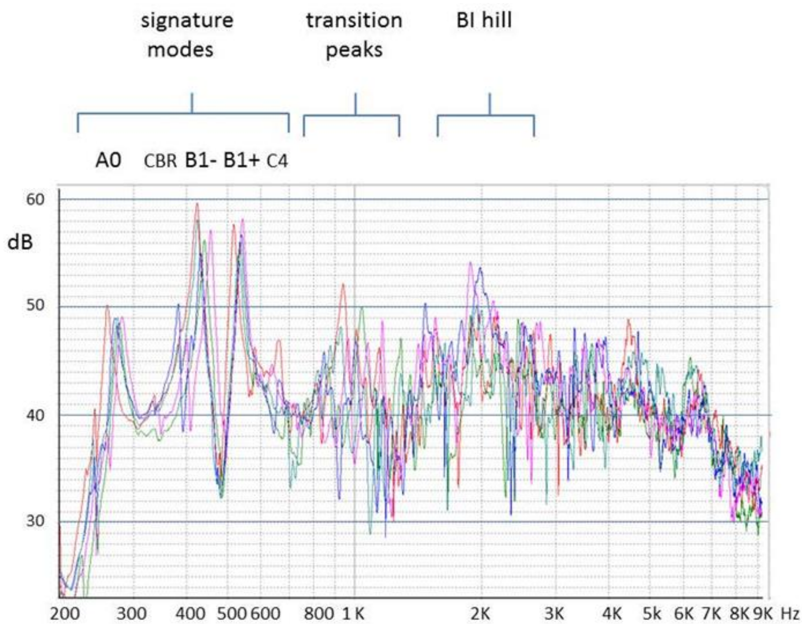

Figure 1 An overlay of the radiated sound from five Golden Period or later Stradivari violins (data courtesy of Joseph Curtin)

Until quite recently, very little was known about the acoustic properties of classic instruments that were considered to be of high quality, or what factors informed such judgements [7, 10]. Knowledge gained through measurement then served as a blue-print for modern makers wishing to make instruments of comparable quality. More recently it has been recognised that fine new instruments can match the acoustic qualities of the old Italian masters. The emphasis has therefore moved towards musicians' and listeners' preferences, regardless of whether they are old or new [8, 9].

Measuring mode shapes and frequencies and using FEA to discover how they depend on the makers choice of materials, graduations of plate thicknesses and arching, plus the role of the strings, bridge, bass bar and soundpost, have provided valuable insights and understanding for makers. This potentially aids them in the making and set-up, allowing them to consistently make instruments of the highest quality, as illustrated in Figure 1 by the large degree of correlation of the acoustic properties between the five high quality Stradivari instruments. However, we are still learning and until fairly recently the nature of the prominent low frequency resonances, let alone the higher frequency modes, were not well understood; nor was the influence of the bridge and island area between the *f*-holes, nor in any detail, the role of bass bar and soundpost. All such components influence a frequency-dependent input filter between the vibrating strings and radiating surfaces of an instrument [4], which determines the perceived timbre and spectral balance over the whole playing range. This topic may well become an important focus in shaping the response and voice of all bowed string instruments.

To gain the necessary understanding of the modes of the violin and related instruments, FEA modelling has been used as a quasi-experimental tool. This has involved changing physical parameters such as plate arching heights, rib strengths and coupling to internal air cavity resonances, often over many orders of magnitude, thereby enhancing our understanding of the important modes of the violin. Such understanding also provides valuable insights into the likely vibrational and acoustical properties of historic instruments, as illustrated later. In addition, FEA analysis has enabled us to understand the rather complicated relationship between the individual free plate modes of vibration and those of the plates in the fully assembled instrument [2] This, to the present authors' knowledge is the only paper that describes how the signature modes of the violin are related to the free plate modes other than by means of a heuristic model. Matters relating to accuracy are not directly relevant since the paper makes it quite clear the intention was to provide understanding rather than accuracy. Nevertheless, the predicted signature mode shapes and mode frequencies are in excellent agreement with observed mode shapes and frequencies, which typically differ by around 10% (less than a tone) from one fine instrument to the next. Despite the complicated transition from free plates to assembled corpus, the individual plates remain the main source of radiated sound in all stringed instruments, as the strings themselves radiated a negligible amount of sound. Free plates, despite the radically different boundary conditions once the box is assembled, continue to be important for makers, largely because it is so much easier to optimise their properties before than after assembly. To a first approximation, the modes of the plates on the fully assembled instruments will be determined by the same elastic parameters and densities of the plates when freely mounted.

