Money, Exchange Rates and Growth

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The model of this paper can easily rationalize the combination of inflationary monetary policy and financial market regulations that typically is viewed as repressing financial systems and hindering economic development. In an economy where countries trade freely in international asset and money markets, monetary and financial market policies affect the real exchange rate of a country in an important way. The response of the real exchange rate determines how policy affects financial market efficiency, and how financial market efficiency affects equilibrium capital formation. Also, when inflation rates are high enough there exist multiple steady state equilibria distinguished by the level of the equilibrium capital stock. The consequences of financial and monetary policies then depend critically on which equilibrium capital stock obtains. For example, low capital stock countries can enhance capital formation by imposing foreign exchange controls. These cause real exchange rate appreciation which reduces the outstanding real value of government liabilities and mitigates the crowding out of private capital.

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1 Introduction

Many emerging economies are characterized by undeveloped and repressed financial systems. These financial systems typically lack (large) equity markets, are dominated by commercial banks that hold high levels of government debt, and are heavily taxed by high rates of debt monetization, inflation, domestic currency reserve requirements, and foreign exchange controls. That underdeveloped financial systems are regulated in such ways seems surprising in light of the widespread belief that there is a systematic positive relationship between financial and economic development, and that well-fuctioning financial markets that are not impeded by regulation and inflation play important allocative functions that promote capital formation.

The model of this paper can easily rationalize the combination of inflationary monetary policy and financial market regulations that typically is viewed as repressing financial systems and hindering economic development. In an economy where countries trade freely in international asset and money markets, monetary and financial market policies affect the real exchange rate of a country in an important way. The response of the real exchange rate to a particular financial market regulation determines how that regulation affects financial market efficiency, and how financial market efficiency affects equilibrium capital formation. For example, if a financial market regulation such as a foreign exchange control causes a large enough real appreciation, the real value of outstanding government liabilitites is reduced and crowding out of private capital formation is inhibited. The banks in the model then appear to be *more* efficient in allocating funds to productive uses. Also, when inflation rates are high enough there exist multiple steady state equilibria distinguished by the level of the equilibrium capital stock. The consequences of financial and monetary policies then depend critically on which equilibrium capital stock obtains. In either equilibrium, some types of financial market regulation enhance capital formation.

The origins of the idea that the efficiency of an economy's financial system is important for its ability to support economic development can be found in Patrick (1966), Cameron (1967), Gurley and Shaw (1967), Goldsmith (1969), McKinnon (1973) and Shaw (1973). Recently, more formal analyses confirm that unregulated private sector financial institutions serve several important allocative functions (Greenwood and Jovanovic (1990), Bencivenga and Smith (1991), Greenwood and Smith (1993) and Schreft and Smith (1994) among many others). These analyses show, for example, how unregulated banks can promote capital formation by mobilizing savings, allocating funds into the most productive uses, and providing insurance against risky returns on investment projects.

Almost all of this formal analysis has been conducted in closed economy environments,[1] yet most developing countries conduct policy in open economy environments, and many policy issues for such countries relate to the effects for growth of adopting financial market regulations that restrict or otherwise influence external financial flows. Developing countries that are financially open confront - and may seek - real exchange rate implications of monetary and financial market policy actions which directly affect international portfolio investment decisions. This paper seeks to develop a framework in which the impact of financial market regulation for the real exchange rate can be evaluated. In addition, and despite the complications that openess seems to imply for the impact of financial market regulation, when countries have achieved growth success despite high levels of such regulation this success has frequently been attributed to the outward orientation of that country's policies. How can high levels of financial market regulation be compatible with growth success for countries that are financially open? The purpose of this paper is to develop a model in which questions such as this can be addressed.

The vehicle that I use to analyze these issues is a two-country, two-currency version of the Diamond (1966) growth model. Two-period lived overlapping generations inhabit each country, while national governments issue local fiat currency and interest-bearing debt, and set constant rates of money growth to finance interest obligations with seignorage revenue. International trade in goods is limited, and real exchange rates need not conform to purchasing power parity due to the presence of non-traded goods. Real exchange rates can therefore be influenced by real, monetary and financial factors in an important way. However, international trade in all assets is unrestricted, all agents have equal access to asset markets, and all markets are perfectly competitive and clear at each date. Thus, this economy can really be viewed as "financially open" despite the fact that trade in goods is limited.

Government debt and private capital of both countries compete in agents' portfolios, while the currency of each country provides special liquidity in interlocation exchange. In particular, the presence of spatial separation both within and across countries limits communication between agents. Limited communication prevents agents from redeeming counterfeitable bonds and claims to capital income in interlocation goods market exchange, while the universal recognizability of either type of currency rationalizes its use as a medium of interlocation exchange. Stochastic shocks to agents' need for liquidity in interlocation exchange then cause agents to diversify their portfolios across both types of currency, as well as interest-bearing bonds and private capital that strictly dominate currency in rate of return.

[1]Two exceptions are Boyd and Smith (1993) and Huybens and Smith (1994).

This motivation for money is modeled explicitly by assuming that there are two symmetric locations within each country, and that agents move between locations domestically and internationally in a stochastic manner. As a result of limited communication, when agents are relocated they carry only currency with them, and, since interlocation exchange requires the currency of the country in which the seller is located, both currencies are held in agents' portfolios to insure against the possibility of relocation. Stochastic relocation plays the role "liquidity" shocks in the Diamond-Dybvig (1983) model, and it is then natural to assume that deposit-taking banks arise in each country to provide insurance against these shocks. To do this they hold diversified portfolios on behalf of young agents comprising both countries'currencies, bonds and capital and set optimal portfolio weights according to depositors' liquidity needs.

In this environment, I consider equilibria in which banks are unregulated as well as equilibria in which they are subject to forms of financial market restrictions. In addition I analyze the equilibrium consequences of alternative rates of debt-monetization. I obtain the following results.

When rates of money growth are sufficiently high, there exist two steady state equilibria in which the value of both currencies is strictly positive. One of these is characterized by a low capital stock and high real and nominal interest rates. In this equilibrium, both countries' governments have positive net liability positions which crowd out private capital. The extent to which either country experiences such crowding out depends on the value of equilibrium capital flows. In the second, high capital stock equilibrium, real and nominal interest rates are low and the value of net government liabilities in each country is negative. Consequently, the equilibrium capital stock of each country is enhanced by government lending to the private banking system.

In a sense, domestic monetary policy and financial market regulations affect equilibrium capital stocks and external capital flows through real exchange rate movements that alter relative incomes. In all cases, the fact that both currencies are imperfectly substitutable in exchange and so held within each country necessitates real exchange rate movements that alter the value of national government liabilities. Consider the case of an increase in the domestic country's money growth rate. This taxes both domestic and foreign holders of that country's currency - all of whom hold the currency only in order to buy domestic goods. For domestic goods market equilibrium, spending is stimulated by a decline in the external value of domestic currency. This raises the domestic value of foreign held domestic currency and so the purchasing power of foreign agents over domestic goods. This effect serves to aggravate the impact of increased debt issues (associated with higher money growth) for the crowding out of private capital in the low capital stock equilibrium. The same real exchange rate movement mitigates the increase in government finance available for private capital formation in the high capital stock equilibrium.

The imposition of a foreign exchange control by a country unambiguously raises the real external value of its currency and so can be rationalized for that purpose. In addition, for low capital stock countries with a sufficiently high money growth and inflation rate, this regulation enhances equilibrium capital formation. Private banks substitute out of foreign currency and into domestic currency, bonds and capital, the returns from which are used to purchase domestic goods. This is offset by a real appreciation which reduces the purchasing power of foreign holders of domestic currency, and by an increase in the capital stock (and the income of interest-earning agents) when money growth rates are high. The real appreciation reduces the domestic value of outstanding government liabilities and resources are freed for higher private capital formation. This exchange control would then raise the relative value of private capital to government debt held within the domestic banking sector and reduce the domestic real and nominal interest rate, both of which observations might typically signal an improvement in that country's financial sector efficiency.

Reserve requirements are also frequently imposed by developing economies on their financial systems in order to either influence their real exchange rate or to expand their inflation tax base. Here, they have an ambiguous real exchange rate effect which can be positive if the money growth rate is sufficiently high, unambiguously reduce the capital stock of a capital poor country, but can raise equilibrium capital formation for a high capital stock country when money growth rates are sufficiently high. Such policies may not be effective devices for influencing real exchange rates, and their consequences for the inflation tax base are ambiguous in general. Interestingly, reserve requirements and ecxhange controls may be viewed as "close substitutes" for the purpose of influencing exchange rates or expanding the inflation tax base. Yet, their consequences for real exchange rates, for the inflation tax base and for growth are very different here.

The remainder of the paper is organized as follows. Section II describes the economic environment and the behaviour of banks. Section III presents conditions for the attainment of general equilibrium, and Section IV analyzes the existence and properties of steady state equilibria. Section V considers the equilibrium consequences of changes in monetary policy, and in financial market regulations, and Section VI concludes.

2 The Economic Environment

I consider a two-country, two-currency economy which is inhabited by an infinite sequence of two-period lived overlapping generations. In each country there are two, symmetric locations and production of a single final good takes place in both locations of each country. While I allow the production technology to differ across the two countries, in many respects they share common characteristics, as I now describe.

2.1 Production

Production in both countries and in each location occurs according to a constant returns to scale production technology. The parameters of the production technology are allowed to differ across the two countries although are identical in the two locations of each country. Specifically, let $F^d(K^d, N^d)$ be the domestic country's production function and let $F^f(K^f, N^f)$ be the foreign country's production function, where $K^d(K^f)$ and $N^d(N^f)$ denote the domestic (foreign) country capital stock and employment level respectively. Then $f^d(k^d)$ $[f^f(k^f)]$ denotes the domestic (foreign) country's intensive production function, where $f^d(k^d) = F^d(K^d/N^d, 1)$ $[f^f(k^f) = F^f(K^f/N^f, 1)]$ and k^d (k^f) is the domestic (foreign) capital-labour ratio.

These intensive production functions are assumed to have the following properties. $\forall k^d > 0$ $[\forall k^f > 0]$, $f^{d'}(k^d) > 0 > f^{d''}(k^d)$ $[f^{f'}(k^f) > 0 > f^{f''}(k^f)]$, $f^d(.)$ $[f^f(.)]$ satisfies the usual Inada conditions, and $f^d(0) = 0$ $[f^f(0) = 0]$ holds. In addition, it is assumed that $\Omega^d(k^d) = k^d/w^d(k^d)$ and $\Omega^f(k^f) = k^f/w^f(k^f)$ share the following property. $\forall k \geq 0$,

$$\partial\Omega(k)/\partial k = \Omega'(k) > 0, \tag{1}$$

This condition holds for $f^d(.)$ $[f^f(.)]$ any CES production function with elasticity of substitution ≤ 1, and so, for example, is satisfied in the case of Cobb-Douglas technology. Finally, $k_1^d > 0$ and $k_1^f > 0$ are given as initial conditions.

At any date t=1,2..., the final good can either be consumed or converted into future capital with a simple linear technology. Specifically, one unit of the good witheld from consumption at t becomes one unit of capital at t+1 with probability one. The capital thus obtained is used in t+1 production and, for simplicity, it is assumed to depreciate completely in the production process.

Both goods and factor markets are assumed to be perfectly competitive within each location and country so that capital and labour are paid their marginal products and factor payments exhaust the value of total output;

$$r_t^d = f^{d'}(k_t^d); t \geq 1, \tag{2a}$$

$$r_t^f = f^{f'}(k_t^f); t \geq 1, \tag{2b}$$

$$w_t^d = f^d(k^d) - f^{d'}(k_t^d)k_t^d = w^d(k_t^d); t \geq 1, \tag{2c}$$

$$w_t^f = f^f(k^f) - f^{f'}(k_t^f)k_t^f = w^f(k_t^f); t \geq 1, \tag{2d}$$

These payments enable consumption and investment activities by factor owners in each country in a manner that I now describe.

2.2 Agents

Each country is populated by an infinite sequence of two-period lived overlapping generations, plus an initial old generation. At each date t=1,2..., young agents are assigned to a specific location within one of the two countries and at each date there is a continuum of ex ante identical young agents with unit mass within each location.

All agents within each country are endowed with a single unit of labour when young, and are retired when old. In addition, each member of the initial old generation in the domestic (foreign) country is endowed with the inherited per capita capital stock k_1^d $[k_1^f]$. Young agents have no other endowments of goods or assets at any date.

At each date, t, in each location of each country agents access the commonly available production technology of that location and produce goods by hiring labour and renting capital. Goods, and so capital, are assumed to be immobile within and across country locations; that is, transportation costs for the good are prohibitive. Hence, only the capital of a given location can be used in the production of goods in that location. In addition, while agents do move between locations (domestically and internationally) as will be described below, the timing of trades which obtains in this economy ensures that only domestic (foreign) labour is used in the production of the domestic (foreign) good. Thus goods and factor market trades are conducted autarkically at the beginning of each period.

For simplicity, agents are assumed to care only about old age consumption, which is denoted simply by c. Specifically, their lifetime utility is given by u(c)=ln(c). Members of the initial old generation therefore simply consume the value of any endowed goods and assets that they hold, while young agents at each date t=1,2... achieve consumption by working when young and saving the value of their real wage in the form of some stores of value. These stores of value are as follows.

2.3 Assets

There are three types of primary asset in this economy which can be held by agents to provide for second period consumption. The first is private capital, produced in both countries by foregoing current consumption as I have described.

In addition, two other primary assets are supplied by government policy. In each country a government issues fiat currency and interest-bearing, default free, one period lived debt. Let M_t^d $[M_t^f]$ denote the time t per capita money supply of the domestic (foreign) country, while B_t^d $[B_t^f]$

denotes the nominal value of outstanding domestic (foreign) country government bonds per capita. The outstanding goods value of money and bonds is measured as follows.

Let p_t^d $[p_t^f]$ be the domestic (foreign) price level at t, and let e_t denote the domestic currency price of a unit of foreign currency at date t - the nominal exchange rate of the domestic country. Then $m_t^d = M_t^d/p_t^d$ $[m_t^f = M_t^f/p_t^f]$ denotes the stock of domestic (foreign) real balances at t, and $b_t^d = B_t^d/p_t^d$ $[b_t^f = B_t^f/p_t^f]$ denotes the real value of outstanding government bonds at t. In addition, $x_t = (e_t p_t^f/p_t^d)$ is the real exchange rate of the domestic country, or the number of domestic goods that must be given up to obtain one unit of the foreign good. Thus $x_t(m_t^f + b_t^f + k_t^f)$ measures the domestic goods value of total outstanding foreign assets at date t.

