Outline of this Lecture

1. Density, Unit weight, and Specific gravity (G_s)
2. Phases in soil (a porous medium)
3. Three phase diagram
4. Weight-volume relationships

For a general discussion we have

Weight W

$$W = Mg$$

density ρ

$$\rho = \frac{M}{V}$$

Unit weight γ

$$\gamma = \frac{W}{V}$$

So that

$$\gamma = \frac{W}{V} = \frac{Mg}{V} = \rho g$$

Unit weight: $\gamma = \rho g$

Unit weight is the product of density and gravity acceleration. It is the gravitational force caused by the mass of material within a unit volume (density) in the unit of Newtons per cubic meter in SI system.

Specific Gravity (G_s)

Specific gravity is defined as the ratio of unit weight (or density) of a given material to the unit weight (or density) of water since

$$G_s = \frac{\gamma}{\gamma_w} = \frac{\rho g}{\rho_w g} = \frac{\rho}{\rho_w}$$

Specific Gravity

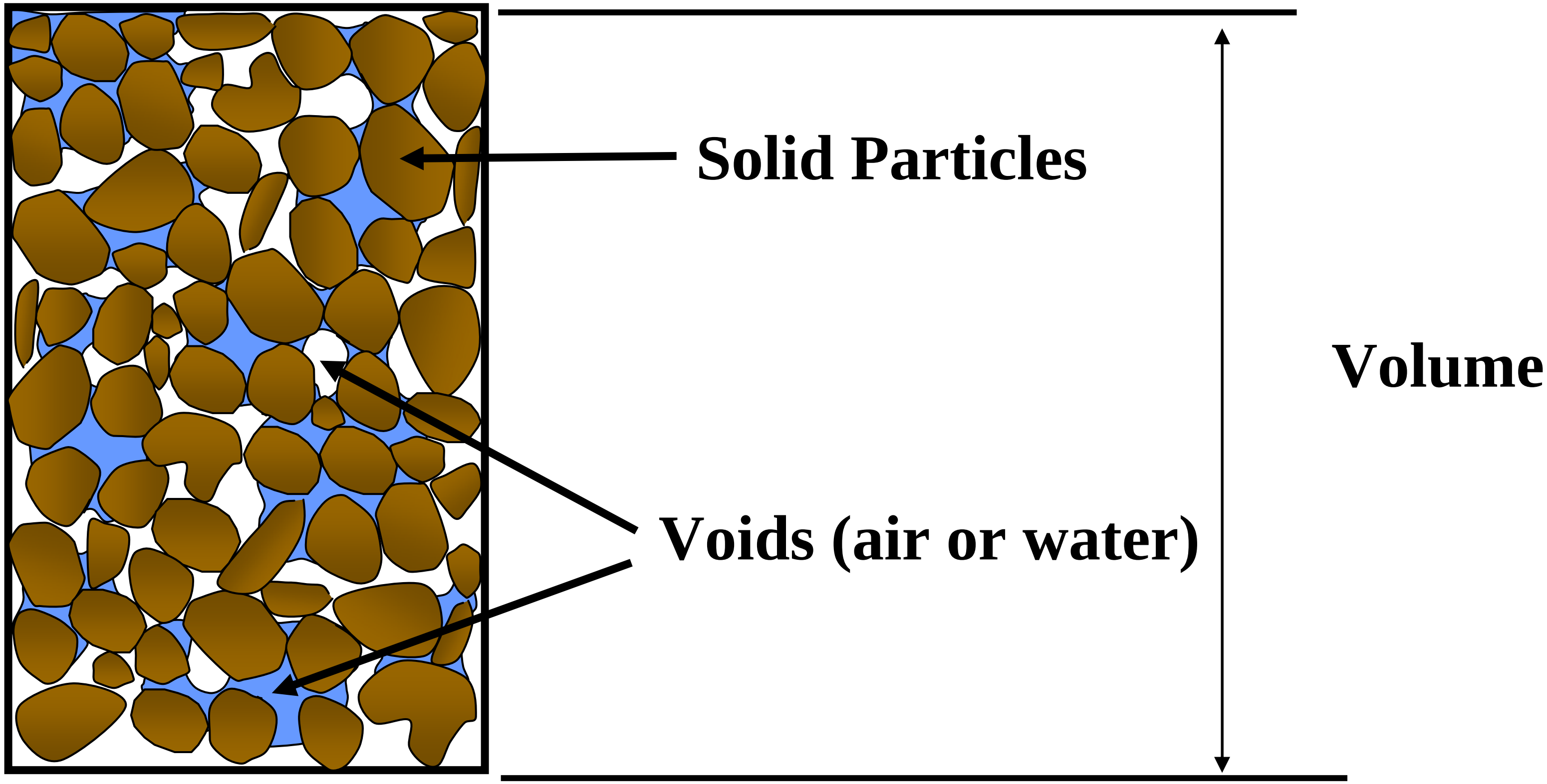
- Expected Value for Gs

Type of Soil	Gs
Sand	2.65 - 2.67
Silty sand	2.67 – 2.70
Inorganic clay	2.70 – 2.80
Soils with mica or iron	2.75 – 3.00
Organic soils	< 2.00