3. From free plates to body shell

In an earlier paper [2], FEA computations were used to investigate the influence of geometric shape, plate thickness and graduation scheme, the arching heights and profiles, and the anisotropic along-and cross-grain orthotropic properties of spruce and maple. It was shown that the modes of freely supported rectangular plates could be smoothly morphed into those of the guitar-shaped plates of the violin. Although changes in geometry result in significant changes in mode shapes and frequencies, the number of modes below a given frequency remains almost unchanged. This follows from the 2-dimensionality of the mode shapes and the relationship between frequency f and wavelength λ of flexural waves on thin flat plates of thickness t, $f\lambda \propto c_L \left({}^{t}/_{\lambda}\right)$, where c_L is the speed of longitudinal waves in the solid. Unlike normal sound waves, the speed of the flexural waves, responsible for radiating sound, is not constant, but is proportional to the ratio of plate thickness to wavelength. Whereas early string instruments tended to use flat plates, later instrument increasingly used more rigid arched plates to support the downward pressure of the strings stretched over the supporting bridge. Higher stiffness could be achieved with lower mass, making it more acoustically and structurally efficient. The important low frequency body shell modes were then increasingly dominated by the arching height rather than plate thickness [2].

Wood such as spruce, typically used for soundboards, has exceptionally large anisotropy of the along- and cross-grain Young's moduli, often as large as 20. Perhaps surprisingly, this anisotropy has only a weak influence on the low frequency, signature modes (e.g. where the bending wavelengths are commensurate with or larger than the plate dimensions)**,** for a given geometric mean of the orthogonal moduli. This is largely because the wave shapes are 2-dimensional and must therefore always involve an average of the along and cross -grain elastic constants [11]. However, recent investigations [4] have shown that, at higher frequencies, the properties of smaller localised regions, like the island area between the *f*-holes, C-holes or slots, are dominated by the weaker cross-grain Young's modulus.

Figure 2 traces the transformation of the modes of the freely supported top and back pates, with the first few modes tuned to rather similar frequencies, as the rib strength supporting their edges is varied from around a millionth (effectively freely supported) to a more typical normal value [3]. This is easily achieved on a computer, but clearly impossible in practice! But such large changes provide very valuable information on how the ribs influence the frequencies and mode shapes of real instruments.

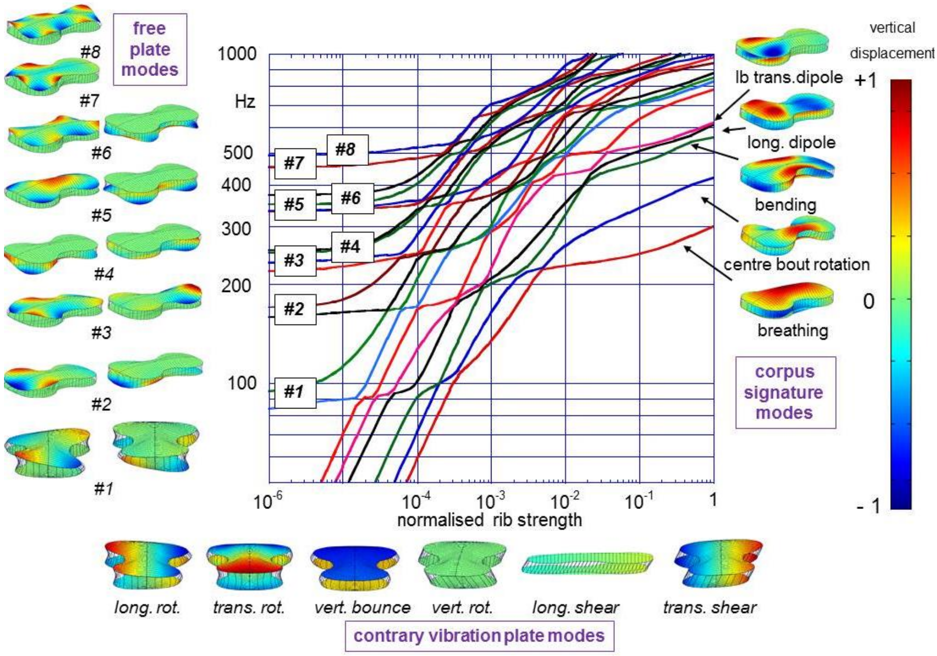

Figure 2 The influence on the modes of the rib-coupled plates, as the rib coupling strength around the plate edges is increased from 10^{-6} to a typical normal value.

Figure 2 also shows many of the generic features common to all shallow, box-like instruments, such as the vihuela and lira da braccio, as well as other members of the violin family. Firstly, almost all the low frequency, initially freely supported, top and back plate modes, grouped on the left-hand side, are dominated by what we might call *flexural wave edge states*, which are unique features of flexural waves [2]. Such modes decay rather quickly towards the inside area of the plate, other than mode #5 (*ring mode*), which has a large amplitude at the centre of the plate. The freely supported top and back plates were tuned to within a semitone or so of each other, by choosing appropriate physical properties on performing FEA computations. The plates of old

Cremonese instruments are usually quite closely matched. It therefore seems very likely that a method for tuning was employed, presumably by tapping and listening. However, there is no contemporary written evidence, nor is enough known about earlier types of instruments to make any useful speculation.

