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Local Whittle estimation in stationary systems: Evidence from US and Canada inflation

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Abstract

This paper uses the bivariate framework introduced by Robinson (2008) to analyse the long run relationship between the monthly inflation rates in the US and Canada. For two stationary long memory time series driven by a common stochastic trend, there may exist a linear combination of the two series with smaller memory parameter. The bivariate model introduces four unknown parameters (two memory parameters, a phase parameter and a cointegration parameter) to be jointly estimated by optimising a local Whittle function. The results indicate the existence of a linear combination between the US and Canada inflation rates that has a long memory less than the two individual series.

JEL Classification: C14, C32.

Keywords: Frequency domain; Local Whittle estimation; Long memory; Semiparametric estimation; US and Canada inflation.

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1 Introduction

In the analysis of a multivariate framework of long memory time series, two main features emerge: the possibility of cointegration and a phase shift that does not need to be zero. This paper concerns with the joint estimation of the memory parameters along with the cointegrating and phase parameters in the bivariate framework developed by Robinson (2008). This procedure is applied to monthly US and Canada inflation rates to examine the long-run equilibrium relationship which consequently has its implications on the interdependence of their monetary policies. The local Whittle estimation is employed where four unknown parameters (two memory parameters, a phase parameter and a cointegration parameter) are introduced. Robinson (2008) introduces an additional parameter to model the phase (γ) between the linear combination between the two series y_t and x_t, $(y_t - \beta x_t)$, and x_t flexibly. Moreover, Robinson (2008) derived the consistency and established the joint asymptotic normality of the estimates under the assumption that the memory parameters lie between zero and $\frac{1}{2}$ and indicated how his results to be applied in statistical inference.

Stationarity of time series was usually associated with the Box-Jenkins modelling methodology with inherent short-memory properties of a series; while lack of statistical evidence for existence of long memory in economic time series made research restricted to the intuitively conventional $I(1)/I(0)$ case. However, recent research in long memory time series modelling has provided enough tools to explore the idea of fractional cointegration empirically. In addition, the term long memory time series includes both stationary and nonstationary series. Since 1990, the mainstream econometric time series literature shows considerable interest in long memory by focusing on unit roots time series. Unit roots series can be perceived as special cases of nonstationary fractional series. The analysis in this paper only covers stationary long memory series. Stationary long memory time series displays a statistically significant dependence between distant observations. This dependence can be formalised by assuming that the autocorrelations decay very slowly, hyperbolically, to zero as a function of the time lag or spectral density displaying a pole at zero frequency. In addition, this dependence structure across time played a vital role in the modelling of macroeconomic and financial data.

As in economic literature, relationships between variables comes in pairs; and hence it is a natural starting point to focus on analysing bivariate relations between stationary long memory time series. One of the first estimation methods in the bivariate framework was the semiparameter narrow-band ordinary least squares (NBLS) regression in the frequency domain, developed in a series of papers by Robinson (1994) and Robinson and Marinucci (2001). The NBLS estimator is an OLS estimator in a spectral regression with a degenerating frequency band around the origin. Robinson (1994) proved the consistency of this estimator in the stationary case. The NBLS estimator reduces the bias in comparison to the OLS estimator, by reducing the effect of correlation between cointegration errors, while the convergence rate of the NBLS estimator depends on the values of the long memory and the bandwidth used in estimation. On the other hand, Lobato (1999) derived a semiparametric two-step estimator of parameters that characterise long memory for a time series vector in the frequency domain. Asymptotic normality of his estimator was established, but did not include Gaussianity condition. The two main methods described above were combined by Marinucci and Robinson (2001) and Christensen and Nielsen (2004), who suggested conducting a fractional cointegration analysis in several steps. First, the memory parameters of the original series are seperately estimated by local Whittle QMLE. Secondly, the narrow band FDLS estimator for the cointegrating vector is calculated, and finally the persistence of the residuals is estimated assuming that the approach is equally valid for residuals. In addition, Velasco (2003) and Hassler et al. (2006) sought to estimate the memory parameter of the equilibrium error by applying semiparametric estimators to the residuals from cointegrating regressions. Nielsen (2007) considered joint estimation of the memory parameters and the cointegrating vector for stationary long memory series in a multivariate framework, but derived its asymptotic distribution only under the long-run exogeneity between the stochastic trend and equilibrium error. Nielsen and Frederiksen (2008) considered a fully modified narrowband least squares estimator that corrects the endogeneity bias of the NBLS, and analysed the estimation of the memory parameters from modified NBLS regression. Shimotsu (2007) also developed a semiparametric estimator for the multivariate stationary framework. He used a more general local form of the spectral density. In general, a joint estimation method for the memory parameters and the cointegrating vector is more preferable. The estimators for the cointegrating parameter considered above are mostly direct in the sense that they do not require estimation of memory parameters. An alternative approach, first introduced by Robinson (2008) in a context of stationary bivariate system, jointly estimate the cointegrating parameters along with the memory parameters or other nuisance parameters which is adopted in this paper.

