# Financial Market Risk Perceptions and the Macroeconomy

## Internet Appendix

Carolin Pflueger* Emil Siriwardane† Adi Sunderam‡

**Abstract**

In this appendix, we provide details about the data construction for all variables used in the main text. We then present a battery of tests and additional analysis demonstrating the robustness of the relationship between the real rate and $PVS_t$. In addition, we show that roughly 90% of the covariation between the real rate and $PVS_t$ stems from the fact that the real rate forecasts future returns on the vol-sorted portfolio. We also relate PVS to objective and subjective measures of expected risk for aggregate macroeconomic variables and the aggregate stock market, showing that PVS is related to expected risk, and that this connection is most evident for subjective measures of risk that reflect both public and private firms. Moreover, we offer complementary VAR and local projection evidence that shocks to risk perceptions, as measured by $PVS_t$, lead to a boom in the real economy. We also document that periods of high risk perceptions coincide with investor outflows from high-volatility mutual funds. Finally, we provide proofs for the propositions contained in the model section of the main text.

*Pflueger: University of Chicago and NBER. E-mail: cpflueger@uchicago.edu
†Siriwardane: Harvard Business School. E-mail: esiriwardane@hbs.edu.
‡Sunderam: Harvard Business School and NBER. E-mail: asunderam@hbs.edu.

# A1 Data Construction

In this section we provide details on how we construct our main variables. We then provide details on the variables used in each table of the main text.

## Construction of $PVS_t$

### Valuation Ratios

Our valuation ratios (book-to-market) derive from the CRSP-COMPUSTAT merged databases. We augment CRSP-COMPUSTAT with the book value data used in Davis, Fama, and French (2000). We provide additional details of our variable construction below, but at a high level our procedure is as follows: for a given firm $f$ on date $t$, we look for a valid value of book equity in COMPUSTAT Quarterly, then COMPUSTAT Annual, and finally the book values contained in Davis, Fama, and French (2000). We assume balance sheet information is known with a one-quarter lag. Finally, we combine the aforementioned book value with the trailing 6-month average of equity market capitalization to form a book-to-market ratio for firm $f$. We have confirmed that our results are not sensitive to these variable definition choices.

*COMPUSTAT Quarterly*: From COMPUSTAT Quarterly (COMPQ). Specifically, we obtain information on all firms (INDFMT = INDL) with a standardized data format (DATAFMT = STD) that report financial information at a consolidated level (CONSOL = C). In order to avoid the well-known survival bias in COMPUSTAT, we only include firms once they have at least 2 years of data.

We define book common equity (BE) according to the standard Fama and French (1993) definition. Specifically, BE is the COMPUSTAT book value of shareholder equity, plus balance-sheet deferred taxes and investment tax credit, minus the book value of preferred stock. We use the par value of preferred stock in COMPQ to estimate the value of preferred stock.

*COMPUSTAT Annual*: When using COMPUSTAT Annual (COMPA) for balance sheet information, we obtain information on all firms (INDFMT = INDL) with a standardized data format (DATAFMT = STD) that report financial information at a consolidated level (CONSOL = C). In order to avoid the well-known survival bias in COMPUSTAT, we only include firms once they have at least 2 years of data. For firms that change fiscal year within a calendar year, we take the last reported date when extracting financial data. This leaves us with one set of observations for each firm (gvkey) in each year.

We define book common equity (BE) according to the standard Fama and French (1993) definition. Specifically, BE is the COMPUSTAT book value of shareholder equity, plus balance-sheet deferred taxes and investment tax credit, minus the book value of preferred stock. Following Fama and French (1993), we use the redemption, liquidation, or par value (in that order) to estimate the value of preferred stock.

*Defining Valuation Ratios*: We then build book-to-market ratios at end of each quarter $t$ as follows:

- The book equity comes from COMPQ, and we assume this data is known with a 3-month lag. This means we add three months to the DATADATE field in COMPQ to define the

"KNOWNDATE". Then at the end of each quarter, we take the book equity on the last available KNOWNDATE. For instance, this means that in June of a given year, we are using the book value of equity from COMPQ as of March in that same year. We prefer this definition because it uses up-to-date balance sheet information, while still allowing for reasonable lags to ensure the information was actually known by market participants at each date.

- If COMPQ does not have a valid book value, we obtain book equity from COMPA, again assuming a one-quarter information lag for balance sheet information. If COMPA also does not have a valid book value for a firm, we check the book equity values from Davis, Fama, and French (2000), which we downloaded from Ken French's website. For the book equity in Davis, Fama, and French (2000), we use their assumption that book values are known as of June 30 of the "Last_Moody_Year" variable.

- For the purposes of computing book-to-market ratios, we use the trailing 6-month average of market capitalization using CRSP Monthly. For instance, in June of a given year we take the average end-of-month market capitalization from January through June of that year. We prefer this definition because it smoothes out any high-frequency movements in equity valuations.

Book-to-market ratios for a given firm then follow naturally. We have also used the Fama and French (1993) definition of book-to-market ratios and obtain very similar results. Fama and French (1993) assume a more conservative lag in terms of when balance sheet is known and also use lagged market capitalization (e.g. in June of a year, use the previous December's market capitalization).

### Volatility Used for Portfolio Sorts

At the end of each quarter, we compute each firm's stock return volatility as the standard deviation of ex-dividend returns (variable RETX) using daily data from the previous two months. We exclude firms that do not have at least 20 observations over this time frame. This approach mirrors the construction of variance-sorted portfolios on Ken French's website.[1]

### Computing PVS

At the end of each quarter $t$, we sort all stocks in the NYSE, AMEX, and NASDAQ into quintiles based on their total volatility. To be included in the portfolio sorts, stocks must have valid return data for quarter $t+1$, meaning they either have three monthly return observations in CRSP monthly or they have valid delisting returns according to Shumway (1997). We enforce this restriction to harmonize forecasting regressions that use $PVS$ at time $t$ to predict returns at $t+1$ on the volatility-sorted portfolios. Total volatility is computed as described above. We then form equal-weighted portfolios based on the quintiles of volatility. Our measure of risk perceptions is defined as:

$$PVS_t \equiv \left(\overline{B/M}\right)_{low\ vol,t} - \left(\overline{B/M}\right)_{high\ vol,t}$$

[1] Our long-short portfolio effective replicates the one on Ken French's website. If we regress our portfolio on his, the point estimate is 0.84, the constant in the regression is statistically indistinguishable from zero, and the $R^2$ is 96%.

In words, $PVS_t$ is the average book-to-market ratio of firms in the low-volatility quintile minus the average book-to-market ratio of firms in the high-volatility quintile. Thus, $PVS_t$ is high when the market valuations of high-volatility firms is large relative to low-volatility firms.

Finally, we define the aggregate book-to-market ratio for our universe of firms as the their total book value divided by their total market capitalization at time $t$.

## A1.1 Table I - Summary Statistics for Volatility-Sorted Portfolios and the Real Rate

The one-year real interest rate is the one-year constant maturity nominal Treasury rate from the U.S. Federal Reserve minus the one-year expectation of inflation (GDP deflator) from the Survey of Professional Forecasters. We linearly detrend the one-year real rate for all of our analysis in the main text, though we show that our analysis is robust to no detrending and alternative detrending methods in Section A2 of this appendix.

## A1.2 Table II - PVS and Investor Risk Perceptions

This table looks at the contemporaneous correlation between $PVS_t$ and several measures of expected risk. Most of the measures are defined in the main text and in the caption in the table. Here, we elaborate on our risk expectation measures that come from analyst forecasts and option prices. Note that our portfolio-level risk measures take the difference between high and low volatility firms.

**IBES Based Measures** Two of the measures we use derive from analyst forecasts of earnings-per-share (EPS) that come from Thompson Reuters IBES data. More specifically, we use the unadjusted summary file from WRDS. Data in IBES is organized by firm $i$, estimation date $d$, earnings announcement date $u$, and earnings type $t$. The two earnings types that we consider are annual and quarterly. We require at least two analyst forecasts for each $(i,d,u,t)$. For this particular cut of the IBES data, we start the sample in 1989. Prior to 1989, the number of high-volatility firms that have a match in IBES fluctuates wildly, but steadily increases from 1989 onward.

For each firm $i$, quarter $t$, earnings announcement date $u$, we first select the last estimation date $d$ that occurs prior to $t$. We then define the quarter $t$ dispersion of firm $i$'s earnings at time $u > t$:

$$\sigma^s_{i,t}(EPS_u) = \frac{\text{Range EPS Forecasts}_{i,d}(u)}{\text{Median EPS Forecasts}_{i,d}(u)}.$$

This is our proxy for analyst time-$t$ expectations of earnings volatility at time $u$. We exclude firms where the median EPS forecast is zero. In addition, because $\sigma_{i,t}(u)$ can be large for low median EPS forecasts, we winsorize it at its 5% and 95% tails.

In the table, we consider two different forecast horizons $u$. First, for quarterly earnings, we select $u$ for each firm such that the earnings announcement corresponds to the next fiscal quarter (fpi = 6). We denote this case by $\sigma^s_{i,t}(EPS_{u=t+1})$. For annual earnings, we choose $u$ such that the earnings announcement corresponds to two fiscal periods from $t$ (fpi = 2). For our annual IBES measure, the average difference between $u$ and $t$ is five quarters, but it can vary depending on fiscal reporting periods and the availability of analyst forecasts. We denote this firm-level measure by $\sigma^s_{i,t}(EPS_{u=t+5})$.

Finally, $\sigma_t(\text{EPS}_{t+1})$ is the median $\sigma^s_{i,t}(EPS_{t+1})$ for high-volatility firms minus the median for low-volatility firms. $\sigma_t(\text{EPS}_{t+5})$ is assembled the same way from $\sigma^s_{i,t}(EPS_{t+5})$ at the firm level. In all cases, our classification of whether a firm is high or low-volatility at time $t$ matches the portfolios used to compute $PVS_t$.

**Option-Based Measure** Our options data derives from the Standardized Volatility Surface from OptionsMetrics on WRDS. For each firm, date, and horizon, the volatility surface contains at-the-money (ATM) put and call options. We define the expected return volatility on date $d$ for horizon $h$ as the average of the put and call implied volatilities. We denote this quantity by $\sigma^{IV}_{i,d}(Ret_{t,t+h})$. For this particularly analysis, we use $h$ = 4 quarters. Option-based measures of expected volatility typically use the entire spectrum of option strike prices (i.e. the VIX). Due to the relative scarcity of out-of-the-money options, especially for the high-volatility firms in our sample, we instead use ATM implied volatilities.

For each firm $i$ and quarter $t$, we find the last available $\sigma^{IV}_{i,d}(Ret_{t,t+4})$ from the OptionMetrics database, requiring that the implied volatility was computed no more than 21 days prior to $t$. To aggregate to the portfolio level, we take median $\sigma^{IV}_{i,t}(Ret_{t,t+4})$ for high-volatility firms minus the median of low-volatility firms. The resulting variable is what we define in the table as $\sigma_t(Ret_{t,t+4})$. This measure begins in 1996Q3 because, prior to this date, we do not have any matches in OptionsMetrics for our high-volatility firms.

**Statistical Forecasting Model for "Objective Measure"** The variable "Objective $\sigma_t(Ret_{t,t+1})$" comes from a simple statistical forecasting model. Define the average realized volatility of high-volatility stocks in the portfolio at time $t$ as $rv_{H,t}$, where each firm's volatility is computed as the daily standard deviation of returns in quarter $t$. $rv_{L,t}$ is the same object for low-volatility firms and $rv_t \equiv rv_{H,t} - rv_{L,t}$. We fit an AR(1) process to $rv_t$ using the full sample of returns. The estimated AR(1) coefficient for this series is 0.92, so $rv_t$ is relatively persistent. The AR(1) model also fits the data well in terms of forecasting, as a simple regression of $rv_{t+1}$ on $rv_t$ yields an $R^2$ of 85%. Finally, the variable Model-Based $\sigma_t(Ret_{t,t+1})$ is defined as the $\mathbb{E}_t[rv_{t+1}]$ that emerges from the AR(1) model.

## A1.3 Table III - The Real Rate and PVS

The caption contains complete details on the variables used in the table.

## A1.4 Table IV - Robustness

### Panel A

Panel A of Table IV in the main text compares $PVS_t$ to other measures of financial market conditions. Most of the variables are described in the caption of the table. Here, we focus on our measure of the time-$t$ expectation of excess aggregate stock market returns from $t$ to $t+4$, denoted $\mathbb{E}_t\,[\text{Mkt-Rf}_{t,t+4}]$. We obtain a statistically optimal measure of expected excess returns following the methodology developed in Kelly and Pruitt (2013). Specifically, we use the three-pass regression filter (3PRF) in Kelly and Pruitt (2013). In particular, we use the entire sample to estimate the 3PRF and assume two latent factors. In our experimentation with the procedure, using two factors balances the desire to have a good in-sample predictor of market returns against overfitting.[2]

The variables that we use as the predictors in the Kelly and Pruitt (2013) procedure are five BM ratios from sorting on each of the following variables: size, BM ratios, cash-flow duration (Weber (2016)), leverage, cash-flow beta with respect to aggregate cash flows, leverage, beta with respect to the aggregate market (using a 5-year window and a 10-year window), and total volatility. We construct BM ratios based on these sorts in the same way we do for $PVS_t$. In addition, we include the aggregate BM ratio, aggregate dividend yield, and CAY from Lettau and Ludvigson (2004). This gives us a total of 43 predictors that we feed into the 3PRF to forecast annual excess market returns. The $R$-squared in the forecasting regression is 14.2%. As a point of comparison, we are able to nearly double the forecasting power (in-sample) of CAY alone, which gives a forecasting R-squared of 7.5%. The sample size for this analysis is 180, and is lower than our main sample ($N = 185$) because the duration sorted portfolios that we include as predictor variables have a shorter sample.

**Panel B**

See the main text and the table caption.

## A1.5 Table V - PVS, the Real Rate, and Future Returns

The table uses $PVS_t$ and the one-year real rate to forecast returns and earnings surprises. Columns (1) and (2) forecast stock returns on the low-minus-high volatility portfolio. Columns (3) and (4) forecast this portfolio's accounting return on equity (ROE), which is defined according to the clean-surplus accounting formula from Cohen et al. (2003). For each firm $i$ and date $t$, we compute future annual ROE based on the next four quarters of financial statements after date $t$. Financial statement information is from the COMPUSTAT quarterly file. Because firms have different reporting periods, the calendar time over which we compute annual ROE differs across firms. Once we compute the future annual ROE for each firm $i$ in quarter $t$, we aggregate to the portfolio level by taking the equal-weighted averages within each volatility quintile. Columns (5) and (6) uses $PVS_t$ and the real rate to forecast excess returns on the CRSP Value-Weighted Index, which we obtained from Ken French's website.

## A1.6 Table VI - PVS and Real Outcomes

See the main text and the table captions.

## A1.7 Table VII - PVS and Good News

We compute the trailing annual ROE (LMH-Vol $\text{ROE}_{t-4\rightarrow t}$) of the low-minus-high volatility portfolio in the same manner as described in Section A1.5. The variable Bank Net $\text{Chargeoffs}_t$ is

[2] We have tried a truly out-of-sample version of the 3PRF and obtain similar conclusions regarding the correlation with the real rate.

computed using data from bank call reports. Because banks have a one-year window over which they can report loan charge-offs, we define Bank Net Chargeoffs$_t$ as the average of the reported charge-off rate at time $t$, $t+1$, $t+2$, and $t+3$, though our results are qualitatively the same if we just use the reported charge-off rate at time $t$. All of the other variables in the table are defined in the caption.

## A1.8 Table VIII - PVS and Revisions in Expectations of Risk

Our analysis of revisions in expected risk builds off the variable construction described in Section A1.2. In row (1), the variable $\sigma_{t+2}(\text{EPS}_{t+3}) - \sigma_t(\text{EPS}_{t+3})$ proxies for the revision in expected earnings volatility for earnings at time $t+3$. Let's start with how we construct $\sigma_t(\text{EPS}_{t+3})$. At time $t$ and for each firm $i$, we find the set of IBES forecasts corresponding to earnings that are three fiscal quarters away (fpi = 9). Again, in calendar time, this ends up corresponding to quarterly earnings realized at $t+3$ for most firms. For this forecast horizon, we then build our dispersion measure, denoted by $\sigma^s_{i,t}(EPS_{u=t+3})$, as in Section A1.2. We also apply the same filters and methodology as described in that section. For each firm, we then hold fixed the date on which earnings will be realized and recompute our dispersion measure at time $t+2$. This delivers us $\sigma^s_{i,t+2}(EPS_{t+3}) - \sigma^s_{i,t}(EPS_{t+3})$, which is the news between time $t$ and $t+2$ about expected earnings volatility at $t+3$. Again, our working assumption here is that our dispersion measure is a good proxy for expected earnings volatility. To aggregate this the portfolio level, we take the median $\sigma^s_{i,t+2}(EPS_{t+3}) - \sigma^s_{i,t}(EPS_{t+3})$ for high-volatility firms in the portfolio at time $t$ minus the median for low-volatility firms. The resulting variable is defined as $\sigma_{t+2}(\text{EPS}_{t+3}) - \sigma_t(\text{EPS}_{t+3})$.

The measure $\sigma^{IV}_{t+3}(Ret_{t+4}) - \sigma^{IV}_t(Ret_{t+4})$ comes from options data. At time $t$ and for each firm $i$, we define $\sigma^{IV}_{i,t}(Ret_{t+4})$ as the implied volatility of stock returns in quarter $t+4$. We use the term structure of option-implied volatilities to compute this measure. Specifically, we take the implied variance of 365-day options at time $t$ and subtract off the implied variance of 273-day options at time $t$, which we then convert to an implied volatility measure.[3] This is a valid approach to estimating the option-implied expected volatility between in quarter $t+4$ so long as there is negligible return autocorrelation at the quarterly frequency. For each firm, $\sigma^{IV}_{i,t+3}(Ret_{t+4})$ is the 90-day implied volatility at time $t+3$. This allows us to construct $\sigma^{IV}_{i,t+3}(Ret_{t+4}) - \sigma^{IV}_{i,t}(Ret_{t+4})$ for each firm, which we then aggregate to the portfolio level by taking the median across high-volatility firms minus the median for low-volatility firms.

Finally, the realized risk measure $\Delta_4\sigma_{t+4}(\text{HML-Vol})$ is constructed as follows. For each firm in the high-volatility quintile at time $t$, we take its realized quarterly stock return volatility at time $t+4$ and subtract its realized quarterly stock return volatility at time $t$ based on daily stock returns within each quarter. We then average this difference across high-volatility firms and then repeat the entire process for low-volatility firms. $\Delta_4\sigma_{t+4}(\text{HML-Vol})$ is the resulting spread between the two groups and it measures the average change in volatility for high-volatility firms minus the average change for low-volatility firms.

[3] Implied volatilities from OptionMetrics standardized volatility surfaces are annualized, so we first translate them to annualized implied variances. We take the 365-day implied variance minus 0.75 times the 273-day variance. This provides an unannualized estimate of return variance between $t+3$ and $t+4$, which we then annualize by multiplying by 4 and then take the square root to arrive at an implied volatility measure.

### A1.9 Table IX - PVS and Implied Volatility Forecast Errors

See the main text and the table caption. For the time-$t$ implied volatility of returns between $t+3$ and $t+4$, we use the same firm-level measure $\sigma_{i,t}^{IV}(Ret_{t+4})$ defined in Section A1.8 of this appendix.

## A2 Robustness: PVS and the Real Rate

The purpose of this section is to conduct several robustness tests to ensure that our statistical inference regarding the relationship between the real rate and $PVS$ is not driven by specific choices in defining our main variables. We begin by discussing alternative methods of filtering the real rate (e.g. using a deterministic versus stochastic trend). We then show that our results are largely unchanged with these alternative filters or if we simply study the raw real rate. We then explore several ways of adjusting the standard errors in our regressions of the real rate on $PVS$ that account for the fact that these variables are persistent. The main takeaway of the section is that there is a robust relationship - both in economic and statistical terms - between the real rate and $PVS_t$. We then explore different variations of our definition of $PVS_t$ in the data and show that the relationship with the real rate is robust. Finally, we explore alternative stock characteristics and show that sorting by volatility is crucial for obtaining a robust relationship between valuation ratios and the real rate. For the remainder of this appendix, we use $R_t$ to denote the raw, i.e. non-detrended, real rate.

### A2.1 Filtering the Raw Real Rate

The top panel of Figure A.1 plots the raw real rate $R_t$ from 1970Q2 to 2016Q2. The downward trend in $R_t$ has received recent attention from many macroeconomists who argue that it reflects a form of economic secular stagnation (e.g. Summers (2015)). In this paper, we do not focus on the longer-run trend in $R_t$, but rather the large cyclical variation around this trend. Our goal is to better understand the determinants of cyclical (i.e., quarterly) movements in the real rate.

To achieve this goal, we need to empirically extract the cyclical component of the real rate. In the main text, we use a simple linear deterministic trend to do so:

$$R_t = \beta_0 + \beta_1 t + r_t \tag{1}$$

Here, the detrended real rate $r_t$ is just the sequence of residuals from the regression. We chose this approach because it is simple and transparent. Still, it is fair to wonder whether a deterministic (downward) linear trend is a plausible model of the economy's real interest rate. No economic theory would predict the real rate to tend towards negative infinity over the next fifty years. A natural alternative that we explore now is to allow for a stochastic drift in the real interest rate. In short, real rates look extremely similar whether we remove a linear or stochastic trend, consistent with the finding that it is extremely difficult to distinguish between deterministic and stochastic trends in finite samples (Campbell and Perron (1991)).[4]

[4] We think of the stochastic or non-stochastic drift as a simple way of controlling for long-run output growth. For example, Holston et al. (2016) embed this type of thinking in their statistical model of the natural rate of interest. They model the natural rate of interest as the sum of two random walks, one of which also drives the stochastic drift of potential output growth.

Specifically, we follow Hamilton (2017) to extract the cyclical component of $R_t$ in the presence of a potentially stochastic drift. For quarterly data, Hamilton (2017) recommends the following regression to achieve the filter:

$$R_t = k_0 + k_1 \times R_{t-8} + k_2 \times R_{t-9} + k_3 \times R_{t-10} + k_4 \times R_{t-11} + \widetilde{r}_t \tag{2}$$

where the cyclical component of $R_t$ is captured by the regression residuals, denoted here by $\widetilde{r}_t$. Importantly, this filtering methodology is relatively agnostic about the underlying trend driving the series.[5] This is particularly useful in our context because, again, we are not interested in understanding longer-run trends in $R_t$. Hamilton (2017) also provides an extensive argument for why regression (2) is superior to the more standard Hodrick-Prescott filter.

The bottom panel of Figure A.1 plots the linearly detrended real rate ($r_t$) and what we call the Hamilton-filtered real rate ($\widetilde{r}_t$). A visual inspection shows that $r_t$ and $\widetilde{r}_t$ are quite similar. That is, linearly detrending and using the Hamilton-filter appear to give similar estimates for the cyclical component of the real rate. A regression of one on the other, run in both levels and first-differences, confirms this intuition:

$$\begin{array}{ccccccc} \widetilde{r}_t & = & \underset{(0.01)}{0.002} & + & \underset{(8.57)}{0.71} & \times & r_t, \qquad R^2 = 0.56 \\ \\ \Delta\widetilde{r}_t & = & \underset{(-0.29)}{-0.01} & + & \underset{(40.45)}{0.99} & \times & \Delta r_t, \qquad R^2 = 0.85 \end{array}$$

where Newey-West $t$-statistics with five lags are listed below point estimates. Both specifications indicate that the linearly detrended real rate is fairly close to the Hamilton-filtered rate. The constant in both regressions is near zero, the point estimate on $r_t$ is near one, and the R-squareds are pretty large. As a result, we focus on the simpler, linearly detrended real rate in the main text and repeat our core analysis on the Hamilton-filtered rate now. To be certain that detrending (in any fashion) is not driving our conclusions, we also show our results using the raw real rate $R_t$ below.

## A2.2 Results Using $\tilde{r}_t$ and the Raw Real Rate

### A2.2.1 The Real Rate and PVS

Table A.1 shows regressions of the form:

$$Y_t = a + b \times PVS_t + \theta' \mathbf{X_t} + \xi_t$$

where $\mathbf{X}_t$ is a vector of control variables and $Y_t$ is either the Hamilton-filtered rate $\tilde{r}_t$ or the raw rate $R_t$. In all cases, we standardize $PVS_t$ to have a mean of zero and variance one. We do the same to the aggregate book-to-market ratio when it is included as a control variable.

**Results with $\tilde{r}_t$** Columns (1)-(6) run the regression for the Hamilton-filtered real rate, $\widetilde{r}_t$. The control variables that we use are the aggregate book-to-market ratio, the output gap, and the inflation rate. For consistency, we extract the cyclical components of these variables using Hamilton

[5] In fact, Hamilton (2017) argues that it is still a useful method for extracting the cyclical component of a series that has a deterministic time trend.

(2017) before including them in the regression. Echoing our results in the main text, the relationship between $PVS$ and $\widetilde{r}_t$ is robust across level and first-difference specifications, and is not altered much by the addition of our control variables. Column (2) adds the aggregate book-to-market ratio as a control to the regression, which has very little effect on both the point estimate on $PVS$, as well as the $R^2$ in the regression. Indeed, a univariate regression of $\widetilde{r}_t$ on the aggregate book-to-market yields an $R^2$ of less than 1%. In terms of economic significance, a one-standard deviation move in $PVS_t$ impacts $\widetilde{r}_t$ nearly three times as much as a one-standard deviation in the aggregate book-to-market. Column (3) of Table A.1 adds the output gap and inflation to the level regression of $\widetilde{r}_t$ on $PVS$. Again, we include these variables to check whether $PVS$ is just picking up on Taylor (1993) rule variables. The Hamilton-filtered rate does load positively and significantly on the output gap, which is what we would expect if the central bank follows some version of a Taylor (1993) rule. The important thing though is that the inclusion of these variables does not impact the point estimate or statistical significance of $PVS$ in the regression. The results using the Hamilton-filtered rate also compared favorably to those using the simple linear detrending in the main text.

**Results with $R_t$** Columns (7)-(9) repeat the analysis for the raw real rate $R_t$. Importantly, in this case, we do also not do any filtering to the control variables – these regressions only use raw variables. Column (7) runs a univariate regression of the raw real rate on $PVS$. The regression coefficient of 1.41 is economically comparable to the point estimate of we get when using the detrended real rate (see Table III in the main text). The R-squared is also comparable to our main results at 0.37. Columns (8) and (9) add the aggregate book-to-market and the output gap and inflation as control variables. While the aggregate book-to-market enters significantly, the R-squared in columns (7) and (8) is almost the same, indicating that the explanatory power of $PVS_t$ for the real rate is much stronger than that of the aggregate book-to-market. A univariate regression of the raw real rate on the raw aggregate book-to-market ratio delivers an $R^2$ of less than 10%, much less than when using $PVS_t$. More importantly, none of our conclusions regarding the relationship between the real rate and $PVS$ are impacted.

#### A2.2.2 The Real Rate and the Aggregate Stock Market

In the previous section, there were some specifications where the point estimate on the aggregate book-to-market ratio was estimated with some measure of statistical precision. Overall though, there is very little evidence suggesting that the valuation of the aggregate stock market contains meaningful information about the dynamics of the real interest rate. For one, the aggregate book-to-market ratio explains a very small amount of variation in the real rate. This is true regardless of how or whether we detrend these variables. Moreover, the relationship is nonexistent when we difference the data and when we linearly detrend the real rate and the aggregate BM ratio, as we do in the main text. In addition, there is ample empirical evidence that variation in the aggregate value of the stock market is largely disconnected from real rate variation (e.g. Campbell and Ammer (1993)). In sum, we do not view the evidence in Section A2.2.1 to reveal a robust link between the real rate and the aggregate BM ratio. In unreported results, we draw similar conclusions if we instead use Shiller's CAPE ratio or CAY from Lettau and Ludvigson (2004).