4. Body shell, bridge and island.

Despite the very strong and complicated influence of the rib coupling on the coupled plate modes, for ribs of normal strength, there are only a small number of relatively simple low frequency modes of the body shell as illustrated in Figure 3. Similar modes are to be expected for all instruments of the violin family, though not necessarily in the same frequency order. Such ordering depends critically on their individual geometry, plate thicknesses, arching and wood properties, but the individual mode shapes and frequencies transform s moothly as these parameters are varied.

Of this set of generic modes, the volume-changing "breathing" mode is acoustically the most important. For members of the violin family, this mode is responsible for almost all the radiated sound over the first 2-octaves of the instrument, either directly or by its coupling to otherwise only weakly radiating modes [3, 12]. The one exception is its coupling to the Helmholtz cavity air mode with air bouncing in and out of the *f*-holes. This important mode is present in all historic and modern instruments with open hole or holes cut into their body shell. For members of the violin family, the vibrations of air through the *f*-holes strongly supports the sound radiated at low frequencies below the *breathing* mode resonance, which would otherwise radiate very weakly at lower frequencies. One of the interesting aspects of our present research on older historic instruments is to investigate how the rose-holes or slots cut into their top plates perform a similar function.

In addition to the formation of the A0 mode, the coupling of the breathing mode to the Helmholtz f-hole resonance also significantly increases the frequency of the breathing mode. As a result, for the violin, its frequency approaches that of an anticlastic bending mode of the body shell (bending with opposite signs along the length and across the width). For a pair of arched plates with different material properties and arching profiles, this results in a strong coupling leading to a pair of modes B1- and B1+**,** illustrated by prominent resonances in Fig. 1, with the breathing and bending mode components of the two normal modes vibrating either in the same or opposite phase relationship.

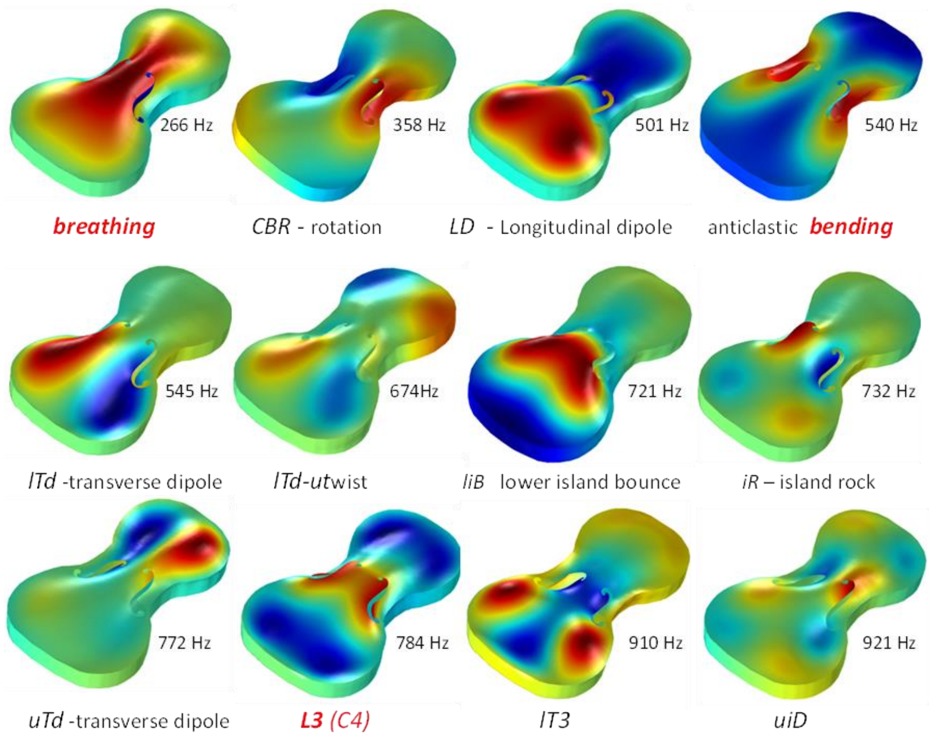

Figure 3. The first 12 computed modes of an empty violin body shell illustrating on a greatly exaggerated scale the vibrations of an empty violin body shell (no neck, bridge, bass bar or soundpost), with a possible nomenclature to describe the symmetry of the modes.

The initial *breathing* mode, responsible for almost all the sound radiation, is now shared between these two modes by amounts determined by the relative frequencies of the two coupled *component* modes and the strength of their coupling [3]. Because the coupling depends on the arching, it is unlikely that it occurred in instruments with flat plates, though types of corpus bending might well couple to other plate modes. High ribs, as in cellos, double basses and members of the viol family, raise the frequencies of the *bending* and *CBR* modes. Instruments such as the viola da gamba, with an arched top but flat back **plate**, have only a single, strongly radiating, *B1-* resonance. Cellos can have a *B1+* mode but it usually has small amplitude and a small breathing component; the *CBR* mode is often strongly coupled to the *B1- breathing*.