Returns to these assets are determined as follows. Capital earns the competitively determined rental rate in each country, while bonds bear a per unit, nominal return guaranteed by the domestic (foreign) government of $I_t^d > 1$ $[I_t^f > 1]$. The *real* return to holding a unit of domestic (foreign) government debt between dates t and t+1, measured in units of domestic (foreign) t+1 goods, is therefore $I_t^d p_t^d/p_{t+1}^d$ $[I_t^f p_t^f/p_{t+1}^f]$. Domestic (foreign) currency, of course, yields the real return p_t^d/p_{t+1}^d $[p_t^f/p_{t+1}^f]$ per unit held between periods t and t+1 in domestic (foreign) t+1 goods units. Currency is thus dominated in rate of return by bonds and - in equilibrium - by capital. However, in an economy in which agents are spatially separated within and across countries they confront limits on interlocation communication and from this fact currency derives liquidity advantages over all other assets. These liquidity advantages can motivate positive holdings of both currencies by all agents, as I now describe.

2.4 Market Interactions and the Timing of Trade

The nature of international and intranational interaction is as follows.

At the beginning of each date t, there is no interlocation movement or communication by any agent. To each location is assigned a continuum of ex ante identical agents with unit mass, each endowed with one unit of labour. Since they care only about (old age) consumption, young agents supply inelastically their single unit of labour to production in return for the competitively determined domestic (foreign) unit real wage of w_t^d $[w_t^f]$. This labour is combined with the private capital of that location which earns the domestic (foreign) rental rate r_t^d $[r_t^f]$ to produce goods, and subsequently factors are paid and goods are traded - again, autarkically within each location.

Young agents then trade their real wage income for some assets. While neither agents nor goods can move during subsequent asset transactions, complete intranational and international communication is possible. Young agents can therefore allocate their savings to both domestic and foreign capital claims, bonds and money.[2] When asset trading concludes, young agents face a positive probability of being relocated either within or across countries - a relocation risk that determines their portfolio allocation choices and which takes the following form.[3]

Let π_d^d $[\pi_f^f]$ be the probability that a resident of the domestic (foreign) country is relocated within his own country and let π_f^d $[\pi_d^f]$ be the probability that a resident of the domestic (foriegn) country is relocated to the foreign (domestic) country. The probability of relocation (domestically or internationally) is constant across periods, known by all agents, is iid across agents in a given country and satisfies $(1-\pi_d^d-\pi_f^d)>0$ $[(1-\pi_f^f-\pi_d^f)>0]$. There is, then, no aggregate randomness within countries and net within country relocations are zero. This need not be the case, however, for relocations across countries. Finally, to keep locations within a country symmetric, I adopt the convention that if residents of lcoation 1 (2) of either country are relocated internationally, they are relocated to location 1 (2) of the other country.

Since goods - and so capital - are immobile, agents who are relocated must carry with them some (other) assets in order to consume in old age. Specifically, agents are assumed to be able to carry *only* currency with them between locations and countries; currency thus has liquidity characteristics that dominate those of bonds or privately issued claims to capital income. This liquidity advantage derives from limited inter-location communication in the presence of spatial separation. At the beginning of each period, constraints on inter-location communication prevent the cross-location exchange of other liabilities issued in the economy for goods. In short, the counterfeitability of other assets prevents their redemption for goods outside the location of their issue. Currency's universal recognizability admits inter-location trade in this environment filling the resulting void in credit markets; currency is the only asset with a non-negotiable claim to goods across locations.[4]

Finally, the currency of the country in which the seller is located is required in all exchange. Consequently, domestically relocated agents require domestic and currency and internationally relocated agents require foreign currency in subsequent (t+1) goods market transactions.

The fact that currency is required in inter-location exchange means that stochastic relocation plays the role of a liquidity preference shock in the Diamond-Dybvig (1983) model. Agents who

[2]It is worth mentioning that while the goods produced by capital - and the capital itself -cannot be transported across countries, I allow generation t young domestic agents to trade in claims to t+1 foreign rental income. These claims are - by agreement - redeemed following t+1 production in domestic goods by domestic agents. Thus domestic agents effectively lend some proceeds of domestic production to their foreign couterparts until asset trade resumes in t+1. The equivalent statements apply to generation t foreign agent capital investments.

[3]The physical relocation risk described here is identical in all respects to that presented in Betts and Smith (1994) and follows also Champ, Smith and Williamson (1992) and Schreft and Smith (1994).

[4]This formulation follows Townsend (1987), Mitsui and Watanabe (1989), Hornstein and Krusell (1993), Champ, Smith and Williamson (1992) and Schreft and Smith (1994) in closed economy environments, and Betts and Smith (1994) in a two-country economy.

are relocated will wish to liquidate prematurely other assets and use the proceeds to acquire the currency of their destination. It is natural to assume, then, that agents seek to insure themselves against relocation risk and Greenwood and Smith (1993) show that this insurance is provided efficiently by unregulated, profit maximizing banks. Here, such banks will arise in each location, accept young agents' deposits, and hold reserves comprising both foreign and domestic currency as well as stocks of private capital and bonds.

Young agents therefore hold assets either directly or indirectly through bank deposits. In the former case, they diversify their asset portfolios subject to the individual uncertainty they face concerning future liquidity needs; in the latter, profit maximizing banks undertake portfolio diversification on their behalf. Once asset market trade is complete, young agents discover whether or not they are to be relocated and their ultimate destination; equivalently, individual portfolio preferences are revealed. If relocated, a young agent either takes any appropriate currency units held directly or withdraws from a bank any deposit return guaranteed in the requisite currency units before moving. If not relocated, agents consume in t+1 the real income derived from directly or indirectly held bonds and private capital.

To summarize, the timing of trade in this economy is as follows. At the beginning of each date t in any location there are some old agents who have moved from elsewhere and carry the currency of the location that they arrive in, some old agents who have not been relocated and hold claims to bond and capital income, and some young agents endowed with labour. Young agents supply labour and old agents private capital to production through competitive factor markets. Production occurs, factors are paid their marginal products, and goods market trade is conducted in which relocated old agents purchase goods with cash while the allocation of final output between consumption and capital investment is determined. Young agents then allocate directly their savings among the competing interest earning assets and currencies, or deposit their wage in banks. Old agents consume the goods they have purchased or their real asset returns. The young then discover whether they are to be relocated, and their ultimate destination; if relocated, they may go to banks and withdraw the appropriate currency or simply move taking with them any requisite currency units designated in their portfolios. This timing of transactions is depicted in Figure 1.

The risk of relocation - of premature asset liquidation - means that young agents will not wish to hold primary assets directly but prefer to have their savings intermediated by unregulated banks. In fact, all savings are intermediated, banks accept young agents' deposits, hold the primary assets in the model directly, and promise returns contingent on a depositor's relocation status and ultimate destination. I now describe the behaviour of these banks.

2.5 The Role of Banks

In each location of each country there are some banks that behave competitively in the sense that they view themselves as being unable to influence the equilibrium returns to assets. On the deposit side, these intermediaries behave as Nash competitors; that is, they announce state contingent deposit returns as a function of relocation status and destination, taking as given the announced return schedules of other banks. Free entry into intermediation means that - in a Nash equilibrium - deposit returns must be chosen such as to maximize the expected utility of a representative young agent for whose deposits banks compete. Announced return schedules must, naturally, satisfy a set of balance sheet constraints.

In the situations considered in this paper, the currency of each country is strictly dominated in rate of return by government bonds (and, in equilibrium, by capital). Domestic and foreign banks therefore hold either currency only in order to accomodate the liquidity needs of agents who are relocated. Banks can also invest in the bonds issued by both governments, and in the capital of both countries. Domestic (foreign) banks can purchase in date t asset market claims on either government, and claims to the income derived from ownership of date t foreign and domestic investment goods.

Let $m^d_{d_t}$ denote per depositor holdings of domestic real balances by domestic banks at t, and let $m^f_{d_t}$ denote per depositor holdings of foreign real balances held by domestic banks at t. The former is measured in units of domestic goods, and the latter in units of foreign goods. Similarly, let $b^d_{d_t}$ [$b^f_{d_t}$] denote the real value of per depositor holdings of domestically (foreign) issued bonds at t, and let $i^d_{d_t}$ [$i^f_{d_t}$] denote the per depositor holdings of domestic (foreign) investment goods by domestic banks at t. Again, domestic (foreign) asset holdings are measured here in domestic (foreign) goods.

Since all savings are intermediated, a representative domestic bank receives a real time t deposit of $w^d(k^d_t)$ per depositor. Thus the bank's balance sheet constraint is, for $t \geq 1$,

$$w^d(k^d_t) = m^d_{d_t} + b^d_{d_t} + i^d_{d_t} + x_t(m^f_{d_t} + b^f_{d_t} + i^f_{d_t}) \tag{3}$$

where x_t converts the value of assets denominated in foreign goods into date t domestic good units.

Such a bank offers a set of state contingent real gross returns on deposits which are denoted as follows. $\rho^d_{d_t}$ is the return delivered to domestic depositors who are relocated domestically, while $\rho^d_{f_t}$ is the real gross return paid to domestic depositors who are relocated abroad. ρ^d_t is the real return guaranteed to domestic depositors who are not relocated. Deposit returns are, of course, constrained by the bank's portfolio composition and the returns on assets that it faces. These constraints are manifested in the following way.

A representative (domestic) bank knows that at each date, t, a fraction π_d^d of its depositors will be relocated domestically. The bank guarantees each such depositor a a per unit real gross return of $\rho_{d_t}^d$ and each of these depositors has deposited $w^d(k_t^d)$. Thus the bank's total per depositor obligation to such individuals is $\pi_d^d \rho_{d_t}^d w^d(k_t^d)$ at date t. Domestically relocated depositors must be given domestic currency to accomplish their transactions and so are paid using the bank's entire holdings of domestic real balances. These balances are then carried into t+1, earning a gross real return of (p_t^d/p_{t+1}^d) between periods t and t+1. The bank therefore faces the budget constraint

$$\pi_d^d \rho_{d_t}^d w^d(k_t^d) \leq m_{d_t}^d (p_t^d/p_{t+1}^d); t \geq 1. \tag{4}$$

It is also true that π_f^d of a representative bank's depositors will be relocated to the foreign country. The bank has promised each such depositor a gross real return of $\rho_{f_t}^d$ for each of the $w^d(k_t^d)$ units deposited. These agents must be paid in foreign currency to accomplish their t+1 transactions and the bank exhausts its holdings of foreign real balances, $m_{d_t}^f$, to meet this obligation. These holdings have a domestic goods value of $x_t m_{d_t}^f$ and are carried into t+1 earning a gross real return of $(e_{t+1}/p_{t+1}^d)(p_t^d/e_t)$. Thus

$$\pi_f^d \rho_{f_t}^d w^d(k_t^d) \leq m_{d_t}^f (e_{t+1}/p_{t+1}^d)(p_t^d/e_t); t \geq 1, \tag{5}$$

must hold.

For domestic agents who are not relocated at t - who comprise a fraction $(1 - \pi_d^d - \pi_f^d)$ of a representative domestic bank's depositors - the guaranteed total real per depositor obligation is $(1 - \pi_d^d - \pi_f^d)\rho_t^d w^d(k_t^d)$. This is financed from the bank's earnings on domestic and foreign bonds and investments in capital. Domestic bonds earn a real gross return of $R_{d_t}^d$ per unit invested, while foreign bonds earn a real gross return of domestic goods unit value $R_{d_t}^f = I_t^f (e_{t+1}/p_{t+1}^d)(p_t^d/e_t)$. Domestic investment in capital goods yields a domestic unit return of $r_{t+1}^d = f^{d'}(k_{t+1}^d)$ per unit withdrawn from current consumption. Foreign investment yields a real return in domestic goods units of $r_{t+1}^f (x_{t+1}/x_t) = f^{f'}(k_{t+1}^f)(x_{t+1}/x_t)$ per unit invested.[5] Consequently, the return constraint relevant to the bank's choice of ρ_t^d is, for t $\geq$1,

$$(1 - \pi_d^d - \pi_f^d)\rho_t^d w^d(k_t^d) \leq b_{d_t}^d R_{d_t}^d + x_t b_{d_t}^f R_{d_t}^f + i_{d_t}^d f^{d'}(k_{t+1}^d) + x_t i_{d_t}^f f^{f'}(k_{t+1}^f)(x_{t+1}/x_t). \tag{6}$$

Competition between banks for the deposits of young agents implies that, in a Nash equilibrium, desposit return shedules and bank portfolio allocations must be chosen to maximize the expected utility of a representative depositor subject to constraints (3)-(6) and non-negativity. In other

[5]At date t, a domestic agent must give up $e_t p_t^f / p_t^d = x_t$ units of domestic goods to obtain a single unit of the foreign investment good. At date t+1, the associated foreign goods value of the return per unit of foreign investment goods held - the t+1 foreign rental rate - is $f^{f'}(k_{t+1}^f)$ which has a domestic goods value at that date of $f^{f'}(k_{t+1}^f)x_{t+1}$.

words, domestic banks will choose $\rho^d_{d_t}, \rho^d_{f_t}, \rho^d_t, m^d_{d_t}, m^f_{d_t}, b^d_{d_t}, b^f_{d_t}, i^d_{d_t}, i^f_{d_t}$ $\forall t \geq 1$ to

(P1) maximize $\pi^d_d ln(\pi^d_d \rho^d_{d_t} w^d(k^d_t)) + \pi^d_f ln(\pi^d_f \rho^d_{f_t} w^d(k^d_t)) + (1-\pi^d_d-\pi^d_f) ln((1-\pi^d_d-\pi^d_f)\rho^d_t w^d(k^d_t))$;

This problem can be transformed as follows.