Even if there is a weak relationship between the real rate and the aggregate value of the stock market in our sample, it is likely unrelated to risk premia. Standard Gordon growth model logic suggests that the aggregate dividend-yield is driven by the risk-free rate $r_f$, the market risk premium $\mathbb{E}\left[r_m - r_f\right]$, and the growth rate of aggregate dividends $g$:[6]

$$D/P = r_f + \mathbb{E}\left[r_m - r_f\right] - g$$

The simple formula immediately illustrates the mechanical relationship between the risk-free rate and the dividend-yield. Of course, $D/P$ and $r_f$ may also correlate if the risk-free rate is also related to the market risk premium or aggregate dividend growth. However, Table V in the main text and Panel A of Table A.2 demonstrate that the real rate contains no forecasting power for excess market returns. Moreover, in Table A.1, in the cases where the aggregate book-to-market enters significantly for the real rate, the point estimate is positive. This is the opposite of what we would expect if risk perceptions drive both the aggregate market and the real risk-free rate. On the contrary, the positive point estimates are consistent with a simple Gordon growth formula above.

Furthermore, Panel A of Table A.2 also demonstrates that both the Hamilton-filtered and raw real rate still forecast returns on the low-minus-high volatility portfolio. In Panel B of Table A.2, we show that the real rate – both the Hamilton-filtered and raw series – has no forecasting power for aggregate real earnings growth or aggregate real dividend growth. In conclusion, the link between the aggregate BM ratio and the real rate appears unrelated to risk perceptions, and statistically unreliable.

### A2.3 Time-Series Inference

The AR(1) coefficients of the Hamilton-filtered rate $\widetilde{r}_t$, the linearly detrended rate $r_t$, and $PVS_t$ are 0.81, 0.85, and 0.88, respectively. While the persistence of $PVS_t$ may appear high, it is useful to keep in mind that it is much less persistent than the aggregate valuation ratios, where persistent regressor biases have found the most attention in asset pricing (Stambaugh (1999)). While $PVS_t$ has a quarterly AR(1) coefficient of 0.88, corresponding to a half-life of about 1.5 years, the aggregate book-to-market has an AR(1) coefficient of 0.98, corresponding to a much longer half-life of around 10 years. This simple comparison already suggests that inference problems from persistent regressors are likely to be much less severe in our setting than for aggregate valuation ratios.

Nonetheless, we use a battery of approaches to formally establish that the relationship between the real rate and $PVS_t$ is not driven by serially correlated regressors. First, we run all our main results in differences, as shown throughout the main text and the appendix. In this section, we explore several ways of adjusting standard errors, GLS, and a bootstrap simulation exercise.

Note: For this particular analysis, we leave $PVS_t$ in its natural units, though for most of the analysis in the main text and in the rest of the appendix we standardize it to have mean zero and variance one.

[6] A similar argument holds for the aggregate book-to-market ratio, but the dividend-price ratio is easier for the purposes of this illustration. As an empirical matter, the two are 98% (60%) correlated in levels (first-differences) for our sample.

### A2.3.1 Standard Error Corrections

Our baseline univariate regression of the linearly detrended real rate ($r_t$) on $PVS$ (standardized to have mean zero and variance one) yields the following estimates:

$$\begin{array}{ccccccc} r_t & = & 0.62 & + & 3.41 & \times & PVS_t \\ & & (4.97) & & (11.26) & & \\ & & [2.63] & & [5.31] & & \end{array}$$

where the parenthesis below the point estimates contain OLS $t$-statistics and the square brackets contain Newey-West $t$-statistics with five lags. The first thing to note from this simple regression is that Newey-West correction still indicates the point estimate on $PVS$ is statistically significant. The second thing to note is that the nonparametric Newey-West correction shrinks the OLS $t$-statistic by a factor of nearly two. This owes in part to the fact that the regression residuals have a first-order autocorrelation of 0.76. We address this persistence directly by using a standard parametric correction based on on the estimated residual autocorrelation. Specifically, we multiply the standard errors in the regression by a factor of $C = (1+\rho)/(1-\rho)$, where $\rho$ is the autocorrelation of the regression residuals. $\rho = 0.76$ means that $C \approx 7.3$, thereby implying that the OLS $t$-statistics need to be divided by a factor of $\sqrt{C} = 2.71$. The parametric correction therefore shrinks the $t$-statistic on $PVS$ from 11.41 to 4.21, so the point estimate is still statistically significant.

For completeness, we repeat the analysis using the Hamilton-filtered real rate $\widetilde{r}_t$. In this case, a univariate regression of $\widetilde{r}_t$ on $PVS_t$ gives:

$$\begin{array}{ccccccc} \widetilde{r}_t & = & 0.59 & + & 3.26 & \times & PVS_t \\ & & (5.09) & & (11.48) & & \\ & & [2.72] & & [6.52] & & \end{array}$$

The first-order autocorrelation of the residuals for this specification is 0.69, implying that the OLS $t$-statistic of 11.48 should be adjusted to 4.92.

The broader takeaway here is that no matter how we adjust our standard errors, we are still able to comfortably reject the null that the point estimate on $PVS$ is equal to zero.

### A2.3.2 Generalized Least Squares (GLS)

For statistically efficiency and to account for the role of outliers, we also estimate the relationship between the linearly detrended real rate and $PVS$ using generalized least squares. This is just a Prais-Winsten regression, which amounts to quasi-differencing the data before running the regression. GLS gives the following estimates:

$$\begin{array}{ccccccc} r_t & = & 0.44 & + & 2.46 & \times & PVS_t \\ & & (1.31) & & (6.16) & & \end{array}$$

where the GLS $t$-statistics are listed below point estimates. We also estimate the same system using the Hamilton-filtered real rate $\widetilde{r}_t$:

$$\begin{array}{ccccccc} \widetilde{r}_t & = & 0.49 & + & 2.57 & \times & PVS_t \\ & & (1.90) & & (6.34) & & \end{array}$$

Regardless of the detrending method, the relationship between the real rate and $PVS$ remains economically and statistically significant when using GLS.

Moreover, if we run the regression using data up until the financial crisis (pre-2009), we get fairly similar point estimates on $PVS$ across simple OLS and GLS estimation methods. For example, when using the Hamilton-filtered real rate, OLS gives a point estimate on $PVS$ of 3.41 and GLS gives a point estimate of 3.25.

#### A2.3.3 Simulation Evidence

Finally, one might be concerned that our results are biased in a Granger-Newbold sense. We use simulations to show that the standard error and $R^2$ from our baseline regression are not just a result of regressor persistence. Specifically, we fit independent AR(1)-GARCH(1,1) models to $r_t$ and $PVS$ and simulate these processes mimicking the persistence properties of $r_t$ and $PVS_t$ and with identical sample length as in the data. In the simulated data, where by construction $r_t$ and $PVS_t$ are unrelated, we regress $r_t$ on $PVS_t$, retaining the Newey-West corrected $t$-statistic (five lags) for $PVS$ and the $R^2$ in the simulated regression. Figure A.2 presents histograms of the simulated $t$-statistics and $R^2$ from this exercise for 10,000 independent simulations. The plot also shows the actual $t$-statistic on $PVS$ and the $R^2$ that we estimate in the data. The $p$-values listed in the plot are just the proportion of simulations where the $t$-statistic (or $R^2$) exceed the actual $t$-statistic we estimate in the data. For both the $t$-statistic and $R^2$, less than 0.5% of simulations can match the regression of the real rate on $PVS$ that we estimate using actual data. Combined with the other analysis in the paper, this tells us that under the null of no relation between $PVS_t$ and $r_t$ it would be highly unlikely to observe the $t$-statistics and $R^2$s that we see in the data. This simulation once again adds to our evidence that the relation between $PVS_t$ and $r_t$ is a real feature of the data and not just an erroneous statistical artifact.

### A2.4 Subsample Stability

Our main sample runs from 1970Q2 through 2016Q2. In this subsection, we study the sub-sample stability of the relationship between the real rate and $PVS_t$. We start by showing that our results are not dependent on the period from 1977 to 1987, a time when the U.S. suffered unusually high inflation and the Federal Reserve – led by Paul Volcker – tightened monetary policy to regain control over inflation. In addition, we expand our sample back to 1953Q2 and show the relationship between $PVS_t$ and the real rate is equally strong in this longer sample. The beginning of this extended sample coincides with the beginning of the series for the constant maturity nominal one-year rate that is available from the St. Louis FRED database. The Survey of Professional Forecasters inflation forecasts are not available prior to 1970Q2, so to construct a one-year real rate series from 1953Q2 to 1970Q2, we use the four-quarter moving average of realized inflation as our measure of expected one-year inflation. This approach for forming expected inflation forecasts is motivated by the findings of Atkeson and Ohanian (2001).[7] To extend $PVS_t$ back to 1953Q2, we use the accounting data from Davis et al. (2000). Specifically, we look for book values from COMPUSTAT

[7] There is a large body of research that studies optimal inflation forecasts, with varying conclusions depending on the subsample of interest. The four-quarter random walk benchmark studied in Atkeson and Ohanian (2001) is surprisingly successful and we use it here due to its simplicity.

quarterly, then COMPUSTAT annual, then Davis et al. (2000), in that order and depending on data availability.

In all cases, we run regressions of the following form, in both levels and first differences:

$$\text{Real Rate}_t = a + b \times PVS_t + \varepsilon_t$$

Table A.3 collects the results of our subsample analysis. For reference, column (1) of the table presents the level-regression results using the baseline sample in the main text. In column (2), we find similar results when we exclude the period from 1977 to 1987, providing some comfort that our results are not dependent on the so-called "Volcker period". Column (3) runs the regression over 1953Q2-2016Q2. For this sample, we use the raw real rate because it appears stationary during this time period.[8] Here, we once again see that the correlation between the real rate and $PVS_t$ is present in the longer sample. In column (4), we focus on the portion of the longer sample that precedes the Volcker period, again confirming a strong link between the real rate and $PVS_t$. Columns (5)-(8) indicate that we obtain similar conclusions when running these regressions in first-differences. Overall, the main takeaway from Table A.3 is that the relationship between $PVS_t$ and the real rate is robust across subsamples.

## A2.5 Decomposing the Real Rate: Inflation Expectations and the Taylor Rule

Our construction of the one-year real interest rate is simply the nominal one-year Treasury rate minus expected one-year inflation from the Survey of Professional Forecasters. Thus, $PVS_t$ can correlate with our real rate variable because it correlates with one of these components. To explore this potential further, we decompose the real rate into its constituent parts and regress both on $PVS_t$. Table A.4 contains the results, and in all regressions, $PVS_t$ is standardized to have zero mean and unit variance. For sake of comparison, we present the results of regressing the detrended real rate and the raw real rate on $PVS_t$ in rows (1)-(2) of the table, respectively. In rows (3) and (4), we decompose the raw real rate into the one-year nominal Treasury bill rate and inflation expectations, so that the difference between the coefficients in row (3) and row (4) equals the coefficient in row (2). This decomposition shows that the correlation between $PVS_t$ and the real rate primarily comes from the nominal rate, not inflation expectations.

In rows (5)-(8), we try to separate movements in the real rate that can be attributed to the Taylor (1993) rule, which sets the real short-term interest rate as a function of inflation and the output gap. Specifically, we decompose the real rate into a Taylor (1993) rule component and a residual. We explore two versions of this decomposition. First, in rows (5) and (6), we use the original monetary policy coefficients on the output gap and inflation from Taylor (1993). Specifically, we compute

$$Taylor1993_t = 0.5 \times (OutputGap_t) + 0.5 \times (Inflation_t - 2) + 2$$

where $Taylor1993_t$ is the real rate that obtains if the central bank follows the Taylor rule exactly. We define the residual as the raw real rate minus $Taylor1993_t$. Rows (5) and (6) show that in this construction the explanatory power of $PVS_t$ for the real rate comes from its explanatory power for

[8] For this sample, the augmented Dickey-Fuller test rejects the null of a random walk with no drift at conventional significance levels.

Taylor (1993) rule residuals. In rows (7) and (8), we do a second version of the decomposition, where we estimate the coefficients on the output gap and inflation. Specifically, we run a regression of the raw real rate on the output gap and inflation and call the fitted value the Taylor rule component. Rows (7) and (8) show that in this construction, the explanatory power of $PVS_t$ for the real rate again comes from its explanatory power for the residuals. These results indicate that $PVS_t$ does not simply capture the reaction of monetary policy along a standard Taylor (1993) rule.

## A2.6 The Real Rate and Alternative Constructions of PVS

In this subsection, we investigate alternative ways of constructing $PVS_t$ and whether these alternatives are also correlated with the real rate. For our baseline measure, we prefer to use a 2-month window to measure volatility because it mirrors the construction of volatility-sorted portfolios on Ken French's website, making the returns from the portfolios that comprise $PVS_t$ directly comparable to his. When computing book-to-market ratios, our approach to averaging market capitalizations is motivated by our assumption that book values are known with at least a one-quarter lag. Thus, smoothing market capitalizations over 6-month windows is designed to roughly match the timing of our accounting data.

Here, we consider the following alternative approaches to constructing $PVS_t$:

1. Measure volatility over a trailing 2-month window and use the last available market capitalization to compute book-to-market ratios ($PVS_{Last}$)

2. Measure volatility over a trailing 2-month window and use the median market capitalization over the same window to compute book-to-market ratios ($PVS_{2M}$)

3. Use the same measurement windows as in the baseline version of $PVS_t$,, but define $PVS_t$, as the market-to-book of high-volatility firms minus the market-to-book of low-volatility firms ($PVS_{MB}$)

4. Use the same measurement windows as in the baseline version of $PVS_t$, but sort stocks into terciles based on volatilities instead of quintiles. $PVS_{Terc}$ is the average book-to-market ratio of firms in the low tercile minus the average book-to-market of the high-volatility tercile.

5. Measure volatility over a 2-year trailing window and use the same approach to book-to-market ratios as in the baseline $PVS_t$

Table A.5 shows the correlation of these various $PVS$ measures with each other and with the one-year real rate (linearly detrended). The table shows the correlations in both levels and first-differences. The first takeaway is that our baseline construction of $PVS$ is highly correlated with all of these alternatives, both in levels and first-differences. Moreover, the correlation between the real rate and $PVS_t$ is largely the same across the different construction approaches. We therefore conclude that the informational content of $PVS_t$ is robust to different construction methods.

## A2.7 The Real Rate and Other Valuation Spreads

### A2.7.1 Univariate Analysis

We now explore alternative explanations for the empirical relationship between the real rate and $PVS_t$. Specifically, we examine the possibility that volatility is simply correlated with another characteristic that is more important for explaining the real rate. We sort stocks along a variety of dimensions and form book-to-market spreads based on the sorting variable. For instance, when examining size as a characteristic, we sort stocks in quintiles based on their market capitalization, then compute the difference between the book-to-market ratio of the smallest (i.e., the lowest quintile of market capitalization) and the largest stocks. Recall that $PVS_t$ is the book-to-market spread that emerges when the characteristic $Y$ is trailing 60-day volatility. We then run the following regression relating the real rate to the spread in book-to-market based on each sort:

$$\text{Real Rate}_t = a + b \times Y_t + \varepsilon_t \tag{3}$$

where $Y_t$ is the book-to-market spread based on sorting on characteristic $Y$. In all cases, we standardize $Y_t$ to have a mean of zero and a variance of one. For reference, column (1) of Table III of the main text runs regression (3) with $PVS_t$ as the explanatory variable. There, we find that a one standard deviation increase in $PVS_t$ is associated with a 1.26 percentage point increase in the one-year real rate and $PVS_t$ alone explains 41% of real rate variation over our main sample.

The results are displayed in Table A.6. In row (1), we relate the real rate to the spread in book-to-market sorting stocks based on the expected duration of their cash flows. If high volatility stocks simply have higher duration cash flows than low duration cash flows, then their valuations should fall more when real rates rise. This is one sense in which low volatility stocks may be more "bond-like" than high volatility stocks (e.g., Baker and Wurgler (2012)). In this case, a mechanical duration effect could explain the relationship between the real rate and $PVS$. To examine this possibility, we follow Weber (2016) and construct the expected duration of cash flows for each firm in our data. We then sort stocks based on this duration measure and calculate the spread in book-to-market between high and low duration stocks. As row (1) shows, the relationship between this duration spread and the real rate is negative. However, it is not consistently statistically significant across specifications and is in general much smaller in magnitude than $PVS$.

Row (2) displays the same exercise when looking at the relative valuations of low-leverage versus high-leverage stocks. We define leverage as the book value of long-term debt divided by the market value of equity. It seems natural to think that high-leverage firms have high volatility, and since these firms effectively are short bonds, their equity may suffer disproportionately from a decrease in the real rate. The positive coefficient in row (2) indicates that this intuition bears out in the data. When the real rate falls, the book-to-market spread between low-and-high leverage firms also falls. In other words, high-leverage firms become cheaper when the real rate falls.

In rows (3)-(5), we sort stocks based on three types of market (CAPM) betas:

1. The first CAPM beta we compute is a two-year rolling beta. In a given quarter, we use the previous twenty-four months worth of monthly return data to compute a CAPM beta. In order to have a valid two-year beta, a firm must have at least 80% of its observations over the previous two years.

2. The second CAPM beta we compute is a "long-run" beta. We first aggregate monthly returns into six-month returns. Then at the end of each quarter we use the previous ten years worth of

data to compute betas from our six-month return series (e.g. 20 observations per regression). Once again, firms must have 80% of their observations in order to have a valid long-run beta.

3. The third CAPM beta we compute uses a two-month window. For each firm, we use daily stock data from the previous two months to compute a high-frequency measure of CAPM beta. We exclude firms that do not have at least 20 observations over this time frame.

In all cases, our benchmark index is the CRSP Value-Weighted Index. For the first two measures of CAPM Beta, all of our individual firm data derives from the CRSP Monthly dataset. We deal with delisted returns as in Shumway (1997) by setting missing delisted returns with codes 400-591 to a value of -30%.

Row (3) indicates that the book-to-market spread based on a two-year CAPM beta is correlated with the real rate. Row (4) sort stocks based on CAPM betas that we compute using long-horizon returns. The motivation for studying longer-run CAPM betas is that long-horizon returns are more plausibly driven by cash flow news rather than discount rate news. Thus, long horizon CAPM betas can be viewed as a measure of aggregate cash flow beta. Row (4) indicates a positive relationship between the book-to-market spread based on long-run CAPM beta in levels, but the relationship is not particularly strong in a statistical sense when moving to first-differences. Row (5) uses our measure of CAPM beta that is computed using daily data over rolling 60-day windows. This construction mimics how we compute volatility (and hence PVS). There is again a positive relationship between 2-month beta and the real rate, but not one that is robust across specifications.

In row (6), we sort stocks on the estimated beta of their cash flows with respect to aggregate cash flows. Specifically, cash flow betas are computed via rolling twelve quarter regressions of quarter-on-quarter EBITDA growth on quarter-on-quarter national income growth. EBITDA is defined as the cumulative sum of operating income before depreciation (series oibdpq from COMPUSTAT quarterly). We require a minimum of 80% of observations in a window to compute a cash flow beta. If high volatility stocks have higher cash flow betas than low volatility stocks, then their valuations should fall more when aggregate growth expectations are low. In this case, our results using $PVS$ could be explained by changes in aggregate growth expectations rather than change in the precautionary savings motive. Row (6) shows that the book-to-market based on cash flow betas is not significantly correlated with the real rate.

Keep in mind that the preceding regressions are all univariate. The relevant question for us is whether $PVS$ is just picking up on the information carried in these various book-to-market spreads. Two pieces of evidence strongly suggest that $PVS$ carries independent information about the real rate. For one, in Table A.7, we run bivariate horse races of $PVS$ against each of these alternative sorting variables. None of these alternative sorting variables drive out $PVS$ from the regression. This is true when running the horse races in levels, first differences, and across different subsamples.

As a second piece of evidence, in row (9) of Table A.6 we run a "kitchen-sink" regression of the following form:

$$\text{Real Rate}_t = a + b_{PVS} \times PVS + \theta^{'} X_t + \varepsilon_t$$

where $X_t$ contains all of the valuation spreads discussed above. Row (9) of the table reports the estimated $b_{PVS}$, its associated standard error, and the adjusted $R^2$ from the regression. The simple takeaway from the kitchen-sink regression is that none of the control variables drive out the explanatory power of $PVS$ for the real rate. The coefficient on $PVS$ remains statistically significant in both the levels and first-differenced specifications, and the point estimate compares favorably to those found in the main text. If anything, including the other control variables increases the economic relationship between $PVS$ and the real rate. These results suggest that the relative valuation of high and low-volatility stocks contains unique information about the real rate.

#### A2.7.2 Horse Races Alternative Stock Characteristics and More Alternative Constructions of PVS

*Alternative Constructions of $PVS_t$*

We first show that we obtain similar results for alternative definitions of $PVS_t$. In row (2) of Table A.7, we recompute $PVS_t$ by value-weighting the book-to-market ratio of stocks within each volatility quintile, as opposed to equal-weighting. In row (3), we obtain similar results sorting stocks on volatility measured over a two-year window, rather than a two-month window. Our baseline result therefore captures changes in the valuation of stocks that historically have been volatile, not changes in the volatility of low-valuation stocks. This distinction is important to our interpretation of $PVS_t$ as a measure of investors' risk perceptions relevant to the macroeconomy.

*Relationship to Other Stock Characteristics*

Rows (4)-(9) of Table A.7 Panel A investigate whether stock return volatility is really the key stock characteristic for the relationship between stock prices and the real rate. In row (4), we run a horse race of $PVS_t$ against the difference in yields between 10-year off-the-run and on-the-run Treasuries, a measure of liquidity premia in the fixed income market (Krishnamurthy (2002), Kang and Pflueger (2015)). The table reports the estimated coefficient on $PVS_t$. The explanatory power of $PVS_t$ for the real rate is unchanged, suggesting that $PVS_t$ subsumes any information about the real rate that is captured in the demand for liquid assets like on-the-run Treasuries.

Next, we test whether volatility simply proxies for another stock characteristic by controlling for book-to-market spreads based on alternative characteristics. These tests help us rule out that the $PVS_t$-real rate relationship captures the pricing of these alternative characteristics, including leverage, growth, and the duration of cash flows. For an alternative characteristic $Y$, we construct a book-to-market spread the same way we construct $PVS_t$. We report the coefficient on $PVS_t$, while controlling for the $Y$-sorted book-to-market spread and the aggregate book-to-market ratio. We consider characteristics $Y$ that capture alternative economic mechanisms through which the real rate might correlate with $PVS_t$: cash flow duration, firm leverage, systematic risk (i.e., CAPM beta), firm size, and value (i.e., book-to-market ratio).

Rows (5)-(9) show that in all cases the regression coefficient on $PVS_t$ is essentially unchanged relative to our baseline results. Row (5) shows that $PVS_t$ is not capturing differences in the duration of cash flows (Weber (2016)) between low- and high-volatility stocks, which would cause their values to move mechanically with interest rates. We draw a similar conclusion when studying leverage sorts in row (6). The results on CAPM beta in row (7) confirm that the relation between $PVS_t$ and the real rate is not simply picking up on aggregate stock market risk, suggesting that investors care about risk factors that are broader than the aggregate stock market.[9] In row (8),

[9] As we discuss in Table A.6, there is a correlation between the real rate and the spread in valuations of beta-sorted portfolios, confirming the intuition that the price of safe assets is high when prices of risky stocks are low. However, the relationship between the real rate and $PVS_t$ is stronger in univariate regressions and in horse races, consistent with our interpretation of total volatility as a more robust measure of an individual stock's risk.

we find that our volatility sorts do not simply proxy for size, despite the fact that smaller firms tend to be more volatile. The value-sorted book-to-market spread is sometimes thought to capture the value of growth options, so the results in row (9) suggests that the relation between $PVS_t$ and the real rate is not driven by growth options. In Table A.8, we use double sorts to show that the relationship between $PVS$ and the real rate is not driven by industry, whether the firm is a dividend payer, as well as the characteristics studied here.

Based on this analysis, we conclude that sorting stocks on volatility is key to our construction of $PVS_t$. From a statistical perspective, it may not be surprising that there exists a cross section of stocks that is correlated with real rates. The economic content of our findings is that volatility, while not a fundamental firm characteristic, is a robust measure of risk. We therefore view these results as supportive of our interpretation of $PVS_t$ as a measure of investor risk perceptions.

*Data Details for Table A.7*

**Value-Weighted Version of $PVS_t$** The value-weighted version of $PVS_t$ is the value-weighted average book-to-market ratio of low-volatility stocks at time $t$ minus the value-weighted average book-to-market ratio of high-volatility stocks at time $t$. The value weights are determined by market capitalizations at the end of quarter $t$.

**$PVS_t$ Based on Two-Year Volatility** The variable "2-Year Volatility" listed under "Alternative Constructions" in the table uses each firm's trailing 2-year volatility to form our volatility-sorted portfolios. We use monthly return data from CRSP to compute this measure of volatility.

**Off-the-Run Minus On-the-Run Treasury Yields** The off-the-run minus on-the-run Treasury yield spread is the difference between the continuously compounded 10-year off-the-run and on-the-run bond yields. On-the-run bond yields are from the monthly CRSP Treasury master file. The off-the-run bond yield is obtained by pricing the on-the-run bond's cash flows with the off the-run bond yield curve of Gürkaynak et al. (2007). For details of the off-the-run spread construction see Kang and Pflueger (2015).

**Other Variables Used in Horse Races** For a description of the other variables used in the horse races, see Section A2.7.1 of this appendix.

### A2.7.3 Double-Sorted Versions of $PVS_t$

In this subsection, we create double-sorted versions of $PVS_t$ as an alternative way to address the possibility that volatility just proxies for another characteristic whose price is correlated with the real rate. More precisely, consider characteristic Y. We construct a Y-neutral version of $PVS_t$ by first grouping stocks at time $t$ based on whether they have above or below median values of characteristic Y. We define "low Y" firms as those firms with below-median values of Y and "high Y" firms are defined analogously. Next, within low-Y firms, we further sort firms into terciles based on volatility. $PVS_t^{L,Y}$ is defined as the average book-to-market ratio of low-volatility and low-Y firms minus the average book-to-market value of high-volatility and low-Y firms. $PVS_t^{H,Y}$ is defined in the same manner, except for high-Y firms. Finally, the Y-neutral version of $PVS_t$

is defined as $(PVS_t^{L,Y} + PVS_t^{H,Y})/2$. This spread measures the difference in valuations of low volatility and high volatility stocks that have similar values of characteristic Y.