Most early instruments had a symmetric body shell structure with its vibrational modes therefore either symmetric (e.g. the *breathing* mode) or antisymmetric (the *CBR* and transverse dipole) modes, as illustrated in Figure 3. Bowing the strings in a transverse direction on a centrally mounted bridge will cause the bridge to rock side to side on its two feet, exciting only the antisymmetric modes, which only radiate weakly at low frequencies. Deliberately introducing some internal asymmetry into the design of stringed instruments was therefore a major development, as the asymmetric rocking of the bridge could then excite the strongly radiating *B1-* breathing and induced *A0* modes [6]. This enhances the warmth and richness of the sound of the violin family instruments over their lowest two octaves. The asymmetry may have been achieved in early instruments by asymmetric graduation of the top plate, but later by the introduction of the offset bass bar and, most importantly, the soundpost [13]. The first literary reference to a soundpost (indeed, the first of any evidence) is *James Soundpost*, a comic musician in Shakespeare's *Romeo and Juliet*, written around 1591. This suggests that soundposts were novel in England at that time, but were presumably introduced somewhat earlier. A reasonable assumption could be around 1550, when Andrea Amati was developing his violin family of instruments. There is no reason to assume that the soundpost was immediately adopted in all instruments that could potentially benefit from their presence: it is far more likely that older and newer technologies and traditions coexisted for some decades.

Another important feature in the design of stringed instruments was the elongated slots, open round holes, fretted rosettes and *f*-holes cut into the top plate. As indicated above, air bouncing in and out through these holes results in a strongly radiating A0 mode, which strongly boosts the sound at low frequencies. For bowed rather than plucked string instruments, the slots, C-holes and *f*-holes also create a highly flexible *island* area between their edges. In older instruments the bridge, supporting the strings, tended to be placed either just above or below the island area, often with strengthening bars underneath the top plate – as in the lira da braccio illustrated later. In modern violins, the standard position is close to the centre of the island area, opposite the notches, midway along the *f*-holes, whereas for early violins and closely related instruments, the bridge was often mounted below the island area. The most commonly offered explanation for this practice is that the vibrating string length was increased to facilitate playing at a lower pitch.

For members of the violin family tall bridges with curved tops optimise the intensity of sound that can be radiated, especially when mounted on the flexible island area with both a bass bar and an offset soundpost wedged between the top and back plates. However, the coupling between the bowed strings and vibrational modes of the bowed strings and the individual body shell vibrations must not be too strong, otherwise it results in the infamous *wolf-note*, producing a croaking or warbling sound, when it becomes impossible to bow a steady note [5]. Optimising the coupling between vibrating string and the body of an instrument is one of the many skills an expert violin maker employs to influence both sound quality and playability of an instrument. This is achieved by choosing an appropriately cut bridge, an appropriate thickness and arching for the top and back plates , especially in the island area, height and profile of the ba

ss bar and soundpost position and choice of strings. All such adjustments af fect the loudness, the playing feel and the timbre.

From an acoustical point of view, the position of the bridge is clearly a major factor in determining the coupling between the vibrating strings and radiating surfaces of the body shell. Mounting at the centre of the island will clearly set up the strongest vibrations originating in this area and coupling to the radiating modes of the upper and lower bouts. This position also allows the strongest possible bowing of the outer two strings resting on a curved bridge top, without the bow hair hitting the inner waist edges. The waist ultimately became very useful in this respect but before the mid-1500s bridge placement does not seem to have been much influenced by the outline. In the more intimate performance spaces of earlier times, when radiated sound power was less important, moving the bridge to different positions below and inside the length of the island could have provided not only a way of adjusting the length of the strings for different pitches, but also the quality of sound produced by the instrument. Furthermore, if there were no soundpost, placing the bridge outside the island would both reduce the *wolf* problem and avoid the island sinking under the force at the bridge feet.

The bridge plays a very important acoustical function in all stringed instruments, ancient and modern. For plucked instruments with the bridge glued to a flat soundboard the bridge had to be low or the torsion would have torn it away from the top plate, as is still true today. Many bowed instruments also had low bridges but the iconography also shows some rather tall ones, particularly on lira da braccio. The much larger resolved downward force of the string tension would, if the bridge were placed at the middle of the island, result in a short life for the instrument, unless the plate was very thick. Placing it at one end of the island, probably with the addition of a transverse bar, would solve both a structural problem and help minimise the potential *wolf* problem. Plucking the strings at an angle to the top plate gives a strong attack to the plucked note from string vibrations perpendicular to the top plate, by coupling strongly to the radiating plate vibrations, and a long ringing sound from the only weakly coupled string vibrations parallel to the plates – just like the sound of a guitar, spinet, harpsichord or piano. Bowing an instrument with a low bridge and no island cannot excite either symmetric or antisymmetric modes effectively, so the sound will be very weak. Presumably, when this was done, there was no expectation or perceived need of a powerful sound.