Let $\gamma^d_{d_t}(\gamma^d_{f_t})$ denote the domestic representative bank's ratio of domestic (foreign) currency to deposits at t; that is

$$\gamma^d_{d_t} \equiv (m^d_{d_t}/w^d(k^d_t)), \quad (7a)$$

$$\gamma^d_{f_t} \equiv (x_t m^f_{d_t}/w^d(k^d_t)) \quad (7b).$$

Similarly, let

$$\beta^d_{d_t} \equiv (b^d_{d_t}/w^d(k^d_t)), \quad (7c)$$

$$\beta^d_{f_t} \equiv (x_t b^f_{d_t}/w^d(k^d_t)), \quad (7d)$$

$$\delta^d_{d_t} \equiv (i^d_{d_t}/w^d(k^d_t)), \quad (7e)$$

and

$$(1-\gamma^d_{d_t}-\gamma^d_{f_t}-\beta^d_{d_t}-\beta^d_{f_t}-\delta^d_{d_t}) \equiv (x_t i^f_{d_t}/w^d(k^d_t)) \quad (7f)$$

The budget constraints (4)-(6) can now be written as

$$\rho^d_{d_t} \leq \frac{\gamma^d_{d_t}(p^d_t/p^d_{t+1})}{\pi^d_d}, \quad (8)$$

$$\rho^d_{f_t} \leq \frac{\gamma^d_{f_t}(e_{t+1}/p^d_{t+1})(p^d_t/e_t)}{\pi^d_f}, \quad (9)$$

$$\rho^d_t \leq \frac{\beta^d_{d_t}R^d_{d_t} + \beta^d_{f_t}R^d_{f_t} + \delta^d_{d_t}f^{d'}(k^d_{t+1}) + (1-\gamma^d_{d_t}-\gamma^d_{f_t}-\beta^d_{d_t}-\beta^d_{f_t}-\delta^d_{d_t})f^{f'}(k^f_{t+1})(\frac{x_{t+1}}{x_t})}{(1-\pi^d_d-\pi^d_f)}, \quad (10)$$

which must each hold $\forall t \geq 1$. A representative domestic bank therefore seeks to solve the problem

(P1') max $\pi^d_d ln(\rho^d_{d_t} w^d(k^d_t)) + \pi^d_f ln(\rho^d_{f_t} w^d(k^d_t)) + (1-\pi^d_d-\pi^d_f) ln(\rho^d_t w^d(k^d_t))$

subject to (8)-(10) and non-negativity, by choice of $\gamma^d_{d_t}, \gamma^d_{f_t}, \beta^d_{d_t}, \beta^d_{f_t}$, and $\delta^d_{d_t}$.

The solution to this problem sets

$$\gamma^d_{d_t} = \pi^d_d; t \geq 1, \quad (11a)$$

$$\gamma^d_{f_t} = \pi^d_f; t \geq 1, \quad (11b)$$

and

$$\beta^d_{d_t} + \beta^d_{f_t} + \delta^d_{d_t} + \delta^d_{f_t} = (1-\pi^d_d-\pi^d_f); t \geq 1, \quad (11c)$$

where (11c) derives from the absence of arbitrage opportunities in asset markets requiring that

$$f^{d'}(k^d_{t+1}) = R^d_{d_t} = R^f_{d_t} x_{t+1}/x_t = f^{f'}(k^f_{t+1}) x_{t+1}/x_t, \tag{12}$$

$\forall t \geq 1$. Notably, the second equality implies that uncovered interest rate parity holds, or $R^d_{d_t}(p^d_{t+1}/p^d_t) = I^d_t = I^f_t(e_{t+1}/e_t)$. As a result, banks are indifferent between the four non-currency assets and only $\gamma^d_{d_t}$ and $\gamma^d_{f_t}$ are determinate. In fact, (10) can be re-written as

$$\rho^d_t \leq (1 - \gamma^d_{d_t} - \gamma^d_{f_t}) f^{d'}(k^d_{t+1})/(1 - \pi^d_d - \pi^d_f); t \geq 1. \tag{13}$$

The problem of banks located in the foreign country is entirely symmetric and so will not be presented here. Competition among foreign banks forces them - in a Nash equilibrium - to choose return schedules and (equivalently) portfolio weights to maximize the expected utility of a young foreign agent. The solution to this problem sets

$$\gamma^f_{f_t} = \pi^f_f; t \geq 1, \tag{14a}$$

$$\gamma^f_{d_t} = \pi^f_d; t \geq 1, \tag{14b}$$

and

$$\beta^f_{f_t} + \beta^f_{d_t} + \delta^f_{f_t} + \delta^f_{d_t} = (1 - \pi^f_f - \pi^f_d); t \geq 1. \tag{14c}$$

3 General Equilibrium

In this section, the determination of general equilibrium for this economy is described. Central to the determination of this equilibrium is a complete description of the behaviour of the government of each country, which now follows.

3.1 Government Activity

At each date, $t \geq 1$, the domestic (foreign) country government has an outstanding money stock of M^d_t (M^f_t) per capita held by the public and an outstanding stock of nominal debt of B^d_t (B^f_t) per capita. The government of each country neither levies taxes nor directly consumes goods, but at each date must generate enough seignorage revenue to service its debt.

The quantities of money held by the initial old generation are exogenously given by $M^d_0 > 0$ and $M^f_0 > 0$ in the domestic and foreign country respectively, while initial bond stocks, $B^d_0 = 0$ and $B^f_0 = 0$. Thus the initial old generation in each country has no bond interest income.

The domestic government therefore faces the budget constraint for $t \geq 2$;

$$B^d_{t-1} I^d_{t-1} = M^d_t - M^d_{t-1} + B^d_t \tag{15}$$

while the foreign government faces an analagous constraint for $t \geq 2$;

$$B^f_{t-1} I^f_{t-1} = M^f_t - M^f_{t-1} + B^f_t \tag{16}$$

These constraints can be re-expressed in the following form:

$$b^d_{t-1} R^d_{d_{t-1}} = m^d_t - m^d_t (p^d_{t-1}/p^d_t) + b^d_t; t \geq 2, \tag{15'}$$

$$b^f_{t-1} R^f_{f_{t-1}} = m^f_t - m^f_t (p^f_{t-1}/p^f_t) + b^f_t; t \geq 2, \tag{16'}$$

while at date t=1, the following must be satisfied:

$$M^d_0 = M^d_1 + B^d_1, \tag{17}$$

$$M^f_0 = M^f_1 + B^f_1. \tag{18}$$

In this context, the following monetary policy choice is considered. At each date, $t \geq 1$, the domestic and foreign country governments conduct monetary policy by choosing (for once and for all) a constant money growth rate. That is,

$$M^d_{t+1}/M^d_t = \sigma^d; t \geq 1,$$

$$M^f_{t+1}/M^f_t = \sigma^f; t \geq 1,$$

where $\sigma^d > 1[\sigma^f > 1]$ are given.

Given this specification of government policies, using the arbitrage condition (12) and the fact that, by definition, $p^d_{t-1}/p^d_t = (m^d_t/m^d_{t-1})(M^d_{t-1}/M^d_t) = (m^d_t/m^d_{t-1})(1/\sigma^d)$ $[p^f_{t-1}/p^f_t = (m^f_t/m^f_{t-1})(M^f_{t-1}/M^f_t) = (m^f_t/m^f_{t-1})(1/\sigma^f)]$, then the following constraints must hold $\forall t \geq 2$;

$$b^d_{t-1} f^{d'}(k^d_t) = m^d_t(\sigma^d - 1)/\sigma^d + b^d_t, \tag{19}$$

$$b^f_{t-1} f^d(k^d_t)(x_{t-1}/x_t) = m^f_t(\sigma^f - 1)/\sigma^f + b^f_t, \tag{20}$$

while (17) and (18) continue to describe the government's behaviour at t=1.

3.2 Asset Market Clearing

There are three independent asset market clearing conditions for this economy. First, the per capita demand for domestic (foreign) real balances by domestic and foreign banks must equal the per capita supply of domestic (foreign) real balances at each date t. The per capita demand for domestic (foreign) real balances at t is just $m^d_{d_t} + m^d_{f_t} = \pi^d_d w^d(k^d_t) + \pi^f_d w^f(k^f_t)x_t$ $[m^f_{f_t} + m^f_{d_t} = \pi^f_f w^f(k^f_t) + \pi^d_f w^d(k^d_t)/x_t)]$ measured in domestic (foreign) goods at t. Thus, money market clearing requires that;

$$m^d_t = \pi^d_d w^d(k^d_t) + \pi^f_d w^f(k^f_t)x_t \tag{21}$$

$$m^f_t = \pi^f_f w^f(k^f_t) + \pi^d_f w^d(k^d_t)/x_t. \tag{22}$$

In order for the international non-currency asset market to clear, (recalling that in equilibrium banks are indifferent between the bonds and the capital of the two countries), the absence of arbitrage condition (12) must hold as well a world market clearing condition. The total per capita supply of bonds and capital investment goods must equal the world per capita demand for bonds and capital goods by the banks of both countries at each date t. The former quantity is just $b_t^d + k_{t+1}^d + x_t(b_t^f + k_{t+1}^f)$ - measured here in domestic goods at date t - while the latter is $(1-\pi_d^d - \pi_f^d)w^d(k_t^d) + (1-\pi_f^f - \pi_d^f)w^f(k_t^f)x_t$. Thus non-currency asset markets clear when

$$b_t^d + k_{t+1}^d + x_t(b_t^f + k_{t+1}^f) = (1-\pi_d^d - \pi_f^d)w^d(k_t^d) + (1-\pi_f^f - \pi_d^f)w^f(k_t^f)x_t; t \geq 1. \tag{23}$$

(21)-(23) together imply that a total asset market clearing condition also will hold in equilibrium;

$$(m_t^d + b_t^d + k_{t+1}^d) + x_t(m_t^f + b_t^f + k_{t+1}^f) = w^d(k_t^d) + x_t w^f(k_t^f); t \geq 1. \tag{24}$$

Finally, two additional asset market conditions must hold and prove very useful in the subsequent analysis. For the domestic country, the supply of investment goods by producers at t - which become date t+1 capital goods - must equal the demand for such investments by the banks of both countries at t. Then, denoting the realized domestic capital investments of the domestic (foreign) bank as $k_{d_{t+1}}^d = i_{d_t}^d$ $[k_{f_{t+1}}^d = i_{d_t}^f]$,

$$\begin{aligned} k_{t+1}^d &= k_{d_{t+1}}^d + k_{f_{t+1}}^d \\ &= w^d(k_t^d) - m_{d_t}^d - m_{d_t}^f x_t - b_{d_t}^d - b_{d_t}^f x_t - k_{d_{t+1}}^f x_t + k_{f_{t+1}}^d, t \geq 1; \end{aligned} \tag{25}$$

must be satisfied in asset market equilibrium. Symmetrically, the following foreign capital market clearing condition must hold;

$$\begin{aligned} k_{t+1}^f &= k_{f_{t+1}}^f + k_{d_{t+1}}^f \\ &= w^f(k_t^f) - m_{f_t}^f - m_{f_t}^d / x_t - b_{f_t}^f - b_{f_t}^d / x_t - k_{f_{t+1}}^d / x_t + k_{d_{t+1}}^f, t \geq 1. \end{aligned} \tag{26}$$

(25) and (26) have an equivalent representation. Specifically, (25) can be re-expressed as

$$\begin{aligned} k_{t+1}^d &= w^d(k_t^d)(1-\pi_d^d - \pi_f^d) - b_{d_t}^d + (b_{f_t}^d + k_{f_{t+1}}^d) - x_t(b_{d_t}^f + k_{d_{t+1}}^f) \\ &= w^d(k_t^d)(1-\pi_d^d - \pi_f^d) - b_t^d + KA_t^d, t \geq 1; \end{aligned} \tag{27}$$

where KA_t^d denotes the net foreign borrowing position of the domestic country - the "capital account". Analagously, (26) can be rewritten as

$$\begin{aligned} k_{t+1}^f &= w^f(k_t^f)(1-\pi_f^f - \pi_d^f) - b_t^f + (b_{d_t}^f + k_{d_{t+1}}^f) - (b_{f_t}^d + k_{f_{t+1}}^d)/x_t \\ &= w^f(k_t^f)(1-\pi_f^f - \pi_d^f) - b_t^f - KA_t^d / x_t, t \geq 1; \end{aligned} \tag{28}$$

where I denote $-KA_t^d/x_t$ by KA_t^f - the capital account of the foreign country expressed in units of foreign country date t goods. These representations of capital market clearing have a standard interpretation. They state that the supply of interest earning assets at t, by the government and date t producers in any location, must equal the demand for those assets by domestic (foreign) banks in that location on behalf of date t young agents, plus the net demand for domestic assets from abroad; net foreign borrowing. They show how private capital and government bonds compete in the asset portfolios of agents while external financing for either country's government debt can substitute for domestic savings.

3.3 Goods Markets

In order for the domestic (foreign) goods market to clear at t, it is necessary that the per capita supply of goods, $f^d(k_t^f)$ $[f^f(k_t^f)]$, equal the per capita demand for goods in any location. Per capita goods demand in a domestic (foreign) country location at t is simply the income of domestic (foreign) residents who were not relocated at t-1, plus the value of real balances carried by agents who were relocated to that location at t-1, plus domestic (foreign) investment by young agents in capital goods at t. In short, it equals consumption spending by the old plus investment spending by the young.

The purchasing power of old agents who were not relocated at t-1 is simply the interest and rental income generated by their intermediated holdings of bonds and capital investments which, in the domestic (foreign) country is $f^{d'}(k_t^d)w^d(k_{t-1}^d)(1-\pi_d^d-\pi_f^d)$ $[f^{f'}(k_t^f)w^f(k_{t-1}^f)(1-\pi_f^f-\pi_d^f)]$ at t. For domestic (foreign) agents who were relocated at t-1, purchasing power derives from their holdings of the entire t-1 stock of domestic (foreign) real balances - m_{t-1}^d $[m_{t-1}^f]$ - which at t has a goods value of $m_{t-1}^d(p_{t-1}^d/p_t^d)$ $[m_{t-1}^f(p_{t-1}^f/p_t^f)$. Thus, the domestic goods market clears at t when

$$\begin{aligned} f^d(k_t^d) &= m_{t-1}^d(p_{t-1}^d/p_t^d) + f^{d'}(k_t^d)w^d(k_{t-1}^d)(1-\pi_d^d-\pi_f^d) + k_{t+1}^d; t \geq 2, \\ &= m_t^d/\sigma^d + f^{d'}(k_t^d)w^d(k_{t-1}^d)(1-\pi_d^d-\pi_f^d) + k_{t+1}^d; t \geq 2, \end{aligned} \tag{29}$$

while the foreign goods market clears at t when

$$\begin{aligned} f^f(k_t^f) &= m_{t-1}^f(p_{t-1}^f/p_t^f) + f^{f'}(k_t^f)w^f(k_{t-1}^f)(1-\pi_f^f-\pi_d^f) + k_{t+1}^f; t \geq 2, \\ &= m_t^f/\sigma^f + f^{f'}(k_t^f)w^f(k_{t-1}^f)(1-\pi_f^f-\pi_d^f) + k_{t+1}^f; t \geq 2. \end{aligned} \tag{30}$$

For $t \geq 2$, (21)-(24), (27) and (28), (29) and (30), the no arbitrage condition (12), and the two government budget constraints (19) and (20) constitute the entire set of equilibrium consitions for this economy. While Walras' Law tells us that only 9 of these are linearly independent, all 11 are displayed here since each will be useful at some point in the analysis.