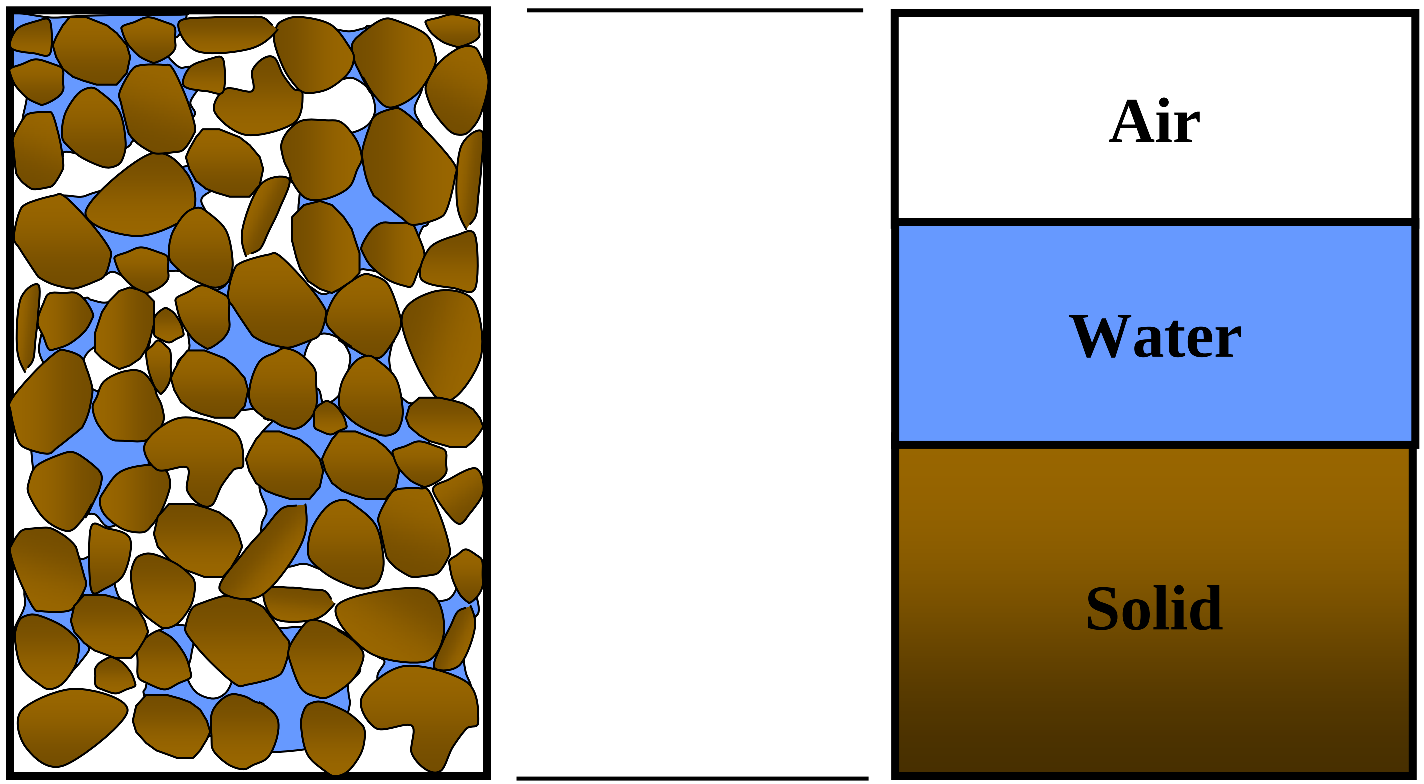
Three Phases of Soils

Naturally occurred soils always consist of solid particles, water, and air, so that soil has three phases: solid, liquid and gas.