5. Historical

Iconography is of major interest to researchers of historic instruments because it can potentially corroborate other sources and, in some instances, may be the only source. It can also be misleading; there could be an element of invention on the part of the artist or simple inaccuracy due to lack of knowledge of musical instruments. Artists sometimes copied earlier works, or conjured notions of traditionally idealised pastoral scenes or ancient Hellenic culture, all of which undermine the reliability of their work as historical evidence. For this reason it is unwise to build an argument on evidence based on sparsely represented instrument types. On the other hand, depictions of some instruments are so common that collectively they can be considered a robust source. For example, from the early 1600s violins and violas da gamba appear frequently and often in fine detail such that, in conjunction with written sources, it is possible to plausibly reconstruct the setup and stringing practice. A method for making historically convincing strings has now been in use for some decades and has a wide acceptance among musicians. Research followed up by a physical reconstruction has been the main way that knowledge of historical instruments has progressed. The methodology presented by the authors offers an additional tool that may help to circumvent a lack of information or ambiguity in the more usual sources.

The Moorish occupation in Spain lasted for nearly 800 years. Spain also had a prosperous Jewish community, the largest in Europe, with many educated and skilled people. It was a very important region for cultural exchange and innovation and continued to be a magnet for musicians and artisans up to the financial decline of the Spanish Crown around 1600. Culture and musical instrument technology also spread by many other means, such as via military expeditions and travelling merchants. For example, the ancient Sutton Hoo lyre and Trossingen lyre appear to be very similar (Figure 4a). They were court instruments, with elaborate ornamentation. The lyre may have arrived in England with the Saxons but similar instruments could also have been introduced by the Romans. Of particular interest, these instruments were made by carving the back, sides and arms from a single piece of wood. The beam between the ends of the arms is jointed and held the tuning pegs. The same approach was taken for making the lira da braccio, except the pair of arms is replaced by a neck and peg box (fig 4c and d) and was bowed instead of plucked. Instruments like these liras are ubiquitous in the iconography from mediaeval times but modern reconstructions are rare. The reason is probably that, although such instruments were common and culturally important, there is no surviving repertoire and nobody really knows how to play them.

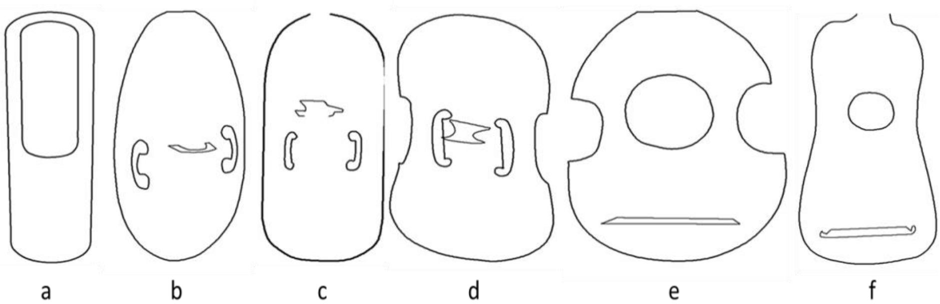

Figure 4. Outline shapes for early instruments (a) Sutton Hoo lyre reconstruction, (b) oval, round-backed lira, (c) from van Eyck altarpiece, 1436, (d) Giovanni Bellini, San Zaccaria altarpiece, 1505, (e) a vihuela with deep C bouts, (f) more typical vihuela from mid 1500s.

There are examples of instruments that have an oval outline and a rounded back, perhaps carved or with bent staves like a lute (Figure 4b and 5 left). These were possibly very like the early kind of oud shown in Spanish iconography. Sometimes they were also bowed. Another ancestral line from the early 1400s, perhaps originating in Spain, was instruments assembled with glue from pieces of wood planed and scraped to thickness, and with the ribs bent to shape. The carved-body liras had little or no waist but those made with bent ribs often had deep C bouts and a fixed bridge (Figure 4e and 5 centre), to which the strings were tied. Instead of a pair of holes as seen in the oval, oud-like, round-backed example (Figure 5, left), there was a round hole some distance from the bridge.

Before 1500 the plucked and bowed forms of the viola were diverging and developing features more advantageous for either plucking or bowing. The plucked type became dominated by the elongated guitar-like shape, with the bridge far down in the lower bout and a round rosette above the waist. By the mid-1500s, *vihuela* meant this type of instrument (Figures 4f and 6, right).

Figure 5. Left, from Cantigas de Santa Maria, 13th century, centre, Spain, late 1400s, rights, Bartolommeo Montagna, Milan 1550

In the bowed versions, a pair of slot-like holes was placed near to the bridge allowing for more rotation of the bridge, similar to liras, which had always had elongated holes and an island. Also merging with the carved lira tradition, the strings were attached, via a tailpiece, to the end of the corpus. This made possible a bridge that could be much taller and could be moved to different positions along the length of the instrument (Figure 6, left).