3.4 The Initial Period

The asset market clearing and no arbitrage conditions take the same form at date 1 as those relevant at all other dates. This is not true, however, for the government budget constraints which are given, at t=1, by (17) and (18). Goods market equilibrium in each country also requires that unique, date 1 market clearing conditions must be satisfied. While the initial capital stocks k_1^d (k_1^f) and initial money stocks - M_0^d (M_0^f) inherited by the initial old domestic (foreign) generation are positive, inherited nominal bond holdings - B_0^d (B_0^f) - are zero. The intial old agents in each country therefore rent the initial capital stock to the production process in their location, and give up their endowed currency holdings, in exchange for output produced in that location. Thus, goods market clearing conditions take the form at date 1;

$$f^d(k_1^d) = M_0^d/p_1^d + f^{d'}(k_1^d)k_1^d + k_2^d, \tag{31}$$

$$f^f(k_1^f) = M_0^f/p_1^f + f^{f'}(k_1^f)k_1^f + k_2^f. \tag{32}$$

4 Steady State Equilibria

This section presents a characterization of steady state equilibria in which the values of all real variables and relative prices are constant at all dates. In particular, in these equilibria $x_t = x, k_t^d = k^d > 0$, and $k_t^f = k^f > 0 \forall t$. In addition, valid equilibria are characterized by x>0 and $I^d > 1$ [$I^f > 1$].

From the no arbitrage condition (12), it is immediate that in such equilibria $R_d^d = f^{d'}(k^d) = f^{f'}(k^f) = R_f^f$. Consequently, the capital stocks of the two countries are related by $k^f = f^{f'^{-1}}(f^{d'}(k^d)) = g(k^d)$. From the government budget constraints, it is apparent that in a steady state equilibrium

$$b^d[f^{d'}(k^d) - 1] = m^d[(\sigma^d - 1)/\sigma^d] \tag{19'}$$

$$b^f[f^{d'}(k^d) - 1] = m^f[(\sigma^f - 1)/\sigma^f]. \tag{20'}$$

Money market clearing in a steady state equilibrium requires that

$$m^d = \pi_d^d w^d(k^d) + \pi_d^f w^f(k^f)x \tag{21'}$$

$$m^f = \pi_f^f w^f(k^f) + \pi_f^d w^f(k^f)/x \tag{22'}$$

Similarly, for non-currency asset markets to clear in any steady state equilibrium, the following world asset market condition must hold:

$$b^d + k^d + x(b^f + k^f) = (1 - \pi_d^d - \pi_f^d)w^d(k^d) + (1 - \pi_f^f - \pi_d^f)w^f(k^f)x \tag{23'}$$

Finally, for goods market clearing in a steady state equilibrium

$$
\begin{aligned}
f^d(k^d) &= (m^d/\sigma^d) + f^{d'}(k^d)w^d(k^d)(1-\pi_d^d-\pi_f^d) + k^d \\
&= \left[\frac{\pi_d^d w^d(k^d) + \pi_d^f w^f(k^f)x}{\sigma^d}\right] + f^{d'}(k^d)w^d(k^d)(1-\pi_d^d-\pi_f^d) + k^d
\end{aligned}
\tag{29'}
$$

holds in the domestic country, while

$$
\begin{aligned}
f^f(k^f) &= (m^f/\sigma^f) + f^{d'}(k^d)w^f(k^f)(1-\pi_f^f-\pi_d^f) + k^f \\
&= \left[\frac{\pi_f^f w^f(k^f) + \pi_f^d w^d(k^d)/x}{\sigma^f}\right] + f^{d'}(k^d)w^f(k^f)(1-\pi_f^f-\pi_d^f) + k^f
\end{aligned}
\tag{30'}
$$

obtains in the foreign country.

Since $k^f = g(k^d)$, the two goods market clearing conditions jointly determine the steady state values of x and k^d, so that the remaining steady state values can be solved for recursively. The money market clearing conditions determine steady state values of m^d and m^f, and the steady state values of b^d and b^f required to achieve asset market clearing are given by the government budget constraints. In addition, once k^d, k^f, b^d and b^f are determined, the equilibrium value of domestic net capital flows KA^d is determined immediately.

It is straightforward to verify that there exist multiple solutions to (29') and (30'), and that there are multiple steady state equilibria with the values of both currencies bounded away from zero ($\infty > x > 0$). This remains true when the special case of Cobb-Douglas production is considered, a case that is particularly simple to analyze and admits more precise statements regarding the quantitative effects of monetary policy and financial market regulatory changes. I therefore assume below that both countries have access to a simple Cobb-Douglas production technology, although most of the results presented generalize.

4.1 Cobb-Douglas Production

Let $f^d(k_t^d) = A^d k_t^{d^\alpha}$ and $f^f(k_t^f) = A^f k_t^{f^\alpha}$,where $A^d(A^f)$ is the domestic (foreign) country's total factor productivity parameter with $A^d \neq A^f$, and $\alpha < 1$ denotes capital's share of total income which is common to the two countries. It is easily verified that these production functions satisfy all of the conditions that were imposed on production technologies in Section 2.2, and they admit a simple differentiation of the two countries' exogenously given productive potential and 'level of development'.

Since, in a steady state equilibrium, $f^{d'}(k^d) = \alpha A^d k^{d(\alpha-1)} = f^{f'}(k^f) = \alpha A^f k^{f(\alpha-1)}$, then

$$
\frac{k^f}{k^d} = \left(\frac{A^f}{A^d}\right)^{(\frac{1}{1-\alpha})}
\tag{33}
$$

$$\frac{w^f(k^f)}{w^d(k^d)} = \frac{(1-\alpha)A^f k^{f^\alpha}}{(1-\alpha)A^d k^{d^\alpha}} = \left(\frac{A^f}{A^d}\right)^{(\frac{1}{1-\alpha})} \tag{34}$$

$$\frac{\Omega^d(k^d)}{\Omega^f(k^f)} = \frac{k^d/w^d(k^d)}{k^f/w^f(k^f)} = (\frac{k^d}{k^f})(\frac{w^f}{w^d}) = 1. \tag{35}$$

Some algebraic manipulation then yields the following goods market clearing conditions - which are the analogues of (29') and (30'):

$$\frac{1}{(1-\alpha)} - \frac{k^{d^{1-\alpha}}}{(1-\alpha)A^d} = \frac{\pi_d^d + \pi_d^f x(\frac{A^f}{A^d})^{(\frac{1}{1-\alpha})}}{\sigma^d} + \alpha A^d k^{d^{\alpha-1}}(1 - \pi_d^d - \pi_f^d) \tag{36}$$

$$\frac{1}{(1-\alpha)} - \frac{k^{d^{1-\alpha}}}{(1-\alpha)A^d} = \frac{\pi_f^f + (\pi_f^d/x)(\frac{A^d}{A^f})^{(\frac{1}{1-\alpha})}}{\sigma^f} + \alpha A^d k^{d^{\alpha-1}}(1 - \pi_f^f - \pi_d^f). \tag{37}$$

These two equations define two loci in (x,k^d) space which determine the steady state values of these two variables. It is evident that, for either market, there are two positive solutions for k^d which will clear that market for any given x. The steady state value of x that clears the domestic goods market at any given capital-labour ratio is

$$x = \left[\frac{\sigma^d(\frac{A^d}{A^f})^{(\frac{1}{1-\alpha})}}{\pi_d^f}\right]\left[\frac{1}{(1-\alpha)} - \frac{k^{d^{1-\alpha}}}{(1-\alpha)A^d} - \frac{\pi_d^d}{\sigma^d} - \alpha A^d k^{d^{\alpha-1}}(1 - \pi_d^d - \pi_f^d)\right], \tag{36'}$$

while that which clears the foreign goods market is

$$x = \left[\frac{\pi_f^d(\frac{A^d}{A^f})^{(\frac{1}{1-\alpha})}}{\sigma^f}\right]\left[\frac{1}{\frac{1}{(1-\alpha)} - \frac{k^{d^{1-\alpha}}}{(1-\alpha)A^d} - \frac{\pi_f^f}{\sigma^f} - \alpha A^d k^{d^{\alpha-1}}(1 - \pi_f^f - \pi_d^f)}\right]. \tag{37'}$$

There are then at most four equilibrium capital-labour ratios.

In addition, from (36) and (37), in any steady state equilibrium

$$\left[\frac{\pi_d^d + \pi_d^f x(\frac{A^f}{A^d})^{(\frac{1}{1-\alpha})}}{\sigma^d}\right] - \left[\frac{\pi_f^f + (\pi_f^d/x)(\frac{A^d}{A^f})^{(\frac{1}{1-\alpha})}}{\sigma^f}\right] = \alpha A^d k^{d^{\alpha-1}}(\pi_d^d + \pi_f^d - \pi_f^f - \pi_d^f) \tag{38}$$

must hold. Thus for any valid equilibrium capital-labour ratio, there exists a unique positive solution for the real exchange rate.

4.2 Characterization of Steady State Equilibria

The slope of the locus defined by (36') is simply given by

$$dx/dk^d = \left[\frac{\sigma^d(\frac{A^d}{A^f})^{(\frac{1}{1-\alpha})}}{\pi_d^f}\right]\left[(1-\alpha)\alpha A^d k^{d^{\alpha-2}}(1 - \pi_d^d - \pi_f^d) - \frac{k^{d^{-\alpha}}}{A^d}\right]. \tag{39}$$

Evidently, then, $dx/dk^d\mid_{36'} \gtreqless 0$ as $k^d \lesseqgtr [(1-\alpha)\alpha A^{d^2}(1 - \pi_d^d - \pi_f^d)]^{\frac{1}{2(1-\alpha)}}$.

Hence, (36') defines a single peaked locus in (x, k^d) space. Furthermore, as $k^d \downarrow 0$, $dx/dk^d \uparrow \infty$, while as $k^d \uparrow \infty$, $dx/dk^d \uparrow -c$.

The location of this locus is determined in part by the capital-labour ratio defined by (38). In addition, the following argument defines the range of (36') over which valid steady state equilibria may obtain. From (36'), it is evident that x > 0 as

$$\frac{1}{1-\alpha} > \pi_d^d/\sigma^d + \frac{k^{d(1-\alpha)}}{A^d(1-\alpha)} + \alpha A^d k^{d^{\alpha-1}}(1-\pi_d^d-\pi_f^d), \tag{40}$$

and when x=0, k^d can take one of the two, positive solutions of

$$k^{d(1-\alpha)} = A^d - \pi_d^d(1-\alpha)A^d/\sigma^d - [\alpha(1-\alpha)A^{d^2}(1-\pi_d^d-\pi_f^d)]k^{d(\alpha-1)}. \tag{41}$$

Thus, for other parameters given, x> 0 for capital-labour ratios that are neither too high nor too low. In fact, as $k^d \downarrow 0$, x $\downarrow -\infty$ while as $k^d \uparrow \infty$, x $\downarrow$ -c.

Consequently, the steady state domestic goods market clearing condition can be represented as locus (36') in Figure 2a.

A symmetric analysis of the foreign goods market clearing condition (37') yields the following characterization. Here, x> (<)0 as

$$\frac{1}{1-\alpha} > (<)\pi_f^f/\sigma^f + \frac{k^{d(1-\alpha)}}{A^d(1-\alpha)} + \alpha A^d k^{d^{\alpha-1}}(1-\pi_f^f-\pi_d^f), \tag{42}$$

while when x $\uparrow \infty$, k^d can approach either of the two positive solutions of

$$k^{d(1-\alpha)} = A^d - \pi_f^f(1-\alpha)A^d/\sigma^f - [\alpha(1-\alpha)A^{d^2}(1-\pi_f^f-\pi_d^f)]k^{d^{\alpha-1}}. \tag{43}$$

It is also evident from (37') that x $\uparrow 0$ as $k^d \downarrow 0$ and as $k^d \uparrow \infty$.

The slope of (37') is given by

$$dx/dk^d = \left[\frac{x^2\sigma^f}{\pi_f^d(\frac{A^d}{A^f})^{\frac{1}{(1-\alpha)}}}\right]\left[\frac{k^{d^{-\alpha}}}{A^d} - (1-\alpha)\alpha A^d k^{d^{\alpha-2}}(1-\pi_f^f-\pi_d^f)\right]. \tag{44}$$

so that $dx/dk^d \mid_{37'} \gtreqless 0$ as $k^d \gtreqless [(1-\alpha)\alpha A^{d^2}(1-\pi_f^f-\pi_d^f)]^{\frac{1}{2(1-\alpha)}}$. Thus, (37') is U-shaped around a single minimum value with asymptotes defined by (43). In addition, it is straightforward to verify that at $k^d = \left[(1-\alpha)\alpha A^{d^2}(1-\pi_f^f-\pi_d^f)\right]^{\frac{1}{2(1-\alpha)}}$, $x \mid 37' > 0$ (see Appendix 1). When x< 0 at sufficiently low and high values of k^d, (37') approaches x=0 from below. In addition, as $k^d \downarrow 0$, $dx/dk^d \uparrow 0$ while when $k^d \uparrow \infty$, $dx/dk^d \uparrow 0$. The foreign goods market clearing condition is depicted as locus (37') in Figure 2b.

Since the domestic goods market clearing locus is centered at the single peak $k^d = [(1-\alpha)\alpha A^{d^2}(1-\pi_d^d-\pi_f^d)]^{\frac{1}{2(1-\alpha)}}$ and the foreign locus above the unique value $k^d = [(1-\alpha)\alpha A^{d^2}(1-\pi_f^f-$

$\pi_d^f)]^{\frac{1}{2(1-\alpha)}}$, there can be *at most* two steady state equilibria with $x > 0$ and $k^d > 0$. The existence of at least one valid steady state equilibrium with x>0 is guaranteed by the following proposition.

Proposition 1.

There exist two steady state equilibria with $x > 0$ and $k^d > 0$. At one of these $f^{d'}(k^d) > 1$ and at the other $f^{d'}(k^d) < 1$.

Proof.

When the domestic capital stock equals its golden rule level, $k^{d^{gr}} = (\alpha A^d)^{(\frac{1}{1-\alpha})}$, such that $\alpha A^d k^{d^{(\alpha-1)}} = 1$, then

$$x = \left[\frac{\sigma^d (A^d/A^f)^{(\frac{1}{1-\alpha})}}{\pi_d^f}\right]\left[\pi_d^d(1 - 1/\sigma^d) + \pi_f^d\right]. \tag{45}$$

Foreign goods market clearing requires that

$$x = \left[\frac{\pi_d^f (A^d/A^f)^{(\frac{1}{1-\alpha})}}{\sigma^f}\right]\left[\frac{1}{\pi_f^f(1 - 1/\sigma^f) + \pi_d^f}\right]. \tag{46}$$

Clearly, $x \mid_{45} > x \mid_{46} > 0$, since

$$\left[\frac{\pi_d^d(\sigma^d - 1) + \pi_f^d}{\pi_d^f}\right] > \left[\frac{\pi_f^d}{\pi_f^f(\sigma^f - 1) + \sigma^f \pi_d^f}\right]. \tag{47}$$

Then there exist two intersections of (36') and (37') at which $x > 0$ and $k^d > 0$, one at which $f^{d'}(k^d) > 1$ and at the other $f^{d'}(k^d) < 1$.