Soil model



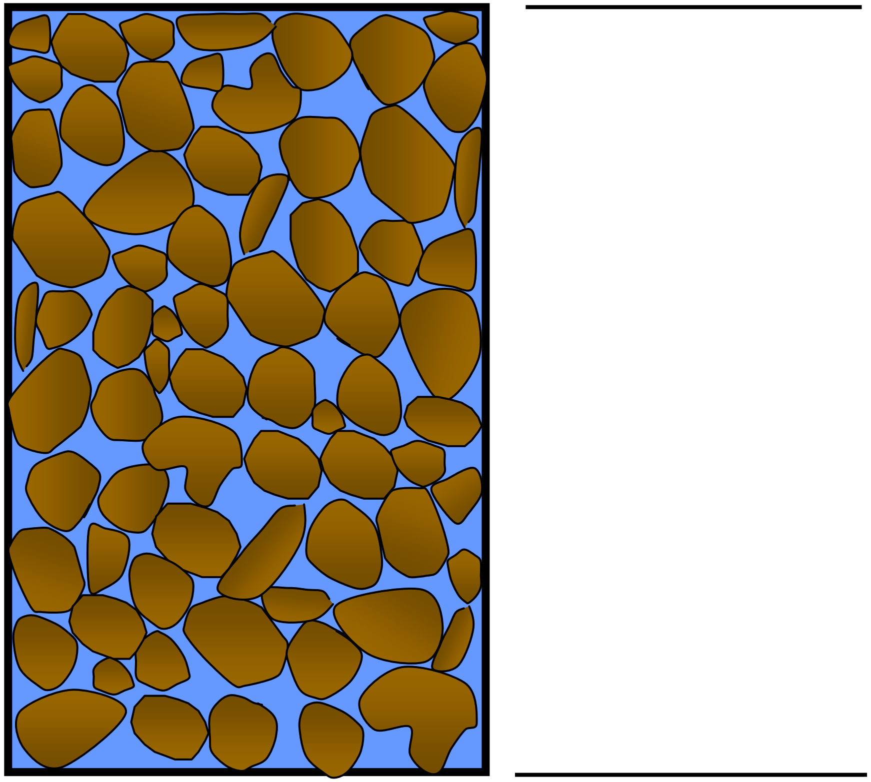
Three Phase Diagram



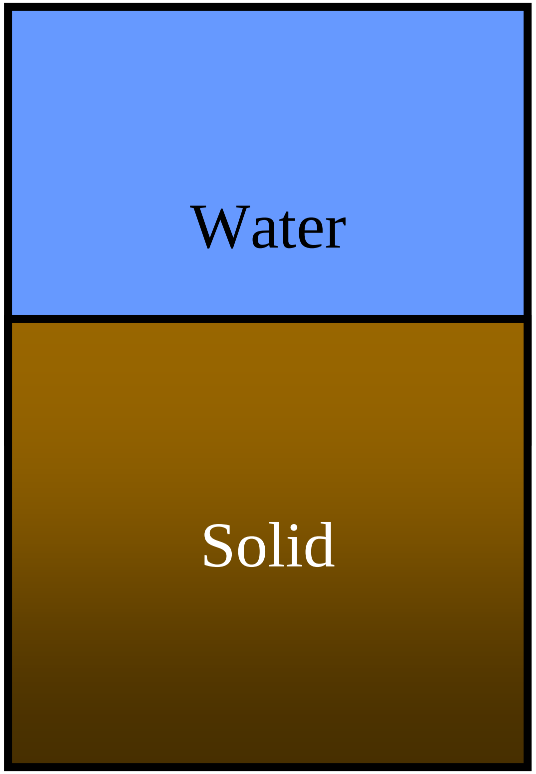
Mineral Skeleton

Idealization:
Three Phase Diagram

Fully Saturated Soils (Two phase)