Many previous empirical studies in economic literature have examined the characteristics of aggregate US and Canada inflation rates. Klein (1976) and Nelson and Schwert (1977) imposed a unit root on the inflation process; while Ball and Cecchetti (1990) and Kim (1993) modelled inflation as a transitory and a permanent component, which is represented as a random walk. On the other hand, Barsky (1987) and Brunner and Hess (1993) argued that inflation was covariance stationary. Hassler and Wolters (1995) found evidence in favour of long memory properties. Furthermore, Doornik and Ooms (2004) used ARFIMA models with different estimation methods in order to model and forecast the long memory characteristics in inflation. This question, regarding the examination and modelling the long memory features in inflation rate series, should take on a new investigation of whether inflation rates are related across countries. Examining such relation has very important implications on the interdependence of domestic monetary policies and the validity of purchasing power parity. As a result, the purpose of this paper is to examine and investigate the interdependence between the inflation rates in US and Canada by studying their long memory properties in a bivariate framework which avoids mathematical complexity of any general multivariate structure. However, unlike the conventional classical approaches discussed in literature, this model allows for the possibility of cointegration and phase shifts.

The rest of the paper is structured as follows. The next section describes the methodology used by presenting Robinson's (2008) bivariate system and demonstrates the local Whittle (or Narrow-Band) estimation. Section 3 reports some simulation results. Section 4 offers an empirical application to analyse the long run equilibrium between the inflation rates in the US and Canada, and finally section 5 concludes.

2 Methodology

Suppose a bivariate jointly covariance stationary process $u_t = (u_{1t}, u_{2t})'$ has a spectral density matrix,

$$f_u(\lambda) \sim \begin{bmatrix} \lambda^{-d_1} & 0 \\ 0 & \lambda^{-d_2} e^{i\gamma \mathrm{sgn}(\lambda)} \end{bmatrix} \begin{bmatrix} \omega_{11} & \omega_{12} \\ \omega_{21} & \omega_{22} \end{bmatrix} \begin{bmatrix} \lambda^{-d_1} & 0 \\ 0 & \lambda^{-d_2} e^{-i\gamma \mathrm{sign}(\lambda)} \end{bmatrix} \text{ as } \lambda \to 0 \quad (2.1)$$

For simplicity, this can be written as,

$$f_u(\lambda) \sim \Lambda^{-1}\Omega\overline{\Lambda}^{-1} \text{ as } \lambda \to 0 \tag{2.2}$$

The parameters d_1, d_2 and γ are unknown real valued and will be collected in vector $\alpha = (d_1, d_2, \gamma)'$, where d_1 and d_2 are the memory parameters and lay in the interval $[0, \frac{1}{2})$ and γ is the phase parameter between u_{1t} and u_{2t} at zero frequency and lies in the interval $\gamma \in (-\pi, \pi]$. The term $\text{sign}(\lambda) = 1$ if $\lambda \geq 0$. The symbol "~" means that for each element, the ratio of real/imaginary parts of the left and right sides tend to 1. In (2.2.2), the over bar denotes the complex conjugate and the parameters and Ω is a 2×2 positive definite matrix.

The spectral density matrix in (2.1) can be written as,

$$f_u(\lambda) \sim \begin{bmatrix} \omega_{11}|\lambda|^{-2d_1} & \omega_{12}\lambda^{-d_1-d_2}e^{-i\gamma\text{sign}(\lambda)} \\ \omega_{21}\lambda^{-d_1-d_2}e^{i\gamma\text{sign}(\lambda)} & \omega_{22}|\lambda|^{-2d_2} \end{bmatrix}. \tag{2.3}$$

From the main diagonal element, it can be deduced that the bivariate series has the memory parameter d_1 and d_2 respectively. On the other hand, the off diagonal elements represent the cross spectrum between the bivariate series. It takes a real value only if ω_{12}, ω_{21} and/or $\gamma = 0$.

For any two time series to be cointegrated and shape a long run equilibrium relationship, they need to share a common stochastic trend with a specific memory parameter. The long run equilibrium relationship is represented in the linear combination that becomes less persistent. Intuitively, most studies focused on the conventional $I(1)/I(0)$ where persistence is reduced from 1 to zero. However, this model is developed where persistence takes values between 0 and $\frac{1}{2}$. Now consider the model that includes the bivariate series $(y_t, x_t)'$,

$$\begin{bmatrix} 1 & -\beta \\ 0 & 1 \end{bmatrix}\begin{bmatrix} y_t \\ x_t \end{bmatrix} = \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix}. \tag{2.4}$$

When $d_1 \neq d_2$ and $\beta = 0$, then y_t and x_t have unequal memories d_1 and d_2 respectively. When $d_1 < d_2$ and $\beta \neq 0$, the bivariate series are said to be cointegrated and the unobservable linear combination $u_{1t} = y_t - \beta x_t$ has a memory of d_1 which is less than the memory for the bivariate series. Robinson's (2008) local Whittle (or narrow-band) estimate $\theta = (d_1, d_2, \gamma, \beta)'$ is considered in this paper where $0 \leq d_1 < d_2 < \frac{1}{2}$ and $\beta \neq 0$.