For example, suppose the characteristic that we are interested in is CAPM-Beta. We then split stocks into low and high beta firms based on the median CAPM-Beta at time $t$. Then within each CAPM-Beta bucket, we compute the difference in book-to-market ratios of low and high volatility stocks. Finally, we average the spread between low- and high-volatility stocks across low and high CAPM-Beta firms. This procedure delivers us a version of $PVS$ that is immunized to CAPM-Beta but differentially exposed to volatility. The sorting variables we use are described in Section A2.7.1 of this appendix. In addition, we construct an industry-neutral version of $PVS_t$ in the same way by first grouping stocks into industries based on their SIC codes and the 48 industry definitions on Ken French's website. We also form $PVS_t$ in the subset of dividend paying and non-dividend paying stocks, where we define a dividend-paying stock at time $t$ as one that has paid a divided any time in the previous two years.[10]

After we build double-sorted versions of $PVS_t$, we run the following regression in both levels and first differences:

$$\text{Real Rate}_t = a + b \times \text{Y-Neutral PVS}_t + \varepsilon_t \tag{4}$$

In all cases, we standardize the double-sorted version of $PVS_t$ (or its first difference) to have a mean of zero and a variance of one. Table A.8 contains the point estimates of $b$, their associated standard errors, and the adjusted $R^2$ from these regressions. Echoing our analysis from Section A2.7.2, we find that all of the double-sorted versions of $PVS_t$ exhibit an economically and statistically significant positive correlation with the real interest rate. By and large, the point estimate on the Y-neutral version of $PVS_t$ is comparable to what we obtain in the main text when using the raw version of $PVS_t$. The fact that the industry-adjusted version of $PVS_t$ continues to explain a large fraction of real rate variation indicates that $PVS_t$ does not just load up on industries that are more exposed to interest rate movements. A similar conclusion holds when looking at dividend versus non-dividend paying stocks. Overall, these facts lend further support of the idea that volatility is the key characteristic underlying the construction of $PVS_t$.

### A2.7.4 Total Volatility vs. Alternative Measures of Risk

As discussed in the main text, we use total volatility of stock returns because it is a robust measure of risk. Intuitively, volatility increases with stocks' exposure to any risk factors that investors care about, and $PVS_t$ captures how much of a price discount investors require for holding risky stocks. To confirm that our results are robust to variations in how we measure risk, we verify that the spread in book-to-market ratios is similar when we sort stocks by their CAPM betas instead of total stock return volatility. The CAPM beta captures systematic risk provided that investors are well-diversified and that investors' aggregate wealth portfolio equals the aggregate stock market. Indeed, we find that $PVS_t$ and the beta-sorted book-to-market spread are 82% correlated in levels and 51% correlated in first-differences. As our preceding analysis shows, the link between the real risk-free rate and the spread in book-to-market ratios when sorting on two-year CAPM betas is similar, albeit weaker, than our baseline results for $PVS_t$.

Of course, investors may care about risk factors other than the aggregate stock market. To allow for a broader set of factors, we sort firms into quintiles based on the volatility of the fitted

[10] We determine dividend yields by looking at the total return and the ex-dividend return in CRSP.

value from a regression of daily stock returns on the Fama and French (1993) factors. To match the construction of our benchmark sorting variable, i.e. total volatility, we use trailing 60-day returns at the end of each quarter $t$. As expected, the resulting book-to-market spread is even more closely correlated with $PVS_t$, with correlations of 87% in levels and 84% in first-differences. Overall, we find that $PVS_t$ is not sensitive to small variations in our measure of risk. We use total volatility as our benchmark sorting variable because it does not require us to take a stand on the underlying risk-factors that investors care about.

# A3 Additional Empirical Results

## A3.1 Monetary Policy Shocks - All announcements

In Section III.A.2 of the main text, we show that monetary policy shocks do not differentially affect high-volatility stocks. In the main text, we exclude unscheduled FOMC meetings because surprise policy changes made outside of regularly scheduled meetings may be driven by financial market conditions. Here we examine the full sample of FOMC meetings for robustness. The results are in Table A.9. As discussed in the paper, if discretionary monetary policy was an omitted variable driving the positive covariance between the observed real rate and $PVS_t$, we should see negative coefficients in Table A.9. Generally the coefficients in Table A.9 are statistically insignificant with inconsistent signs. Using daily returns, we find a positive correlation that is borderline statistically significant for some specifications. However, this is the opposite of what we would expect if monetary policy acted as an omitted variable for our findings in Table III in the main paper. Instead, a positive correlation is consistent with the Fed scheduling additional meetings to cut interest rates in times and stabilizing high-volatility stocks in times of market turmoil. Consistent with this interpretation, in untabulated results we find that the positive correlation is entirely driven by surprise changes in 2001. In that year, the Fed cut rates aggressively outside of regularly scheduled meetings in the aftermath of the technology bubble.

## A3.2 Decomposing Comovement between the Real Rate and $PVS_t$

Section III.B of the paper establishes that both the real rate and $PVS_t$ forecast future returns on volatile stocks. In this subsection, we use a present-value decomposition to argue that this return predictability has implications for interpreting the contemporaneous correlation between the real rate and $PVS_t$. Vuolteenaho (2002) derives the following relation tying a firm $i$'s log book-to-market ratio to its future log return and log accounting return (ROE):

$$\theta_{i,t} = r_{i,t+1} - e_{i,t+1} + \rho\theta_{i,t+1} + \nu_{it}$$

where $\theta_i$ is the log book-to-market of firm $i$, $r_{i,t+1}$ is its log stock return, and $e_{i,t+1}$ is the log ROE. $\rho$ is a log-linearization constant and $\nu_{i,t}$ is an approximation error, such that $\theta_{i,t} \approx r_{i,t+1} - e_{i,t+1} + \rho\theta_{i,t+1}$. To map this expression to the current setting, we define the log version of $PVS_t$, denoted by $pvs_t$, as follows:

$$pvs_t \equiv \left[\frac{1}{N_{L,t}} \sum_{i \in \text{Low Vol}_t} \theta_{i,t}\right] - \left[\frac{1}{N_{H,t}} \sum_{i \in \text{High Vol}_t} \theta_{i,t}\right]$$

where, for example, $N_{L,t}$ is the number of firms in the low vol portfolio at time $t$. The Vuolteenaho (2002) decomposition then implies that:

$$\begin{aligned}
pvs_t &\approx r_{t+1}^{PVS} - e_{t+1}^{PVS} + \rho \times pvs_{t+1} \\
r_{t+1}^{PVS} &\equiv \left[\frac{1}{N_{L,t}} \sum_{i \in \text{Low Vol}_t} r_{i,t+1}\right] - \left[\frac{1}{N_{H,t}} \sum_{i \in \text{High Vol}_t} r_{i,t+1}\right] \\
e_{t+1}^{PVS} &\equiv \left[\frac{1}{N_{L,t}} \sum_{i \in \text{Low Vol}_t} e_{i,t+1}\right] - \left[\frac{1}{N_{H,t}} \sum_{i \in \text{High Vol}_t} e_{i,t+1}\right]
\end{aligned} \tag{5}$$

In addition, we assume that $pvs_t$ follows an AR(1) process, $pvs_{t+1} = a + \phi pvs_t + \xi_{t+1}$. Next, combining the AR-process with Equation (5), plus some rearranging yields:

$$\begin{aligned}
Cov(\text{Real Rate}_t, pvs_t) \approx (1-\rho\phi)^{-1} \times [&Cov\left(\text{Real Rate}_t, r_{t+1}^{PVS}\right) - Cov\left(\text{Real Rate}_t, e_{t+1}^{PVS}\right) \\
&+ \rho Cov(\text{Real Rate}_t, \xi_{t+1})]
\end{aligned}$$

Dividing both sides by $Cov(\text{Real Rate}_t, pvs_t)$ delivers a simple covariance decomposition:

$$1 = \Psi_r - \Psi_e + \Psi_\xi \tag{6}$$

where $\Psi_r \equiv (1-\rho\phi)^{-1} \times Cov\left(\text{Real Rate}_t, r_{t+1}^{PVS}\right)/Cov(\text{Real Rate}_t, pvs_t)$, and so forth.

Equation (6) states that covariation between today's real rate and $pvs_t$ can arise for three reasons: (i) today's real rate forecasts future returns to the volatility-sorted portfolio, $r^{PVS}$; (ii) today's real rate forecasts future cash flows on the same portfolio, $e^{PVS}$; or (iii) today's real rate forecasts future innovations in tomorrow's $pvs$.

To operationalize the decomposition, we need to first estimate $\phi$ and $\rho$. We fit a simple AR(1) for $pvs$ and find that $\phi = 0.88$ for quarterly data. With regards to $\rho$, we consider a range of values from 0.9 to 0.97.[11] All of the other components needed for the covariance decomposition are estimated from simple covariances in the data, namely one-quarter ahead forecasting regressions of returns and ROEs on $PVS_t$.[12]

For all of the ranges of $\rho$ that we consider, $\Psi_r$ is never less than 70% and approaches 100% for larger values of $\rho$. Moreover, for all of the ranges of $\rho$ considered in Vuolteenaho (2002), $\Psi_r$ is never below 90%. This is rather unsurprising given that the real rate does not forecast future ROE for the low-minus-high volatility portfolio. We therefore conclude that a large majority of the covariation (around 90%) between $PVS_t$ and the real rate can be attributed to the real rate forecasting future returns on the volatility-sorted portfolio. Put differently, $PVS_t$ and the real rate correlate because discount rate shocks to high-volatility stocks coincide with shocks to the real rate. This result is consistent with the interpretation of $PVS_t$ as a measure of risk perceptions rather than expected cash flows.

[11] Vuolteenaho (2002) sets $\rho = 0.967$ for annual data. We use a range of values to get a sense of how sensitive our decomposition is to the approximation constant.

[12] Note that in estimating $Cov\left(\text{Real Rate}_t, e_{t+1}^{PVS}\right)$ by forecasting future ROE with $PVS_t$, we are imposing that investors have rational expectations of the cash flows of high-volatility versus low-volatility firms. As discussed later in Section A3.5, this assumption is justified by the fact that $PVS_t$ does not forecast surprises in ROE based on analyst forecasts, nor does it correlate with analyst expectations of cash flows. To be clear, it could be that movements in the expected return of high-versus-low volatility stocks are still driven by behavioral forces and irrational expectations of risk. Indeed, this is precisely what we find in the main text.

## A3.3 Return Forecasting for Other Asset Classes

Next, we show that $PVS_t$ captures common variation in the compensation investors demand for holding volatile securities within several different asset classes, consistent with the idea that it is a broad measure of risk perceptions relevant to the macroeconomy.

We use test asset portfolios from He et al. (2017), which cover six asset classes: U.S. corporate bonds, sovereign bonds, options, credit default swaps (CDS), commodities, and currencies.[13] Within each asset class, we form a portfolio that is long the lowest-volatility and short the highest-volatility portfolio in the asset class, where volatility is measured with a 5-year rolling window of prior monthly returns. The first three columns in Table A.10 contain summary statistics on the volatility-sorted portfolios in each asset class. In contrast to equities, the average returns of long-short portfolios are negative for several asset classes, showing that the low-volatility premium in U.S. equities (Ang et al. (2006)) is not a systematic feature of all asset classes.

The second set of columns in Table A.10 shows that both $PVS_t$ and the real interest rate forecast quarterly returns on volatility-sorted portfolios for many asset classes. The top row shows our results for U.S. equities. The remaining rows show economically and statistically significant evidence that $PVS_t$ and the real rate forecast long-short returns within three other asset classes: U.S. corporate bonds, options, and CDS. There is also a positive, marginally significant correlation between $PVS_t$ and sovereign bond returns, and a positive but insignificant correlation between $PVS_t$ and commodity returns. We obtain similar results forecasting annual returns.

These regressions show that both $PVS_t$ and the real rate reflect common variation in the compensation investors demand for holding volatile securities across a variety of asset classes. To quantify the strength of this common variation, we compute for each asset class $c$ the correlation $\rho_c$ between the low-minus-high volatility return in $c$ and the average return of the low-minus-high volatility trade in all other asset classes excluding $c$. For example, $\rho_c$ for $c =$ options computes the correlation of the return on the volatility trade in options and the average return of the trade across all asset classes except options. The average $\rho_c$ is 0.42, comparable to common variation in value and momentum strategies across asset classes (Asness et al. (2013)).

### A3.3.1 Data Construction - Other Asset Classes

In Table A.10, we use both $PVS_t$ and the one-year real rate to forecast returns on the low-minus-high volatility trade in other asset classes. To do so, we use the test assets from He et al. (2017), henceforth HKM. We focus on the following asset classes from HKM: equities, U.S. corporate bonds, sovereign bonds, options, credit default swaps (CDS), commodities, and foreign exchange (FX). We refer the reader to HKM for more detail on each of these portfolios.

Within each asset class, we form a portfolio that is long the low volatility portfolio in that asset class, and short the high-volatility portfolio. For each portfolio in each asset class, we compute the volatility at each quarter using the trailing 5-year history of monthly portfolio returns, requiring a minimum of four years of data. We are constrained to use monthly data because HKM do not have daily asset class data. For example, suppose we want to form the low-minus-high volatility

[13] For U.S. stocks, He et al. (2017) use the Fama-French 25 portfolios. We use our own volatility-sorted portfolios for consistency and because this induces a bigger spread in volatility. We obtain qualitatively similar results with the Fama-French 25.

portfolio for U.S. corporate bonds in quarter $t$.[14] We then compute the volatility of each of the 10 HKM corporate bond portfolios over the previous 5 years. We then go long the portfolio with the lowest trailing volatility and short the portfolio with the highest volatility. We hold this long-short portfolio for one quarter, and then repeat the process. Denote the returns to this long-short strategy as $LMHV_t^c$, where the superscript $c$ denotes the asset class we are studying and the subscript denotes time of the return. The forecasting regressions in Panel B of the table use $PVS_t$ or Real Rate$_t$ to forecast $LMHV_{t+1}^c$ for several different $c$.

## A3.4 Robustness: PVS and Real Outcomes

### A3.4.1 Evidence from VARs

The observed real rate mixes two components – the natural rate of interest and discretionary monetary policy – as we show in Eq. (14) in the paper. We would expect these two components to have offsetting correlations with real investment:

- The natural rate of interest is positively correlated with investment through risk perceptions according to our model.
- Discretionary monetary policy tends to push towards a negative correlation between the real risk-free rate and real investment, as an exogenous increase in the real interest rate is generally thought to be contractionary (e.g. Christiano et al. (1999)).

Consistent with this interpretation, the evidence in Section III.A.2 suggests that it is possible to separate these forces using a measure of risk perceptions like $PVS_t$. Figure III in the main paper isolates the first channel, showing that investment increases following an increase in $PVS_t$, conditional on holding the real risk-free rate constant. We now further disentangle the two channels driving the real rate-investment relationship. When we estimate a standard VAR that includes both $PVS_t$ and the real rate, we find that a contractionary monetary policy shock increases the real risk-free rate and decreases investment, while a shock to $PVS_t$ increases the real risk-free rate and increases investment, exactly as expected.

We estimate a VAR that is as simple and transparent as possible, while following a common set of recursiveness assumptions, similar to Sims (1980), Bernanke and Mihov (1998) and Gilchrist and Zakrajšek (2012). We use the following strategy for measuring dynamic effects:

$$Y_t = \sum_{i=1}^{k} B_i Y_{t-i} + \sum_{i=1}^{k} C_i P_{t-i} + A^y v_{y,t} \tag{7}$$

$$P_t = \sum_{i=0}^{k} D_i Y_{t-i} + \sum_{i=0}^{k} G_i P_{t-i} + A^p \begin{bmatrix} v_{PVS,t} \\ v_{MP,t} \end{bmatrix}. \tag{8}$$

Here, $Y_t$ is a vector of quarterly non-policy variables, consisting of unemployment, the investment-to-capital ratio, and detrended inflation. $P_t$ is a vector of policy variables consisting of $PVS_t$ and the detrended real rate. Eq. (7) describes a set of structural relationships in the economy, where macroeconomic variables depend on lagged values of macroeconomic and policy variables. Eq.

[14]This corresponds to US_bond11 through US_bond20 in HKM's data.

(8) describes the stance of monetary policy conditional on contemporaneous macroeconomic variables. Our baseline estimation uses $k = 1$ lag.

We estimate the structural policy shocks under the restriction that $v_{PVS,t}$ does not respond to $v_{MP,t}$ contemporaneously, but $v_{MP,t}$ may respond to $v_{PVS,t}$, consistent with the Federal Reserve actively monitoring macroeconomic and financial variables. It is plausible that investors' risk perceptions shift gradually over time and do not jump in response to monetary policy actions. Indeed, this identification restriction is supported by our analysis of monetary policy shocks in the main text.[15] Following Bernanke and Mihov (1998), structural innovations in the real rate and $PVS_t$ shocks are assumed to affect output, inflation, and precautionary savings demand with a lag.

As a baseline, the left panel of Figure A.3 shows responses to an unexpected tightening by the Federal Reserve. Consistent with the long literature on monetary policy shocks, summarized in Christiano et al. (1999), unemployment increases and and inflation decreases after a one-standard-deviation shock to the real interest rate. The effect on the investment-to-capital ratio is not statistically different from zero. Interestingly, $PVS_t$ does not respond to monetary policy shocks with tight 95% confidence intervals, consistent with our prior finding that monetary policy does not act as an omitted variable driving the relation between the observed real rate and $PVS_t$.

The right panel of Figure A.3 shows that a positive $PVS_t$ shock (corresponding to a decline in risk perceptions) significantly decreases unemployment and increases real investment, despite being associated with a similar increase in the real rate as the MP shock. The contrasting responses across the left and right panels in Figure A.3 are exactly what we would expect if $PVS_t$ isolates risk perceptions.

Shocks to $PVS_t$ are both statistically significant and quantitatively important for unemployment and investment, as shown by forecast error variance decompositions. Ten quarters after the shock, $PVS_t$ shocks explain 14% of variation in the unemployment rate and 39% of the variation in investment-to-capital ratios. It is intuitive that risk perceptions shocks should matter most for real investment, since it is the interface between financial market attitudes towards risk and the real economy. For comparison, the monetary policy shocks explain 17% of variation in unemployment and only 5% of variation in the investment-to-capital ratio.

**Robustness** Figure A.4 shows that impulse responses look similar to Figure A.3 if we use the pre-crisis sample. Figure A.5 shows that our findings are not dependent on the specific identification assumption. We see that unemployment and investment responses are similar if we make the alternative identification assumption that $PVS_t$ is faster than the real rate.

Figure A.6 shows that again the conclusion is similar if instead of estimating a VAR(1) we include additional lags in our estimation and base the impulse responses on a VAR(4). Finally, Figure A.7 shows that we obtain similar results if we replace the unemployment rate by the output gap. Of course, the output gap responses have the opposite signs of the unemployment responses in our baseline specification, because the output gap decreases in recessions, whereas unemployment increases. So, our results linking shocks to risk perceptions and the real economy are not specific

[15] This identification restriction is not crucial to our findings. As we show below, our conclusions are unchanged if instead we make the opposite identification assumption that $PVS_t$ responds to the real rate contemporaneously, but the real rate reacts to risk perceptions with a lag. This second identification assumption is different from saying that the Fed does not pay attention to the stock market. It merely requires that the Fed historically did not react instantaneously to the cross-sectional valuation spread newly documented in this paper. Impulse responses are also robust to excluding the post-crisis period and to including additional lags.

to a particular measure of economic activity.

#### A3.4.2 Evidence from Jordà (2005) Local Projections

In Section III.C of the main text, we use Jordà (2005) local projections to show that an increase in $PVS_t$ forecasts a boom in investment, an expansion of output, and a decline in unemployment. In that analysis, we control for lagged outcome variables and the real interest rate. Here, we explore the robustness of those local projections by running the following sequence of regressions:

$$y_{t+h} = a + b^h_{PVS} \times PVS_t + b^h_{RR} \times RealRate_t + b^h_y \times y_t + b^h_{mkt} \times \text{Agg BM}_t + b^h_{cp} \times CP_t + \varepsilon_{t+h}$$

where $h$ is the forecast horizon. $y_{t+h}$ is either the investment-to-capital at time $t+h$, the real output gap at $t+h$, or the change in the unemployment rate between $t$ and $t+h$. In this regression, we also control for the aggregate book-to-market ratio ($\text{Agg BM}_t$) and the Cochrane and Piazzesi (2005) bond risk-factor, the latter of which we construct using quarterly data and forward rates from Gürkaynak et al. (2007). We include the aggregate book-to-market ratio and the Cochrane and Piazzesi (2005) factor to test whether $PVS_t$ reflects redundant information embedded in measures of financial market activity from aggregate stock and bond markets.

Figure A.8 displays the results of these local projections. The main thing to notice is the magnitude of the response of the macroeconomy to a risk perceptions shock is very similar in these specifications compared to those shown in Figure III of the main text. Following an increase in $PVS_t$, investment and the output gap both rise and unemployment falls, even when controlling for the value of the aggregate stock market and the Cochrane and Piazzesi (2005) factor. These results therefore suggest that $PVS_t$ contains information about the real side of the economy that are not contained in these alternative financial market indicators.

In Table A.11 we present regression evidence for forecast horizon $h = 1$. We run our baseline local projections adding different control variables including the aggregate book-to-market ratio, the 10-year minus 1-year Treasury yield spread, the Cochrane and Piazzesi (2005) bond risk-factor, the Gilchrist and Zakrajšek (2012) credit spread, and the Baker et al. (2016) policy uncertainty variable. $PVS_t$ continues to have forecasting power for future macroeconomic outcomes in all cases.

#### A3.4.3 Firm-Level Investment

Our motivating model from Section II.A of the main text suggests that a decline in risk perceptions should disproportionately affect real investment at high-risk firms. To examine this prediction, we run firm-level regressions in Compustat data of investment on indicators for the firm's volatility quintile, $PVS_t$, and the interactions between $PVS_t$ and the quintile dummies:

$$\frac{CAPX_{i,t\to t+4}}{A_{i,t}} = a_i + a_t + \sum_{q=1}^{5} b_q \cdot 1^q_{it} + b_{PVS} \times PVS_t + \sum_{q=2}^{5} b_{q,pvs} \cdot 1^q_{it} \times PVS_t + b_{CF}\frac{CF_{i,t\to t+4}}{A_{i,t}} + \varepsilon_{i,t+4}.$$

where $1^q_{it}$ is an indicator that firm $i$ is in volatility quintile $q$ at time $t$. $a_i$ and $a_t$ are firm and time fixed effects, respectively. The variable $CAPX_{i,t\to t+4}/A_t$ captures investment for the firm from time $t$ to $t+4$ and $CF_{i,t\to t+4}/A_t$ controls for the cash flows of the firm over the same period. The coefficient of interest in the regression is the interaction between the firm's volatility quintile and

$PVS_t$. Table A.12 reports the regression results: as predicted by the model, the investment of higher-volatility firms is more sensitive to $PVS_t$ than the investment of lower-volatility firms. This result is also robust across pre- and post-2000 subsamples.

#### A3.4.4 Private versus Public Firms

Our measure of risk perceptions derives from the pricing of volatile stocks. These firms generally account for a small portion of the value of the aggregate stock market, which explains why $PVS_t$ has a low correlation with the valuation of the aggregate stock market. However, given that volatile firms are a small part of the market, it is perhaps surprising that movements in their price can impact aggregate economic outcomes like unemployment and real investment. The resolution of this apparent tension is that private firms make up a significant part of the overall real economy, and they behave more like high-volatility public firms than low-volatility public firms.

More specifically, previous studies have found that private firms make up roughly 50% of aggregate non-residential fixed investment, 70% of private-sector employment, 60% of sales, and 50% of pre-tax profits (Davis et al. (2007) Asker et al. (2014) and Zwick and Mahon (2017)). We see a similar pattern in our data. In Figure A.9, we plot the imputed share of U.S. investment coming from private firms. We compute private firm investment as the difference between aggregate investment and the investment of publicly traded firms, which we measure in COMPUSTAT.[16] The figure shows that private firms account for roughly half of aggregate investment, and their share is relatively stable over time.

Moreover, private firms are more similar to high-volatility public firms than low-volatility public firms. A first simple way to make this point is to compare firm characteristics. Asker et al. (2014) show that private firms are smaller, less profitable, and invest more than public firms. In Table A.13, we show that a similar pattern holds when comparing high-volatility public firms to low-volatility public firms. Over our COMPUSTAT quarterly sample (1982Q1-2016Q2), the median high-volatility public firm is much smaller, with \$32 million in nominal assets compared to \$1323 million in assets for low-volatility public firms. In terms of profitability, the median high-volatility public firm had an annual return on assets (ROA) of 0.1%, whereas the median low-volatility public firm had an ROA of 11.8%. And, in terms of investment, the median high-volatility public firm invested at a rate of 7.4% compared to 5.3% for low-volatility public firms. Thus, much like private firms, high-volatility public firms are smaller, less profitable, and invest more than low-volatility public firms.

A second way to make the point that private firms are more like high-volatility firms is to examine their investment behavior. As described in Section V in the main text, aggregate investment is much more correlated with the investment rates of high-volatility stocks (79% correlation) compared to low-volatility stocks (35% correlation). Because private firm investment is such a large share of aggregate investment, these correlations reinforce the notion that private firms are more like high-volatility firms. See Table A.14 for these correlations.

[16] After 1990, we use COMPUSTAT quarterly to compute total public-firm investment, as it provides more up-to-date accounting information. In each quarter $t$ and for each firm $f$, we take the last observation in the data within a year of $t$. We then compute total investment for publicly traded firms by summing over all firms. Prior to 1990, we compute aggregate investment using COMPUSTAT annual because coverage is poor in COMPUSTAT quarterly. We chose 1990 as the cutoff because this was the date when total public-firm investment from COMPUSTAT quarterly and COMPUSTAT annual converged to roughly the same level.

More directly, in Figure A.10 we decompose the response of aggregate investment to a PVS shock into the portion driven by private-firm investment and public-firm investment. We do so by stripping out COMPUSTAT investment from aggregate investment. This analysis mirrors the Jorda (2005) local projections from Section III.C of the paper and Section A3.4.2 of this appendix. The main takeaway from the figure is that private-firm investment is a key part of the response to a shock to PVS, consistent with the idea that private firms behave more like volatile public firms.

Overall, this analysis suggests that private firms are an important component of the real economy, and PVS likely captures the risk perceptions of this subset of firms. PVS does not forecast the aggregate stock market – yet still forecasts aggregate investment and economic expansions – because the aggregate stock market is tilted towards safer "bond-like" stocks that are fundamentally different than private firms.

## A3.5 Additional Analysis of Expectations

### A3.5.1 Contemporaneous Cash-Flow Expectations

In Section II.D of the main paper, our regression analysis indicates that $PVS_t$ is highly correlated with several measures of expected risk. One concern with these results is that expectations of risk may comove with expectations of the future cash flows. We deal with this potential issue in the main text by directly controlling for contemporaneous cash-flow expectations in our regressions. We now show more directly through a series of univariate regressions that $PVS_t$ is only weakly correlated with cash-flow expectations. Specifically, in Table A.15 Panel A, we run contemporaneous regressions of $PVS_t$ on expectations of future cash flows constructed from the Thompson Reuters IBES data set of equity analyst forecasts. For each stock, we construct the consensus analyst forecast of ROE. We then compute the difference between the median forecast for high-volatility stocks and the median forecast for low-volatility stocks. We regress $PVS_t$ on this spread in expected cash flows. In column (1), we use analyst forecasts for the next quarter, in column (2), we examine annual forecasts, and in column (3) we use analyst forecasts for long-term growth.[17] We standardize both $PVS_t$ and the explanatory variables. The sample for these regressions is shorter because IBES data is only reliable for our cross section after the early 1990s, and the number of observations varies across columns because different forecasts are available starting at different dates. We find similar results if we restrict the sample to the common period where all variables are available.