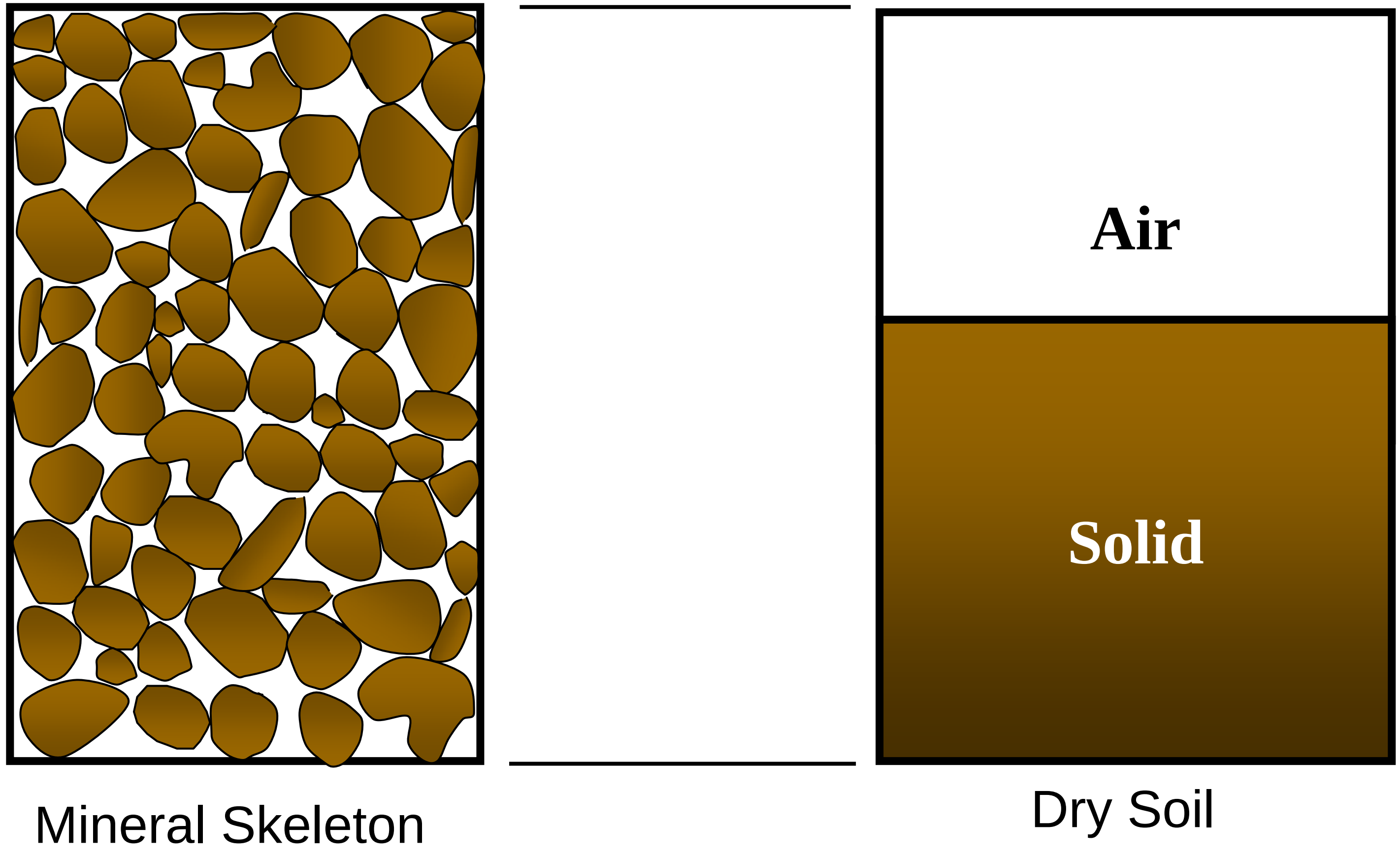


Mineral Skeleton

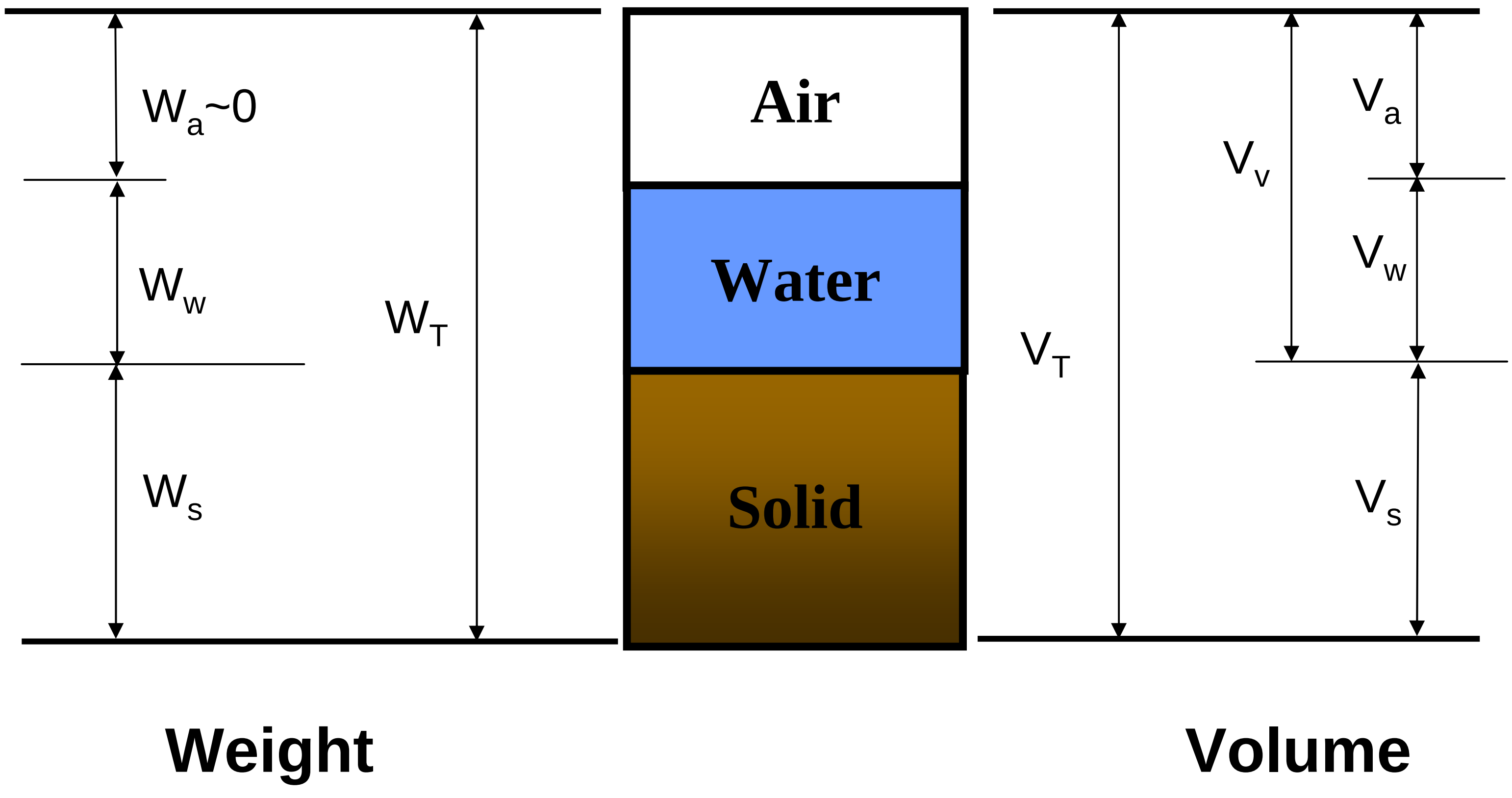