Figure 6. Viola da braccio, Italian, 1544 (left), vihuela, Luis de Milan frontispiece, 1535 (right)

The naming of instruments from this time is fraught with confusion. Vihuela is simply the Spanish word for viola and the two were, at first, interchangeable. Vihuela or viola denoted a category of instruments with flat, wooden plates, bent ribs and an elongated neck. A viola da mano was held like a guitar and plucked with fingers or a quill, whereas the lira, viola da braccio and viols were played with a bow and held in various ways. A lira da braccio might take the oval, carved or deep-waisted form. A viola da braccio differs only in not having the one or two un-fingered strings of the lira and usually only four strings. A viola da gamba, held on the knees or between the legs could also take a large variety of forms, few of which survived long into the 16th century. These names do not directly define the precise construction of these instruments, which probably concerned the musicians less than their particular musical usage. Rather similar looking instruments might be employed for a variety of purposes, while some that were significantly different could be employed for the same purpose. We can see in the iconography bridges with flat tops and different amounts of curvature. When flat, the playing style may have been like a hurdy-gurdy, all strings sounding at once, but with the possibility of fingering a melody on one or more outer strings. Liras with some bridge curvature and five to seven strings would suit an arpeggiated style, while violas da braccio, with only 4 strings could probably allow single, melodic notes.

Arched plates appear in the iconography from around 1500. The Bartolommeo Montagna picture from 1500 (Figure 5, right) shows a lira da braccio with an arched top. If it also had an arched back, which cannot be seen, it would have almost had the essential design features of a violin and the potential to benefit from an offset soundpost, though is unlikely that it had one. The ribs are carved, which we can infer from the way they are scalloped, so it is from the carved lira tradition, but with elements from the bent rib, viola tradition.

An overview of the 16th century suggests a trend towards greater depth in instrument sound. New types of bass strings became available which were more flexible and stretchy, making it practical to use higher tensions or lower pitches without increasing the vibrating length. It may have been plucked instruments, such as lutes and vihuelas, which first took advantage of this innovation since no major design changes were needed. The direction taken with bowed instruments may well have not been driven by a desire for greater depth at all. Measurements demonstrate that the island area provided a means of getting more high frequency energy into the corpus by exciting centrally antisymmetric modes (Figure 7, 4th from left). Antisymmetric modes, while poor low frequency radiators, radiate well at higher frequencies, where the radiated energy becomes highly directional, but proportional to the vibrational amplitudes of the body shell. The technology that allowed for massively improved low frequency response in bowed string instruments was coupling of centrally antisymmetric modes to symmetric *breathing*, or volume-change modes (as described in section 4). The asymmetry enforced by the offset bass bar and soundpost made this possible.

6. Measurements and computations on historic instruments

In this section, we present preliminary measurements and computations made on a number of modern reconstructions of historic instruments. We were able to find examples that are, arguably, representative of the instrument types and features that we have discussed above. We were interested in generic types so that we could apply FEA modelling and experimental modal analysis to explore generic behaviours such as for corpora with flat plates but different outlines and soundboards with and without islands. This approach provides insight into the acoustic function and evolution of historic instrument at the same time as enhancing knowledge of violin acoustics. As this was a preliminary study it was decided to do the FEA modelling without attached neck for the sake of simplicity. Work with violins with and without necks shows that the inertia of the neck can lower the frequency of the bending mode and that neck/ fingerboard modes can interact with corpus modes splitting them into a mirrored pair. The authors believe that it is good practice to try to understand a simpler state and add the effects of substructures at a later stage. The effects of the presence of a neck can be seen in the measured data.

6a. Viola da gamba

The authors have performed experimental modal analysis on two violas da gamba and have conducted a preliminary generic FEA investigation. We will briefly describe the measurements on just one of the instruments. This was made by Stoppani in the early 1980s using a model after the John Rose family from around 1600. It has a flat back with no cross-bars and an arched top made of 5 bent staves - a construction used by a number of English makers around that time. There is a small bass bar and a soundpost. The corpus length is 43.5 cm, like a large viola, but with twice the rib height at 8.8 cm. A flat back double bass has a very similar construction with rather similar modal behaviour. As with the cello and double bass, the taller ribs significantly increase the frequency while reducing the amplitude of the corpus *bending* modes. In this gamba, there is no evidence for the usual *CBR* mode always seen in the violin and viola, though the generic corpus bending modes can still be found at much higher frequencies (relative to the breathing mode) but coupled with higher order plate modes. As with all arched plates, the rib edges adjoining the top plate can move inwards and outwards, as the out-of-plane flexural vibrations strongly induce in-plane longitudinal radial and peripheral stretching and compressions around the edges.

There are two regions of significant volume changes from the flexural vibrations, which will result in strong radiation. One is the *A0* mode at around 140 Hz and the other is a single *B1 breathing* mode at around 250 Hz. There is no *B1+* mode.