A local Whittle estimation is considered which employs Fourier frequencies in the neighbourhood of the origin. To begin with, the discrete Fourier transform (dFt) and the periodogram of a time series W_t are defined and evaluated at frequency λ as

$$w_j(\lambda) = \frac{1}{\sqrt{2\pi n}}\sum_{t=1}^{n} W_t e^{-it\lambda_j} \tag{2.5}$$

$$I_w(\lambda) = w_j(\lambda)w_j^*(\lambda) = n^{-1}(\sum_{t=1}^{n} W_t e^{-it\lambda})(\sum_{t=1}^{n} W_t e^{-it\lambda})' \tag{2.6}$$

where $w_j^*(\lambda)$ is the conjugate transpose of $w_j(\lambda)$. The Whittle function, $Q(\theta, \Omega)$, is approximated to the (negative) log-likelihood function is

$$Q(\theta, \Omega) = \frac{1}{m}\sum_{j=1}^{m}(\log\left|\Lambda^{-1}\Omega\overline{\Lambda}^{-1}\right| + tr[\Omega^{-1}Re(\Lambda I(\lambda_j)\overline{\Lambda})]) \tag{2.7}$$

To find the local Whittle estimator, function (2.7) is minimised with respect to the unknown parameters θ and Ω. The first step is to concentrate (2.7) with respect to the parameter Ω solving the resulting first order condition for Ω and then substituting the result back into (2.7). The solution of the first order condition with respect to Ω gives

$$\hat{\Omega}(\theta) = \frac{1}{m}\sum_{j=1}^{m}\Lambda_j Re(I(\lambda_j)\bar{\Lambda}_j$$

By substituting $\hat{\Omega}(\theta)$ into $Q(\theta,\Omega)$, this yields the concentrated likelihood function $R(\theta)$ in terms of the four parameters,

$$R(\theta) = \log\det\{\hat{\Omega}(\theta)\} - 2(\sum_{s=1}^{2} d_s)\frac{1}{m}\sum_{j=1}^{m}\log|\lambda_j| \qquad (2.8)$$

The local Whittle estimator of the parameter of interest, θ, can then be defined in terms of the concentrated likelihood

$$\hat{\theta} = \arg\min_{\theta\in\Theta} R(\theta) \qquad (2.9)$$

The space of the true parameter θ is the compact set $\Theta \in \mathbb{R}^4$. The consistency and asymptotic properties of the local Whittle estimator $\hat{\theta}$ was also established in Robinson (2008).

3 Finite sample simulations

In this section, the finite sample behaviour of LW estimator is investigated by conducting a Monte Carlo study. The following four generating mechanisms for u_{1t} and u_{2t} are considered.

Model A:

$$u_{1t} = (1-L)^{-d_1}\varepsilon_{1t} \qquad u_{2t} = (1-L)^{-d_2}\varepsilon_{2t}$$

Model B:

$$u_{1t} = (1-L)^{-d_1}\eta_{1t} \qquad u_{2t} = (1-L)^{-d_2}\varepsilon_{2t}$$

$$\eta_{1t} = 0.5\eta_{1,t-1} + \varepsilon_{1t}$$

Model C:

$$u_{1t} = (1-L)^{-d_1}\varepsilon_{1t} \qquad u_{2t} = (1-L)^{-d_2}\eta_{2t}$$

$$\eta_{2t} = 0.5\eta_{2,t-1} + \varepsilon_{2t}$$

Model D:

$$diag\{(1-L)^{d_1}, (1-L)^{d_2}\}(1-0.5L)u_t = \sqrt{R}\varepsilon_t$$

where $\varepsilon_t = (\varepsilon_{1t}, \varepsilon_{2t})'$ is bivariate independently and identically distributed with mean zero and unit variance, ρ is the correlation between ε_{1t} and ε_{2t}, and

$$R = \begin{bmatrix} 1 & 2\rho \\ 2\rho & 4 \end{bmatrix}.$$

Based on the above generating mechanisms, the process y_t in (2.4) is generated with $\beta = 1$. The data is generated (for all simulations) with two sets of memory parameters. Firstly the memory parameters used are $(d_1, d_2) = (0.05, 0.4)$ which is close to many practical situations and supported by the empirical application reported in the next section, and then $(d_1, d_2) = (0.2, 0.3)$ which indicates a weaker form of fractional cointegration where the two memory parameters are very close. Model A has no short-run dynamics, unlike Models B and C where short-run dynamics are introduced to u_{1t} and u_{2t} respectively. Model D satisfies the spectral density function adopted in this paper in (2.1) to (2.3). The elements of the main diagonal for R are 1 and 4, while the off-diagonal elements is 2ρ, thus the phase parameter is set as $\gamma = (d_2 - d_1)\frac{\pi}{2}$. For the Monte Carlo study, 10000 replications for sample sizes n are used where $n = 128$ and $n = 512$ are chosen. The former sample size is chosen to be close to the application in the next section. The bandwidth parameters chosen are $m = n^{0.5}$ and $m = n^{0.4}$ to examine the robustness of the LW estimator due to changes in the bandwidth. The Monte Carlo bias and root mean squared error (RMSE) results of the local Whittle estimator for all above models are reported in Tables 1 and 2. Simulations are performed using Ox 6.0 and TSM 4.35.