As one would expect, expectations of future cash flows are positively correlated with $PVS_t$. When investors expect high cash flows for high-volatility stocks, $PVS_t$ tends to be high. However, the correlation is quite weak – across the three specifications, expectations of cash flows explain at most 15% of the variation in $PVS_t$. Mechanically, this means that the remaining 85% of variation in $PVS_t$ must be explained by variation in expectations of future returns (Campbell and Shiller (1988)). This accords with our results in Section A3.2, where we concluded that nearly 90% of the comovement between the real rate and $PVS_t$ arises because the real rate forecasts future returns to volatility-sorted stocks. The main takeaway here is that variation in $PVS_t$ is primarily driven by investor expectations of returns, not their expectations of cash flows.

[17] IBES defines long-term growth as the "expected annual increase in operating earnings over the company's next full business cycle", a period ranging from three to five years.

### A3.5.2 Forecasting Revisions in Expectations of Future Cash Flows

In Section III.B of the main text we document that $PVS_t$ negatively forecasts future returns on a portfolio that is long low-volatility stocks and short high-volatility stocks. We concluded from that analysis that $PVS_t$ reflects changes in the cost-of-capital at high-volatility firms: when $PVS_t$ is low, high-volatility firms need to offer investors higher returns on capital because investors perceive these firms as especially risky. An alternative explanation for these forecasting results is that investors have biased beliefs about future cash flows. If investors are overly optimistic about the future earnings of volatile stocks, they will bid up the prices of those stocks and hence $PVS_t$. When high future earnings are not realized, $PVS_t$ will fall as investors revise their beliefs downwards and realized returns on high-volatility stocks will be low. This behavioral story would match the fact that high values of $PVS_t$ forecasts future returns on high-volatility stocks.

To examine the possibility that investors have biased beliefs about expected cash flows, we once again use analysts forecasts from the Thompson Reuters IBES data set. We define a stock's quarterly ROE surprise as the difference between its realized ROE and the analyst consensus ROE forecast. The annual ROE surprise is the average surprise over the previous four quarters. Row (1) of Table A.15 Panel B shows that there is no evidence that $PVS_t$ forecasts earnings surprises. In row (2), we examine revisions in expectations of future cash flows. We study how analyst expectations for quarterly earnings at quarter $t+3$ evolve from quarter $t$ to $t+2$. We choose these horizons based on data availability in IBES. Row (2) shows that $PVS_t$ does not forecast revisions in expected earnings. These results reiterate the point that $PVS_t$ is largely driven by expectations of risk, and not by incorrect beliefs about the future cash flows of volatile firms.

### A3.5.3 PVS and Direct Measures of Expected Risk

In Section II.D of the main text, we showed that $PVS_t$ is negatively correlated with direct measures of expected risk: (i) subjective expectations of earnings volatility of high-volatility stocks (relative to low-volatility) from analyst forecasts; (ii) expected return volatility of high-volatility stocks based on option prices; (iii) objective expectations of return volatility for high-volatility stocks, where objective expectations are defined from the perspective of a statistical forecasting model; (iv) the percent of loan officers loosening lending standards, which is plausibly related to their subjective expectations of risk; (v) small business optimism; and (vi) the Baker et al. (2016) economic policy uncertainty measure. Our main finding is that $PVS_t$ is highly correlated with subjective measures of risk and weakly correlated with objective measures of risk from statistical forecasting models.

We complement the evidence in Section II.D with additional results in Table A.16, where we relate $PVS_t$ to objective and subjective measures of expected risk for aggregate macroeconomic variables and the aggregate stock market. Overall, Table A.16 further supports the conclusion that $PVS_t$ is related to expected risk, and that this connection is most evident for subjective measures of risk that reflect both public and private firms. The first set of columns in Table A.16 uses univariate regressions to investigate the link between $PVS_t$ and other measures of expected risk. The second set of columns links the one-year real interest rate with the same measures of expected risk. In row (1), we build a measure of the expected volatility of the aggregate stock market. From 1986 onward, we follow Bloom (2009) and use the VXO implied volatility index of the S&P 100, which is highly correlated with the popular VIX index. Options data is not available prior to 1986, so we use the one-step ahead forecast from fitting an AR(1) model to the within-quarter realized volatility of the aggregate stock market, which we scale to create a smooth series when the VXO becomes available. The regression results show that $PVS_t$ is lower when the expected volatility of the aggregate stock market is high. However, the relationship is not statistically significant, emphasizing the importance of using a measure of risk that does not overweight low-volatility public firms.

In row (2), we construct an objective measure of risk designed to reflect the whole economy, not just those firms that dominate the aggregate stock market. Specifically, we an fit ARMA(1,1)-GARCH(1,1) model to industrial production growth and then define the objective expectation of risk as the one-period forecast of volatility from the GARCH component of the model. We find a negative relationship between the expected risk of industrial production and $PVS_t$, though this relation is not statistically insignificant. In untabulated results, we observe similar patterns when using GDP growth or consumption growth. Using a similar measure of macroeconomic risk, Hartzmark (2016) finds a statistically significant relation with interest rate over a sample period that includes the Great Depression. Our finding of a weak result during our post-war sample emphasizes the importance of relating $PVS_t$ to subjective measures of macroeconomic risk.

In row (3), we again find little relation between $PVS_t$ and the macroeconomic uncertainty index from Jurado et al. (2015), again emphasizing the need to relate $PVS_t$ to subjective measures of risk. Jurado et al. (2015) define the uncertainty of a macroeconomic series as the conditional volatility of the purely unforecastable component of that series. They employ sophisticated econometric techniques to compute uncertainty measures for a wide range of macroeconomic and financial series, and then combine them into a single aggregate index of macroeconomic uncertainty.

Rows (2) and (3) focus on objective measures of macroeconomic risk, whereas asset prices and macroeconomic activity should reflect subjective expectations over a broad cross-section of private and public firms. Motivated by this observation, in row (4) of Table A.16 we create an index of macroeconomic uncertainty using forecast dispersion of growth rates in real GDP, industrial production, real private fixed nonresidential investment, and corporate profits. Specifically, we obtain 1-quarter, 2-quarter, and 4-quarter forecasts from the Survey of Professional Forecasters for 1985 onwards, which is when real growth rates were asked for instead of imputed. We then apply a Hodrick-Prescott filter to each series, standardize, and take a cross-sectional average to arrive at our SPF Macroeconomic Uncertainty index. A univariate regression of $PVS_t$ on this SPF Macroeconomic Uncertainty index confirms that it is low when macroeconomic survey uncertainty is high. The point estimate in the regression is measured precisely and is large relative to the other measures in the table. Similarly, SPF Macroeconomic Uncertainty is negatively correlated with the real rate, though less than $PVS_t$ itself (Table III of the main text). Overall, these results reinforce the idea that $PVS_t$ reflects subjective expectations of risk that are relevant for macroeconomy, in part because $PVS_t$ captures risk perceptions relevant to private and public firms, and not just the risks of low-volatility public firms that are relatively overweighted in the aggregate stock market.

### A3.5.4 Subjective Expectations of Risk and Realized Risk

In both Section II.D of the main text and Section A3.5.3 of this appendix, we showed that $PVS_t$ is more correlated with subjective expectations of risk than objective expectations. In Section IV.B of the main text, we argued that this fact is consistent with our other evidence that risk expectations may not be fully rational. To be clear, this is not to say that subjective expectations of risk that drive $PVS_t$ are completely disconnected from reality. Indeed, the model of diagnostic expectations in the main text is based on the assumption that subjective expectations reflect a kernel of truth. To show that this is the case, we build on our finding from Section II.D that $PVS_t$ is correlated with subjective expectations of risk that are embedded in the options of high-volatility firms. In particular, we test whether option-implied volatilities are rooted in reality by checking whether they forecast subsequent realized volatilities. We do so via the following panel forecasting regression of future firm-level realized volatility on current implied volatility:

$$\text{Realized Volatility}_i(t+k,t+h) = a + b \times \text{IV}_{i,t}(t+k,t+h) + \varepsilon_{i,t}$$

where Realized Volatility$_i(t+k,t+h)$ is the realized volatility of firm $i$ from $t+k$ to $t+h$. $IV_{i,t}$ is the implied volatility of firm $i$ measured at time $t$ for returns from $t+k$ to $t+h$.[18] Table A.17 contains the results of these regressions, which again are run only for firms that are classified as high-volatility as of time $t$. In column (1), we test whether time-$t$ implied volatilities based on options with a one-year maturity forecast realized volatility over the subsequent year. Column (2) of the table runs the panel forecasting regression with industry-by-time fixed effects to account for any potential variance risk premiums embedded in the options of firms in the same industry. Columns (3) and (4) focus on the sample preceding the 2008-09 financial crisis. The point estimates in the pre-crisis sample are generally higher than their full-sample counterparts, likely reflecting dislocations in the options markets during the crisis. Columns (5)-(8) run similar regressions for $k = 3, h = 4$. In all cases, the point estimates and their standard errors indicate that implied volatilities forecast for future realized volatility.

There are two key takeaways from these predictive regressions. First, expectations of volatility embedded in the options of high-volatility firms do reflect information about future realized volatility. As the paper shows, these expectations are also a key driver of movements in PVS. Second, however, these expectations are biased estimates of future volatility. To see why, note that if option-implied volatilities were an unbiased estimate of future realized volatility, these regressions would deliver a constant of zero and a point estimate on implied volatilities of one. However, in all of our specifications, we can comfortably reject this null hypothesis.

In the main text, we show that the degree of bias in risk expectations has predictable temporal patterns. In particular, we use PVS to forecast implied volatility forecast errors, defined as the difference between realized volatility and implied volatility. We find that when PVS is high, realized volatility consistently exceeds expected volatility from options, especially for high-volatility stocks. Taken together then, our analysis paints a simple picture. PVS is at least partially driven by expectations of risk. These expectations are based in some truth, as evidenced by the fact that they forecast future volatility. At the same time, these expectations are often biased in the sense that periods where PVS is high are reliably followed by periods in which realized volatility exceeds expected volatility.

[18] We obtain implied volatilities at the firm level from the standardized volatility surface produced by OptionMetrics. For a given maturity, we compute the implied volatility for each firm by averaging all available strikes in the standardized volatility surface dataset. We chose this route for its convenience and simplicity, though a more precise approach would build VIX-like measures at the firm level.

#### A3.5.5 Realized Risk, Expected Risk, and Good News

In Table A.18 we directly examine the link between measures of realized and expected risk and macroeconomic news that we assumed in the model. For compactness, we include all macroeconomic news measures in every column.[19] In column (1), we show that the realized volatility of high-volatility stocks falls relative to low-volatility stocks following good news. Columns (2)-(4) repeat the analysis using three different measures of expected risk. Column (2) shows that analysts' perceived risk of high-volatility firms declines when there is positive news about GDP and corporate profit growth. The same general pattern emerges when we use the percentage of banks loosening lending standards (column 3) and the NFIB Small Business Optimism Index (column 4) as our measures of expected risk.

#### A3.5.6 Measurement of Revisions in Expected Risk

In the main text, we use options to study revisions from quarter $t$ to $t+3$ in the expected volatility of stock returns that will be realized between $t+3$ and $t+4$. In particular, we test whether $PVS_t$ can forecast these revisions, which we infer from the implied volatility embedded in option prices.

To formalize our approach, first define the time-$t$ conditional variance of returns between $t+k$ and $t+h$, denoted by $R_{t+k,t+h}$, as:

$$\begin{aligned}\mathbb{V}_t\left(R_{t+k,t+h}\right) &\equiv \mathbb{E}_{t+k}\left[R^2_{t+k,t+h}\right] - \mathbb{E}^2_{t+k}\left[R_{t+k,t+h}\right] \\ &= \mathbb{E}_t\left[\mathbb{V}_{t+k}\left(R_{t+k,t+h}\right)\right] + \mathbb{V}_t\left(\mathbb{E}_{t+k}\left[R_{t+k,t+h}\right]\right) \end{aligned} \tag{9}$$

where the second equality follows from the law of total variance.

Next, define the news about variance between $t$ and $t+k$ as:

$$\begin{aligned}\eta_{t+k} &\equiv \mathbb{E}_{t+k}\left[\mathbb{V}_{t+k}\left(R_{t+k,t+h}\right)\right] - \mathbb{E}_t\left[\mathbb{V}_{t+k}\left(R_{t+k,t+h}\right)\right] \\ &= \mathbb{V}_{t+k}\left(R_{t+k,t+h}\right) - \mathbb{E}_t\left[\mathbb{V}_{t+k}\left(R_{t+k,t+h}\right)\right] \end{aligned} \tag{10}$$

Our approach in the main text effectively focuses on the following object:

$$\theta_{t+k} \equiv \mathbb{V}_{t+k}\left(R_{t+k,t+h}\right) - \mathbb{V}_t\left(R_{t+k,t+h}\right),$$

which we can easily construct using option prices at time $t$ and $t+k$. $\theta_{t+k}$ is not exactly the same as the news about expected variance, but is close. To concretely relate the two, substitute Eq. (9) into Eq. (10) and rearrange to get:

$$\theta_{t+k} = \eta_{t+k} - \mathbb{V}_t\left(\mathbb{E}_{t+k}\left[R_{t+k,t+h}\right]\right) \tag{11}$$

In the data, we use $PVS_t$ to forecast $\theta_{t+k}$. However, to ensure our point estimates on $PVS_t$ are not biased in this regression, we should also control for $\mathbb{V}_t\left(\mathbb{E}_{t+k}\left[R_{t+k,t+h}\right]\right)$. Thus, the remaining task is to construct $\mathbb{V}_t\left(\mathbb{E}_{t+k}\left[R_{t+k,t+h}\right]\right)$ in the data. To do so, let's focus on the case where $k=3$ and $h=4$, as we do in the main text. Next, notice that we can write:

$$\mathbb{V}_t(\mathbb{E}_{t+3}[R_{t+3,t+4}]) = \mathbb{E}_t\left\{\mathbb{E}^2_{t+3}[R_{t+3,t+4}]\right\} - \mathbb{E}^2_t[R_{t+3,t+4}] \tag{12}$$

[19]The fact that the real GDP surprise does not come in significant in columns (1) and (3) is a product of the multivariate regression. In univariate regressions, it comes in negative and significant.

We can form an estimate of $\mathbb{E}_{t+3}[R_{t+3,t+4}]$ by regressing $R_{t+3,t+4}$ on $PVS_{t+3}$. In turn, the square of the fitted value from this forecasting regression provides an estimate of $\mathbb{E}_t\left\{\mathbb{E}^2_{t+3}[R_{t+3,t+4}]\right\}$. Similarly, we can construct an estimate $\mathbb{E}^2_t[R_{t+3,t+4}]$ based on the square of the fitted value from a regression of $R_{t+3,t+4}$ on $PVS_t$. Combining the two yields an proxy for $\mathbb{V}_t\left(\mathbb{E}_{t+k}\left[R_{t+k,t+h}\right]\right)$, which we then add as a control to our forecasting regression. The results are presented below:

$$
\begin{array}{ccccccccccc}
\theta_{t+3} & = & a & + & b_1 & \times & PVS_t & + & b_2 & \times & \mathbb{V}_t\left(\mathbb{E}_{t+3}\left[R_{t+3,t+4}\right]\right) \\
 & & 0.05 & & 0.47 & & & & 0.004 & & \\
 & & (0.28) & & (2.99) & & & & (5.05) & &
\end{array}
$$

where point estimates are listed below the coefficients and $t$-statistics based on Newey-West standard errors with five lags are in parenthesis. As is clear from the regression, controlling for the time-$t$ variance of expected returns at $t+3$ does not change the main conclusion that $PVS_t$ forecasts revisions in risk. Moreover, if we just regress $\theta_{t+3}$ onto $PVS_t$, the point estimate is basically unchanged at 0.48. With this in mind, and to keep the exposition as simple as possible, in the main text we focus on predicting revisions in volatility as opposed to variance. In addition, we do not control for the time-$t$ variance of expected returns at $t+3$.

#### A3.5.7 Forecasting Negative Returns

We next examine return forecasts as a complementary way of assessing whether the expectations of risk underlying $PVS_t$ are rational. We study the profitability of strategies that sell put options because their returns depend directly on the accuracy of investors' expectations of risk. Under rational expectations, riskier strategies should always have higher expected returns. Assuming that options on high-volatility stocks are riskier than options on low-volatility stocks, rational investors should always require higher expected returns for selling puts on high-volatility firms. In contrast, if investors underestimate risk when $PVS_t$ is high, as our previous results suggest, then expected returns to selling puts on high-volatility firms may be lower than returns to selling puts on low-volatility firms at these times.

We compute the returns to selling puts using data from OptionMetrics, following the procedure of Jurek and Stafford (2015). For each firm $i$ and quarter $t$, this procedure finds the set of out-of-the-money put options with the lowest maturity greater than 182 days. From this set, we then select the put option that is closest-to-the-money and require that the delta of the option is at least -0.4 to account for differences in volatility across firms and time. We sell this option at the best bid price, hold it for one quarter, then buy it at the best offer price.[20] At the portfolio level, we take the equal-weighted average of high-volatility firm returns minus the equal-weighted average of low-volatility firm returns.

Panel A of Figure A.11 plots the realized returns to this strategy at time $t+1$ as a function of $PVS_t$, as well as the fitted value from the forecasting regression and the 95% confidence interval for the fitted value. We label forecast dates with significantly negative expected returns. The figure shows that conditional expected returns were significantly negative in 2000q1 and 2000q2, as indicated by the 95% confidence interval falling below zero. This suggests that when $PVS_t$

[20]Following Jurek and Stafford (2015), we also assume the put writing strategy is twice levered. Leverage only affects the level of returns, not our return forecasting results. The assumed amount of leverage is well within the Chicago Board Options Exchange (CBOE) margin requirements for single name options.

is high, investors underestimate risk and therefore charge too little when selling put options on volatile firms.

The option price data is available for a relatively short sample, so there are only two quarters in which we forecast negative expected returns. Reassuringly, Figure A.11 Panel B shows that the periods when we forecast negative returns to selling puts on volatile stocks coincide with periods when we forecast negative excess returns to holding volatile stocks themselves. Taken together, the evidence on return predictability suggests that investors sometimes underestimate risk. At these times, volatile stocks are too expensive and puts on volatile stocks are too cheap. Subsequently, investors realize that they under-estimated risk and revise their expectations of risk upward. The prices of volatile stocks then fall, and the prices of puts on volatile stocks rise. Investors underestimate risk enough that during the quarters with the highest values of $PVS_t$, we forecast significantly negative returns to selling puts on volatile stocks and to holding volatile stocks.

# A4 Model Appendix

In this appendix, we provide proofs for the model propositions.

## A4.1 Risk

Consumption growth is described by the following process:

$$\begin{aligned} \Delta c_{t+1} &= \varepsilon_{t+1}, & (13)\\ \varepsilon_{t+1} &= \exp(a - b\varepsilon_t)\eta_{t+1}, & (14) \end{aligned}$$

where $\eta_{t+1}$ is iid standard normal, so the conditional variance of log output growth is $\mathbb{V}_t(\varepsilon_{t+1}) = \exp(a - b\varepsilon_t)$.

Taking the comparative static in the vicinity of $\varepsilon_t = 0$ then gives Proposition 2a:

$$\frac{d\mathbb{V}_t(\varepsilon_{t+1})}{d\varepsilon_t} = -\exp(a)b < 0. \qquad (15)$$

## A4.2 Real Risk-Free Rate

The stochastic discount factor can be written as:

$$\begin{aligned} M_{t+1} &= \beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}, & (16)\\ &= \beta\exp(-\gamma\varepsilon_{t+1}). & (17) \end{aligned}$$

The time-$t$ log real risk-free rate is then given by the asset pricing Euler equation:

$$\begin{aligned} 1 &= \mathbb{E}_t\left[\exp(r_{ft})M_{t+1}\right], & (18)\\ &= exp(r_{ft})\beta\exp\left(\frac{1}{2}\gamma^2\mathbb{V}_t(\varepsilon_{t+1})\right), & (19) \end{aligned}$$

implying that

$$r_{ft} = -\ln(\beta) - \frac{1}{2}\gamma^2\mathbb{V}_t(\varepsilon_{t+1}). \qquad (20)$$

Taking the comparative static of $r_{ft}$ in the vicinity of $\varepsilon_t = 0$ gives:

$$\frac{dr_{ft}}{d\varepsilon_t} = -\frac{1}{2}\gamma^2 \frac{d\mathbb{V}_t(\varepsilon_{t+1})}{d\varepsilon_t}. \tag{21}$$

Substituting in for $\frac{d\mathbb{V}_t(\varepsilon_{t+1})}{d\varepsilon_t}$ from equation (15) gives Proposition 2d:

$$\frac{dr_{ft}}{d\varepsilon_t} = \frac{1}{2}\gamma^2 \exp(a)b > 0. \tag{22}$$

## A4.3 Risky Returns

The marginal return to capital in firm $i$ is the marginal benefit of an additional unit of investment divided by the marginal cost:

$$\begin{aligned} R_{it+1} &= \left(\frac{dY_{it+1}}{dK_{it+1}}\right) / \left(\frac{d\Phi_{it}}{dI_{it}}\right), & (23) \\ &= \exp\left(s_i\varepsilon_{t+1} - \frac{1}{2}s_i^2\mathbb{V}_t(\varepsilon_{t+1})\right) / \phi'\left(\frac{I_{it}}{K_{it}}\right). & (24) \end{aligned}$$

Taking the expectation conditional on information known at time $t$ shows:

$$\mathbb{E}_t[R_{it+1}] = 1/\phi'\left(\frac{I_{it}}{K_{it}}\right). \tag{25}$$

Substituting for $R_{it+1}$ into the asset pricing Euler equation, $1 = \mathbb{E}_t[M_{t+1}R_{it+1}]$, gives:

$$\begin{aligned} 1 &= \frac{\mathbb{E}_t\left[M_{t+1}\exp\left(s_i\varepsilon_{t+1} - \frac{1}{2}s_i^2\mathbb{V}_t(\varepsilon_{t+1})\right)\right]}{\phi'\left(\frac{I_{it}}{K_{it}}\right)}, & (26) \\ &= \frac{\beta\exp\left(\frac{1}{2}\left((\gamma - s_i)^2 - s_i^2\right)\mathbb{V}_t(\varepsilon_{t+1})\right)}{\phi'\left(\frac{I_{it}}{K_{it}}\right)}, & (27) \end{aligned}$$

so the log expected return on capital must equal:

$$\begin{aligned} \ln(\mathbb{E}_t[R_{it+1}]) &= \ln\left(1/\phi'\left(\frac{I_{it}}{K_{it}}\right)\right), \\ &= -\ln\beta - \frac{1}{2}\left((\gamma - s_i)^2 - s_i^2\right)\mathbb{V}_t(\varepsilon_{t+1}). & (28) \end{aligned}$$

Combining this with the expressions for the real risk-free rate (20) gives Eq. (6) in the main paper:

$$\ln(\mathbb{E}_t[R_{it+1}]) - r_{ft} = \gamma s_i \mathbb{V}_t(\varepsilon_{t+1}). \tag{29}$$

Taking the difference of the expression (28) for high- vs. low-volatility firms gives:

$$\ln[\mathbb{E}_t R_{Ht+1}] - \ln[\mathbb{E}_t R_{Lt+1}] = \gamma(s_H - s_L)\mathbb{V}_t(\varepsilon_{t+1}). \tag{30}$$

We then take the comparative static of (30) with respect to $\varepsilon_t$ in the vicinity of $\varepsilon_t = 0$ and apply the chain rule with (15) to obtain Proposition 2c:

$$\frac{d(\ln[\mathbb{E}_t R_{Ht+1}] - \ln[\mathbb{E}_t R_{Lt+1}])}{d\varepsilon_t} = -\gamma(s_H - s_L)\exp(a)b. \tag{31}$$

## A4.4 Valuation Ratios and $PVS_t^{model}$

We next solve for book-to-market ratios and $PVS_t^{model}$. Because of our assumption that each firm produces only for one period, firm $i$'s market-to-book ratio equals:

$$\begin{aligned}\frac{V_{it}-D_{it}}{K_{it+1}} &= \frac{\mathbb{E}_t\left[M_{t+1}D_{it+1}\right]}{K_{it+1}} &(32)\\ &= \mathbb{E}_t\left[M_{t+1}\exp\left(s_i\varepsilon_{t+1}-\frac{1}{2}s_i^2\mathbb{V}_t\left(\varepsilon_{t+1}\right)\right)\right] &(33)\end{aligned}$$

With the expression for expected returns (25) and the asset pricing Euler equation (26) it follows that the book-to-market ratio equals the expected return:

$$\begin{aligned}\frac{K_{it+1}}{V_{it}-D_{it}} &= \frac{K_{it+1}}{\mathbb{E}_t\left[M_{t+1}D_{it+1}\right]}=\frac{1}{\phi'\left(\frac{I_{it}}{K_{it}}\right)}, &(34)\\ &= \mathbb{E}_t\left[R_{it+1}\right]. &(35)\end{aligned}$$

We can therefore write $PVS_t^{model}$ as:

$$\begin{aligned}PVS_t^{model} &= \ln\left(\frac{K_{Lt+1}}{V_{Lt}-D_{Lt}}\right)-\ln\left(\frac{K_{Ht+1}}{V_{Ht}-D_{Ht}}\right), &(36)\\ &= -\left(\ln\left[\mathbb{E}_tR_{Ht+1}\right]-\ln\left[\mathbb{E}_tR_{Lt+1}\right]\right). &(37)\end{aligned}$$

Proposition 2b then follows directly from Proposition 2c.

## A4.5 Real Investment

Finally, we use the functional form for $\phi$ to solve for real firm investment. Because adjustment costs are assumed to be quadratic, we have:

$$\phi'\left(\frac{I_{it}}{K_{it}}\right) = 1+\frac{I_{it}}{K_{it}}. \quad (38)$$

Equating the log return on real investment (25) with the log return required by risk-averse investors (28) then gives:

$$\begin{aligned}inv_{it} &= \ln\left(1+\frac{I_{it}}{K_{it}}\right), &(39)\\ &= \ln\beta+\frac{1}{2}\left(\left(\gamma-s_i\right)^2-s_i^2\right)\mathbb{V}_t\left(\varepsilon_{t+1}\right), &(40)\\ &= \ln\beta-\gamma\left(s_i-\frac{\gamma}{2}\right)\mathbb{V}_t\left(\varepsilon_{t+1}\right).\end{aligned}$$

Taking the comparative static with respect to $\varepsilon_t$ in the vicinity of $\varepsilon_t=0$ and applying the chain rule gives:

$$\begin{aligned}\frac{dinv_{it}}{d\varepsilon_t} &= \frac{1}{2}\left(\gamma^2-2\gamma s_i\right)\frac{d\mathbb{V}_t\left(\varepsilon_{t+1}\right)}{d\varepsilon_t}, &(41)\\ &= -\frac{1}{2}\left(\gamma^2-2\gamma s_i\right)\exp(a)b, &(42)\\ &= \gamma\left(s_i-\frac{\gamma}{2}\right)\exp(a)b. &(43)\end{aligned}$$

Propositions 2e and 2f then follow.

## A4.6 Model with Diagnostic Beliefs

While the model in the main text features rational expectations, in the data we find that $PVS_t$ forecasts revisions in expected risk. In this section, we augment the model with the diagnostic expectations of Gennaioli and Shleifer (2010, 2018); Bordalo et al. (2018) to rationalize this additional evidence.