6b. Lira da braccio

This instrument on which measurements were made is a reconstruction based on many examples found in the iconography. It appears to have been made by a competent, though probably not professional, craftsman. Due to a variety of setup issues it was decided to perform the modal analysis without strings, with an equivalent driving point position close to where the left foot of the bridge would traditionally have been. It serves as an illustrative example of a given type of instrument, as described above and shown in Figure 4, c and d. Of particular interest is the very large island area in the top plate bounded by long slots with two transverse bars positioned under the soundboard 2 to 3 cm, one above and one below the island area.

Surprisingly, these bars have a relatively small influence on the low frequency mode shapes and frequencies - much less so than holes or outline features that determine the boundaries within which the flexural waves have to fit. Although the island area occupies a much larger proportion of the top plate than that of the violin, there is still a close similarity with several of the modes of the violin, illustrated in Figure 3.

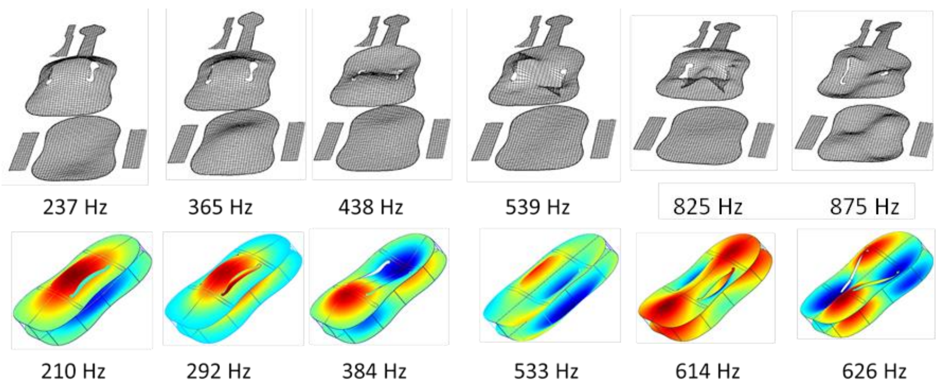

Figure 7. Comparison of experimental modal analysis measurements on a Lira da braccio with unknown elastic properties and FEA in vacuo computations using guessed elastic properties.

Although the elastic properties of the measured lira da braccio were unknown and had to be guessed and that these preliminary FEA computations were made *in vacuo*, Figure 7 illustrates relatively strong agreement between the computed and measured frequencies and shapes for some, but not all these first few modes. By modifying the parameters used in the FEA computations a much closer comparison will almost certainly be achieved. In both experimental and FEA computations many of the higher frequency modes are observed to be strongly localised in either the top or back plates. This is because the two plates will resonate with nearly identical modes shapes but at different frequencies. Only when their frequencies are closely matched and the excited modes share common symmetry elements will the vibrations in both top and back plates be strongly excited together. As with the violin family, this tends to happen more easily at lower than higher frequencies, when the vibrational modes can also be primarily excited in either the upper of lower bouts of either the top or back plates.

Interestingly, the measured *A0 f*-hole resonance at 237 Hz has no obvious single *B1- breathing* mode with plates vibrating in opposite directions to drive it. In contrast to the violin, there is a relatively strong *anti-breathing* mode (both plates moving in the same direction) measured at 365 Hz and computed at 292 Hz. Because the top and back plates have significantly different thicknesses and other properties, this mode has a net volume change, which might be responsible for exciting the observed A0 resonance. In addition to measuring mode shapes and frequencies, in both the experimental and computer modelled investigation, it is easy to compute the net volume change of the empty shell when driven from a given point. The rate of change of volume is directly related to the monopole radiation source responsible for radiating sound uniformly when the size of the instrument is less than the acoustic wavelength. This information can then be used to provide a realistic estimate of how the instrument would probably have sounded in the lower frequency range.

6c. The Vihuela

Iconographic evidence reveals a great many instruments with shallow boxes and deep C bouts, like the vihuela, which was sometimes plucked and sometimes bowed (viola da mano and viola d'arco, in Italian). We have investigated the modes of a copy of such an instrument made by an able student in 1980. The depth and size of the C bouts is nearing the extremity of what we observe in the iconography. There is a round hole at the middle of the waist with a parchment rosette.

As an example of the kind of modes that would be expected on an instrument of this shape, Figure 8 shows the first eight modes computed for the *in vacuo* modes of a generic model of the vihuela. The model has the same geometry as the measured instrument, with anisotropic elastic constants chosen to give reasonably close matching of the freely supported plate frequencies.