Table 1: Simulation Results for bias and RMSE where $\rho = 0$

	Model A						Model B						Model C						Model D					
	Bias			RMSE			Bias			RMSE			Bias			RMSE			Bias			RMSE		
	d_1	d_2	β	d_1	d_2	β	d_1	d_2	β	d_1	d_2	β	d_1	d_2	β	d_1	d_2	β	d_1	d_2	β	d_1	d_2	β
$d_1 = 0.05, d_2 = 0.4$																								
$n = 128$																								
$m = n^{0.4}$	0.19	0.13	0.22	0.26	0.23	0.31	0.34	0.27	0.43	0.39	0.35	0.44	0.15	0.12	0.17	0.14	0.11	0.16	0.06	0.04	0.09	0.18	0.12	0.15
$m = n^{0.5}$	0.14	0.09	0.17	0.24	0.18	0.25	0.26	0.24	0.31	0.28	0.31	0.42	0.11	0.10	0.14	0.16	0.10	0.10	0.03	0.02	0.07	0.09	0.07	0.12
$n = 512$																								
$m = n^{0.4}$	0.16	0.11	0.18	0.14	0.11	0.25	0.28	0.21	0.33	0.36	0.35	0.38	0.10	0.09	0.12	0.11	0.09	0.12	0.04	0.03	0.05	0.13	0.09	0.08
$m = n^{0.5}$	0.12	0.08	0.15	0.15	0.14	0.21	0.25	0.20	0.27	0.32	0.25	0.34	0.08	0.08	0.10	0.07	0.06	0.09	0.00	0.00	0.02	0.06	0.05	0.03
$d_1 = 0.2, d_2 = 0.3$																								
$n = 128$																								
$m = n^{0.4}$	0.65	0.42	0.35	0.74	0.55	0.49	0.86	0.81	0.76	0.84	0.78	0.59	0.44	0.35	0.40	0.33	0.25	0.31	0.18	0.15	0.23	0.31	0.27	0.29
$m = n^{0.5}$	0.61	0.43	0.31	0.67	0.50	0.45	0.83	0.84	0.73	0.79	0.77	0.54	0.41	0.30	0.36	0.32	0.27	0.35	0.16	0.13	0.25	0.28	0.25	0.20
$n = 512$																								
$m = n^{0.4}$	0.53	0.36	0.27	0.63	0.51	046	0.77	0.69	0.71	0.80	0.73	0.52	0.39	0.32	0.37	0.35	0.24	0.25	0.17	0.16	0.19	0.25	0.21	0.25
$m = n^{0.5}$	0.49	0.33	0.24	0.61	0.42	0.40	0.70	0.67	0.65	0.73	0.76	0.48	0.46	0.34	0.39	0.28	0.20	0.22	0.10	0.11	0.21	0.24	0.19	0.15