Fully Saturated

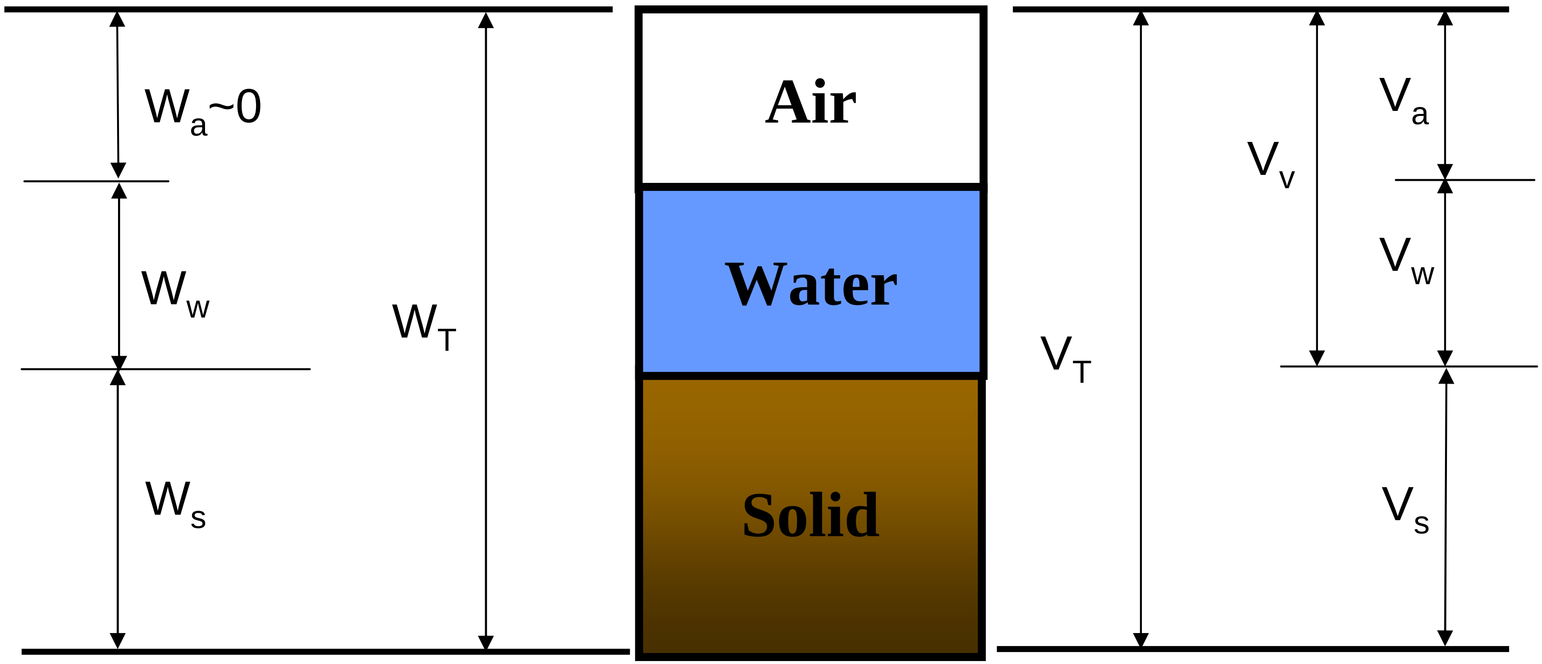
Dry Soils (Two phase) [Oven Dried]