The key properties of subjective expectations of risk that we are trying to capture are that (i) they fall after good news, and (ii) they fall too far, so that there are predictable upward revisions. We now show that one can account for these features of the data by assuming that investors update using diagnostic expectations, overweighting states of the world that are representative. Following the assumptions in Gennaioli and Shleifer (2018, Chapter 5), under diagnostic expectations and the subjective perceived time-$t$ conditional mean and variance of $\varepsilon_{t+1}$ are:

$$\mathbb{E}_t^{\theta}(\varepsilon_{t+1}) = 0, \tag{44}$$

$$\mathbb{V}_t^{\theta}(\varepsilon_{t+1}) = \frac{\mathbb{V}_t(\varepsilon_{t+1})}{1+\theta(1-\exp(-b\varepsilon_t))}, \tag{45}$$

where $\mathbb{V}_t(\varepsilon_{t+1})$ continues to denote the objective conditional variance.[21] For $\theta > 0$, Eq. (45) implies that investors tend to underestimate macroeconomic risk following a positive $\varepsilon_t$ shock and overestimate risk following a negative $\varepsilon_t$ shock. In our model, objective risk falls after a positive consumption surprise, but subjective risk falls even more. Thus, diagnostic beliefs capture the over-extrapolation we document in the data.

### A4.6.1 Results for Model with Diagnostic Beliefs

Assuming that preferences and the firm's problem are the same as in Section II in the main paper, the equilibrium under diagnostic expectations is characterized by the same equations as before (Eqs. (6), (10), and (11)) in the main paper, simply replacing objective risk $\mathbb{V}_t(\varepsilon_{t+1})$ with subjective risk $\mathbb{V}_t^{\theta}(\varepsilon_{t+1})$.[22] Similarly, the comparative statics in Proposition 2 that capture key elements of risk-centric theories have the same signs as before and are amplified by a factor of $(1+\theta)$. In other words, diagnostic expectations strengthen risk-centric economic fluctuations because investors' expectations of risk overreact to recent news.

Finally, we show that diagnostic expectations lead to predictable revisions in investor expectations of risk. We assume that at the end of period $t$ investors learn the true volatility and revise

[21] This result follows from Proposition 1 in Gennaioli and Shleifer (2018, Chapter 5) under the following assumptions: The representativeness of state $\varepsilon_{t+1}$ is given by $\frac{h(\varepsilon_{t+1}|\varepsilon_t)}{\bar{h}(\varepsilon_{t+1})}$, where $h$ is the likelihood function and $\bar{h}(\varepsilon_{t+1})$ is the reference likelihood. As in Gennaioli and Shleifer (2018, Chapter 6), we assume that agents' reference distribution is the distribution at the state in the absence of news, i.e. $\bar{h}(\varepsilon_{t+1}) = h(\varepsilon_{t+1}|\varepsilon_t = 0)$. The distorted likelihood $h^{\theta}$ equals $h^{\theta}(\varepsilon_{t+1}|\varepsilon_t) = h^{\theta}(\varepsilon_{t+1}|\varepsilon_t)\left(\frac{h(\varepsilon_{t+1}|\varepsilon_t)}{\bar{h}(\varepsilon_{t+1})}\right)^{\theta} Z$, where $Z$ is a constant ensuring that the likelihood of different states integrates up to one. The parameter $\theta$ indexes the degree of belief distortion, where $\theta = 0$ corresponds to rational expectations and $\theta > 0$ implies that agents overweight representative states.

[22] To ensure that subjective expected total factor productivity is equalized across firms, we continue to assume that the representative investor perceives firm $i$'s total factor productivity $Z_{i,t+1} = \exp\left(s_i\varepsilon_{t+1} - \frac{1}{2}s_i^2\mathbb{V}_t^{\theta}(\varepsilon_{t+1})\right)$.

their beliefs to $\mathbb{V}_t(\varepsilon_{t+1}) = exp(a - b\varepsilon_t)$. The following proposition gives the relationship between the revision in beliefs and $PVS_t^{model}$.

**Proposition 3:** Suppose we have two types of firms $H$ and $L$ with $s_H > s_L > \frac{\lambda}{2}$ and that investors have diagnostic beliefs ($\theta > 0$). In the neighborhood of $\varepsilon_t = 0$, high values of $PVS_t^{model}$ forecast positive revisions in expected risk:

$$\frac{d(\mathbb{V}_t[\varepsilon_{t+1}] - \mathbb{V}_t^{\theta}[\varepsilon_{t+1}])}{dPVS_t^{model}} = \frac{\theta}{1+\theta}\frac{1}{\gamma(s_H - s_L)} > 0.$$

Proposition 3 formalizes the intuition in classical risk-centric accounts of the business cycle that expectations of risk contain an element of overreaction (Keynes (1937), Minsky (1977)). Following a good shock, investors lower their subjective expectations of risk too much, resulting in a value of $PVS_t^{model}$ that is too high. They then predictably revise their beliefs back up, so high values of $PVS_t^{model}$ forecast positive revisions in expectations of risk. Proposition 3 shows that the model with diagnostic expectations can rationalize the finding that $PVS_t$ positively forecasts revisions in expected risk (Table VIII) and volatility forecast errors (Table IX). These findings cannot be explained by the rational model with $\theta = 0$. A simple calculation shows that our empirical results imply reasonable magnitudes for the belief distortion parameter, $\theta$. Rows (1) and (4) of Table II suggest that subjective expectations of risk move about twice as much in response to $PVS_t$ as objective expectations, which implies that we need $\theta \approx 1$, in line with the estimates of Bordalo et al. (2018) and Bordalo et al. (2018).

### A4.6.2 Derivations for Model with Diagnostic Beliefs

Our derivation of the subjective distribution follows Gennaioli and Shleifer (2018), Chapters 5 and 6. Their Proposition 1 in Chapter 5 states the following:

> Suppose that $ln\tilde{X}|I_0 \sim N\left(\mu_0, \sigma_0^2\right)$ and $ln\tilde{X}|I_{-1} \sim N\left(\mu_{-1}, \sigma_{-1}^2\right)$. Then, provided $(1+\theta)\sigma_{-1}^2 - \theta\sigma_0^2 > 0$, the distorted density $h^{\theta}(\tilde{X}|I_0)$ is also lognormal with mean $\mu_0(\theta)$ and variance $\sigma_0^2(\theta)$ given by:
>
> $$\mu_0(\theta_0) = \mu_0 + \frac{\theta\sigma_0^2}{\sigma_{-1}^2 + \theta\left(\sigma_{-1}^2 - \sigma_0^2\right)}(\mu_0 - \mu_{-1}), \tag{46}$$
>
> $$\sigma_0^2(\theta) = \sigma_0^2\frac{\sigma_{-1}^2}{\sigma_{-1}^2 + \theta\left(\sigma_{-1}^2 - \sigma_0^2\right)} \tag{47}$$

As in Gennaioli and Shleifer (2018), Chapter 6, we assume that agents' reference distribution is the distribution at the state vector in the absence of news. That is, the reference distribution for $\varepsilon_{t+1}$ before learning $\varepsilon_t$ is the distribution at the conditional average of $\varepsilon_t$, i.e. at $\mathbb{E}_{t-1}(\varepsilon_t) = 0$. This gives $\mu_{-1} = 0$, $\sigma_{-1}^2 = \exp(a)$, $\mu_0 = 0$, and $\sigma_0^2 = \exp(a - b\varepsilon_t)$.[23] Substituting into the Proposition

[23] We follow Bordalo et al. (2018) in considering the distribution at the conditional average of $\varepsilon_t$ as the reference distribution for simplicity and tractability. Alternatively, we could also consider the case where the reference distribution equals the conditional distribution of $\varepsilon_{t+1}$ conditional on knowing $\varepsilon_{t-1}$. This would give $\mu_{-1} = 0$ and $\sigma_{-1} = \sqrt{exp(a + \frac{1}{2}b^2\sigma_{t-1}^2)}$. This would make the solution more complicated but preserve the main qualitative feature that the subjective variance $\mathbb{V}_t^{\theta}(\varepsilon_{t+1})$ reacts more to $\varepsilon_t$ than the objective variance $\mathbb{V}_t(\varepsilon_t)$.

gives the subjective mean and variance for $\varepsilon_{t+1}$ after having learned $\varepsilon_t$:

$$\begin{aligned}
\mathbb{E}_t^{\theta}\left(\varepsilon_{t+1}\right) &= 0, && (48)\\
\mathbb{V}_t^{\theta}\left(\varepsilon_{t+1}\right) &= \exp\left(a-b\varepsilon_t\right)\frac{1}{1+\theta\left(1-\exp(-b\varepsilon_t)\right)}, && (49)\\
&= \frac{\mathbb{V}_t\left(\varepsilon_{t+1}\right)}{1+\theta\left(1-\exp(-b\varepsilon_t)\right)}. && (50)
\end{aligned}$$

The subjective variance is therefore distorted relative to the objective variance by a factor of $\frac{1}{1+\theta(1-\exp(-b\varepsilon_t))}$.[24]

The derivations for the real risk-free rate (20), log expected excess returns (29), and log real firm investment (40) go through with $\mathbb{V}_t\left(\varepsilon_{t+1}\right)$ replaced by $\mathbb{V}_t^{\theta}\left(\varepsilon_{t+1}\right)$ everywhere. This proves Proposition 1′.

To prove Proposition 2′a, we find the comparative static of $\mathbb{V}_t^{\theta}\left(\varepsilon_{t+1}\right)$ with respect to $\varepsilon_t$ in the vicinity of $\varepsilon_t = 0$:

$$\begin{aligned}
\frac{d\mathbb{V}_t^{\theta}\left(\varepsilon_{t+1}\right)}{d\varepsilon_t} &= -\left(1+\theta\right)b\exp(a)\\
&= \left(1+\theta\right)\frac{d\mathbb{V}_t\left(\varepsilon_{t+1}\right)}{d\varepsilon_t}. && (51)
\end{aligned}$$

To show Proposition 2′b, note that Propositions 2b through f were all proved using the chain rule for $\frac{d\mathbb{V}_t(\varepsilon_{t+1})}{d\varepsilon_t}$. Replacing $\frac{d\mathbb{V}_t(\varepsilon_{t+1})}{d\varepsilon_t}$ by $\frac{d\mathbb{V}^{\theta}{}_t(\varepsilon_{t+1})}{d\varepsilon_t}$ throughout shows that the comparative statics in Proposition 2b through f scale up by a factor $1+\theta$ under diagnostic expectations.

To prove Proposition 3, we apply the chain rule with respect to $PVS_t^{model}$ to take the derivative in the vicinity of $\varepsilon_t = 0$:

$$\begin{aligned}
\frac{d\left(\mathbb{V}_t\left(\varepsilon_{t+1}\right)-\mathbb{V}_t^{\theta}\left(\varepsilon_{t+1}\right)\right)}{dPVS_t^{model}} &= \frac{d\left(\mathbb{V}_t\left(\varepsilon_{t+1}\right)-\mathbb{V}_t^{\theta}\left(\varepsilon_{t+1}\right)\right)}{d\varepsilon_t}\frac{1}{\frac{dPVS_t^{model}}{d\varepsilon_t}} && (52)\\
&= \theta\exp(a)b\frac{1}{\gamma(s_H-s_L)\left(1+\theta\right)\exp(a)b} && (53)\\
&= \frac{\theta}{1+\theta}\frac{1}{\gamma(s_H-s_L)} > 0. && (54)
\end{aligned}$$

This completes the model proofs.

[24]Technically the proposition only applies if $1+\theta\left(1-exp(-b\varepsilon_t)\right) > 0$, or if the variance does not increase excessively. We follow Bordalo et al. (2018) in imposing this condition, which holds with probability one in the perfectly rational limit with $\theta \to 0$.

# References


Ang, A., R. J. Hodrick, Y. Xing, and X. Zhang (2006). The cross-section of volatility and expected returns. *Journal of Finance 61*(1), 259–299.

Asker, J., J. Farre-Mensa, and A. Ljungqvist (2014). Corporate investment and stock market listing: A puzzle? *Review of Financial Studies 28*(2), 342–390.

Asness, C. S., T. J. Moskowitz, and L. H. Pedersen (2013). Value and momentum everywhere. *Journal of Finance 68*(3), 929–985.

Atkeson, A. and L. E. Ohanian (2001). Are Phillips curves useful for forecasting inflation? *Federal Reserve Bank of Minneapolis Quarterly Review 25*(1), 2–11.

Baker, M. and J. Wurgler (2012). Comovement and predictability relationships between bonds and the cross-section of stocks. *Review of Asset Pricing Studies 2*(1), 57–87.

Baker, S. R., N. Bloom, and S. J. Davis (2016). Measuring economic policy uncertainty. *Quarterly Journal of Economics 131*(4), 1593–1636.

Bernanke, B. S. and K. N. Kuttner (2005). What explains the stock market's reaction to federal reserve policy? *Journal of Finance 60*(3), 1221–1257.

Bernanke, B. S. and I. Mihov (1998). Measuring monetary policy. *Quarterly Journal of Economics 113*(3), 869–902.

Bloom, N. (2009). The impact of uncertainty shocks. *Econometrica 77*(3), 623–685.

Bordalo, P., N. Gennaioli, R. La Porta, and A. Shleifer (2018). Diagnostic expectations and stock returns. *Journal of Finance*, forthcoming.

Bordalo, P., N. Gennaioli, and A. Shleifer (2018). Diagnostic expectations and credit cycles. *Journal of Finance 73*(1), 199–227.

Campbell, J. Y. and J. Ammer (1993). What moves the stock and bond markets? A variance decomposition for long-term asset returns. *Journal of Finance 48*(1), 3–37.

Campbell, J. Y. and P. Perron (1991). Pitfalls and opportunities: What macroeconomists should know about unit roots. *NBER Macroeconomics Annual 6*(Chapter 3), 141 – 220.

Campbell, J. Y. and R. J. Shiller (1988). The dividend-price ratio and expectations of future dividends and discount factors. *Review of Financial Studies 1*(3), 195–228.

Christiano, L. J., M. Eichenbaum, and C. L. Evans (1999). Monetary policy shocks: What have we learned and to what end? In J. B. Taylor and M. Woodford (Eds.), *Handbook of Macroeconomics*, Volume 1, pp. 65–148. Elsevier.

Cochrane, J. H. and M. Piazzesi (2005). Bond risk premia. *American Economic Review 95*(1), 138–160.

Cohen, R. B., C. Polk, and T. Vuolteenaho (2003). The value spread. *Journal of Finance 58*(2), 609–641.

Davis, J. L., E. F. Fama, and K. R. French (2000). Characteristics, covariances, and average returns: 1929 to 1997. *Journal of Finance 55*(1), 389–406.

Davis, S. J., J. Haltiwanger, R. Jarmin, J. Miranda, C. Foote, and E. Nagypal (2007). Volatility and dispersion in business growth rates: Publicly traded versus privately held firms. *NBER macroeconomics annual 21*, 107–180.

Fama, E. F. and K. R. French (1993). Common risk factors in the returns on stocks and bonds. *Journal of Financial Economics 33*(1), 3–56.

Gennaioli, N. and A. Shleifer (2010). What comes to mind. *Quarterly Journal of Economics 125*, 1399–1433.

Gennaioli, N. and A. Shleifer (2018). *A Crisis of Beliefs: Investor Psychology and Financial Fragility*. Princeton University Press.

Gilchrist, S. and E. Zakrajšek (2012). Credit spreads and business cycle fluctuations. *American Economic Review 102*(4), 1692–1720.

Gorodnichenko, Y. and M. Weber (2016). Are sticky prices costly? Evidence from the stock market. *American Economic Review 106*(1), 165–199.

Gürkaynak, R. S., B. Sack, and J. H. Wright (2007). The US treasury yield curve: 1961 to the present. *Journal of Monetary Economics 54*(8), 2291–2304.

Hamilton, J. D. (2017). Why you should never use the Hodrick-Prescott filter. *Review of Economics and Statistics*, forthcoming.

Hartzmark, S. M. (2016). Economic uncertainty and interest rates. *Review of Asset Pricing Studies 6*(2), 179–220.

He, Z., B. Kelly, and A. Manela (2017). Intermediary asset pricing: New evidence from many asset classes. *Journal of Financial Economics 126*(1), 1–35.

Holston, K., T. Laubach, and J. Williams (2016). Measuring the natural rate of interest: International trends and determinants. Working paper, Federal Reserve Bank of San Francisco.

Jordà, O. (2005). Estimation and inference of impulse responses by local projections. *American Economic Review 95*(1), 161–182.

Jurado, K., S. C. Ludvigson, and S. Ng (2015, March). Measuring uncertainty. *American Economic Review 105*(3), 1177–1216.

Jurek, J. W. and E. Stafford (2015). The cost of capital for alternative investments. *Journal of Finance 70*(5), 2185–2226.

Kang, J. and C. E. Pflueger (2015). Inflation risk in corporate bonds. *Journal of Finance 70*(1), 115–162.

Kelly, B. and S. Pruitt (2013). Market expectations in the cross-section of present values. *Journal of Finance 68*(5), 1721–1756.

Keynes, J. M. (1937). The general theory of employment. *Quarterly Journal of Economics 51*(2), 209–223.

Krishnamurthy, A. (2002). The bond/old-bond spread. *Journal of Financial Economics 66*(2), 463–506.

Lettau, M. and S. C. Ludvigson (2004). Understanding trend and cycle in asset values: Reevaluating the wealth effect on consumption. *American Economic Review 94*(1), 276–299.

Minsky, H. P. (1977). The financial instability hypothesis: An interpretation of keynes and an alternative to "standard" theory. *Nebraska Journal of Economics and Business 16*(1), 5–16.

Nakamura, E. and J. Steinsson (2018). High frequency identification of monetary non-neutrality: The information effect. *Quarterly Journal of Economics 133*(3), 1283–1330.

Romer, C. D. and D. H. Romer (2004). A new measure of monetary shocks: Derivation and implications. *American Economic Review 94*(4), 1055–1084.

Shumway, T. (1997). The delisting bias in CRSP data. *Journal of Finance 52*(1), 327–340.

Sims, C. A. (1980). Macroeconomics and reality. *Econometrica 48*, 1–48.

Stambaugh, R. F. (1999). Predictive regressions. *Journal of Financial Economics 54*, 375–421.

Summers, L. H. (2015). Demand side secular stagnation. *American Economic Review 105*(5), 60–65.

Taylor, J. B. (1993). Discretion versus policy rules in practice. *Carnegie-Rochester Conference Series on Public Policy 39*, 195–214.

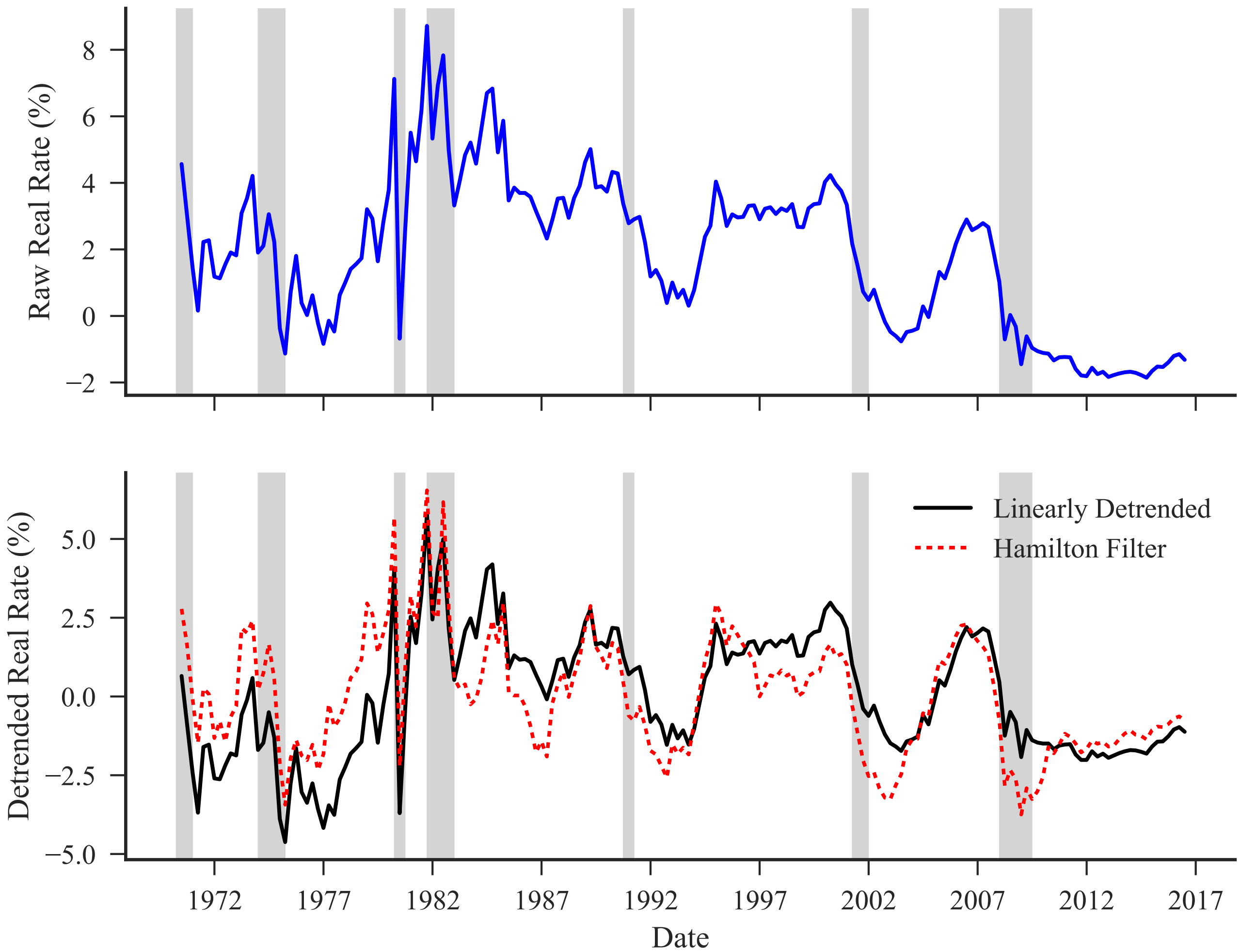
Vuolteenaho, T. (2002). What drives firm-level stock returns? *Journal of Finance 57*(1), 233–264.

Weber, M. (2016). Cash flow duration and the term structure of equity returns. Working paper, University of Chicago.

Zwick, E. and J. Mahon (2017). Tax policy and heterogeneous investment behavior. *American Economic Review 107*(1), 217–48.

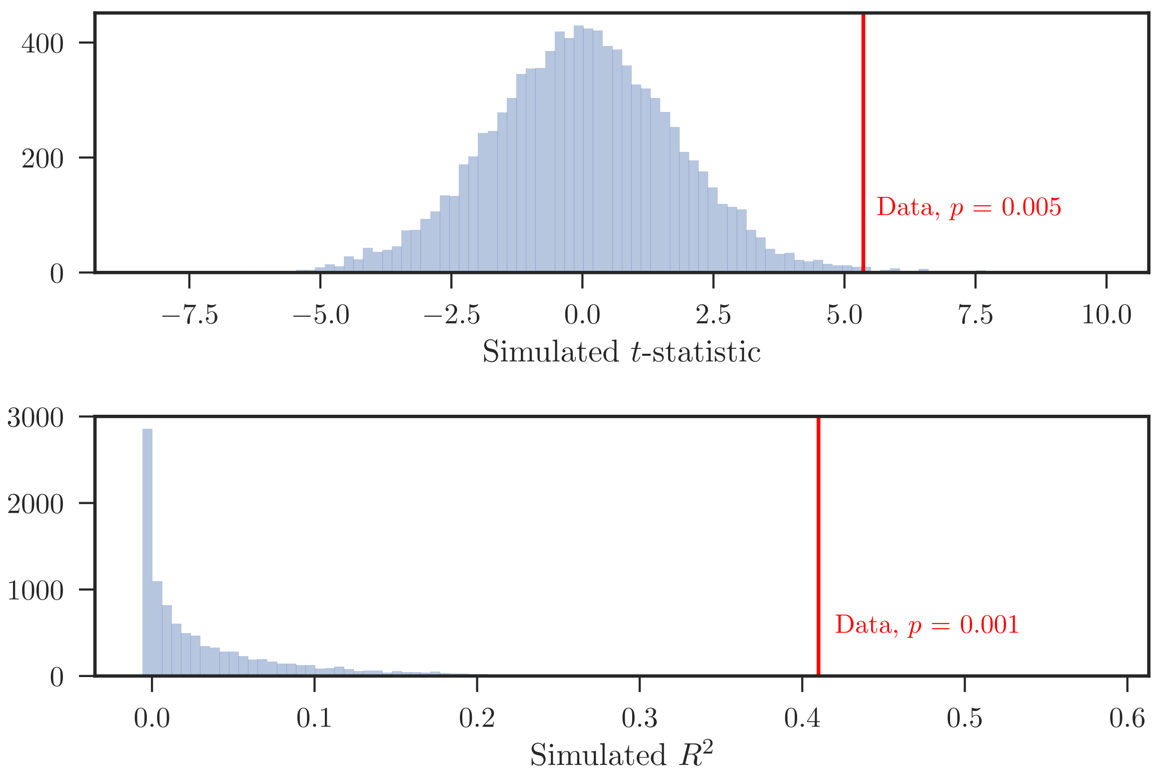
# APPENDIX FIGURES

Figure A.1: Comparing Filtering Methods for the Real Rate



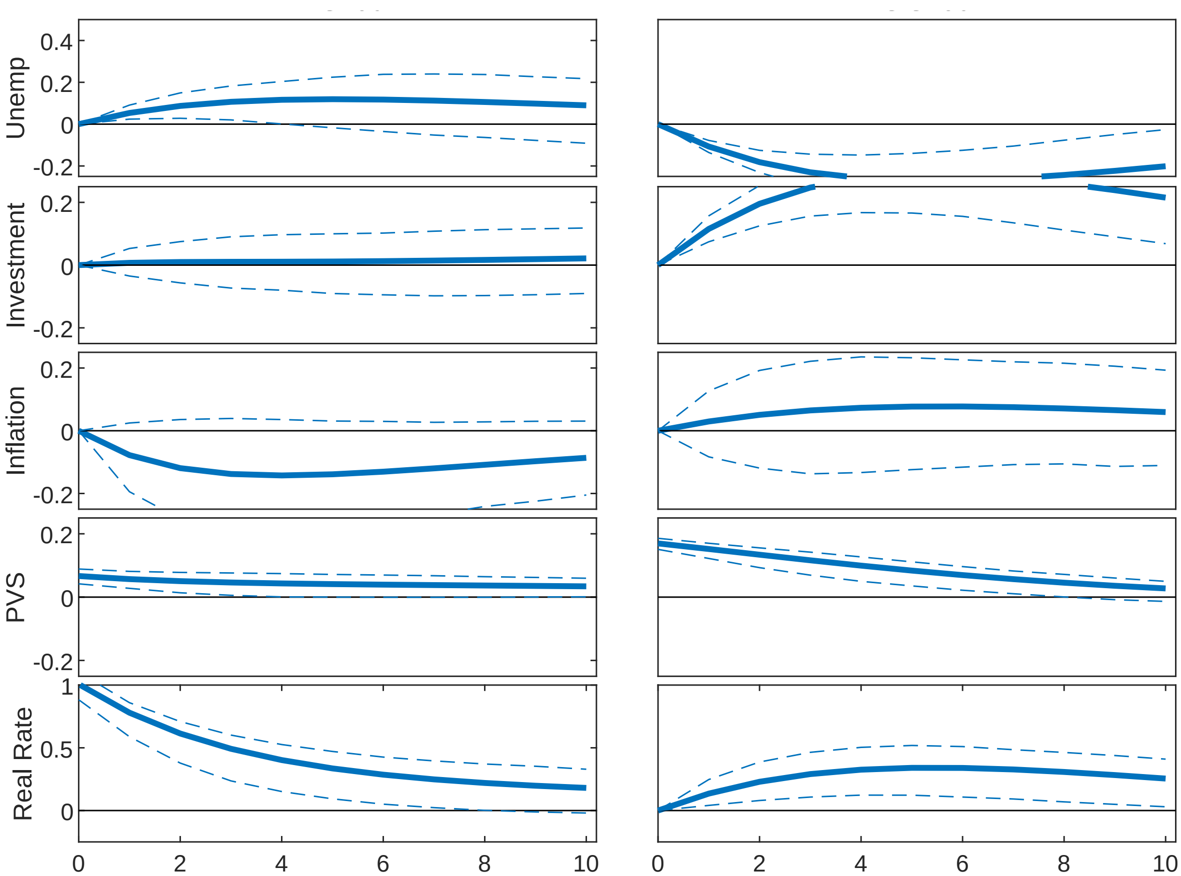

*Notes*: The top panel of the figure plots the raw one-year real rate. The raw real rate is the one-year Treasury bill rate net of one-year survey expectations of the inflation (the GDP deflator) from the Survey of Professional Forecasters, expressed in percentage terms. The bottom panel of the figure compares two different methods for extracting the cyclical component of the real rate. The first just uses a deterministic time trend. The second uses the methodology of Hamilton (2017), with full details in Section A2.1. Data is quarterly and spans 1970Q2-2016Q2. Shaded bars indicate NBER recessions.