Thus there exists at least one steady state equilibrium with $I^d > 1$ $[I^f > 1]$. At one of the two intersections of (36') and (37') with $x > 0$, $\underline{k}^d < k^{d^{gr}}$ and $f^{d'}(k^d) > 1 \Rightarrow I^d > 1$ and $I^f > 1$. At the second such intersection $\bar{k}^d > k^{d^{gr}}$ and $f^{d'}(k^d) < 1$, and this is a valid equilibrium for the economy iff $f^{d'}(\bar{k}^d) = \alpha A^d \bar{k}^{d^{\alpha-1}} > \max\,(1/\sigma^d, 1/\sigma^f)$.

There is a large class of economies for which $\bar{k}^d$ is a valid steady state equilibrium capital stock. This is verified by observing that to rule out the existence of a high capital stock steady state equilibrium requires that $(36') \mid f^{d'}(k^d) = max(1/\sigma^d, 1/\sigma^f) \geq (37') \mid f^{d'}(k^d) = max(1/\sigma^d, 1/\sigma^f) >$

0. Thus there exists a unique steady state equilibrium with $\min(I^d, I^f) > 1$ iff, for $\sigma^d < \sigma^f$:

$$\left[\frac{\sigma^d}{\pi_d^f}\right]\left[\frac{1-\alpha\sigma^d}{1-\alpha} - \frac{(1-\pi_f^d)}{\sigma^d}\right] \geq \left[\frac{\pi_f^d}{\sigma^f}\right]\left[\frac{1}{\frac{1-\alpha\sigma^d}{1-\alpha} + \pi_f^f\left[\frac{1}{\sigma^d} - \frac{1}{\sigma^f}\right] - \frac{(1-\pi_d^f)}{\sigma^d}}\right] > 0, \tag{48}$$

and for $\sigma^d > \sigma^f$:

$$\left[\frac{\sigma^d}{\pi_d^f}\right]\left[\frac{1-\alpha\sigma^f}{1-\alpha} - \frac{(1-\pi_f^d)}{\sigma^f} + \pi_d^d\left[\frac{1}{\sigma^f} - \frac{1}{\sigma^d}\right]\right] \geq \left[\frac{\pi_f^d}{\sigma^f}\right]\left[\frac{1}{\frac{1-\alpha\sigma^f}{1-\alpha} - \frac{(1-\pi_d^f)}{\sigma^f}}\right] > 0. \tag{49}$$

It is evident that a necessary condition for uniqueness is that the money growth rate be less than $1/\alpha$ in both countries. For any economy in which this does not hold, $\min(I^d, I^f) > 1$ at $\bar{k}^d$. Thus, for sufficiently high money growth rates there exist both a high capital stock and a low capital stock steady state equilibrium.

The slopes of the domestic and foreign goods market clearing loci at each of the two (potential) steady state equilibria are established by the following proposition.

Proposition 2.

a) At $\bar{k}^d$, $dx/d\bar{k}^d \mid_{36'} < 0$ and $dx/d\bar{k}^d \mid_{37'} > 0$.

b) At $\underline{k}^d$, $dx/d\underline{k}^d \mid_{36'} > 0$ and $dx/d\bar{k}^d \mid_{37'} < 0$.

Proof See Appendix 2.

The two goods market clearing loci and steady state equilibria are represented in Figure 3.

4.3 Steady State Welfare

The level of welfare in the domestic country at any steady state equilibrium is given by
$U^d = \pi_d^d ln\left[\frac{w^d(k^d)}{\sigma^d}\right] + \pi_f^d ln\left[\frac{w^d(k^d)}{\sigma^f}\right] + (1-\pi_d^d-\pi_f^d) ln\left[f^{d'}(k^d)w^d(k^d)\right]$.
Then evidently, $dU/dk^d \geq 0$ iff $\alpha \geq \frac{(1-\pi_d^d-\pi_f^d)}{(2-\pi_d^d-\pi_f^d)}$. The high capital stock equilibrium, though characterized by a low marginal product of capital may yield higher expected utility than the low capital stock equilibrium. This is because although interest income declines in k for any $\alpha < 0.5$, real wage income and hence the real income of currency holders is always increasing in k. When a sufficiently high fraction of the population derives consumption from real balances rather than from interest income, expected utility is increasing in k.

4.4 Remarks

The existence of multiple non-trivial steady state equilibria in closed economy monetary growth models in which the value of money is positive is quite rare. One exception is Schreft and Smith (1994) who present a single currency economy similar in some respects to the current model. In this two-currency economy, there can be multiple non-trivial steady state equilibria in which the value of both currencies is strictly positive. This potential multiplicity of equilibria reflects the endogeneity of each government's net liability position.

At $\underline{k}^d$, $f^{d'}(\underline{k}^d) > 1$, $I^d > \sigma^d$ $[I^f > \sigma^f]$, and each government has a positive net lending position $b^d > 0$ $[b^f > 0]$ as can be seen from inspecting the government budget constraints. In this case, the total value of each government's net liability position - $m^d + b^d$ $[m^f + b^f]$ - is strictly positive. Then, the world supply of bonds and money competes for available savings with world private capital. World government 'lending' tends to 'crowd out' private capital formation, and the steady state capital stock of each country is low at the high nominal interest rate equilibrium.

This effect can be mitigated somewhat for one of the two countries by borrowing from abroad to enhance private capital formation, as is evident from the savings-investment conditions (52) and (54). However, when one country is a net borrower in international capital markets, the other is a net lender; the allocation of "crowding out" of private capital across countries obviously depends on the value of net capital flows while the crowding out of world capital is independent of such conditions.

At $\bar{k}^d$, $I^d < \sigma^d$ $[I^f < \sigma^f]$, and each government is a net lender to the private sector, or $b^d < 0[b^f < 0]$. In this case, money creation provides for government financing of private capital formation in both countries, enabling the attainment of a higher capital stock at a low nominal interest rate equilibrium. Again, international capital flows can either enhance or somewhat mitigate this effect either country.

5 Monetary Policy and Financial Market Regulations

In the environment described above, the fundamental determinants of steady state capital stocks and the real exchange rate are the relocation probabilities of the two countries - the exogenously determined portfolio preferences, the relative values of their total factor productivity, and the value of each country's money growth rate. The consequences of changes in money growth rates, and of imposing foreign exchange controls and reserve requirements are as follows.

5.1 Effects of an increase in σ^d

An increase in the money growth rate of the domestic country shifts only the domestic goods market clearing locus, leaving unchanged the foreign goods market clearing condition. From (36'),
$(\partial x/\partial\sigma^d)(\sigma^d/x) = 1 + (\frac{\pi_d^d}{\pi_d^f})(\frac{A^d}{A^f})^{\frac{1}{1-\alpha}}(\frac{1}{x}) > 0.$
Thus the locus defined by (36') shifts up as illustrated in Figure 4a.

It is apparent from Figure 4a that while there is an unambiguous rise in the real exchange rate of the domestic country (a real depreciation of the domestic country's currency), the effects for the equilibrium capital stock depend on the initial steady state situation. If the two countries originate from the low capital stock equilibrium, the steady state consequence of raising the rate of domestic money growth is to *reduce* the domestic (and foreign) capital stock. Conversely, from an initially high capital stock equilibrium, the consequence of such a policy is to raise the capital stock further. The intuition for these results is as follows.

At either equilibrium, the rise in σ^d taxes domestic and foreign holders of domestic real balances and so reduces demand for domestic goods. This is offset by a rise in the domestic country's real exchange rate - which raises the purchasing power of foreign agents in the domestic economy who hold domestic currency units - and by a fall in the equilibrium capital stock (which induces a relatively large increase in bond and capital interest income) at the low capital stock equilibrium, or a rise in the equilibrium capital stock (which has a relatively large impact on investment demand) at the high capital stock equilibrium.

At $\underline{k}^d$, the fall in steady state capital stocks reflects aggravated "crowding out" of private investment in the domestic country and - in fact - in the world economy. The rise in σ^d permits an increase in domestic government borrowing, an opportunity that is enhanced by the increase in x which raises the real demand for domestic currency. The value of net domestic government liabilities therefore tends to rise and crowd out domestic capital formation. While foreign country policy is unchanged, the real exchange rate movement reduces the foreign goods value of foreign government net liabilities. In the new steady state equilibrium, therefore, the total value of foreign issued assets from both the private and public sector is lower. Effectively, foreign savers finance the domestic country's increased government indebtedness; foreign country lending to the domestic country unambiguously increases.

At $\bar{k}^d$, both governments are net lenders to the private sector. The increase in domestic money growth allows $b^d/w^d(k^d)$ to become *more* negative, permitting a higher rate of financing of domestic capital formation, and the equilibrium capital stocks of both countries rise. However, while the increase in the domestic country's capital stock is financed primarily by higher domestic country government lending, the foreign country's higher capital stock must be financed by higher net borrowing from abroad since the foreign goods value of foreign government liabilities declines at the new real ecxhange rate.[6]

5.2 The Effects of Foreign Exchange Controls

Consider now the consequences of a domestic country attempt to raise the external value of its currency by imposing (or tightening) a foreign exchange control that restricts the amount of foreign currency that its residents may hold. Such a policy may be represented by imposing a binding constraint on the optimization problem of domestic banks of the form:

$$\gamma^d_{f_t} \leq \bar{\gamma}^d_f \in (0, \pi^d_f); \forall t \geq 1.$$

This upper bound on foreign currency holdings means that domestic banks will maximize (P1') subject to (13)-(15), non-negativity and the exchange control.

The outcome of banks' optimization then sets

$$\begin{aligned}
\gamma^d_{f_t} &= \bar{\gamma}^d_f; \forall t \geq 1, \\
\gamma^d_{d_t} &= \pi^d_d \frac{(1-\bar{\gamma}^d_f)}{(1-\pi^d_f)} > \pi^d_d; \forall t \geq 1, \\
1 - \gamma^d_{d_t} - \gamma^d_{f_t} &= (1 - \pi^d_d - \pi^d_f)\frac{(1-\bar{\gamma}^d_f)}{(1-\pi^d_f)} > (1 - \pi^d_d - \pi^d_f); \forall t \geq 1.
\end{aligned}$$

Hence banks reduce the portfolio weight assigned to foreign currency according to the foreign exchange control and substitute into both interest-bearing assets and domestic currency.

The two steady state goods market clearing conditions are now:

$$x = \frac{\sigma^d(\frac{A^d}{A^f})^{(\frac{1}{1-\alpha})}}{\pi^f_d}\left[\frac{1}{(1-\alpha)} - \frac{k^{d^{1-\alpha}}}{(1-\alpha)A^d} - \frac{\pi^d_d(1-\bar{\gamma}^d_f)}{\sigma^d(1-\pi^d_f)} - \alpha A^d k^{d^{\alpha-1}}(1-\pi^d_d-\pi^d_f)\frac{(1-\bar{\gamma}^d_f)}{(1-\pi^d_f)}\right] \quad (50)$$

$$x = \frac{\bar{\gamma}^d_f(\frac{A^d}{A^f})^{(\frac{1}{1-\alpha})}}{\sigma^f}\left[\frac{1}{\frac{1}{(1-\alpha)} - \frac{k^{d^{1-\alpha}}}{(1-\alpha)A^d} - \frac{\pi^f_f}{\sigma^f} - \alpha A^d k^{d^{\alpha-1}}(1-\pi^f_f-\pi^f_d)}\right], \quad (51)$$

in the domestic and foreign country respectively. The effects of a tightening of the foreign exchange control - or of its imposition - for the steady state real exchange rate and capital stock(s) are as follows.

[6] The effects of an increase in σ^f are entirely symmetric to those presented in this section and is represented graphically in Figure 4b. In this case, the real exchange rate of the domestic country of course falls while the capital stock implications and their interpretation remain unchanged.

It is apparent that both the domestic and foreign goods markets are affected by a tightening of the foreign exchange control. In fact, the value of $\bar{\gamma}_f^d$ is a fundamental determinant of the steady state real exchange rate. From (50),

$$\partial x/\partial\bar{\gamma}_f^d = \frac{\sigma^d(\frac{A^d}{A^f})^{\frac{1}{1-\alpha}}}{\pi_d^f}\left[\frac{\pi_d^d}{(1-\pi_f^d)\sigma^d} + \frac{\alpha A^d k^{d^{\alpha-1}}(1-\pi_d^d-\pi_f^d)}{1-\pi_f^d}\right] > 0, \tag{52}$$

so that the domestic goods market locus shifts down when a foreign exchange control is implemented or tightened (a reduction in $\bar{\gamma}_f^d$. In addition, from the foreign goods market clearing condition,

$$\partial x/\partial\bar{\gamma}_f^d = x/\bar{\gamma}_f^d > 0, \tag{53}$$

such that the foreign goods market locus also shifts down.

These effects are illustrated in Figure 5. The real exchange rate of the domestic country unambiguously falls as a result of the imposition or tightening of an exchange control. Thus, countries can use such controls to influence their real exchange rates in a predictable way in this environment. In addition, foreign exchange controls introduce or aggravate any asymmetry in the location of the two goods market clearing loci, since the peak of the domestic locus shifts to the right. It can be easily verified that the conditions for there to exist two steady state equilibria continue to hold, however.

The effect for the steady state capital stock is not unambiguous, but depends on the size of σ^d. Totally differentiating (50) and (51), using Proposition 2, and some algebra yields the following result.

Proposition 3.

At $\underline{k}^d[\bar{k}^d]$, $(dk^d/d\bar{\gamma}_f^d)\gtrless 0$ iff

$$\left[\frac{\sigma^d\left(\frac{A^d}{A^f}\right)^{\frac{1}{(1-\alpha)}}}{\pi_d^f}\right]\left[\frac{\alpha A^d k^{d^{\alpha-1}}(1-\pi_d^d-\pi_f^d)}{1-\pi_f^d} + \frac{\pi_d^d}{\sigma^d(1-\pi_f^d)}\right] \lesseqgtr \left[\gtrless\right] 1$$

iff $$\frac{\pi_d^f\left(\frac{A^d}{A^f}\right)^{\frac{1}{(1-\alpha)}} - \frac{\pi_d^d}{1-\pi_f^d}}{1-\frac{\pi_d^d}{1-\pi_f^d}} \lesseqgtr \left[\gtrless\right] I^d.$$

For low capital stock economies, equilibrium capital formation can be enhanced by the imposition or tightening of a foreign exchange control $(d\underline{k}^d/d\bar{\gamma}_f^d) < 0$ when the nominal interest rate - effectively σ^d - is sufficiently high, while for high capital stock economies the converse is true. High inflation low captal stock countries can enhance their capital formation by imposing os tightening restrictions on the value of foreign exchange held by domestic residents.