Three Phase Diagram

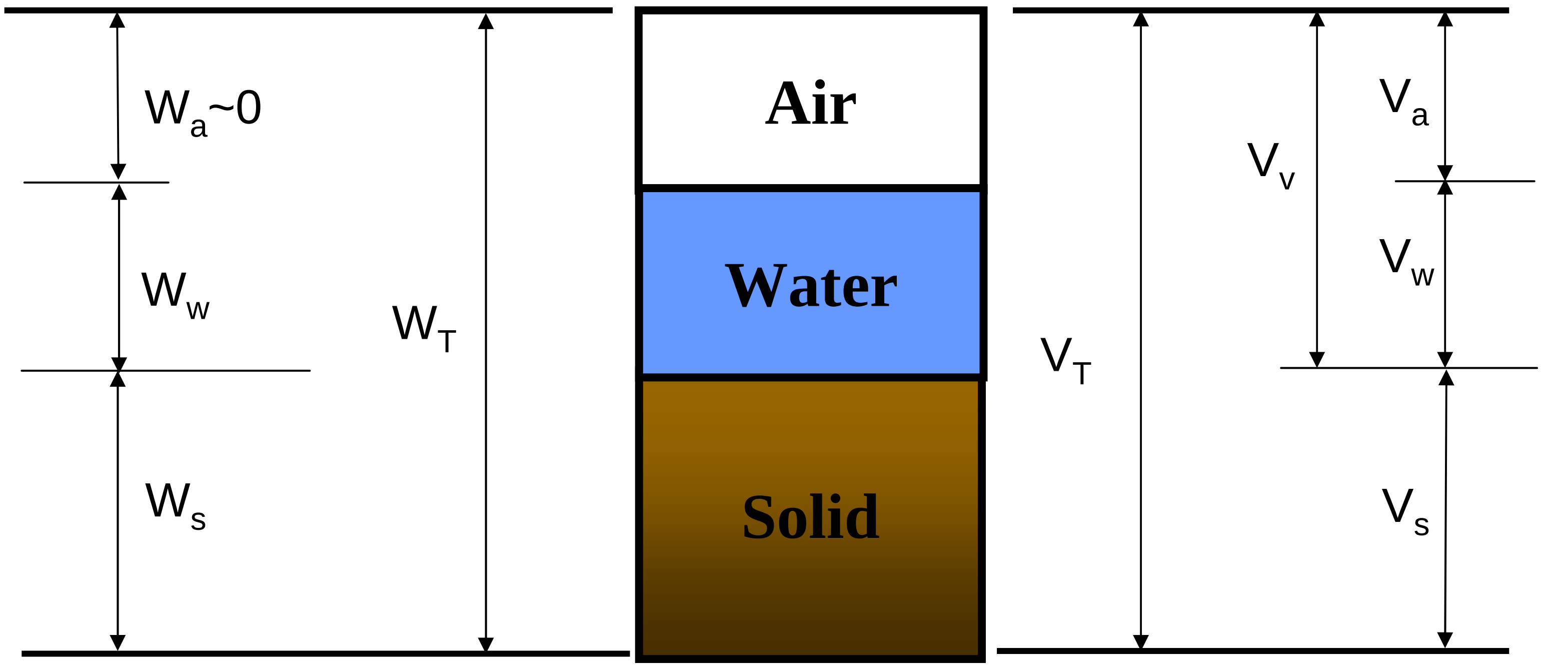


Phase relationship: the phase diagram



Wt: total weight
Ws: weight of solid
Ww: weight of water
Wa: weight of air = 0

Vt: total volume
Vs: volume of solid
Vw: volume of water
Vv: volume of the void



Vt = Vs + Vv = Vs + Vw + Va;

It is convenient to assume the volume of the solid phase is unity (1) without lose generality.

Mt = Ms + Mw; and

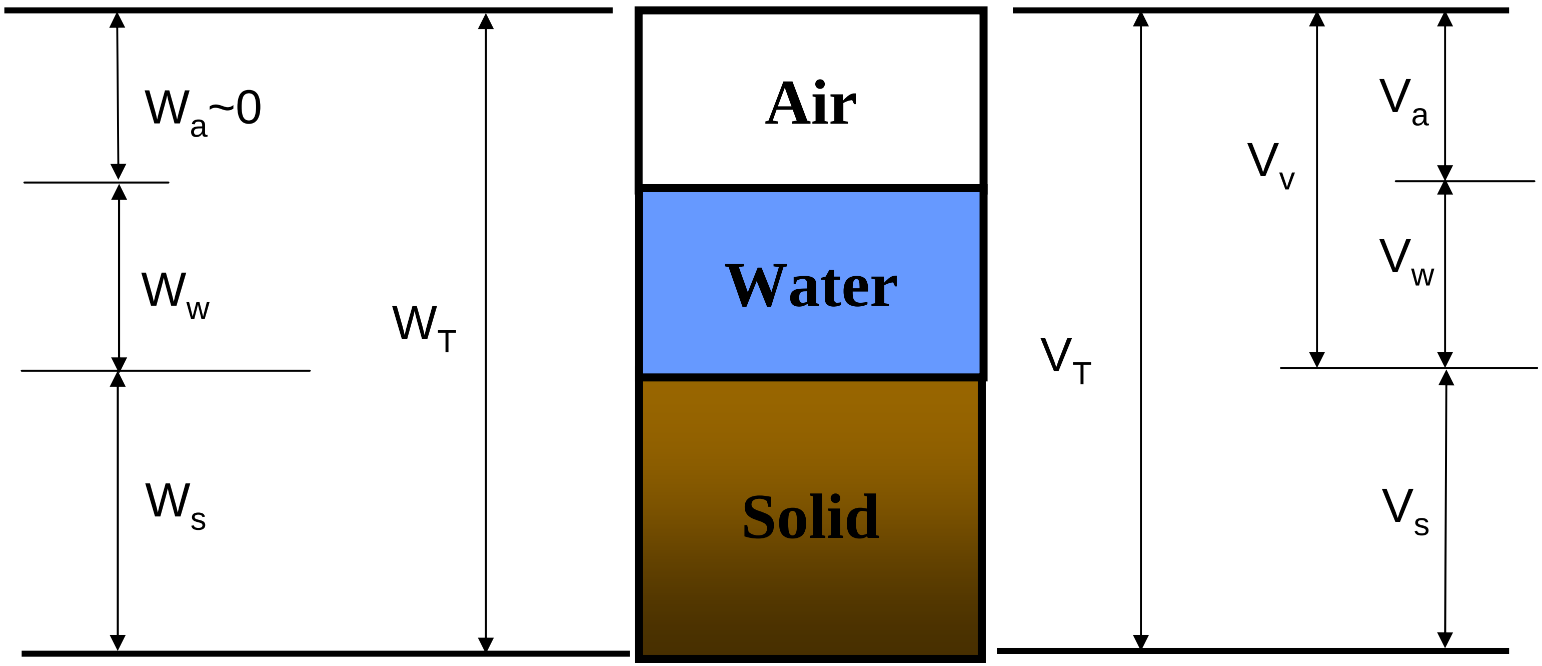
Wt = Ws + Ww, since W=Mg

Void ratio: e = Vv/Vs; Porosity n = Vv/Vt

$$e = \frac{V_v}{V_s} = \frac{V_v}{V_t - V_v} = \frac{V_v / V_t}{1 - V_v / V_t} = \frac{n}{1-n}$$

$$n = \frac{V_v}{V_t} = \frac{V_v}{V_s + V_v} = \frac{V_v / V_s}{1 + V_v / V_s} = \frac{e}{1+e}$$

Apparently, for the same material we always have e > n. For example, when the porosity is 0.5 (50%), the void ratio is 1.0 already.