Figure A.2: Simulated $t$-statistics and $R^2$



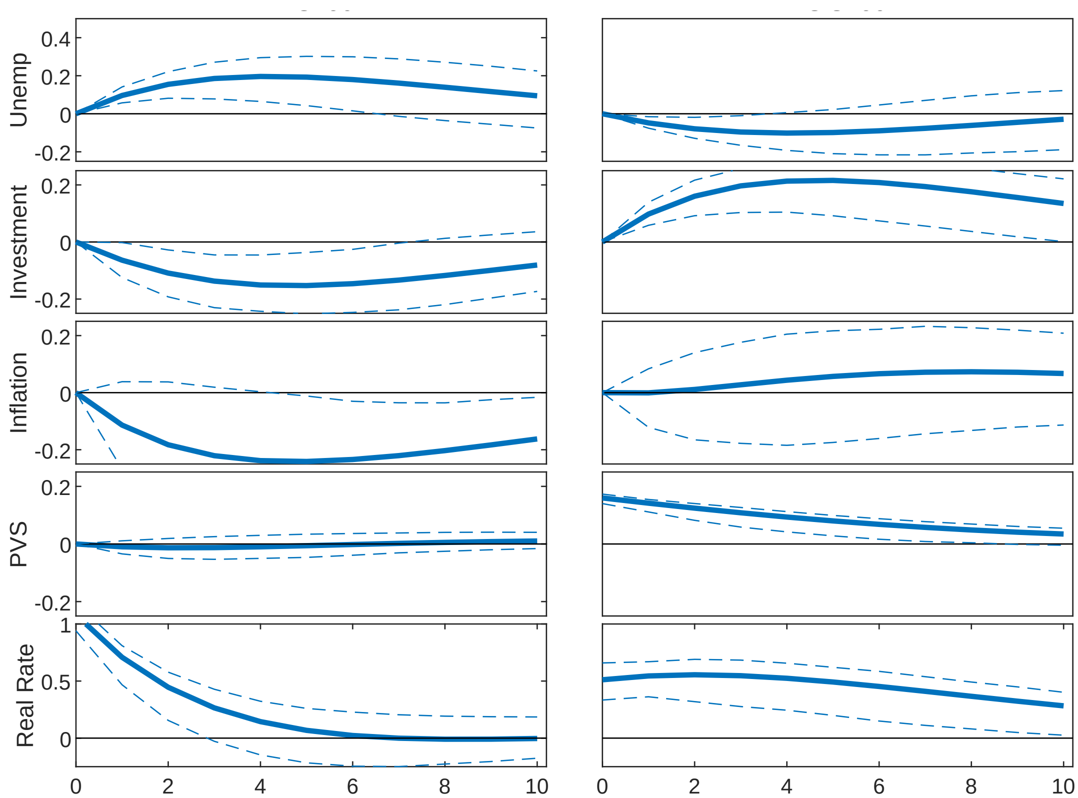

*Notes*: This figure plots simulated $t$-statistics and $R^2$ for a univariate regression of the real rate on $PVS$. We independently fit AR(1)-GARCH(1,1) processes to each series and simulate each 10,000 times. Within each simulation, we regress the real rate on $PVS$, saving the Newey-West $t$-statistic (with five lags) and the $R^2$. The top panel of the figure shows the distribution of the $t$-statistics from this procedure and the bottom panel shows the $R^2$. The red bar shows the actual estimate of each statistic in the data. The $p$-values listed in the plot are computed as the proportion of simulations that have a $t$-statistic (or $R^2$) that exceeds the actual value in the data. The one-year real rate is the one-year Treasury bill rate net of one-year survey expectations of the inflation (the GDP deflator) from the Survey of Professional Forecasters, expressed in percent and linearly detrended. See the Section A1 for details on how we construct $PVS$.

Figure A.3: Impulse Responses to Monetary Policy and PVS Shocks (Traditional VAR)



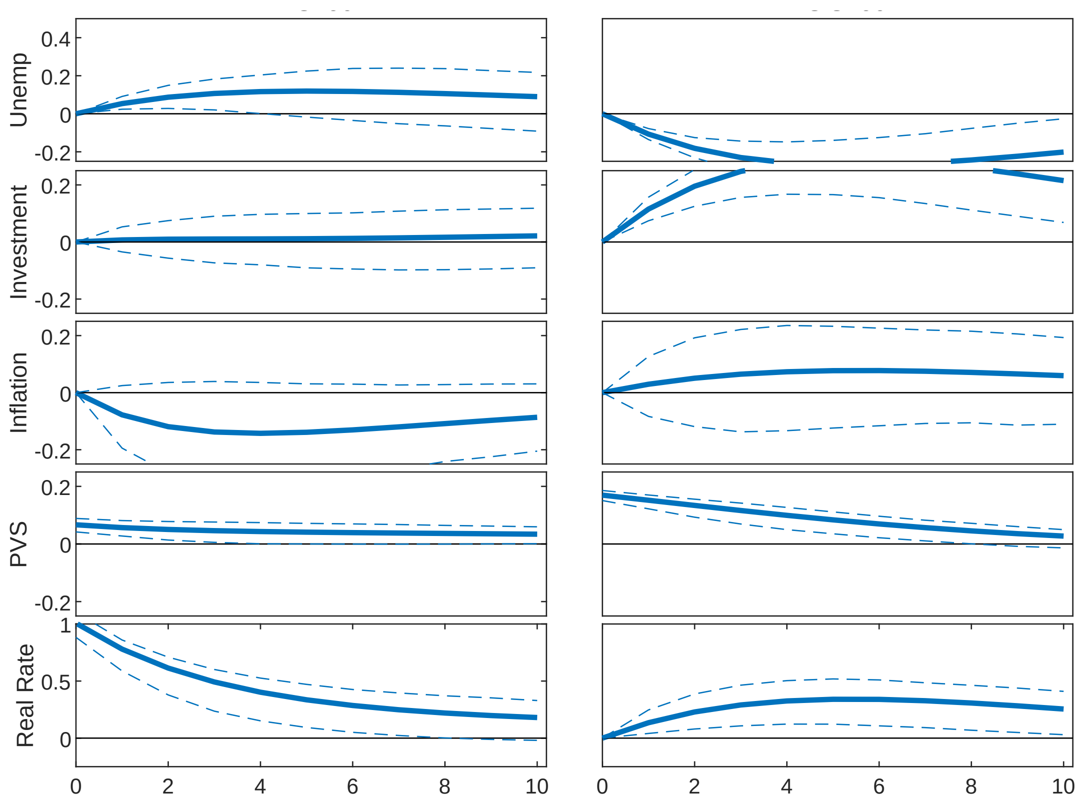

*Notes*: This figure plots impulse responses to monetary policy shocks (left panel) and PVS shocks (right panel). Impulse responses to one-standard deviation shocks are estimated from a five-variable VAR(1) in unemployment, the investment-capital ratio, inflation, PVS, and the linearly detrended real rate with one lag using quarterly data 1970Q-2016Q2. Unemployment is the civilian unemployment rate (UNRATE). The investment-capital ratio is computed as private nonresidential fixed investment (PNFI) divided by the previous year's current-cost net stock of fixed private nonresidential assets (K1NTOTL1ES000). Following Bernanke and Mihov (1998), structural innovations in the real rate are assumed to affect output, inflation, and PVS with a lag. PVS shocks are assumed to affect output and inflation with a lag, but have a contemporaneous effect on the real rate. Dashed lines denote 95% confidence bands, generated by simulating 1000 data processes with identical sample length as in the data from the estimated VAR dynamics.

Figure A.4: Impulse Responses Pre-Crisis



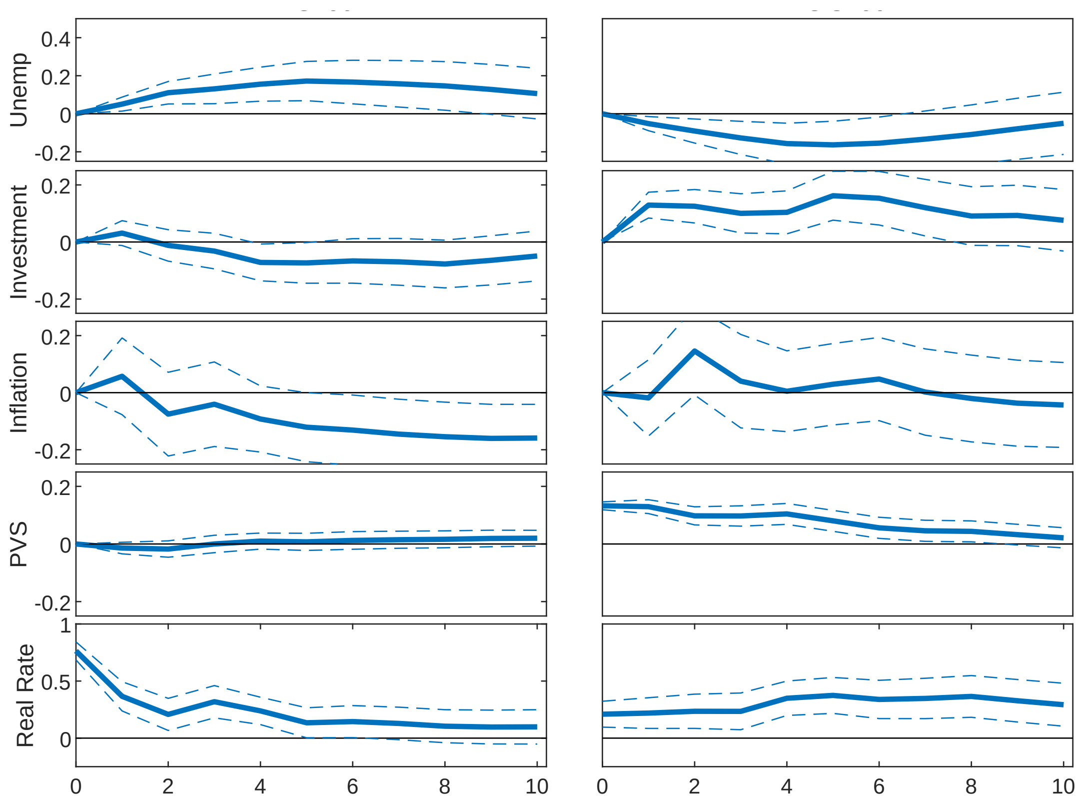

*Notes*: This figure plots impulse responses to monetary policy shocks (left panel) and *PVS* shocks (right panel). It corresponds to Figure A.3 of this appendix, but uses the pre-crisis sample that ends in 2008Q4.

Figure A.5: Impulse Responses Alternative Ordering



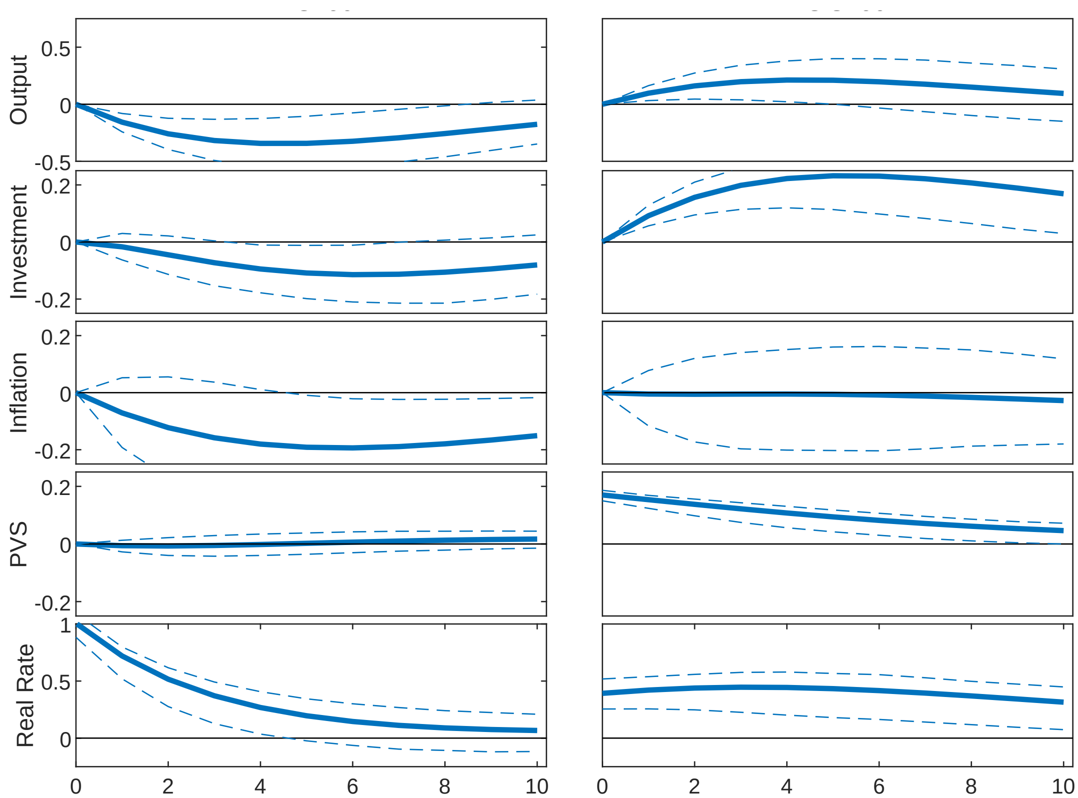

*Notes*: This figure plots impulse responses to monetary policy shocks (left panel) and $PVS$ shocks (right panel). It differs from Figure A.3 of this appendix in that here we construct impulse responses under the assumption that $PVS_t$ reacts to the real rate immediately, but the real rate reacts to $PVS_t$ with a lag.

Figure A.6: Impulse Responses for VAR(4)



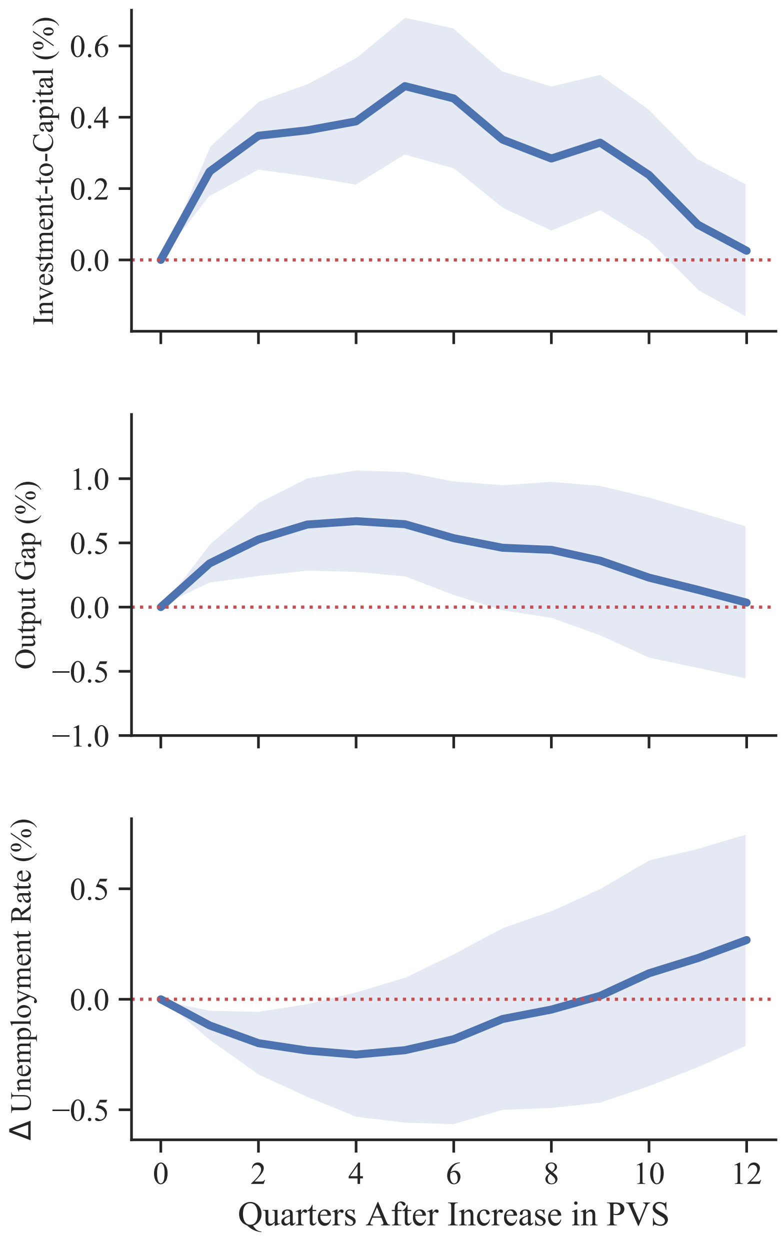

*Notes*: This figure plots impulse responses to monetary policy shocks (left panel) and *PVS* shocks (right panel). It differs from Figure A.3 of this appendix in that impulse responses are based on a VAR(4) instead of a VAR(1).

Figure A.7: Impulse Responses with Output Gap



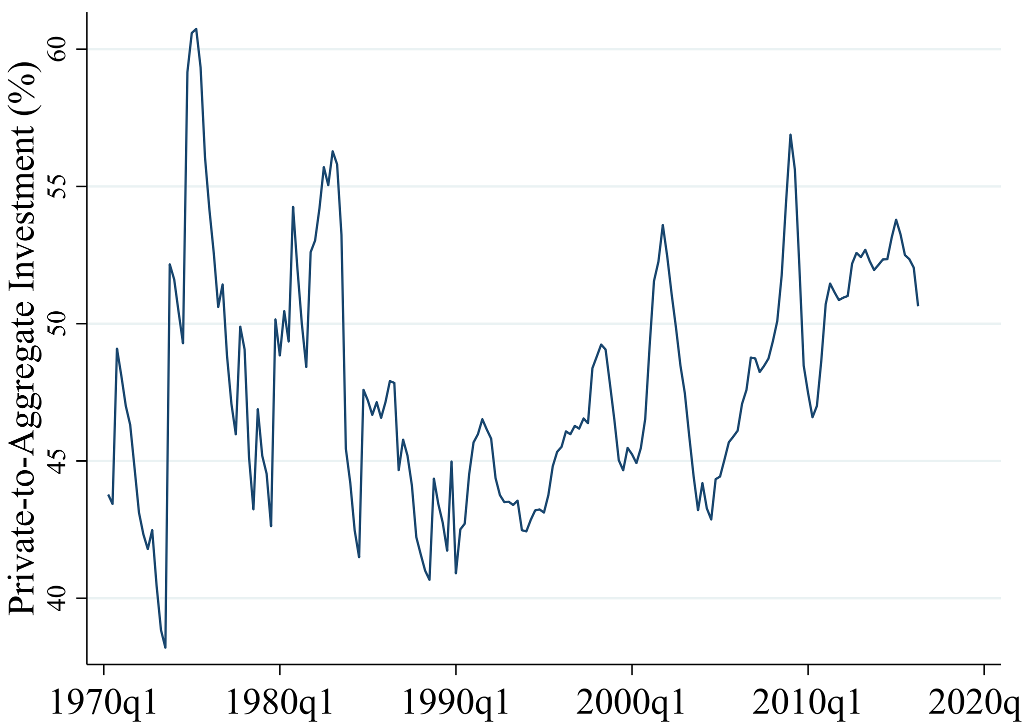

*Notes*: This figure plots impulse responses to monetary policy shocks (left panel) and *PVS* shocks (right panel). It differs from Figure A.3 of this appendix in that it uses the output gap instead of the unemployment rate.

Figure A.8: Impulse Responses of the Macroeconomy to PVS Shocks (Local Projections)



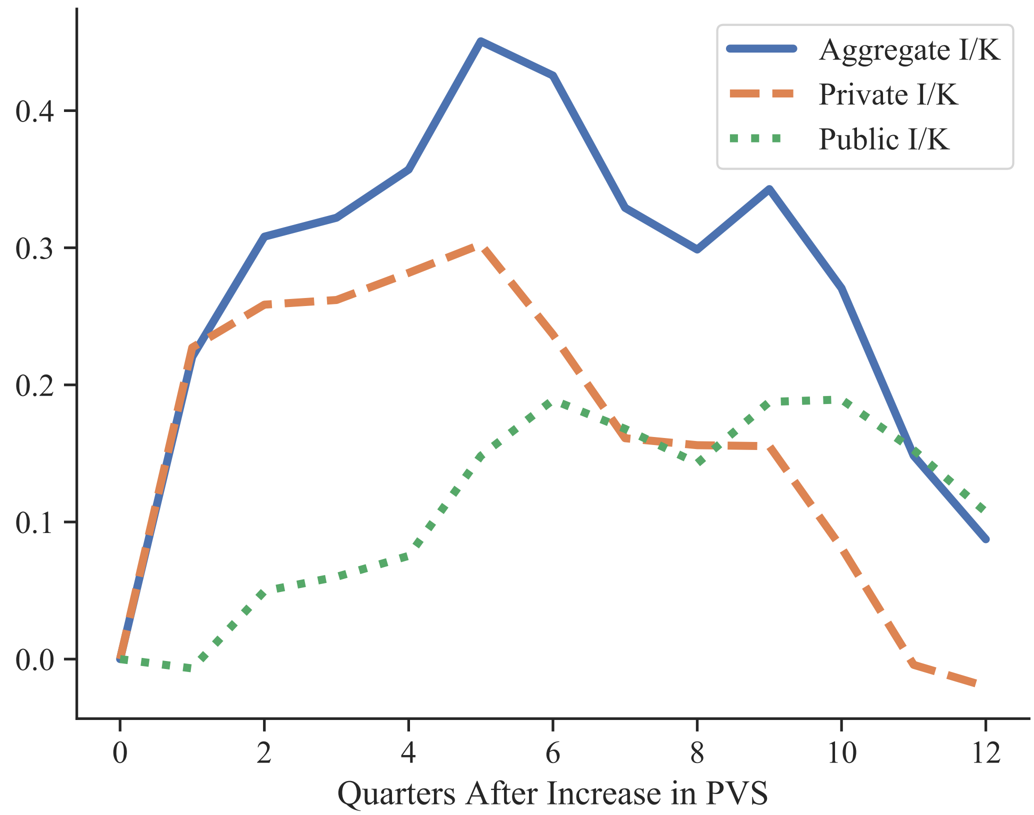

*Notes*: This figure plots the estimated impulse response (and its associated 95% confidence band) of several macroeconomic variables to a one-standard deviation shock to $PVS_t$ using local projections. We compute impulse responses using Jordà (2005) local projections of each macroeconomic outcomes onto $\text{PVS}_t$. In all cases, we run regressions of the following form: $y_{t+h} = a + b^h_{PVS} \times PVS_t + b^h_{RR} \times RealRate_t + b^h_y \times y_t + b^h_{mkt} \times \text{Agg BM}_t + b^h_{cp} \times CP_t + \varepsilon_{t+h}$, where Agg $\text{BM}_t$ is the aggregate book-to-market ratio and $CP_t$ is the Cochrane and Piazzesi (2005) bond risk-factor. We consider three different macroeconomic outcomes for the $y$-variable. The first is the investment-to-capital ratio, defined as the level of real private nonresidential fixed investment (PNFI) divided by the previous year's current-cost net stock of fixed private nonresidential assets (K1NTOTL1ES000). The second is the real output gap, defined as the percent deviation of real GDP from real potential output. The third is the change in the U.S. civilian unemployment rate. When forecasting the investment-capital ratio, $y_{t+h}$ is the level of the investment-capital ratio at time $t+h$. For the output gap, $y_{t+h}$ is the level of the output gap at time $t+h$. Finally, for the unemployment rate, $y_{t+h}$ is the change in the unemployment rate between $t$ and $t+h$, and $y_t$ is the change between $t-1$ and $t$. All macroeconomic variables come from the St. Louis FRED database and are expressed in percentage points. $PVS_t$ is defined as in the main text. The real rate is the one-year Treasury bill rate net of one-year survey expectations of the inflation (the GDP deflator) from the Survey of Professional Forecasters, expressed in percent and linearly detrended. For all regressions, we use Newey-West standard errors with five lags. Data is quarterly and spans 1970Q2-2016Q2.

Figure A.9: The Share of Aggregate Investment from Private Firms




*Notes*: This figure plots the share of U.S. fixed non-residential investment (excluding government investment) coming from private firms. Aggregate U.S. investment is defined as the level of real private nonresidential fixed investment (PNFI), which we obtain from the St. Louis FRED database. We measure the level of investment from publicly traded firms using data on U.S. listed firms from COMPUSTAT. After 1990, we use COMPUSTAT quarterly to compute total public-firm investment, as it provides more up-to-date accounting information. In each quarter $t$ and for each firm $f$, we take the last observation in the data within a year of $t$. We then compute total investment for publicly traded firms by summing over all firms. Prior to 1990, we compute aggregate investment using COMPUSTAT annual because coverage is poor in COMPUSTAT quarterly. We chose 1990 as the cutoff because this was the date when total public-firm investment from COMPUSTAT quarterly and COMPUSTAT annual converged to roughly the same level. The level of private-firm investment is defined as aggregate investment minus public-firm investment. The figure then plots private firm investment scaled by aggregate investment, expressed in percentage points. Data is quarterly and spans 1970Q2-2016Q2.