The intuition for this result is as follows. When a foreign exchange control is imposed, banks substitute into domestic currency and interest bearing assets. This raises the real income derived from these sources and hence spending on domestic goods, which necessitates a fall in the real exchange rate (a real appreciation) of the domestic country and a rise (fall) in capital formation for domestic goods equilibration at the low (high) capital stock equilibrium to reduce spending relative to output. Spending in the foreign goods market is reduced by the foreign exchange control's imposition. Thus, for foreign goods market equilibration, a real appreciation of the domestic currency and a fall (rise) in capital formation is required at the low (high) capital stock steady state. Thus, the larger is the domestic relative to the foreign goods market locus shift, the greater is the tendancy for the capital stock to rise (fall) at the low (high) capital stock equilibrium and the larger is the real exchange rate appreciation.

Under what conditions will the capital stock rise at $\underline{k}^d$? When the domestic money growth rate and nominal interest rate are sufficiently high. High money growth rates represent a high tax on domestic currency holdings and reduce proportion of goods demand accounted for by currency. For any given capital stock, the real appreciation required to equilibrate the domestic market is therefore increasing in σ^d as shown in (52).

What is the mechanism for higher capital formation? Any real appreciation of the domestic currency reduces the domestic goods value of outstanding domestic currency and bonds, tending to offset the positive effect of the exchange control for the supplies of these assets. When the real exchange rate movement is large enough, the former effect dominates the latter, reducing the value of outstanding government liabilities and enabling higher equilibrium rates of capital formation.

Thus, the government of a capital poor country that conducts inflationary monetary policy can achieve a lower real exchange rate, higher capital stock, and substitute private capital for government debt in financial markets through the imposition of an exchange control.

5.3 The Effects of Reserve Requirements

Consider now the consequences of an attempt to raise the external value of a currency - or the inflation tax base - by imposing (or increasing) a binding reserve requirement on private banks. This policy may be represented by a binding constraint on the optimization problem of domestic banks of the form:

$$\gamma^d_{d_t} \geq \hat{\gamma}^d_d \in (0, \pi^d_d); \forall t \geq 1.$$

This lower bound on domestic currency holdings means that domestic banks maximize (P1') subject to (16)-(18), non-negativity and (97).

The outcome of banks' optimization then sets

$$
\begin{aligned}
\gamma^d_{d_t} &= \hat{\gamma}^d_d; \forall t \geq 1, \\
\gamma^d_{f_t} &= \pi^d_f \frac{(1-\hat{\gamma}^d_d)}{(1-\pi^d_d)} < \pi^d_f; \forall t \geq 1, \\
1-\gamma^d_{d_t}-\gamma^d_{f_t} &= (1-\pi^d_d-\pi^d_f)\frac{(1-\hat{\gamma}^d_d)}{(1-\pi^d_d)} < (1-\pi^d_d-\pi^d_f); \forall t \geq 1.
\end{aligned}
$$

Hence banks raise the portfolio weight assigned to domestic currency according to the reserve requirement and substitute out of both interest-bearing assets and foreign currency.

The two steady state goods market clearing conditions are now:

$$
x = \frac{\sigma^d(\frac{A^d}{A^f})^{(\frac{1}{1-\alpha})}}{\pi^f_d}\left[\frac{1}{(1-\alpha)} - \frac{k^{d^{1-\alpha}}}{(1-\alpha)A^d} - \frac{\hat{\gamma}^d_d}{\sigma^d} - \alpha A^d k^{d^{\alpha-1}}(1-\pi^d_d-\pi^d_f)\frac{(1-\gamma^d_d)}{(1-\pi^d_d)})\right], \tag{54}
$$

$$
x = \frac{\pi^d_f(1-\gamma^d_d)(\frac{A^d}{A^f})^{(\frac{1}{1-\alpha})}}{\sigma^f(1-\pi^d_d)}\left[\frac{1}{\frac{1}{(1-\alpha)} - \frac{k^{d^{1-\alpha}}}{(1-\alpha)A^d} - \frac{\pi^f_f}{\sigma^f} - \alpha A^d k^{d^{\alpha-1}}(1-\pi^f_f-\pi^f_d)}\right], \tag{55}
$$

in the domestic and foreign country respectively. The effects of an increase in the reserve requirement - or of its imposition - for the steady state real exchange rate and capital stock(s) are as follows.

It is apparent that both the domestic and foreign goods markets are affected by the reserve requirement. In fact, the value of $\hat{\gamma}^d_d$ is a fundamental determinant of the steady state real exchange rate. From (54),

$$
\partial x/\partial\hat{\gamma}^d_d = \frac{\sigma^d(\frac{A^d}{A^f})^{\frac{1}{1-\alpha}}}{\pi^f_d}\left[-\frac{1}{\sigma^d} + \frac{\alpha A^d k^{d^{\alpha-1}}(1-\pi^d_d-\pi^d_f)}{1-\pi^d_d}\right] \gtreqless 0, \tag{56}
$$

so that the domestic goods market locus can shift in either direction when a reserve requirment is implemented or raised. From (55),

$$
\partial x/\partial\hat{\gamma}^d_f = -x/(1-\hat{\gamma}^d_d) < 0, \tag{57}
$$

so the foreign goods market locus shifts down.

The consequences of a reserve requirement for the real exchange rate are therefore ambiguous, and depend on the relative impact of the reserve requirment for the real incomes of domestic currency holders and interest- earning asset holders. The requirement raises the income of domestic currency holders and so tends to reduce the real exchange rate for the domestic goods market to clear. However, it also reduces the real income of bond and capital owners since banks reduce their holdings of these assets which acts to raise the real exchange rate. The lower (higher) is the money growth rate, the lower (higher) is the tax on currency holders and the less of domestic output they account for. When a reserve requirement is combined with a very high rate of money growth it is possible that a real depreciation will result.[7]

The capital stock implications of a reserve requirement are also ambiguous, and this is because the requirement may either raise or reduce the goods value of outstanding domestic government liabilities. Totally differentiating (54) and (55) and some algebra yields

$$\left(\frac{dk^d}{d\hat{\gamma}_d^d}\right)\left[\left(\frac{\frac{k^{d^{-\alpha}}}{A^d}-\alpha(1-\alpha)A^d k^{d^{\alpha-2}}(1-\pi_f^f-\pi_d^f)}{\frac{1}{1-\alpha}-\frac{k^{d^{1-\alpha}}}{(1-\alpha)A^d}-\frac{\pi_f^f}{\sigma^f}-\alpha A^d k^{d^{\alpha-1}}(1-\pi_f^f-\pi_d^f)}\right)+\left(\frac{\frac{k^{d^{-\alpha}}}{A^d}-\frac{\alpha(1-\alpha)A^d k^{d^{\alpha-2}}(1-\pi_d^d-\pi_f^d)(1-\hat{\gamma}_d^d)}{(1-\pi_d^d)}}{\frac{1}{1-\alpha}-\frac{k^{d(1-\alpha)}}{(1-\alpha)A^d}-\frac{\hat{\gamma}_d^d}{\sigma^d}-\frac{\alpha A^d k^{d^{\alpha-1}}(1-\pi_d^d-\pi_f^d)(1-\hat{\gamma}_d^d)}{1-\pi_d^d}}\right)\right]$$
$$=\left[\frac{\sigma^d\left(\frac{A^d}{A^f}\right)^{\frac{1}{1-\alpha}}}{\pi_d^f x}\right]\left[\frac{\alpha A^d k^{d^{\alpha-1}}(1-\pi_d^d-\pi_f^d)}{1-\pi_d^d}-1/\sigma^d\right]+\left(\frac{1}{(1-\hat{\gamma}_d^d)}\right).$$

It is straightforward to verify that at $\underline{k}^d[\bar{k}^d]$, $(dk^d/d\hat{\gamma}_d^d) < (\gtrless)0$. In particular, for high capital stock economies, equilibrium capital formation will be enhanced by the imposition or raising of a reserve requirement when the money growth rate is sufficiently high, while for low capital stock countries it is unambiguously reduced. The intuition for this result is as follows.

When the requirement is imposed, banks substitute out of foreign currency and interest bearing assets thereby tending to reduce private capital formation directly. In addition the reserve requirement admits high levels of government debt at $\underline{k}^d$, and so crowding out of private capital, and high levels of government lending to the private financial sector at $\bar{k}^d$ which mitigates the fall in equilibrium investment. Finally, as argued above the real exchange rate may appreciate little or not at all, preventing a subtstantial decline in government borrowing (lending) from this source. The capital stock unambiguously declines at $\underline{k}^d$. At $\bar{k}^d$, the rise in government lending offsets the direct negative impact of the requirement for capital formation when the money growth rate is high enough to prevent a large real appreciation. An example of the consequences of imposing a reserve requirement is illustrated in Figure 6.

Thus, the government of a high capital stock country that conducts highly inflationary monetary policy can raise its capital stock through the imposition of a reserve requirement while capital poor countries will always reduce capital formation with such a measure. No predictable real exchange rate movement results, except to the extent that it will be neither very large and positive nor very large and negative. It is this limit on the real exchange rate impact of the requirment that allows higher capital formation for high capital stock countries; and this, in turn, is the result of the

[7]In addition, reserve requirements introduce or aggravate any asymmetry in the location of the two goods market clearing loci, since the peak of the domestic locus shifts to the left. It can be easily verified that the conditions for there to exist two intersections of (101) and (102) at $x > 0$ continue to hold, however.

international portfolio reallocation consequences of any such restriction.

6 Conclusion

In this paper I have analysed an economy in which monetary policy, conditions in financial markets, and relative productivities across countries determine both the equilibrium real exchange rate and equilibrium capital stocks. When money growth rates in the two economies are sufficiently high, the economy has multiple non-trivial steady state equilibria in which the value of both currencies is strictly positive.

Of the two non-trivial steady state equilibria that can obtain in a Cobb-Douglas version of this economy, one is characterized by high capital stocks and a low nominal interest rate, and governments of both countries are net lenders to their respective banking systems thereby contributing to private capital formation. The relative levels of government lending in the two countries (and so rates of debt monetaization) determine which of the two countries is a net foreign borrower. In this equilibrium, increases in the rate of money creation by either government actually enhance opportunities for capital formation in that country; the rise in money growth in conjunction with the higher real exchange rate of that country enables higher government lending to the private sector, and so higher equilibrium capital stocks. The capital stock of the 'inflating' country rises due to greater government financing, while that of the foreign country increases through financing obtained from abroad.

The second steady state equilibrium for this economy has a low capital stock for both countries, a high world real interest rate, and the government of each country is a net borrower from its private banking system. In this equilibrium, government debt competes with private capital in the asset portfolios of private agents, crowding out private investment. Here, increases in money creation rates by either country again raise that country's real exchange rate but reduce further its equilibrium capital stock by allowing for higher levels of government indebtedness. The equilibrium capital stock of the foreign country also falls, but this corresponds to an outflow of savings to help finance the domestic country's higher government debt level.

The analysis suggests some rationales for the imposition of exchange controls and reserve requirements, especially when countries complement such measures with inflationary monetary policy. An exchange control can be used to unambiguously reduce the domestic country's real exchange rate, while reserve requirements constitute a less effective device for achieving a real appreciation. More generally, the two types of regulation have very different aggregate consequences.

Exchange controls can actually improve a low capital stock country's capital formation performance when that country's money growth rate is sufficiently high. The real exchange rate movement induced by the policy is critical for this to occur. A sufficiently large real appreciation reduces the outstanding value of domestic government liabilities and so admits higher resource utilization in private capital formation. This real "income" effect of the control supports the direct portfolio substitution out of foreign exchange and into interest earning assets as well as domestic currency. A reserve requirement, by contrast, unambiguously reduces the equilibrium capital stock of a capital poor country, but will raise that of a high capital stock country which conducts highly inflationary monetary policy. In this case, it is limitations on the appreciation of the real exchange rate that admits increased private investment. When the real exchange rate decline is small enough, the outstanding value of net government lending can rise following the imposition of reserve requirement, thus enhancing government financed private capital formation. This is despite the portfolio reallocation out of private capital formation that is caused by the requirement.

In this economy, where a country's currency and other assets are held abroad, financial market regulations need not have the expected real exchange rate effect, nor - in fact - the anticipated impact for the domestic inflation tax base. The effects of such regulations for these variables and for capital formation depend on the relative purchasing powers of holders of domestic currency and of interest-earning assets in a two-country world, and this depends critically on prevailing rates of money growth. In addition, the effects of all policies depend crucially on *which* steady state an economy originates in.

While many of these results have been obtained in an environment in which some quite restrictive assumptions have been made, on preferences and technology in particular, I conjecture that many of them will continue to hold in some form under more general conditions. Furthermore, analysis of other financial market regulations - such as deposit rate ceilings and capital controls, monetary policies such as open market operations and inflation rate targeting, and of alternative exchange rate regimes is easily conducted in the model.

Of particular interest is the analysis of a fixed exchange rate regime in this environment. It is immediate from the steady state versions of the no arbitrage conditions that when one of the two countries is forced to endogenize its money growth rate such as to maintain a fixed nominal rate of exchange between the two currencies, then in a steady state equilibrium this requires that it set its money growth rate to be equal to that of the "leader" country. Independent changes in the domestic country's money growth rate are no longer possible, however changes in the money growth rate of the foreign country must then be matched one-for-one by changes in the domestic country's money growth rate. Consequently, both goods market loci shift (the domestic one upwards and the foreign one downwards) as a result of money growth rate changes, which therefore have less potent real exchange rate effects and larger (negative) capital stock effects under fixed than under flexible exchange rates. This suggests that real exchange rates may be less variable under fixed exchange rates than under the flexible exchange rate regime considered here - as is true empirically - while real aggregate performance may be more volatile.

Finally, many of the results obtained here are empirically testable. They predict that we should observe some systematic long-run correlations of variables such as the real exchange rate, capital stock, and the capital account with the money growth rate of a country, as well as specific permanent effects for these variables following changes in financial market regime. The results also suggest that there exist important relationships between real exchange rate movements and the relative value of government to privately issued debt held in a country's financial system. These relationships reflect the "transmission mechanism" for monetary and financial market policy to an economy's growth performance identified in this paper, and empirical evaluation of the validity of this mechanism is left to future work.

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Appendix 1.