Degree of saturation: S =Vw/Vv x 100%

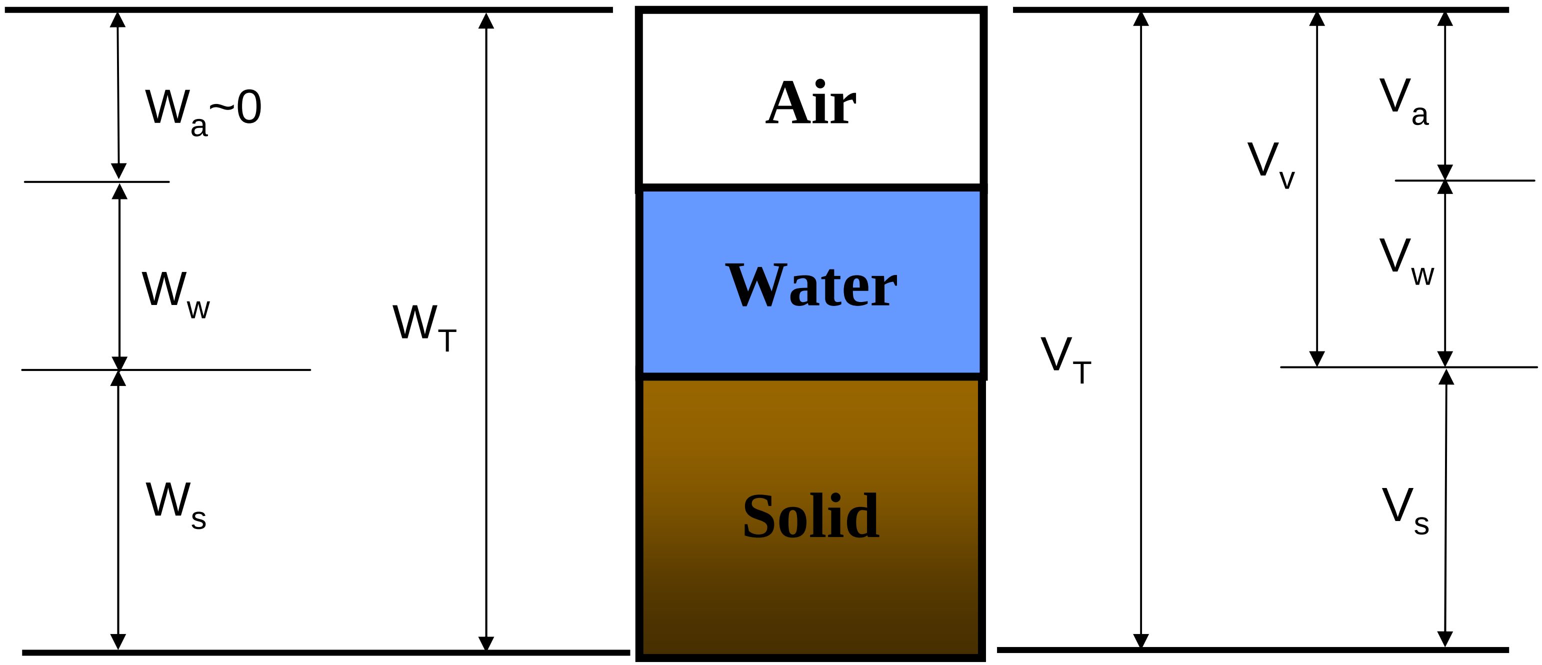
Saturation is measured by the ratio of volume.

Moisture content (Water content): *w* = Ww/Ws,

Ww – weight of water, Ws – weight of solid

Water content is measured by the ratio of weight.

So that *w* can be greater than 100%.



Degree of saturation: S =Vw/Vv x 100%

Saturation is measured by the ratio of volume.

Moisture content: *w* = Ww/Ws, Ww=Vwγ_w

Ww – weight of water, Ws – weight of solid

Water content is measured by the ratio of weight.

Definition of 3 types of unit weight

Total unit weight (moisture unit weight, wet unit weight) γ :

$$\gamma = \frac{W_t}{V_t} = \frac{W_s + W_w}{V_t}$$

Dry unit weight γ_d :

$$\gamma_d = \frac{W_s}{V_t}, \quad \because V_t > V_s \quad \therefore \gamma_d < \gamma_s$$

Saturated unit weight (when saturation S=1) γ_{sat} :

$$\gamma_{sat} = \frac{W_t}{V_t}$$

Moisture unit weight γ:

$$\gamma = \frac{W_t}{V_t} = \frac{W_s + W_w}{V_t}$$

Solid unit weight γ_s

$$\gamma_s = \frac{W_s}{V_s}$$

dry unit weight γ_d

$$\gamma_d = \frac{W_s}{V_t}, \quad \because V_t > V_s \quad \therefore \gamma_d < \gamma_s$$

Since

$$\gamma_d = \frac{W_s}{V_t} = \frac{W_t - W_w}{V_t} = \frac{W_t}{V_t} - \frac{W_w}{V_t} = \gamma - \frac{W_w W_s}{W_s V_t} = \gamma - w\gamma_d$$

$$so \quad that \quad \gamma_d + w\gamma_d = \gamma \quad and \quad \gamma_d = \frac{\gamma}{1 + w}$$

From the original form of the dry unit weight

$$\gamma_d = \frac{\gamma}{1+w}$$

By taking the Taylor expansion and truncated at the first order term:

$$\gamma_d = \frac{\gamma}{1+w} = \gamma(1-w+w^2-w^3+w^4-w^5+\ldots) \approx \gamma(1-w)$$

Because the moisture content *w* is a number always smaller than one, i.e., w<1.