Table 2: Simulation Results for bias and RMSE where $\rho = 0.5$

	Model A						Model B						Model C						Model D					
	Bias			RMSE			Bias			RMSE			Bias			RMSE			Bias			RMSE		
	d_1	d_2	β	d_1	d_2	β	d_1	d_2	β	d_1	d_2	β	d_1	d_2	β	d_1	d_2	β	d_1	d_2	β	d_1	d_2	β
$d_1 = 0.05, d_2 = 0.4$																								
$n = 128$																								
$m = n^{0.4}$	-0.25	-0.19	0.27	0.34	0.26	0.33	-056	-034	0.46	0.43	0.38	0.46	0.20	0.15	0.21	0.16	0.17	0.24	0.14	0.06	0.15	0.21	0.15	0.16
$m = n^{0.5}$	-0.17	-0.12	0.24	0.28	0.16	0.29	-0.43	-0.31	0.37	0.35	0.37	0.43	0.15	0.10	0.15	0.22	0.13	0.18	0.09	0.05	0.09	0.14	0.09	0.11
$n = 512$																								
$m = n^{0.4}$	-0.21	-0.17	0.25	0.18	0.17	0.29	-0.35	-0.26	0.37	0.41	0.32	0.34	0.16	0.11	0.19	0.15	0.14	0.16	0.06	0.04	0.11	0.15	0.07	0.09
$m = n^{0.5}$	-0.17	-0.13	0.19	0.18	0.20	0.28	-0.29	-0.29	0.32	0.36	0.27	0.32	0.16	0.13	0.15	0.11	0.08	0.12	0.02	0.01	0.05	0.08	0.09	0.08
$d_1 = 0.2, d_2 = 0.3$																								
$n = 128$																								
$m = n^{0.4}$	-0.60	-0.46	0.42	0.68	0.57	0.54	-0.74	0.78	0.81	0.81	0.80	0.68	0.36	0.43	0.41	0.54	0.46	0.38	0.31	0.26	0.20	0.25	0.26	0.32
$m = n^{0.5}$	-0.44	-0.51	0.37	0.62	0.52	0.47	-0.65	0.73	0.76	0.84	0.75	0.65	0.44	0.25	0.28	0.41	0.43	0.42	0.27	0.17	0.21	0.15	0.29	0.24
$n = 512$																								
$m = n^{0.4}$	-0.47	-0.52	0.31	0.57	0.46	0.47	-0.68	0.75	0.74	0.76	0.75	0.67	0.42	0.35	0.35	0.45	0.40	0.35	0.19	0.15	0.14	0.18	0.16	0.21
$m = n^{0.5}$	-0.56	-0.45	0.29	0.55	0.45	0.48	-0.59	0.66	0.66	0.70	0.71	0.70	0.39	0.29	0.22	0.38	0.36	0.39	0.15	0.16	0.15	0.21	0.16	0.23

Table 3: Simulation Results for median bias and MAD where $\rho = 0$

	Model A						Model B						Model C						Model D					
	Bias			MAD			Bias			MAD			Bias			MAD			Bias			MAD		
	d_1	d_2	β	d_1	d_2	β	d_1	d_2	β	d_1	d_2	β	d_1	d_2	β	d_1	d_2	β	d_1	d_2	β	d_1	d_2	β
$d_1 = 0.05, d_2 = 0.4$																								
$n = 128$																								
$m = n^{0.4}$	0.048	0.046	0.050	0.05	0.046	0.06	0.067	0.054	0.07	0.061	0.051	0.08	0.042	0.043	0.03	0.041	0.044	0.04	0.029	0.032	0.03	0.045	0.044	0.04
$m = n^{0.5}$	0.041	0.027	0.041	0.06	0.037	0.05	0.053	0.049	0.07	0.049	0.054	0.07	0.036	0.041	0.05	0.045	0.038	0.04	0.024	0.034	0.03	0.048	0.036	0.03
$n = 512$																								
$m = n^{0.4}$	0.045	0.042	0.043	0.04	0.045	0.05	0.057	0.049	0.06	0.066	0.061	0.06	0.042	0.029	0.04	0.042	0.036	0.04	0.034	0.061	0.03	0.049	0.028	0.03
$m = n^{0.5}$	0.040	0.037	0.045	0.04	0.043	0.04	0.055	0.051	0.05	0.064	0,057	0.06	0.037	0.031	0.04	0.035	0.035	0.03	0.026	0.019	0.02	0.034	0.024	0.04
$d_1 = 0.2, d_2 = 0.3$																								
$n = 128$																								
$m = n^{0.4}$	0.092	0.074	0.065	0.10	0.083	0.07	0.102	0.103	0.09	0.107	0.103	0.08	0.073	0.068	0.07	0.065	0.053	0.06	0.046	0.041	0.03	0.062	0.058	0.06
$m = n^{0.5}$	0.090	0.076	0.060	0.09	0.081	0.08	0.105	0.102	0.09	0.093	0.112	0.09	0.072	0.064	0.07	0.063	0.052	0.07	0.049	0.047	0.05	0.055	0.057	0.05
$n = 512$																								
$m = n^{0.4}$	0.086	0.065	0.054	0.09	0.073	0.07	0.095	0.086	0.09	0.109	0.094	0.07	0.069	0.057	0.06	0.064	0.051	0.04	0.038	0.045	0.03	0.056	0.051	0.06
$m = n^{0.5}$	0.082	0.062	0.047	0.09	0.075	0.08	0.094	0.095	0.08	0.098	0.097	0.06	0.075	0.066	0.07	0.054	0.054	0.05	0.041	0.042	0.05	0.054	0.047	0.04