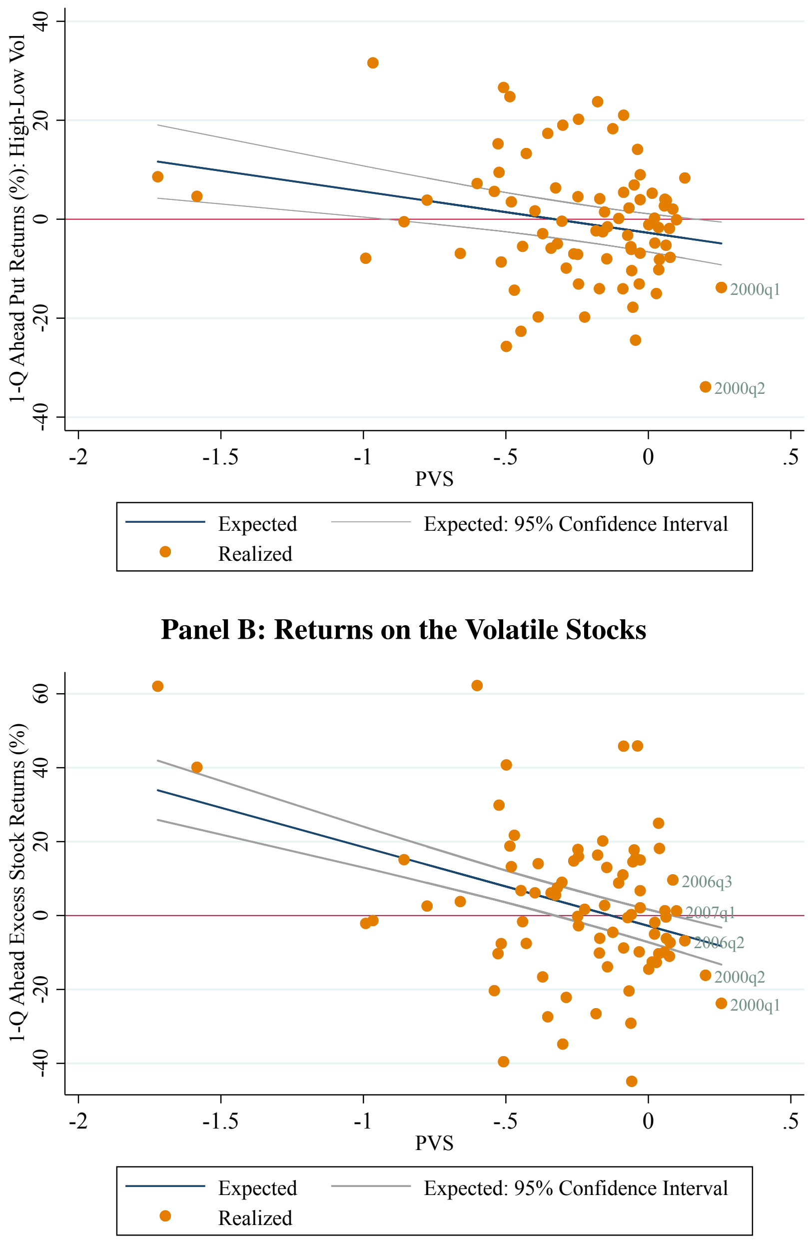
Figure A.10: Decomposing the Aggregate Investment Response to PVS Shocks




*Notes*: This figure plots the estimated impulse responses of aggregate investment, investment by private firms, and investment by public firms to a one-standard deviation shock to PVS using local projections. We compute impulse responses using Jordà (2005) local projections of each variable onto $PVS_t$ via regressions of the following form: $y_{t+h} = a + b^h_{PVS} \times PVS_t + b^h_{RR} \times RealRate_t + b^h_y \times y_t + \varepsilon_{t+h}$, where $y_t$ is the aggregate investment rate at time $t$. Aggregate investment is defined as the level of real private nonresidential fixed investment (PNFI) divided by the previous year's current-cost net stock of fixed private nonresidential assets (K1NTOTL1ES000). Public-firm investment is total investment from COMPUSTAT (CAPX + R&D) divided by the previous year's current-cost net stock of fixed private nonresidential assets. Private-firm investment is the difference between aggregate investment and public-firm investment. Data is quarterly and spans 1970Q2-2016Q2.

Figure A.11: $PVS_t$ and Negative Returns

**Panel A: Returns to Selling Puts on Volatile Stocks**




*Notes*: Both panels of this figure relate $PVS_t$ to future returns. In Panel A, we form a portfolio that sells out-of-the-money put options on high-volatility firms and buys out-of-the-money put options on low-volatility firms. In Panel B, we instead forecast excess returns on high-volatility stocks alone (i.e., not the long-short portfolio underlying $PVS_t$). In both cases, realized returns are depicted by orange dots in the graph. In addition, we forecast returns at $(t+1)$ with $PVS_t$ at time $t$ and plot the fitted value from the regression in blue. The gray bands are the 95% confidence interval for the fitted value in the regression and are based on Newey-West standard errors with five lags. In instances where the upper bound of the 95% confidence interval is negative – meaning expected returns are negative and statistically significant – we label the realized return with the date of the forecast. $PVS_t$ is the difference between the average book-to-market (BM) ratio of low-volatility stocks and the average BM-ratio of high-volatility stocks. The internet appendix contains details on variable construction. For both panels, data is quarterly and runs from 1996Q1 to 2016Q2.

# APPENDIX TABLES

Table A.1: Real Rate Variation (Alternative Filters and Raw Series)

| Dependent. Variable: | Hamilton-Filtered Real Rate ($\tilde{r}_t$) | | | | | | Raw Real Rate | | |
|---|---|---|---|---|---|---|---|---|---|
| | Levels | | | First-Differences | | | Levels | | |
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
| $PVS_t$ | 1.20** | 1.44** | 1.32** | 0.39** | 0.40** | 0.32** | 1.41** | 1.36** | 1.20** |
| | (0.18) | (0.18) | (0.22) | (0.13) | (0.13) | (0.12) | (0.25) | (0.20) | (0.25) |
| Aggregate BM | | 0.51** | 0.50** | | 0.02 | 0.15* | | 0.57** | 0.74 |
| | | (0.14) | (0.20) | | (0.08) | (0.09) | | (0.24) | (0.48) |
| Output Gap | | | 0.27** | | | 0.47** | | | 0.30** |
| | | | (0.06) | | | (0.16) | | | (0.15) |
| Inflation | | | 0.15* | | | -0.01 | | | 0.02 |
| | | | (0.08) | | | (0.09) | | | (0.13) |
| Constant | 0.00 | 0.00 | 0.00 | -0.02 | -0.02 | -0.02 | 1.86** | 1.86** | 2.20** |
| | (0.19) | (0.18) | (0.15) | (0.06) | (0.06) | (0.05) | (0.28) | (0.27) | (0.70) |
| Adj. $R^2$ | 0.42 | 0.47 | 0.59 | 0.11 | 0.10 | 0.20 | 0.37 | 0.43 | 0.49 |
| $N$ | 185 | 185 | 185 | 184 | 184 | 184 | 185 | 185 | 185 |

*Notes*: This table reports regression estimates of the one-year real rate on the spread in book-to-market (BM) ratios between high volatility and low volatility stocks ($PVS_t$). For all NYSE, AMEX, and NASDAQ firms in CRSP, we compute volatility at the end of each quarter using the previous sixty days of daily returns. We then form equal-weighted portfolios based on the quintiles of volatility. Within each quintile, we compute the average book-to-market (BM) ratio. Section A1 contains full details on how we compute BM ratios. $PVS_t$ is defined as the difference in BM ratios between the bottom and top quintile portfolios. Aggregate BM is computed by summing book equity values across all firms and divided by the corresponding sum of market equity values. The output gap is the percentage deviation of real GDP from the CBO's estimate of potential real GDP. Inflation is the annualized percentage four-quarter growth in the GDP price deflator from the St. Louis Fed (GDPDEF). The one-year real rate is the one-year Treasury bill rate net of one-year survey expectations of the inflation (the GDP deflator) from the Survey of Professional Forecasters, expressed in percent. In columns (1)-(6), we follow Hamilton (2017) in extracting the cyclical component of the real rate and use it in the regression. We do the same for inflation, the aggregate BM ratio, and the output gap. See Section A2.1 for more details on the procedure. Columns (7)-(9) use the raw real rate. We do not present first-difference analysis of the raw real rate because this maps directly to the first-difference analysis of the linearly detrended real rate that we present in the main text. Standard errors are reported under point estimates and computed using Newey-West (1987) with five lags. In each set of regressions, we normalize PVS (or its first difference) to have a mean of zero and a variance of one. * indicates a p-value of less than 0.1 and ** indicates a $p$-value of less than 0.05. Data is quarterly and spans 1970Q2-2016Q2.

Table A.2: The Real Rate and Future Returns (Alternative Filters and Raw Series)

***Panel A:*** *Return Forecasting*

| | Vol-Sorted Ret$_{t\to t+1}$ | | Mkt-Rf$_{t\to t+1}$ | |
|---|---|---|---|---|
| | (1) | (2) | (3) | (4) |
| Hamilton-Filtered Real Rate ($\tilde{r}_t$) | 1.49** | | -0.19 | |
| | (0.58) | | (0.39) | |
| Raw Real Rate | | 1.17** | | -0.24 |
| | | (0.46) | | (0.27) |
| Adj. $R^2$ | 0.03 | 0.03 | -0.00 | -0.00 |
| $N$ | 184 | 184 | 184 | 184 |

***Panel B****: Aggregate Earnings and Dividend Growth Forecasting*

| Dep. Variable: | $g^E_{t,t+1}$ | | $g^E_{t,t+4}$ | | $g^D_{t,t+1}$ | | $g^D_{t,t+4}$ | |
|---|---|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
| $\tilde{r}_t$ | -4.32 | | -11.45 | | 0.10 | | -0.06 | |
| | (6.34) | | (7.60) | | (0.51) | | (0.59) | |
| $R_t$ | | -3.68 | | -7.45 | | -0.64 | | -0.75 |
| | | (4.10) | | (5.07) | | (0.39) | | (0.48) |
| Adj $R^2$ | 0.00 | 0.00 | 0.06 | 0.04 | -0.00 | 0.04 | -0.01 | 0.07 |
| $N$ | 184 | 184 | 181 | 181 | 184 | 184 | 181 | 181 |

*Notes*: **Panel A** of this table uses the one-year real interest rate to forecast returns on either the low-minus-high volatility equity portfolio or the excess returns on the aggregate stock market. For all NYSE, AMEX, and NASDAQ firms in CRSP, we compute volatility at the end of each quarter using the previous sixty days of daily returns. We then form equal-weighted portfolios based on the quintiles of volatility. Volatility-sorted returns are returns on the lowest minus highest volatility quintile portfolios. Vol-Sorted Ret in the forecasting regression corresponds to returns on this low-minus-high volatility portfolio. When forecasting the aggregate stock market, we use the excess return of the CRSP Value-Weighted index obtained from Ken French's website. For quarterly regressions, standard errors are computed using Newey-West (1987) with two lags. **Panel B** of the table reports forecasting regressions of real aggregate earnings growth ($g^E$) or real aggregate dividend growth ($g^D$) using the one-year real rate. Real earnings and real dividends come from Robert Shiller's website. The one-year real rate is the one-year Treasury bill rate net of one-year survey expectations of the inflation (the GDP deflator) from the Survey of Professional Forecasters, expressed in percent. In the table, $\tilde{r}_t$is the cyclical component of the real rate, extracted using Hamilton (2017). See Section A2.1 for more details on the procedure. $R_t$ is simply the raw real rate. Standard errors are reported below point estimates and computed using Newey-West (1987) with two lags for quarterly regressions and five lags for annual. * indicates a p-value of less than 0.1 and ** indicates a p-value of less than 0.05. For both panels, all regressions have a constant, but we omit the estimates to save space. Data is quarterly and spans 1970Q2-2016Q2. Growth rates and returns and expressed in percentage terms.

Table A.3: Subsample Analysis of $PVS_t$ and the Real Rate

| Dep. Variable: | One-Year Real Rate | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | Levels | | | | First-Differences | | | |
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
| PVS | 1.26** | 1.08** | 1.05** | 0.53** | 0.40** | 0.24** | 0.29** | 0.24** |
| | (0.24) | (0.28) | (0.20) | (0.11) | (0.14) | (0.10) | (0.10) | (0.09) |
| Subsample | Main | Main, Ex. Volcker | Long | Pre-1977 | Main Sample | Main, Ex. Volcker | Long | Pre-1977 |
| $N$ | 185 | 145 | 253 | 95 | 184 | 143 | 252 | 94 |
| Adj. $R^2$ | 0.41 | 0.30 | 0.26 | 0.21 | 0.13 | 0.13 | 0.08 | 0.11 |

*Notes*: This table reports regression estimates of the one-year real rate on the contemporaneous spread in book-to-market (BM) ratios between low- and high-volatility stocks ($PVS_t$). For all NYSE, AMEX, and NASDAQ firms in CRSP, we compute volatility at the end of each quarter using the previous sixty days of daily returns. We then form equal-weighted portfolios based on the quintiles of volatility. Within each quintile, we compute the average book-to-market (BM) ratio. The Appendix contains full details on how we compute BM ratios. $PVS_t$ is defined as the difference in BM ratios between the bottom (BM Low Vol) and top quintile (BM High Vol) portfolios. The one-year real rate is the one-year Treasury bill rate net of one-year survey expectations of the inflation (the GDP deflator) from the Survey of Professional Forecasters, expressed in percent. For the columns listed as "Main Sample" and "Ex. Volcker", we linearly detrend the one-year real rate using the sample 1970Q2-2016Q2. Standard errors are listed below each point estimate in parentheses and are computed using Newey-West (1987) with five lags. * indicates a $p$-value of less than 0.1 and ** indicates a $p$-value of less than 0.05. In the table, $PVS_t$ is standardized to have mean zero and variance one. This is true in both the levels regression and the first-differenced regressions. Data is quarterly and the subsamples are as follows: (i) "Main" is from 1970Q2-2016Q2 and corresponds to the sample used in the main text; (ii) "Main, Ex. Volcker" is 1970Q2-1976Q4 and 1987Q4-2016Q2. It is the main example but excludes the period 1977Q1-1986Q4; (iii) "Long" is the period 1953Q2-2016Q2. To extend PVS back to 1953, we use the book equity data from Davis, Fama, and French (2000). In addition, to compute the one-year real rate prior to 1970Q2, we take the 1-year nominal rate minus the four-quarter moving average of inflation; and (iv) "Pre-1977" is 1953Q2-1976Q4.

Table A.4: Decomposition of the Real Interest Rate and $PVS_t$

| | One-Year Real Rate Decomposition | Regression on PVS | | |
|---|---|---|---|---|
| | | $b$ | $se(b)$ | Adj. $R^2$ |
| (1) | Baseline Detrended Real Rate | 1.26 | 0.24 | 0.41 |
| (2) | Baseline Raw Real Rate | 1.41 | 0.25 | 0.37 |
| (3) | Nominal 1-Year Rate | 1.90 | 0.59 | 0.27 |
| (4) | Expected Inflation | 0.49 | 0.41 | 0.06 |
| (5) | Fixed Taylor Rule Implied Rate (Taylor, 1993) | 0.36 | 0.33 | 0.04 |
| (6) | Residual | 1.05 | 0.39 | 0.23 |
| (7) | Fitted Taylor Rule Implied Rate | 0.22 | 0.20 | 0.04 |
| (8) | Residual | 1.19 | 0.30 | 0.33 |

*Notes*: This table reports univariate regressions of several variables on $PVS$. Section A1 of the internet appendix contains full details on how we compute $PVS_t$ , defined as the difference in book-to-market ratios between low and high volatility stocks. In Row (1), the dependent variable in the regression is the linearly detrended one-year real rate. The dependent variable in Row (2) is the raw one-year real rate. The one-year real rate is the one-year Treasury bill rate net of one-year survey expectations of the inflation (the GDP deflator) from the Survey of Professional Forecasters, expressed in percent. Rows (3) and (4) decompose the raw one-year real rate into the one-year nominal rate and expected inflation. Rows (5) and (6) decompose the raw one-year real rate into a Taylor (1993) rule component and a residual component. The Taylor (1993) rule component is defined as $Taylor1993_t = 0.5 \times (OutputGap) + 0.5 \times (Inflation - 2) + 2$. The output gap is the percentage deviation of real GDP from the CBO's estimate of potential real GDP. Inflation is the annualized percentage four-quarter growth in the GDP price deflator from the St. Louis Fed (GDPDEF). The Taylor (1993) rule residual used in Row (5) is then Raw Real Rate$_t - Taylor1993_t$. Rows (7) and (8) use the same decomposition, where the fitted Taylor rule is defined as the fitted value from a regression of the raw real rate on the output gap and inflation. The Fitted Taylor Rule residual in Row (8) is the residual from the aforementioned regression. Standard errors are computed using Newey-West (1987) with five lags. Data is quarterly and the full sample spans 1970Q2-2016Q2. In all cases, $PVS_t$ is standardized to have a mean of zero and a variance of one.

Table A.5: Robustness of Different Approaches to Constructing PVS

| | $X$-Variable | $PVS_t = a + b \times X_t$ | | | $\text{RealRate}_t = a + c \times X_t$ | | |
|---|---|---|---|---|---|---|---|
| | | $b$ | $se(b)$ | $R^2$ | $c$ | $se(c)$ | $R^2$ |
| (1) | $PVS$ | | | | 1.26 | 0.24 | 0.41 |
| (2) | $PVS_{Last}$ | 0.86 | 0.08 | 0.73 | 0.86 | 0.24 | 0.19 |
| (3) | $PVS_{2M}$ | 0.92 | 0.10 | 0.84 | 1.01 | 0.29 | 0.26 |
| (4) | $PVS_{MB}$ | 0.85 | 0.10 | 0.73 | 1.33 | 0.16 | 0.45 |
| (5) | $PVS_{Terc}$ | 0.99 | 0.01 | 0.99 | 1.31 | 0.23 | 0.44 |
| (6) | $PVS_{2Yr}$ | 0.94 | 0.05 | 0.88 | 1.40 | 0.21 | 0.51 |

*Notes*: This table shows the regressions of $PVS_t$ onto variants of itself and then also regressions of the one-year real rate onto variants of $PVS$. $PVS_t$ refers to the primary variable used in the paper. $PVS_{Last}$ uses a 2-month window to measure volatility and uses the most recent market value when constructing book-to-market ratios. $PVS_{2M}$ uses a 2-month window to measure volatility and uses the median market capitalization over the previous 2-months to compute book-to-market ratios. $PVS_{MB}$ is the average market-to-book of high-volatility stocks minus the average of low-volatility stocks. The construction of this variable mirrors that of $PVS_t$, though we winsorize individual market-to-book ratios at their 5% tails to mitigate the impact of outliers. To compute $PVS_{Terc}$, we first sort stocks into terciles, as opposed to quintiles, based on their trailing two-month volatility at the end of each quarter. $PVS_{Terc}$ is the average book-to-market ratio of the low-tercile volatility firms minus the average of the high-tercile volatility firms, where we again mimic the construction of our original PVS variable for book-to-market ratios. $PVS_{2Yr}$ is the version of PVS that uses a 2-year window to measure volatility, as opposed to a 2-month window. The variable RealRate is the one-year nominal Treasury rate minus the one-year ahead inflation expectation from the Survey of Professional Forecasters, linearly detrended and expressed in percentage points. In the table, $PVS_t$ and all of its variants have been standardized to have a mean of zero and variance of one. Standard errors are computed using Newey-West (1987) with five lags. Data is quarterly and runs from 1970Q2 to 2016Q2 ($N = 185$).

Table A.6: The Real Rate and Valuation of Other Characteristic-Sorted Portfolios - Univariate Results

| | | Real $\text{Rate}_t = a + b \times X_t + \varepsilon_t$ | | | | | |
|---|---|---|---|---|---|---|---|
| | | Levels | | | First-Differences | | |
| | | $b$ | $se(b)$ | Adj. $R^2$ | $b$ | $se(b)$ | Adj. $R^2$ |
| *Univariate:* | | | | | | | |
| (1) | Duration | **-0.69** | 0.26 | 0.12 | -0.22 | 0.13 | 0.03 |
| (2) | Leverage | **0.54** | 0.24 | 0.07 | **0.17** | 0.08 | 0.02 |
| (3) | Beta | **1.20** | 0.21 | 0.37 | 0.13 | 0.08 | 0.01 |
| (4) | LR Beta | **1.12** | 0.18 | 0.32 | *0.13* | 0.07 | 0.01 |
| (5) | 2M-Beta | 0.35 | 0.25 | 0.03 | **0.36** | 0.14 | 0.11 |
| (6) | CF Beta | -0.02 | 0.29 | -0.01 | -0.04 | 0.07 | -0.00 |
| (7) | Size | **-1.12** | 0.19 | 0.32 | **-0.25** | 0.11 | 0.05 |
| (8) | Value | **0.69** | 0.21 | 0.12 | **0.18** | 0.09 | 0.02 |
| *Kitchen-Sink:* | | | | | | | |
| (9) | PVS | **2.04** | 0.57 | 0.59 | **0.46** | 0.16 | 0.17 |

*Notes*: This table reports regression estimates of the one-year real rate on the book-to-market spreads of portfolios formed on various sorting characteristics. Rows (1)-(8) run the following regression, in both levels and first-differences: Real $\text{Rate}_t = a + b \times$ Y-Sorted BM $\text{Spread}_t + \varepsilon_t$, where Y-Sorted BM $\text{Spread}_t$ is the spread in book-to-market ratios between stocks sorted on characteristic Y. Our main variable of interest in the study is the spread in book-to-market ratios between high volatility and low volatility stocks ($PVS_t$). For all NYSE, AMEX, and NASDAQ firms in CRSP, we compute volatility at the end of each quarter using the previous sixty days of daily returns. We then form equal-weighted portfolios based on the quintiles of volatility. Within each quintile, we compute the average book-to-market (BM) ratio. $PVS_t$ is defined as the difference in BM ratios between the bottom and top quintile portfolios. We form book-to-market spreads in the same fashion for other sorting variables. The sorting variables we use are: (1) Duration (Weber (2016)); (2) Leverage, measured as long-term debt from COMPUSTAT divided by market equity; (3) CAPM Beta, measured using monthly data over rolling 5 year windows; (4) Long-Run (LR) CAPM Beta, measured using semi-annual data over a rolling ten year window; (5) 2M-Beta, computed at the end of each quarter using the previous sixty days of daily returns; (6) Cashflow (CF) Beta, which is measured by regressing EBITDA growth on national income growth; (7) market capitalization; and (8) book-to-market ratios themselves (value). Spreads are always between the high quintile and the low quintile of the sorting variable. In Row (9), we run a kitchen-sink regression of the real rate on $PVS_t$ plus all of the book-to-market spreads in rows (1)-(8) and report the estimated coefficient on $PVS_t$. The real rate is the one-year Treasury bill rate net of one-year survey expectations of the inflation (the GDP deflator) from the Survey of Professional Forecasters, expressed in percent and linearly detrended. In all cases, we normalize all book-to-market spreads (or their first-difference) to have mean zero and variance one. Standard errors are computed using both Newey-West (1987) with five lags. Italicized point estimates indicates a $p$-value of less than 0.1 and bold point estimates indicate a $p$-value of less than 0.05. Data is quarterly and spans 1970Q2-2016Q2.

Table A.7: Horse Races and Alternative Constructions of PVS

| | | Levels | | | | | | First-Differences | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Full | | | Pre-Crisis | | | Full | | | Pre-Crisis | | |
| | | $b$ | $se(b)$ | $R^2$ | $b$ | $se(b)$ | $R^2$ | $b$ | $se(b)$ | $R^2$ | $b$ | $se(b)$ | $R^2$ |
| (1) | Baseline | **1.26** | 0.25 | 0.41 | **1.50** | 0.21 | 0.47 | **0.44** | 0.16 | 0.13 | **0.63** | 0.17 | 0.21 |
| *Alternative Constructions:* | | | | | | | | | | | | | |
| (2) | Value-Weight | **1.12** | 0.25 | 0.32 | **1.42** | 0.24 | 0.41 | **0.31** | 0.13 | 0.08 | **0.40** | 0.15 | 0.10 |
| (3) | 2-Yr Volatility | **1.42** | 0.23 | 0.52 | **1.62** | 0.20 | 0.54 | **0.26** | 0.11 | 0.05 | **0.43** | 0.10 | 0.10 |
| *Horse-Races:* | | | | | | | | | | | | | |
| (4) | Liquidity | **1.39** | 0.22 | 0.46 | **1.57** | 0.21 | 0.50 | **0.37** | 0.17 | 0.15 | **0.56** | 0.19 | 0.22 |
| (5) | Duration | **1.17** | 0.28 | 0.41 | **1.31** | 0.27 | 0.48 | **0.44** | 0.14 | 0.12 | **0.61** | 0.14 | 0.20 |
| (6) | Leverage | **1.50** | 0.25 | 0.43 | **1.64** | 0.23 | 0.47 | **0.57** | 0.20 | 0.14 | **0.74** | 0.22 | 0.21 |
| (7) | CAPM Beta | **1.26** | 0.23 | 0.41 | **1.46** | 0.20 | 0.47 | **0.32** | 0.12 | 0.15 | **0.50** | 0.12 | 0.23 |
| (8) | Size | **1.08** | 0.46 | 0.41 | **1.42** | 0.41 | 0.46 | **0.61** | 0.25 | 0.13 | **0.73** | 0.29 | 0.20 |
| (9) | Value | **1.52** | 0.31 | 0.43 | **1.70** | 0.26 | 0.47 | **0.69** | 0.22 | 0.16 | **0.79** | 0.24 | 0.22 |

*Notes*: This table reports a battery of robustness exercises for the relationship between $PVS_t$ and the real rate documented in Table III in the main text. Specifically, we report time-series regression results of the following form: Real Rate$_t = a + b \times PVS_t + \theta X_t + \varepsilon_t$, where $PVS_t$ is the average book-to-market ratio of low-minus-high volatility stocks. We run this regression in levels and in first differences and, in each case, we standardize $PVS_t$ (or its first-difference) to have a mean of zero and variance of one over the full sample. $X_t$ is a one of several control variables. For all specifications, the table reports the estimated coefficient on $PVS_t$. Row (1) repeats our baseline result from Table III in the main text, columns (2) and (6). Row (2) uses value weights instead of equal weights when forming $PVS_t$. Row (3) constructs $PVS_t$ using the past two years of return volatility, as opposed to the past two months. In rows (4)-(9), we run horse races of $PVS_t$ against several other variables. Row (4) controls for the spread between off-the-run and on-the-run Treasury yields (Krishnamurthy (2002)). In rows (5)-(9), we control for the book-to-market spread based on other characteristic sorts. The CAPM beta is based daily stock returns over a rolling two-month window. See the internet appendix for a description of each characteristic, details on variable construction, and alternative CAPM betas. The one-year real rate is the one-year Treasury bill rate net of one-year survey expectations of the inflation (the GDP deflator) from the Survey of Professional Forecasters, expressed in percent and linearly detrended. The listed standard errors are computed using Newey-West (1987) with five lags. Italic point estimates indicates a $p$-value of less than 0.1 and bold indicates a $p$-value of less than 0.05. Data is quarterly and the full sample spans 1970Q2-2016Q2, while the pre-crisis sample ends in 2008Q4.