$k^d = w^d(k^d)f^{d'}(k^d)(1-\pi_f^f-\pi_d^f) \Leftrightarrow k^f = w^f(k^f)f^{f'}(k^f)(1-\pi_f^f-\pi_d^f)$. Then, from (30'), $f^f(k^f) = w^f(k^f + f^{f'}(k^f)k^f = (m^f/\sigma^f) + 2f^{f'}(k^f)w^f(k^f)(1-\pi_f^f-\pi_d^f) = \left[\frac{\pi_f^f w^f(k^f)+\pi_f^d w^d(k^d)/x}{\sigma^f}\right] + 2f^{f'}(k^f)w^f(k^f)(1-\pi_f^f-\pi_d^f) \Leftrightarrow w^f(k^f)\left[1+f^{f'^2}(k^f)(1-\pi_f^f-\pi_d^f) - 2f^{f'}(k^f)(1-\pi_f^f-\pi_d^f) - \frac{\pi_f^f}{\sigma^f}\right] = \frac{\pi_f^d w^d(k^d)}{x\sigma^f}$. It is evident that $1+f^{f'^2}(k^f) - 2f^{f'}(k^f) + [2f^{f'}(k^f) - f^{f'^2}(k^f)][\pi_f^f+\pi_d^f] - \pi_f^f/\sigma^f = (1-f^{f'})^2(1-\pi_f^f-\pi_d^f) - \pi_f^f/\sigma^f + (\pi_f^f+\pi_d^f) > 0 \Rightarrow \infty > 1/x > 0$ at the minimum of (37').

Appendix 2: Proof of Proposition 2

First note that $dx/d\bar{k}^d \mid_{36'} \gtreqless 0 \Leftrightarrow \bar{k}^d \lesseqgtr \left[(1-\alpha)\alpha A^{d^2}(1-\pi_d^d-\pi_f^d)\right]^{\frac{1}{2(1-\alpha)}} \Leftrightarrow \bar{k}^d \lesseqgtr f^{d'}(\bar{k}^d)w^d(\bar{k}^d)(1-\pi_d^d-\pi_f^d) \Leftrightarrow \Omega^d(\bar{k}^d) \lesseqgtr f^{d'}(\bar{k}^d)(1-\pi_d^d-\pi_f^d)$.

Analagously, $dx/d\bar{k}^d \mid_{37'} \gtreqless 0 \Leftrightarrow \bar{k}^d \gtreqless \left[(1-\alpha)\alpha A^{d^2}(1-\pi_f^f-\pi_d^f)\right]^{\frac{1}{2(1-\alpha)}} \Leftrightarrow \bar{k}^d \gtreqless f^{d'}(\bar{k}^d)w^d(\bar{k}^d)(1-\pi_f^f-\pi_d^f) \Leftrightarrow \Omega^d(\bar{k}^d) \gtreqless f^{d'}(\bar{k}^d)(1-\pi_f^f-\pi_d^f) \Leftrightarrow \bar{k}^f \gtreqless f^{f'}(\bar{k}^f)w^f(\bar{k}^f)(1-\pi_f^f-\pi_d^f)$.

a). At $\bar{k}^d$, $f^{d'}(\bar{k}^d) < 1$ so that $m^d+b^d+x(m^f+b^f) = \left[(\pi_d^d w^d(\bar{k}^d)+\pi_d^f w^f(\bar{k}^f)x)\left(\frac{\sigma^d f^{d'}(\bar{k}^d)-1}{\sigma^d(f^{d'}(\bar{k}^d)-1)}\right)\right] + \left[(\pi_f^f w^f(\bar{k}^f)x+\pi_f^d w^d(\bar{k}^d))\left(\frac{\sigma^f f^{d'}(\bar{k}^d)-1}{\sigma^f(f^{d'}(\bar{k}^d)-1)}\right)\right] < 0$. Asset market clearing then requires that $(\bar{k}^d + x\bar{k}^f) > w^d(\bar{k}^d) + xw^f(\bar{k}^f) \Leftrightarrow \bar{k}^d\left(1+x\left(\frac{A^f}{A^d}\right)^{\frac{1}{1-\alpha}}\right) > w^d(\bar{k}^d)\left(1+x\left(\frac{A^f}{A^d}\right)^{\frac{1}{1-\alpha}}\right) \Leftrightarrow \Omega^d(\bar{k}^d) > 1$. Hence, $\Omega^d(\bar{k}^d) > \max\ (f^{d'}(\bar{k}^d)(1-\pi_d^d-\pi_f^d), f^{d'}(\bar{k}^d)(1-\pi_f^f-\pi_d^f))$, and $dx/d\bar{k}^d \mid_{36'} < 0$ and $dx/d\bar{k}^d \mid_{37'} > 0$ hold at $\bar{k}^d$.

b). At $\underline{k}^d$, $\Omega^i < 1$ and $\underline{k}^i < w^i(\underline{k}^i)$, i=d,f, holds. Assume that $\underline{k}^i \geq w^i(\underline{k}^i)f^{i'}(\underline{k}^i)(1-\pi_i^i-\pi_j^i)$, i,j=d,f. Then from (66) and (68), $\underline{k}^i \geq f^i(\underline{k}^i) - m^i/\sigma^i - \underline{k}^i \Leftrightarrow \underline{k}^i \geq \frac{f^i(k^i)-(m^i/\sigma^i)}{2}$. Hence, $\underline{k}^i \geq \left[\frac{w^i(k^i)+f^{i'}(k^i)k^i-m^i/\sigma^i}{2}\right]$. This implies the following inequality: $w^i(\underline{k}^i) > \left[\frac{w^i(k^i)-m^i/\sigma^i}{2-f^{i'}(k^i)}\right] \Rightarrow w^i(k^i) > \frac{m^i}{\sigma^i}\left[\frac{1}{f^{i'}(k^i)-1}\right]$. From goods market clearing, we therefore know that $w^i(\underline{k}^i) = f^i(\underline{k}^i) - \underline{k}^i f^{i'}(\underline{k}^i) = m^i/\sigma^i + k^i(1-f^{i'}(\underline{k}^i)) + w^i(\underline{k}^i)f^{i'}(\underline{k}^i)(1-\pi_i^i-\pi_j^i) > \frac{m^i}{\sigma^i}\left[\frac{1}{f^{i'}(k^i)-1}\right] \Leftrightarrow \underline{k}^i(1-f^{i'}(\underline{k}^i)) + w^i(\underline{k}^i)f^{i'}(\underline{k}^i)(1-\pi_i^i-\pi_j^i) > \frac{m^i}{\sigma^i}\left[\frac{2-f^{i'}(k^i)}{f^{i'}(k^i)-1}\right]$. Then $k^i(2-f^{i'}(\underline{k}^i)) > \frac{m^i}{\sigma^i}\left[\frac{2-f^{i'}(k^i)}{f^{i'}(k^i)-1}\right] \Leftrightarrow \underline{k}^i > m^i/\sigma^i\left[\frac{1}{f^{i'}(k^i)-1}\right]$. But then $f^i(\underline{k}^i) = m^i/\sigma^i + k^i + w^i(\underline{k}^i)f^{i'}(\underline{k}^i)(1-\pi_i^i-\pi_j^i) < k^i[f^{i'}(\underline{k}^i)-1] + k^i + w^i(\underline{k}^i)f^{i'}(\underline{k}^i)(1-\pi_i^i-\pi_j^i) = k^i f^{i'}(\underline{k}^i) + w^i(\underline{k}^i)f^{i'}(\underline{k}^i)(1-\pi_i^i-\pi_j^i)$ and this implies that $w^i(\underline{k}^i) < w^i(\underline{k}^i)f^{i'}(\underline{k}^i)(1-\pi_i^i-\pi_j^i)$ which contradicts $w^i(\underline{k}^i) > k^i$. Hence, $\underline{k}^i < w^i(\underline{k}^i)f^{i'}(\underline{k}^i)(1-\pi_i^i-\pi_j^i)$ must hold and $dx/d\underline{k}^d \mid_{36'} > 0$ and $dx/d\underline{k}^d \mid_{37'} < 0$. **QED.**

FIGURE 1: Timing of Transactions

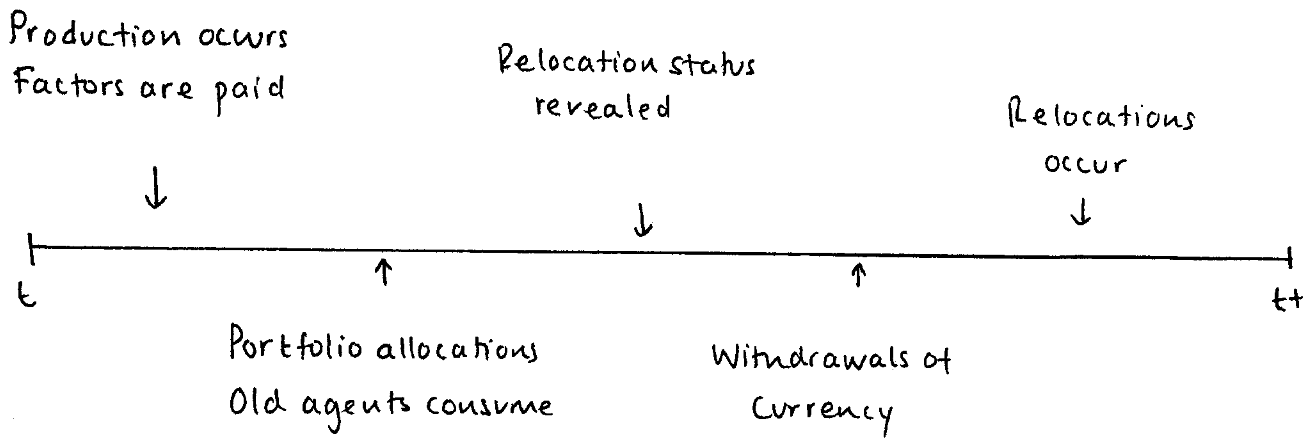
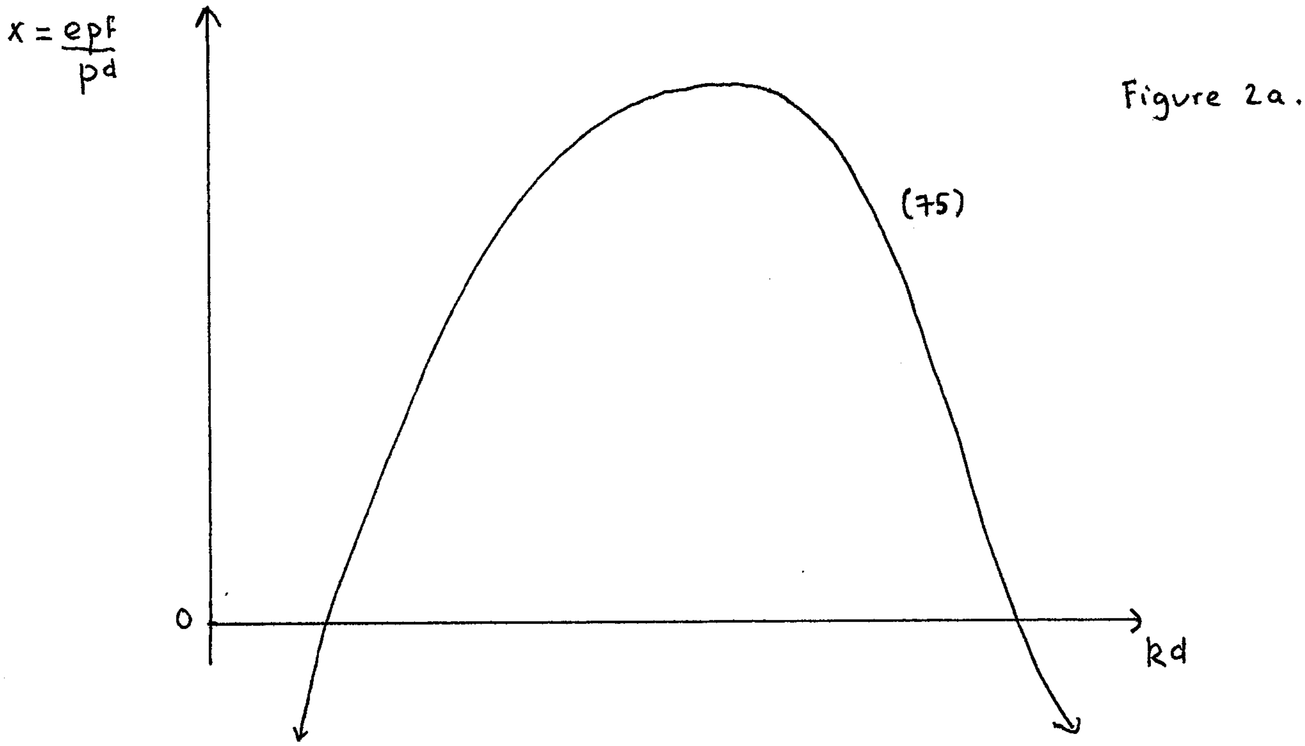