Table 4: Simulation Results for median bias and MAD where $\rho = 0.5$

	Model A						Model B						Model C						Model D					
	Bias			MAD			Bias			MAD			Bias			MAD			Bias			MAD		
	d_1	d_2	β	d_1	d_2	β	d_1	d_2	β	d_1	d_2	β	d_1	d_2	β	d_1	d_2	β	d_1	d_2	β	d_1	d_2	β
$d_1 = 0.05, d_2 = 0.4$																								
$n = 128$																								
$m = n^{0.4}$	-0.051	-0.04	0.057	0.06	0.056	0.06	-0.08	-0.065	0.07	0.064	0.071	0.07	0.053	0.042	0.05	0.045	0.048	0.05	0.045	0.003	0.04	0.055	0.042	0.04
$m = n^{0.5}$	-0.046	-0.04	0.054	0.05	0.048	0.05	-0.07	-0.063	0.05	0.075	0.068	0.07	0.042	0.041	0.04	0.051	0.045	0.05	0.038	0.008	0.04	0.043	0.038	0.03
$n = 512$																								
$m = n^{0.4}$	-0.053	-0.03	0.057	0.04	0.046	0.05	-0.07	-0.26	0.06	0.071	0.062	0.06	0.047	0.040	0.05	0.045	0.044	0.04	0.037	0.012	0.05	0.063	0.046	0.03
$m = n^{0.5}$	-0.046	-0.04	0.048	0.04	0.052	0.05	-0.06	-0.29	0.06	0.067	0.052	0.07	0.049	0.038	0.04	0.041	0.037	0.04	0.032	0.004	0.08	0.019	0.014	0.02
$d_1 = 0.2, d_2 = 0.3$																								
$n = 128$																								
$m = n^{0.4}$	-0.093	0.07	0.074	0.09	0.086	0.08	-0.09	0.101	0.08	0.121	0.095	0.08	0.064	0.074	0.07	0.085	0.073	0.06	0.061	0.055	0.05	0.054	0.046	0.06
$m = n^{0.5}$	-0.075	0.07	0.066	0.09	0.083	0.07	-0.08	0.098	0.09	0.104	0.099	0.08	0.072	0.053	0.05	0.082	0.075	0.07	0.059	0.049	0.04	0.052	0.052	0.05
$n = 512$																								
$m = n^{0.4}$	-0.078	-0.08	0.063	0.08	0.074	0.08	-0.06	0.093	0.09	0.091	0.097	0.09	0.073	0.063	0.06	0.078	0.073	0.07	0.048	0.045	0.04	0.046	0.047	0.05
$m = n^{0.5}$	-0.087	-0.09	0.057	0.08	0.075	0.06	-0.07	0.089	0.11	0.090	0.092	0.09	0.068	0.059	0.06	0.079	0.065	0.07	0.043	0.042	0.05	0.042	0.050	0.05

For Model A, the values of the bias are high for almost all the specifications. The RMSE decreases for all the parameters for a larger bandwidth. The bias and RMSE of d_1 are higher than those of d_2. In Model B, the biases and RMSE are found to be larger when there is no short run dynamics. However, both bias and RMSE decreases for larger bandwidth and sample size chosen. For Model C, the LW estimator appears to perform better than Model A, as the bias and RMSE are lower. Finally, the simulation for Model D works very well and produces unbiased estimates with very low bias and RMSE compared to the other models. In general, for all models, when the memory parameters are closer $(d_1, d_2) = (0.2, 0.3)$, even for larger n, the bias is more severe; however matters improve for larger bandwidth. On the other hand, for $(d_1, d_2) = (0.05, 0.4)$ the sizes of bias and RMSE are better on average.

In tables 1 and 2, the values of the mean bias are very high, which might indicate that the first moment of the estimator does not exist. As a result, the median bias and the median absolute deviation (MAD) are reported in tables 3 and 4 instead of the mean bias and the root mean square error (RMSE). The median bias for Model A is very low. In Model B, the biases and RMSE are found to be larger when there is no short run dynamics. In addition, the LW estimator appears to perform better in Model C than in Model A, as the median bias and MAD are lower over the different bandwidths. Finally, the simulation for Model D works very well and produces unbiased estimates with very low bias and RMSE compared to the other models. In general, both median bias and MAD decreases for all the parameters as the bandwidth increases. The median bias and MAD of d_1 are higher than those of d_2.

Overall, it seems difficult to draw exact conclusions about the effect of ρ, as it only causes the bias to change sign but does not change the size of bias or RMSE and it has no significant effect on the performance of the LW estimator. On the other hand, relatively larger bandwidth appears to be preferable as the LWE works best.

4 Application to the US and Canada inflation rates

Consumer price indices of the United States and Canada are originally examined. Monthly inflation rates (116 observations) are calculated based on the CPI measure of the US and Canada. This data measures the inflation rate for each month as the percentage increase from the same month of the previous year. The empirical analysis has been carried out using the monthly US and Canada inflation rates for the time period of January 2001 to August 2010. The series was obtained from the USA Federal Reserve Bank and the Bank of Canada respectively.

Figure 1 provides graphs of the inflation rates, correlograms and periodograms respectively. The inflation rates in United States and Canada show the same trend movements which increased steadily with some oscillations to mid 2008 where the trend sharply declined. This similarity in the patterns between the US and Canada inflation rates, though not the levels, can lead to a potential cointegration relation between the two series. The corresponding correlograms exhibit the typical hyperbolic decline associated with long memory processes, while the periodograms in figure 1 confirm the presence of long memory features in the inflation series.