Table A.8: The Real Rate and Double-Sorted Versions of PVS

| | | Real $\text{Rate}_t = a + b \times$ Y-Neutral $\text{PVS}_t + \varepsilon_t$ | | | | | |
|---|---|---|---|---|---|---|---|
| | | Levels | | | First-Differences | | |
| | Characteristic Y | $b$ | $se(b)$ | Adj. $R^2$ | $b$ | $set(b)$ | Adj. $R^2$ |
| (1) | Duration | **0.81** | 0.20 | 0.16 | **0.36** | 0.12 | 0.10 |
| (2) | Leverage | **1.16** | 0.22 | 0.35 | **0.38** | 0.13 | 0.12 |
| (3) | 2M-Beta | **1.30** | 0.22 | 0.43 | **0.24** | 0.09 | 0.04 |
| (4) | Size | **1.23** | 0.24 | 0.39 | **0.38** | 0.14 | 0.12 |
| (5) | Value | **1.10** | 0.22 | 0.31 | **0.34** | 0.13 | 0.09 |
| (6) | Industry-Adjusted | **1.15** | 0.21 | 0.34 | **0.29** | 0.11 | 0.06 |
| (7) | Div. Payers | **1.22** | 0.18 | 0.38 | **0.27** | 0.09 | 0.06 |
| (8) | Non-Div. Payers | **0.63** | 0.26 | 0.10 | **0.31** | 0.13 | 0.08 |

*Notes*: This table reports a battery of robustness exercises for our main results. Specifically, we report time-series regression results of the following form, in both levels and first-differences: Real $\text{Rate}_t = a + b \times$ Y-Neutral $\text{PVS}_t + \varepsilon_t$. For rows (1)-(5), the variableY-Neutral $\text{PVS}_t$ is constructed by sorting all NYSE, AMEX, and NASDAQ firms in CRSP into two bins based on the median value of characteristic Y at time $t$. Within the high-Y (above median) firms, we further sort firms into terciles based on their volatility over the previous sixty days. Within each tercile, we compute the average book-to-market (BM) ratio between the low and high-volatility firms. We repeat this procedure for firms in the low-Y bucket. Y-Neutral $\text{PVS}_t$ is then defined as ($\text{BM}_t$ of Low-Volatility − $\text{BM}_t$ of High-Volatility with High Y)/2 + ($\text{BM}_t$ of Low-Volatility − $\text{BM}_t$ of High-Volatility with Low Y)/2. In row (6), we compute an industry-adjusted version of $PVS_t$ by first sorting stocks into industries based on their SIC code and the 48 industry definitions on Ken French's website. Within each industry $i$ we sort firms into quintiles based on their trailing 60-day volatility and then define $PVS_{i,t}$ as the average BM ratio of low-volatility firms in industry $i$ minus the average BM ratio of high-volatility firms in industry $i$. The industry-adjusted $PVS_t$ is defined as the equal weighted $PVS_{i,t}$ across all 48 industries. In row (7), we construct $PVS_t$ only for the set of firms who have paid a dividend over the past twenty-four months. Row (8) repeats the exercise for the set of firms that have not paid a dividend over the past twenty-four months. See Section A2.7.3 of this appendix for more details on how we construct each of these versions of $PVS_t$. In all cases, we standardize $PVS_t$ (or its first difference) to have a mean of zero and a variance of one. The one-year real rate is the one-year Treasury bill rate net of one-year survey expectations of the inflation (the GDP deflator) from the Survey of Professional Forecasters, expressed in percent and linearly detrended. Standard errors are computed using Newey-West (1987) with five lags. Italicized point estimates indicates a $p$-value of less than 0.1 and bold point estimates indicate a $p$-value of less than 0.05. Data is quarterly and the full sample spans 1970Q2-2016Q2.

Table A.9: Volatility-Sorted Returns and Monetary Policy Surprises - Including Unscheduled FOMC Dates

$$\text{Vol-Sorted Ret}_{t\to t+1} = a + b \times \text{MP Shock}_{t\to t+1} + \varepsilon_{t\to t+1}$$

| | Quarterly Data | | | | Daily Data | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | All | | Scheduled | | All | | Scheduled | | Sample | |
| MP Shock | $b$ | $se(b)$ | $b$ | $se(b)$ | $b$ | $se(b)$ | $b$ | $se(b)$ | Start | End |
| Romer and Romer (2004) | 0.75 | 1.55 | 0.71 | 1.55 | 0.27 | 0.27 | 0.27 | 0.27 | 1970Q1 | 1996Q4 |
| Bernanke and Kuttner (2005) | -2.94 | 14.18 | -1.65 | 21.18 | 5.55 | 3.96 | -1.08 | 2.14 | 1989Q2 | 2008Q2 |
| Gorodnichenko and Weber (2016) | -1.14 | 18.64 | 1.60 | 52.57 | 13.34 | 6.13 | 3.67 | 3.73 | 1994Q1 | 2009Q4 |
| Nakamura and Steinsson (2018) | 1.46 | 23.91 | 12.83 | 58.14 | 18.74 | 8.79 | 5.29 | 4.86 | 1995Q1 | 2014Q1 |

*Notes*: This table reports regressions of volatility-sorted returns onto monetary policy shocks. For all NYSE, AMEX, and NASDAQ firms in CRSP, we compute volatility at the end of each quarter using the previous sixty days of daily returns. We then form equal-weighted portfolios based on the quintiles of volatility. Volatility-sorted returns are returns on the lowest minus highest volatility quintile portfolios. Quarterly return regressions aggregate daily monetary policy shocks by summing over all shocks within a quarter. The Romer and Romer (2004) shock is the change in the intended Federal Funds rate inferred from narrative records around monetary policy meetings, after controlling for changes in the Federal Reserve's information. The Bernanke and Kuttner (2005) shock is derived from the price change in Federal Funds future contracts relative to the day before the policy action. The Gorodnichenko and Weber (2016) shock is derived from the price change in Federal Funds futures from 10 minutes before to 20 minutes after an FOMC press release. The Nakamura and Steinsson (2018) shock is the unanticipated change in the first principal component of interest rates with maturity up to one year from 10 minutes before to 20 minutes after an FOMC news announcement. Columns listed as "All" include all policy changes and "Scheduled" includes only changes that occurred at regularly scheduled policy meetings. In restricting the analysis to regularly scheduled meetings, we exclude quarters after 1993Q4 where the Federal Reserve made policy changes outside of scheduled meetings. Prior to 1994, policy changes were not announced after meetings so the distinction between scheduled and unscheduled meetings is not material. Robust standard errors are reported.

Table A.10: $PVS_t$, the Real Rate, and Future Returns to Volatile Assets in Other Asset Classes

| | | | | Forecasting Low-High Vol $\text{Ret}_{t\to t+1}$ with | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | $PVS_t$ | | | Real $\text{Rate}_t$ | | |
| Asset Class | $N$ | Mean | Volatility | $b$ | $se(b)$ | $R^2$ | $b$ | $se(b)$ | $R^2$ |
| U.S. Stocks | 184 | 2.7 | 29.6 | **5.34** | 1.04 | 0.13 | **1.57** | 0.56 | 0.04 |
| U.S. Corporate Bonds | 136 | -3.1 | 8.9 | **2.36** | 0.70 | 0.27 | *0.51* | 0.27 | 0.03 |
| Sovereign Bonds | 50 | -10.9 | 19.5 | *2.89* | 1.60 | 0.09 | 0.46 | 0.78 | -0.02 |
| Options | 88 | -16.0 | 17.8 | **1.92** | 0.80 | 0.03 | *1.07* | 0.56 | 0.02 |
| CDS | 31 | -7.0 | 6.4 | **1.77** | 0.40 | 0.47 | **0.77** | 0.31 | 0.11 |
| Commodities | 89 | 10.3 | 35.4 | 1.19 | 2.44 | -0.01 | -0.34 | 1.33 | -0.01 |
| FX | 120 | 1.2 | 10.8 | -0.20 | 0.33 | -0.01 | -0.57 | 0.38 | 0.02 |

*Notes*: This table reports summary statistics and forecasting results for portfolios sorted on volatility in other asset classes. For U.S. stocks, the low-minus-high vol return is defined as in Panel A. For other asset classes, we use the portfolios in He et al. (2017) as test assets. Within each asset class and in each quarter, we sort the test portfolios based on their trailing 5-year monthly volatility. We then form a new portfolio that is long the lowest-volatility portfolio and short the highest-volatility portfolio within each asset class. For U.S. stocks, we use our own low-minus-high volatility portfolio based on all CRSP stocks. The reported mean and the volatility are annualized and in percentage terms. The columns under "Forecasting Low-High Vol $\text{Ret}_{t\to t+1}$" report the point estimate, $t$-statistic, and adjusted $R^2$ from forecasting one-quarter ahead returns on the low-minus-high volatility trade within each asset class using $PVS_t$ or Real $\text{Rate}_t$. Standard errors are based on Newey-West (1987) standard errors with two lags. Italicized point estimates indicates a $p$-value of less than 0.1 and bold point estimates indicate a $p$-value of less than 0.05. The real rate is the one-year Treasury bill rate net of one-year survey expectations of the inflation (the GDP deflator) from the Survey of Professional Forecasters, expressed in percentage points and linearly detrended. $PVS_t$ is the average book-to-market ratio of low-minus-high volatility stocks. We standardize $PVS_t$ to have mean zero and variance one for our full sample (1970Q2-2016Q2). Quarterly return data from He et al. (2017) ends in 2012 and data availability varies with asset class. All returns are expressed in percentage points.

Table A.11: Robustness: PVS and Real Outcomes

| | | Investment-Capital Ratio | | Output Gap | | Unemployment | |
|---|---|---|---|---|---|---|---|
| | | $b$ | $se(b)$ | $b$ | $se(b)$ | $b$ | $se(b)$ |
| (1) | Baseline | 0.22 | 0.05 | 0.29 | 0.09 | -0.11 | 0.03 |
| (2) | Agg B/M | 0.24 | 0.04 | 0.34 | 0.08 | -0.12 | 0.04 |
| (3) | Yield Spread | 0.22 | 0.05 | 0.30 | 0.08 | -0.13 | 0.03 |
| (4) | Cochrane-Piazzesi | 0.23 | 0.05 | 0.28 | 0.08 | -0.11 | 0.03 |
| (5) | GZ Spread | 0.14 | 0.07 | 0.20 | 0.13 | -0.08 | 0.06 |
| (6) | Policy Uncertainty | 0.22 | 0.04 | 0.31 | 0.07 | -0.13 | 0.06 |

*Notes*: This table compares other measures of financial conditions to $PVS_t$, the average book-to-market ratio of low-minus-high volatility stocks. This table presents regression evidence for forecast horizon $h = 1$. We run our baseline local projections adding different control variables including the aggregate book-to-market ratio, the 10-year minus 1-year Treasury yield spread, the Cochrane and Piazzesi (2005) bond risk-factor, the Gilchrist and Zakrajšek (2012) credit spread, and the Baker et al. (2016) policy uncertainty variable. $PVS_t$ continues to have forecasting power for future macroeconomic outcomes in all cases.

Table A.12: PVS and Firm-Level Outcomes

| Dependent Variable | $\% \; CAPX^{Ann}_{i,t+4}/A_{i,t}$ | | | | | |
|---|---|---|---|---|---|---|
| | *Full Sample* | | *1983Q1-1999Q4* | | *2000Q1-2016Q2* | |
| | (1) | (2) | (3) | (4) | (5) | (6) |
| $\% \; CF^{Ann}_{i,t+4}/A_{i,t}$ | 0.08** | 0.07** | 0.09** | 0.09** | 0.05** | 0.05** |
| | (0.00) | (0.00) | (0.01) | (0.01) | (0.00) | (0.00) |
| $PVS_t$ | 0.63** | 0.00 | 0.47** | 0.00 | 0.33** | 0.00 |
| | (0.10) | (0.00) | (0.16) | (0.00) | (0.06) | (0.00) |
| $PVS_t \times 1^{q=2}_{it}$ | 0.15** | 0.17** | 0.19** | 0.17* | 0.13** | 0.13** |
| | (0.04) | (0.04) | (0.09) | (0.10) | (0.04) | (0.03) |
| $PVS_t \times 1^{q=3}_{it}$ | 0.24** | 0.29** | 0.30** | 0.29** | 0.27** | 0.28** |
| | (0.04) | (0.05) | (0.11) | (0.12) | (0.05) | (0.05) |
| $PVS_t \times 1^{q=4}_{it}$ | 0.28** | 0.38** | 0.45** | 0.46** | 0.36** | 0.38** |
| | (0.06) | (0.06) | (0.14) | (0.14) | (0.05) | (0.06) |
| $PVS_t \times 1^{q=5}_{it}$ | 0.16* | 0.33** | 0.37** | 0.43** | 0.42** | 0.45** |
| | (0.09) | (0.08) | (0.15) | (0.16) | (0.07) | (0.08) |
| FE | $i$ | $(i,t)$ | $i$ | $(i,t)$ | $i$ | $(i,t)$ |
| $R^2$ | 0.57 | 0.59 | 0.59 | 0.59 | 0.68 | 0.69 |
| # of Firms | 9,356 | 9,356 | 6,792 | 6,792 | 5,604 | 5,604 |
| $N$ | 315,333 | 315,333 | 155,080 | 155,080 | 160,073 | 160,073 |

*Notes*: This table studies how firm-level investment interacts with PVS. We measure firm $i$'s investment at time $t$ as the running four-quarter total CAPX (denoted $CAPX^{Ann}_{i,,t}$) divided by the book value of assets at time $t-4$ (denoted $A_{i,t-4}$). $CF^{Ann}_{i,t}$ is the running four-quarter total cash flow for the firm, computed as depreciation and amortization plus income before extraordinary items. Both are winsorized at their 1% tails. We run regressions of the form: $CAPX^{Ann}_{i,t+4}/A_{i,t} = FE + \sum_{q=2}^{5} a_q \cdot 1^q_{it} + b_1 \times CF^{Ann}_{i,t+4}/A_{i,t} + \sum_{q=1}^{5} c_q \times 1^q_{it} + d_2 \times PVS_t + \sum_{q=2}^{5} d_q \times PVS_t \times 1^q_{it} + \varepsilon_{i,t+4}$, where $1^j_{it}$ is an indicator function for whether firm $i$ is in volatility-quintile $j$ at time $t$. $PVS_t$ is average book-to-market ratio of low-minus-high volatility stock and in all regressions is standardized to have mean zero and variance one for the period 1970q2-2016q2. FE is a set of fixed effects as indicated in the table. We use all firms in the CRSP-COMPUSTAT merged database where the value of book assets is greater than $10 million. We exclude financial firms and firms with negative investment. Standard errors are listed below point estimates and are double-clustered by firm and by quarter. * indicates a $p$-value of less than 0.1 and ** indicates a $p$-value of less than 0.05. The full sample runs from 1983Q1-2016Q2. The total size of the subsamples does not match the full sample because we drop fixed-effect groups of size one.

Table A.13: Characteristics of Low versus High-Volatility Firms

| | | Low-Vol Firms | High-Vol Firms |
|---|---|---|---|
| Total Assets ($ mm) | Mean | 10,821 | 722 |
| | Median | 1,323 | 32 |
| | | | |
| ROA (%) | Mean | 12.2 | -6.9 |
| | Median | 11.8 | 0.01 |
| | | | |
| Investment Rate (%) | Mean | 6.9 | 13.9 |
| | Median | 5.3 | 7.4 |

*Notes*: This table shows statistics on the nominal size (total assets), profitability (return on assets, or ROA), and investment rates of firms sorted into volatility quintiles. At each date $t$, we compute the trailing 60-day volatility of each firm in the CRSP-COMPUSTAT merged database and them sort stocks into quintiles based on their volatility. ROA is defined as the trailing four-quarter sum of earnings before operating income before depreciation, scaled by the book value of assets at $t-4$. The investment rate for each firm is defined as the trailing four-quarter sum of CAPX and R&D, scaled by the book value of assets at $t-4$. We winsorize ROA and investment rates at their 1% tails to mitigate the impact of outliers. Data is quarterly and spans 1983Q1-2016Q2.

Table A.14: High-Volatility and Low-Volatility Firm Investment

| | Aggregate I/K | Low-Vol | Medium-Vol | High-Vol |
|---|---|---|---|---|
| Aggregate I/K | 1 | 0.35 | 0.59 | 0.79 |
| Low-Vol | 0.35 | 1 | 0.87 | -0.08 |
| Medium-Vol | 0.59 | 0.87 | 1 | 0.27 |
| High-Vol | 0.79 | -0.08 | 0.27 | 1 |

*Notes*: This table shows the correlation of aggregate investment (private nonresidential fixed investment divided by the aggregate capital stock) with the investment rates of firms sorted into volatility terciles. At each date $t$, we compute the trailing 60-day volatility of each firm in the CRSP-COMPUSTAT merged database and them sort stocks into terciles based on their volatility. The investment rate for each firm is defined as the trailing four-quarter sum of CAPX and R&D, scaled by the book value of assets at $t-4$. The investment rate within each tercile is the average rate across firms in that tercile. Data is quarterly and spans 1990Q1-2016Q2. We start in 1990Q1 because the level of total investment in COMPUSTAT quarterly data aligns with total investment from COMPUSTAT annual from that point forward.

Table A.15: PVS and Investor Expectations of Cash Flows

***Panel A**: Contemporaneous Relationship with Expectations of Cash Flows*

| Dependent Variable | $PVS_t$ | | |
|---|---|---|---|
| | (1) | (2) | (3) |
| *High-Minus-Low Volatility Stocks:* | | | |
| $\mathbb{E}_t\left[\mathrm{ROE}_{t+1}\right]$ | 0.29** | | |
| | (0.12) | | |
| $\mathbb{E}_t\left[\mathrm{ROE}_{t+1\to t+4}\right]$ | | 0.30** | |
| | | (0.13) | |
| $\mathbb{E}_t$ [Long-Term Growth] | | | 0.35* |
| | | | (0.20) |
| Adj. $R^2$ | 0.10 | 0.11 | 0.15 |
| $N$ | 102 | 110 | 110 |

***Panel B**: PVS, Cash-Flow Surprises, and Future Revisions in Expectations*

$$Y = a + b \times PVS_t + \varepsilon$$

| | | $b$ | $se(b)$ | Adj. $R^2$ | $N$ |
|---|---|---|---|---|---|
| | *Expected Cash Flows:* | | | | |
| (1) | ROE Surprise$_{t+1\to t+4}$ | 0.13 | 0.13 | 0.00 | 94 |
| (2) | $\mathbb{E}_{t+2}\left[\mathrm{ROE}_{t+3}\right] - \mathbb{E}_t\left[\mathrm{ROE}_{t+3}\right]$ | -0.09 | 0.10 | -0.00 | 102 |

*Notes*: **Panel A** of this table shows contemporaneous regressions of $PVS_t$ on investor expectations of cash flows. In column (1), for each firm $i$ and date $t$, we use the time-$t$ expectation of quarterly accounting return on equity (ROE) at time $t+1$, denoted $\mathbb{E}_t\left[\mathrm{ROE}_{i,t+1}\right]$, from the Thompson Reuters IBES dataset. At the portfolio level, $\mathbb{E}_t\left[\mathrm{ROE}_{t+1}\right]$ is the cross-sectional median for high-volatility stocks minus the median for low-volatility stocks, where stocks are designated as high or low volatility at time $t$ based on their past 60 days of realized returns. In column (2), we mirror the expected ROE measure in column (1) but instead use the annual ROE forecast from IBES for the next fiscal year. Column (3) again follows the same approach, but instead uses the "long-term growth" estimate provided by IBES. $PVS_t$ is the average book-to-market ratio of low-minus-high-volatility stocks. We include a constant in all regressions and all variables are standardized to have mean zero and unit variance. Newey-West (1987) standard errors with five lags are listed below point estimates. **In Panel B**, we use $PVS_t$ to forecast future revisions in expected cash flows and risk. In row (1), we forecast the median return on equity (ROE) surprise for low-volatility stocks minus the median ROE surprise for high-volatility stocks, where ROE surprises are computed using Thomson Reuters IBES data. The time horizon for our ROE surprises is time $t+1$ to $t+4$. In row (2), we compute revisions in expected $\mathrm{ROE}_{t+3}$ based on the Thompson Reuters IBES database of analyst forecasts. Specifically, for each firm $i$ and date $t$, we use the median forecast of ROE time $t+3$, denoted $\mathbb{E}_{t+2}\left[\mathrm{ROE}_{i,t+3}\right]$. For each $(i,t)$, we choose the shortest forecast horizon $h$ such that the quarterly earnings are at least two fiscal quarters away, which in calendar time is generally between 3 and 4 quarters from date $t$. For each firm $i$, we then define the revision in expected ROE at time $(t+2)$ as $\mathbb{E}_{t+2}\left[\mathrm{ROE}_{i,t+3}\right] - \mathbb{E}_t\left[\mathrm{ROE}_{i,t+3}\right]$. At the portfolio level, $\mathbb{E}_{t+2}\left[\mathrm{ROE}_{t+3}\right] - \mathbb{E}_t\left[\mathrm{ROE}_{t+3}\right]$ is the cross-sectional median revision for high-volatility stocks minus the median revision for low-volatility stocks. In all cases, data is quarterly and depends on data availability, though the full sample for $PVS_t$ spans 1970Q2 to 2016Q2.

Table A.16: PVS, the Real Rate, and Other Measures of Expected Risk

| | $X$-variable | $N$ | $PVS_t = a + b \times X_t$ | | |
|---|---|---|---|---|---|
| | | | $b$ | $se(b)$ | $R^2$ |
| (1) | $\sigma_t\left(\text{Aggregate Stock Market Ret}_{t+1}\right)$ | 185 | -0.23 | 0.15 | 0.05 |
| (2) | $\sigma_t\left(\text{Industrial Production Growth}_{t+1}\right)$ | 184 | -0.10 | 0.23 | 0.00 |
| (3) | Macro Uncertainty $\text{Index}_t$ | 185 | 0.07 | 0.23 | -0.00 |
| (4) | SPF Macro Uncertainty Index | 126 | -0.44 | 0.17 | 0.27 |

*Notes*: This table different measures of economic risk with $\text{PVS}_t$ and the one-year real interest rate. $\sigma_{t+1}$ (Agg. Stock Market) in Row (1) is the expected volatility of the aggregate stock market. From 1986 onward, it is the time $t$ value of the VXO option implied volatility index from the CBOE. To fill in the data prior to 1986, we fit an AR(1) model to within-quarter realized volatility of the CRSP Value-Weighted index. We then use the one-step ahead forecast made at time $t$ from the AR(1) model, reindexed to create a smooth series when appending the VXO after 1986. In row (2), we fit an ARMA(1,1)-GARCH(1,1) to industrial production growth and then use the one-step ahead GARCH volatility forecast as our time-$t$ measure of expected risk. Row (3) uses the macroeconomic uncertainty index from Jurado et al. (2015). Finally, in row (4) we create an index of macroeconomic uncertainty using forecast dispersion of growth rates in real GDP, industrial production, real private fixed nonresidential investment, and corporate profits. We obtain 1-quarter, 2-quarter, and 4-quarter forecasts come from the Survey of Professional Forecasters for 1985 onwards, which is when real growth rates were asked for instead of imputed. We then HP filter each series, standardize, and take a cross-sectional average to arrive at SPF Macro Uncertainty index. The first set of regressions in the table shows the results of a univariate regression of $\text{PVS}_t$ on each of these expected risk measures. The second set of regressions in the table shows the results of a univariate regression of the real rate on contemporaneous values of each variable. The one-year real rate is the one-year Treasury bill rate net of one-year survey expectations of the inflation (the GDP deflator) from the Survey of Professional Forecasters, expressed in percent and linearly detrended. In all regressions, we standardize both PVS and the expected risk measures to have mean zero and variance one in order to facilitate comparison of magnitudes. Standard errors are computed using Newey-West (1987) with five lags. Data is quarterly and the full sample spans 1970Q2-2016Q2.

Table A.17: Subjective Expectations and Realized Risk

| Dep. Variable | Realized Volatility$(t+k,t+h)$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | $k=0,h=4$ | | | | $k=3,h=4$ | | | |
| | Full Sample | | Pre-Crisis | | Full Sample | | Pre-Crisis | |
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
| $\mathrm{IV}_t(t+k,t+h)$ | 0.71** | 0.46** | 0.94** | 0.36** | 0.26** | 0.19** | 0.51** | 0.27** |
| | (0.08) | (0.06) | (0.09) | (0.10) | (0.08) | (0.07) | (0.14) | (0.09) |
| Constant | 0.36** | | 0.16* | | 0.77** | | 0.55** | |
| | (0.06) | | (0.08) | | (0.07) | | (0.12) | |
| FE | | $(\mathrm{ind}\times t)$ | | $(\mathrm{ind}\times t)$ | | $(\mathrm{ind}\times t)$ | | $(\mathrm{ind}\times t)$ |
| $R^2$ | 0.21 | 0.72 | 0.41 | 0.86 | 0.02 | 0.65 | 0.11 | 0.75 |
| $N$ | 1,213 | 935 | 254 | 171 | 1,178 | 902 | 248 | 167 |

*Notes*: This table uses implied volatilities at the firm-level to forecast realized volatility for high-volatility firms. For each firm $i$ and time $t$, we use the term structure of implied volatilities at time $t$ to back out what the implied volatility of returns is for the horizon $t+k$ to $t+h$, under the assumption that quarterly returns are not autocorrelated. We then run the following panel regression:

$$\text{Realized Volatility}_i(t+k,t+h) = a + b\times \mathrm{IV}_{i,t}(t+k,t+h)+\varepsilon_{i,t}$$

We use all firms in the CRSP-OptionMetrics merged database. High-volatility firms are defined as in Table 1 of the main text. The row FE indicates whether a fixed effect was included in the regression and industries are defined using the 30 industry definitions from Ken French's website. Standard errors are listed below point estimates and are double-clustered by firm and by quarter. * indicates a $p$-value of less than 0.1 and ** indicates a $p$-value of less than 0.05. The full sample runs from 1996Q1-2016Q2 and the pre-crisis sample runs from 1996Q1-2006Q2. The size of the subsamples that include fixed effects do not match their counterparts without fixed-effects because we drop fixed-effect groups of size one.

Table A.18: Realized Risk, Expected Risk, and Good News

| Dependent Variable | $\Delta_4$HML-Realized Vol | $\Delta_4\sigma_t$ (EPS$_{t+5}$) | $\Delta_4$(% Banks Loose) | $\Delta_4$Small Business Opt. |
|---|---|---|---|---|
| | (1) | (2) | (3) | (4) |
| Real GDP Surprise$_{t-4\to t}$ | -0.12 | -0.38** | -0.03 | 0.36** |
| | (0.09) | (0.13) | (0.13) | (0.11) |
| Corporate Profit Surprise$_{t-4\to t}$ | -0.44** | -0.14** | 0.54** | 0.16* |
| | (0.13) | (0.07) | (0.11) | (0.08) |
| LMH-Vol ROE$_{t-4\to t}$ | -0.05 | 0.04 | 0.14** | 0.06 |
| | (0.09) | (0.07) | (0.07) | (0.08) |
| $\Delta_4$Bank Net Chargeoffs$_t$ | 0.27** | 0.31** | -0.23** | -0.17** |
| | (0.08) | (0.08) | (0.08) | (0.06) |
| Adj. $R^2$ | 0.34 | 0.43 | 0.47 | 0.27 |
| $N$ | 158 | 106 | 101 | 158 |

*Notes*: This table reports univariate regressions of four-quarter changes of various measures of realized and expected risk on: (1) the surprise in real GDP growth, defined as realized real GDP growth from time $t-4$ to $t$ minus the expected annual growth forecast at time $t-4$ made by the Survey of Professional Forecasters; (2) the four-quarter change in analysts' expected risk for high-volatility versus low-volatility firms as described in Table II in the main texgt; (3) the trailing annual ROE of the low-minus-high volatility portfolio; and (4) the four-quarter change in bank net chargeoff rate, taken from bank call reports. In terms of our risk measures, in column (1), we use the change in the average realized stock return volatility of high-volatility firms minus that of low-volatility firms. In column (2), we use the change in expected analyst uncertainty over earnings (see Table II in the main text for a complete description). In column (3), we use the change in the net percent of U.S. banks loosening lending standards, taken from the Federal Reserve Senior Loan Officer Opinion Survey (SLOOS). In column (4), we use the change in the NFIB Small Business Optimism index. The operator $\Delta_4 Z_t$ denotes $Z_t$ - $Z_{t-4}$ for variable $Z$. In each regression, we include a constant and standardize all variables to have mean zero and variance one. In all cases, Newey-West (1987) standard errors with five lags are listed below point estimates. Data is quarterly and depends on data availability.