These computations emphasise the difference between the modes of instruments having an island area, formed between slots, and instruments with an open hole between the waists. A major effect of the low rib height and deep C-bouts is to lower the frequencies of the corpus bending modes permitting coupling to the low order plate modes and to the rigid body and bending modes of the neck structure. There are now two *breathing* and two *anti-breathing* modes with volume changes largely localised within the upper and lower bouts. Because of differences in top and back plate properties all such modes will have a net volume change, which could excite the *A0 f*-hole air resonance (not shown here because the computation at present are *in vacuo*).

In addition, there is a mode surrounding the round hole, which is closely similar to the *C4* mode of the violin, with both longitudinal dipole modes and transverse dipole modes in both the lower and upper bouts. The attached neck is likely to significantly affect the low frequency modes localised in the upper bout.

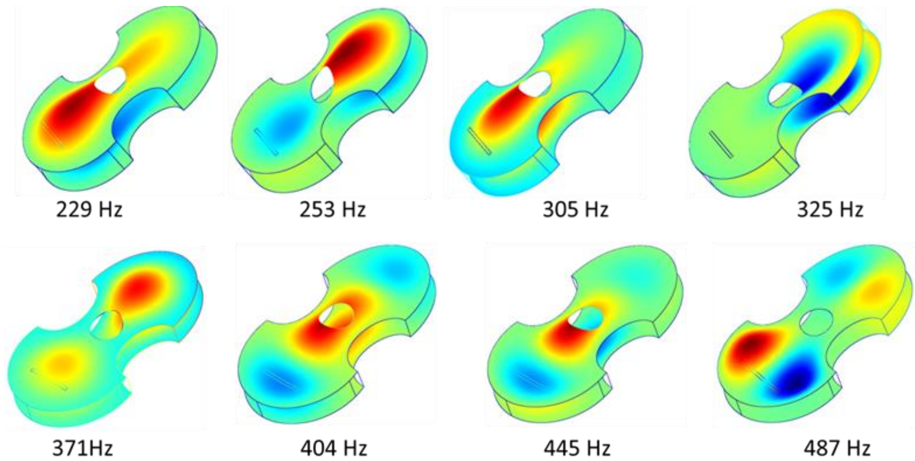

Figure 8. The first *in vacuo* eight computed body shell modes of a generic vihuela model without attached neck and peg arm.

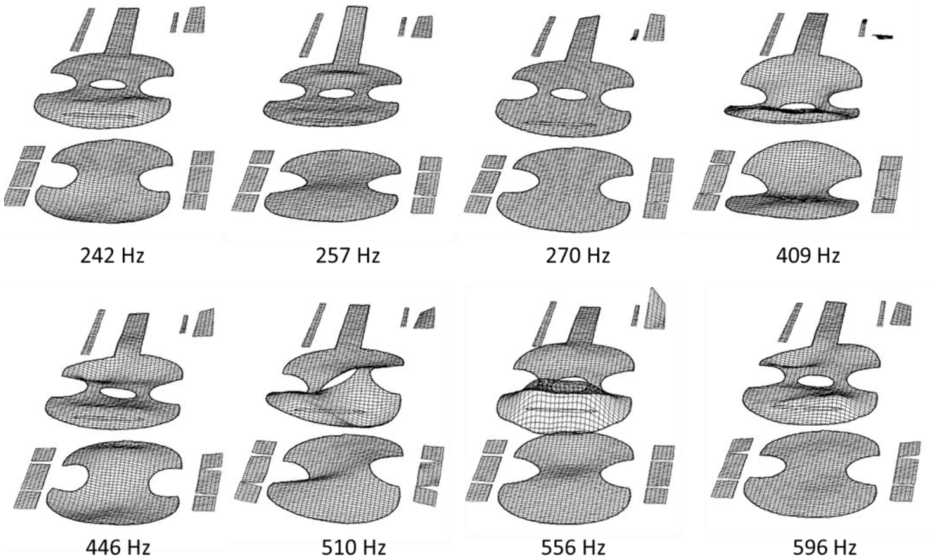

Figure 9. The first eight measured modes of the vihuela.

Figure 9 shows modal amplitudes measured for the lowest frequency modes of the vihuela. The agreement between the computed generic model predictions and measured modes is not as good as those of the lira da braccio. This is probably because these preliminary computations lacked the rather heavy neck and were performed *in vacuo*, which would have changed the frequencies of any volume-changing breathing mode, but not significantly the modes shapes. In addition, the measured vihuela had two strong strengthening bars above and below the open hole, which had not initially been discovered because access to the inside was limited by the parchment rosette covering the hole.

6. Conclusions

Significant progress has been made towards understanding the vibrational and acoustic properties of a number of different designs of historic instruments, through applying our increasingly detailed understanding of the violin family. While experimental modal analysis provides the means to fully characterise the acoustic and vibrational properties of historic instruments, computer aided finite element analysis offers valuable insights into the observed properties. Due to the rarity and fragility of surviving instruments modal analysis is normally performed on modern copies and sometimes the only source of information is iconographic. The longer term aim is to support people researching and building reconstructions to explore the possible sounds and playing characteristics of less well understood historic instruments.

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