Figure 1: The inflation rate, the correlogram (ACF) and the periodogram of USA and Canada respectively

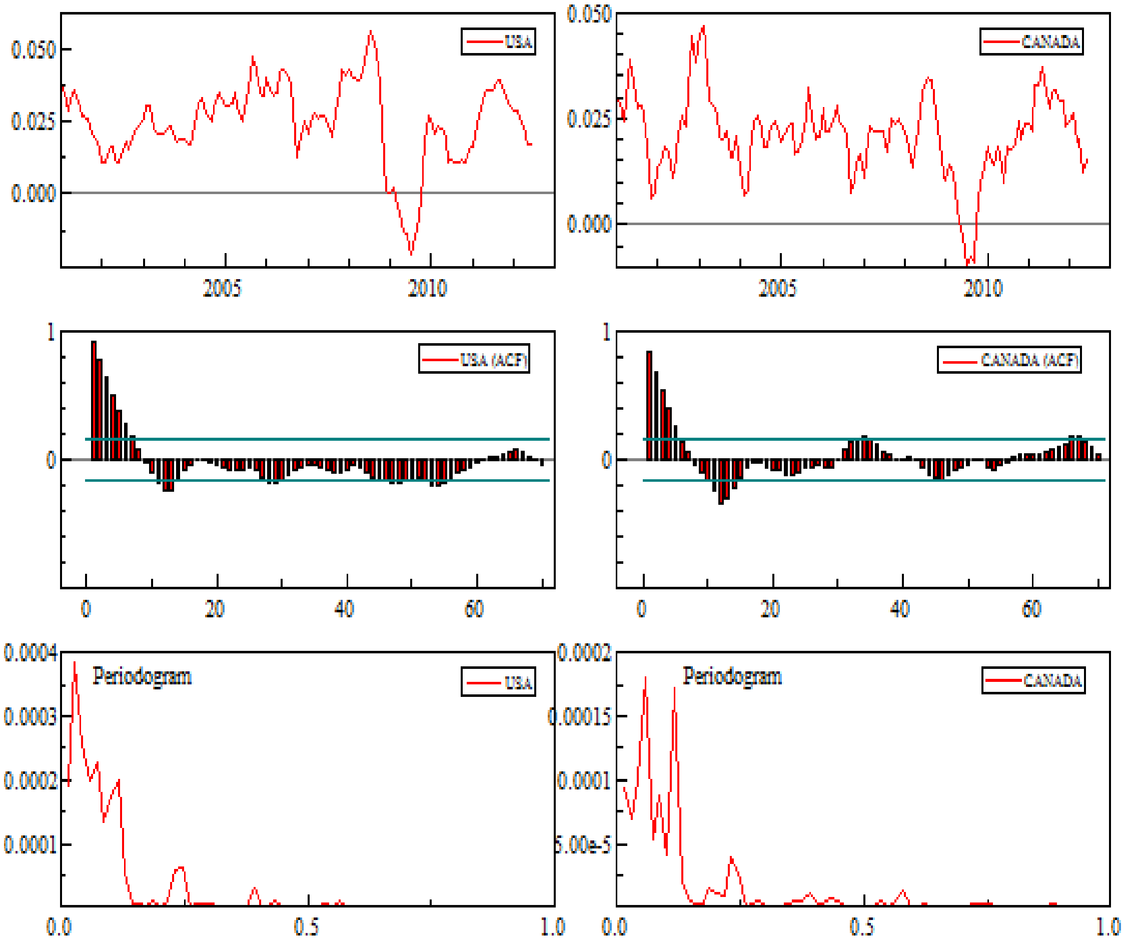

Table 5: Descriptive Statistics and Unit Root Tests

USA					
	Obs.	Mean	S.D.	Min.	Max.
	116	0.024371	0.014422	-0.021	0.056
	Skewness	Kurtosis	J.B.	ADF	KPSS
	-0.76105	4.0196	16.223*	-3.817	0.483
Canada					
	Obs.	Mean	S.D.	Min.	Max.
	116	0.020147	0.010036	-0.009	0.047
	Skewness	Kurtosis	J.B.	ADF	KPSS
	0.23046	4.1825	7.7850**	-5.912	0.365

Note: * and ** denote statistical significance of J.B. at the 1% and 5% levels respectively. The critical values of ADF unit root tests are -2.54, -1.94, -1.61 at 1%, 5%, 10% levels of significance. The critical values for KPSS test are 0.784, 0.521 and 0.437 at 1%, 5%, 10% levels of significance.

Table 6: The estimates of the LM parameters

	USA		Canada	
$m = n^{0.45}$				
	$\hat{d}_{GPH}$	$\hat{d}_{LW}$	$\hat{d}_{GPH}$	$\hat{d}_{LW}$
	0.473	0.488	0.318	0.274
	(0.147)	(0.112)	(0.098)	(0.073)
$m = n^{0.5}$				
	$\hat{d}_{GPH}$	$\hat{d}_{LW}$	$\hat{d}_{GPH}$	$\hat{d}_{LW}$
	0.448	0.426	0.291	0.236
	(0.172)	(0.124)	(0.109)	(0.080)

Note: The numbers in the parenthesis are standard errors.