FIGURE 2: Goods Market Clearing Loci



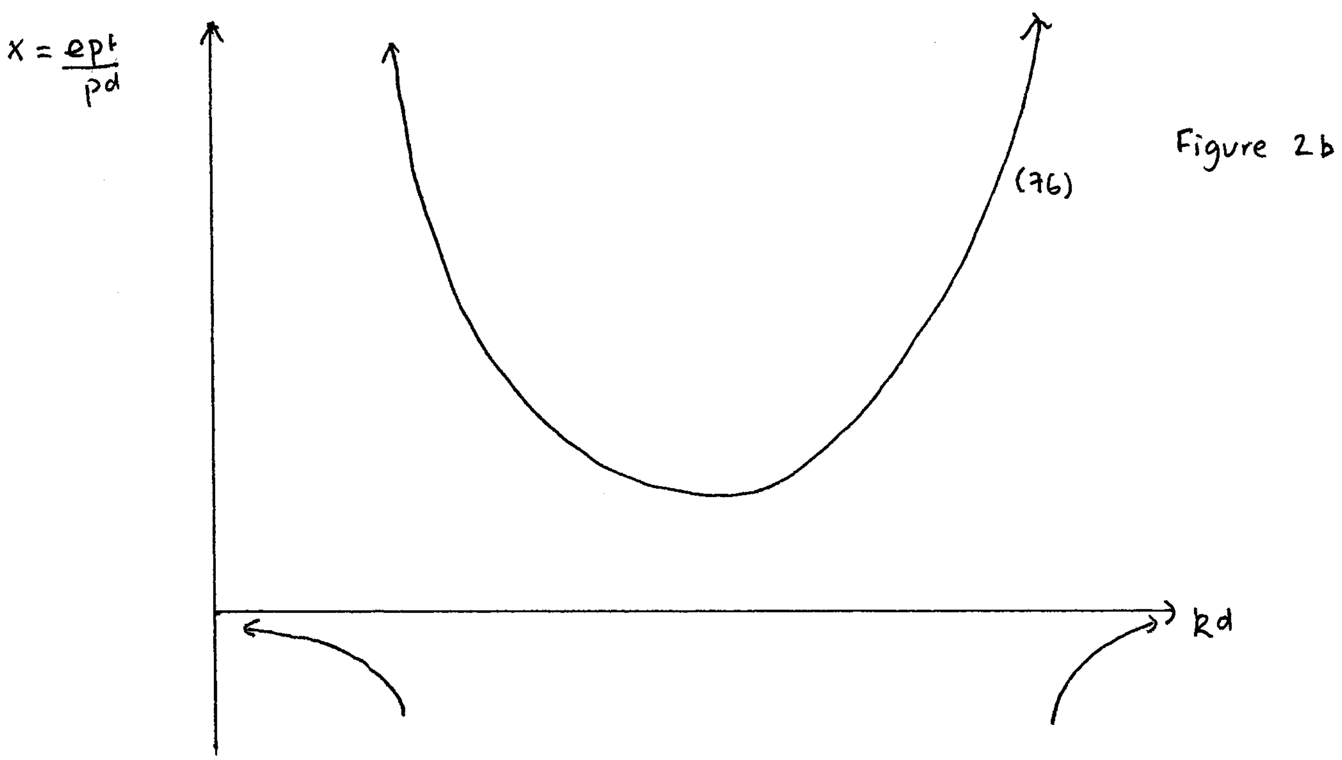


FIGURE 3: Steady State Equilibria

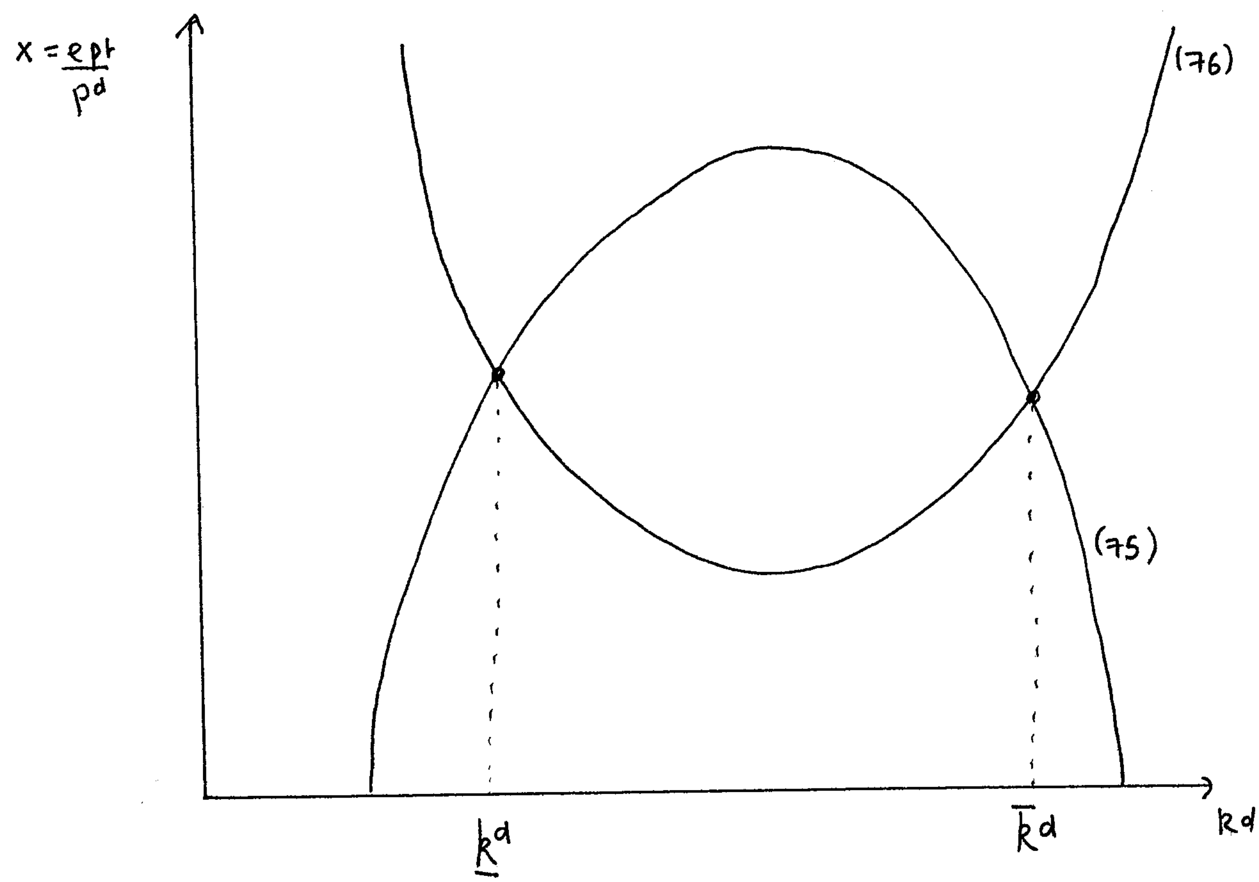


FIGURE 4: Money Growth Rate Changes

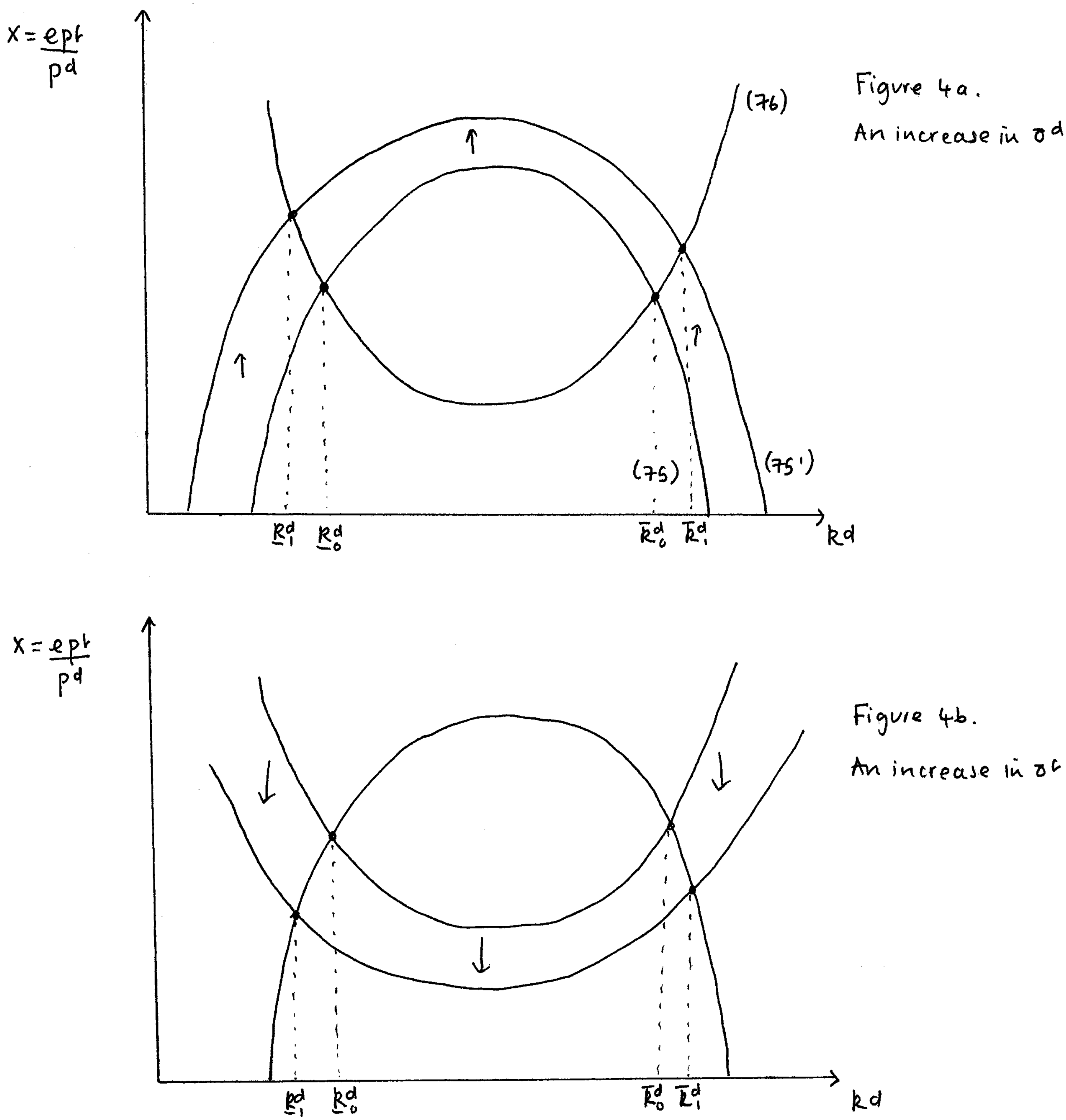
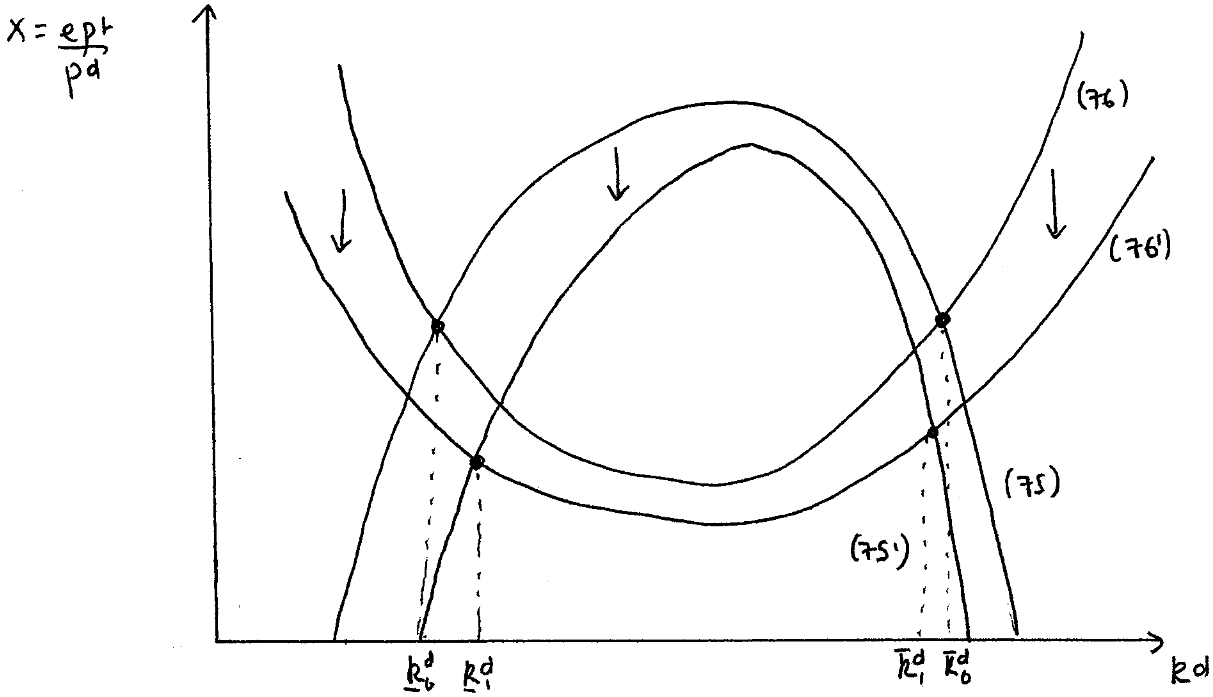
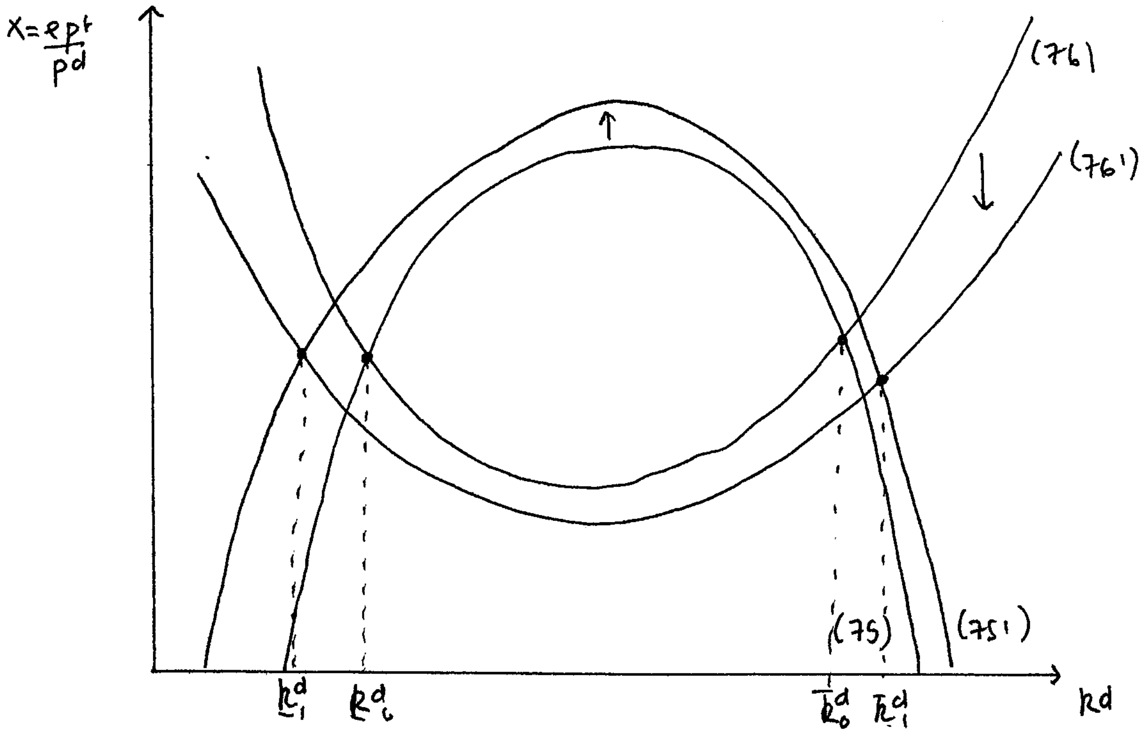


FIGURE 5: Exchange Controls



Imposition / Tightening by domestic country.

FIGURE 6: Reserve Requirements



Imposition or Raising by domestic country.