Table 5 reports several descriptive statistics along with two unit root tests, including mean, standard deviation, skewness, kurtosis, Jarque-Bera statistic, ADF and KPSS. The US inflation rate averaged 2.4%, while the Canada inflation rate averaged 2%[1]. The values in the table give some information about the distribution of the US and Canada inflation rates. Both skewness and kurtosis statistics indicate that distributions are not normal. According to JB statistic, it is very clear that there are significant departures from normality.

The next step of the analysis is to examine the unit root properties of the inflation rates using Augmented Dickey-Fuller (ADF) and Kwiatkowski, Phillips, Schmidt and Shin (KPSS) unit root tests. The results are presented in table 5 and suggest that both series can be represented by stationary long memory processes. However, these unit root tests, especially the ADF, do not take account of the possible long memory properties of the series. Therefore, two semiparametric methods are employed to examine the long memory properties of the data.

The GPH and LW estimators for long memory parameters are reported in table 6 for different bandwidths $m = n^{0.45}$ and $n^{0.5}$ where all the estimates can be seen to be statistically significant. The results show that the memory estimates are not sensitive to the bandwidth choice, although they decrease as the bandwidth increases (the memory estimates vary from 0.42 to 0.48 and 0.23 to 0.31 for US and Canada respectively) and that inflation rates exhibits stationary long memory properties. Consequently, if there exists a stable relationship between the inflation rates, a stationary fractional cointegration would be expected.

Now, consider the bivariate model in (2.4),

$$\pi_{USA,t} - \beta\pi_{CAN,t} = u_{1t}$$
$$\pi_{CAN,t} = u_{2t}$$

when $\beta \neq 0$, the two series $\pi_{USA,t}$ and $\pi_{CAN,t}$ are said to be cointegrated where the linear combination u_{1t} has a memory of d_1 which less than the memory of the original two series.

[1] The Bank of Canada aims to keep inflation rate at the 2% midpoint of an inflation-control target range of 1-3 %.

Table 7: Application to the US and Canada inflation rates

$m = n^{0.45}$				
	$\hat{d}_1$	$\hat{d}_2$	$\hat{\beta}$	$\hat{\gamma}$
	0.072	0.356	1.165	0.206
	(0.033)	(0.1138)	(0.298)	(0.081)
$m = n^{0.5}$				
	$\hat{d}_1$	$\hat{d}_2$	$\hat{\beta}$	$\hat{\gamma}$
	0.056	0.328	1.149	0.281
	(0.029)	(0.156)	(0.352)	(0.110)

Note: Standard errors are reported in the parentheses.

Table 7 reports the joint local Whittle estimation including the estimates of the four unknown (two memory, phase and cointegration) parameters, while the standard errors are represented in parentheses. The results indicate that all the coefficients are statistically significant for both bandwidths $m = n^{0.45}$ and $n^{0.5}$, respectively. The estimate of memory parameter d_2 indicates that the inflation rates can be described as stationary long memory series confirming the results in table 12. In addition, the estimate of d_1 for the unknown linear combination appears to have less memory than d_2. Moreover, the estimate of the cointegrating parameter β is close to unity reflecting a cointegration relationship between the US and Canada inflation rates. In particular, the LW estimates of the cointegration coefficient are significantly higher than unity for bandwidths $m = n^{0.45}$ and $n^{0.5}$, implying that the long-run rate of inflation in the US is higher than that in Canada.

5 Conclusion

One contribution of this paper is to apply the theoretical framework in Robinson (2008) to inflation rates which allows for a new parameter, the phase shift, to the bivariate model. The possibility of existence of long memory features in the inflation rates was initially examined, then the relationship between the monthly US and Canada inflation rates was analysed using the analysis in Robinson (2008). This approach is preferable to other conventional methods as it allows for the possibility of phase shifts along with cointegration. The four unknown parameters were jointly estimated using local Whittle estimation.

The main finding is that the monthly US and Canada inflation rates exhibit the properties of stationary long memory series confirming the presence of long memory in macroeconomic time series which is consistent with the results reported in Hassler and Wolters (1995) and Doornik and Oooms (2004). The local Whittle estimate gives evidence to a fractional cointegration relationship between the US and Canada inflation rates, with the estimate of the cointegrating parameter, β, is higher than unity. This implies that the long-run rate of inflation in the US is higher than that in Canada. Furthermore, this link between inflation rates in the US and Canada has its vital implications on the interdependence of monetary policies in both countries and the validity of purchasing power parity. As the US and Canada have differing rates of inflation, and the relative price of goods is linked to the exchange rate through the purchasing power parity theory. The relative prices of goods should change and the value of US dollar may decline against the Canadian dollar.

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