Thus, the dry unit weight γ_d can be approximated as:

$$\gamma_d = (1-w)\gamma$$

Relationships among *S, e, w, and* G$_s$

$$S = \frac{V_w}{V_v}, then \quad V_w = SV_v = Se, \qquad given \quad V_s = 1$$

A simple way to get Das, Equation 3.18

$$w = \frac{W_w}{W_s} = \frac{\gamma_w V_w}{\gamma_s V_s} = \frac{\gamma_w eS}{G_s \gamma_w} = \frac{eS}{G_s}, \quad \because V_s = 1, \quad \therefore V_v = 1$$

$$thus \quad Se = wG_s$$

When the soil is 100% saturated (S=1)
we have, Equation 3.20

$$e = wG_s$$

Relationships among γ, *n*, *w*, *and* G_s

$$W_s = \gamma_s V_s = G_s \gamma_w (1-n), \qquad given \quad V_s = 1-n$$

$$W_w = wW_s = wG_s \gamma_w (1-n)$$

So that the dry unit weight γ_d is

$$\gamma_d = \frac{W_s}{V_t} = \frac{G_s \gamma_w (1-n)}{1-n+n} = G_s \gamma_w (1-n)$$

And the moist unit weight γ is

$$\gamma = \frac{W_t}{V_t} = \frac{W_s + W_w}{V_t} = \frac{G_s \gamma_w (1-n) + wG_s \gamma_w (1-n)}{1}$$

$$= (1+w) G_s \gamma_w (1-n) = G_s \gamma_w (1-n)(1+w)$$

Relationships among γ, *n*, *w*, *and* G_s (cont.)

When S=1 (fully saturated soil)

$$\gamma_{sat} = \frac{W_s + W_w}{V_t} = \frac{G_s\gamma_w(1-n) + n\gamma_w}{1} = [G_s(1-n) + n]\gamma_w$$

the moisture content *w* when S=1 can be expressed as

$$w = \frac{W_w}{W_s} = \frac{n\gamma_w}{G_s\gamma_w(1-n)} = \frac{n}{G_s(1-n)} = \frac{e}{G_s}$$

$$recall \quad Se = wG_s, thus \quad e = wG_s \qquad \because S = 1$$

Weight-Volume Relationships (Table 3.1)

Table 3.1 Various Forms of Relationships for γ, γ_d, and γ_{sat}

Moist unit weight (γ)		***Dry unit weight*** (γ_d)		***Saturated unit weight*** (γ_{sat})	
Given	**Relationship**	**Given**	**Relationship**	**Given**	**Relationship**
w, G_s, e	$\frac{(1+w)G_s\gamma_w}{1+e}$	γ, w	$\frac{\gamma}{1+w}$	G_s, e	$\frac{(G_s+e)\gamma_w}{1+e}$
S, G_s, e	$\frac{(G_s+Se)\gamma_w}{1+e}$	G_s, e	$\frac{G_s\gamma_w}{1+e}$	G_s, n	$[(1-n)G_s+n]\gamma_w$
w, G_s, S	$\frac{(1+w)G_s\gamma_w}{1+\frac{wG_s}{S}}$	G_s, n	$G_s\gamma_w(1-n)$	G_s, w_{sat}	$\left(\frac{1+w_{sat}}{1+w_{sat}G_s}\right)G_s\gamma_w$
w, G_s, n	$G_s\gamma_w(1-n)(1+w)$	G_s, w, S	$\frac{G_s\gamma_w}{1+\left(\frac{wG_s}{S}\right)}$	e, w_{sat}	$\left(\frac{e}{w_{sat}}\right)\left(\frac{1+w_{sat}}{1+e}\right)\gamma_w$
S, G_s, n	$G_s\gamma_w(1-n)+nS\gamma_w$	e, w, S	$\frac{eS\gamma_w}{(1+e)w}$	n, w_{sat}	$n\left(\frac{1+w_{sat}}{w_{sat}}\right)\gamma_w$
		γ_{sat}, e	$\gamma_{sat}-\frac{e\gamma_w}{1+e}$	γ_d, e	$\gamma_d+\left(\frac{e}{1+e}\right)\gamma_w$
		γ_{sat}, n	$\gamma_{sat}-n\gamma_w$	γ_d, n	$\gamma_d+n\gamma_w$
		γ_{sat}, G_s	$\frac{(\gamma_{sat}-\gamma_w)G_s}{(G_s-1)}$	γ_d, S	$\left(1-\frac{1}{G_s}\right)\gamma_d+\gamma_w$
				γ_d, w_{sat}	$\gamma_d(1+w_{sat})$

The 3rd column is a special case of the 1st column when S = 1.

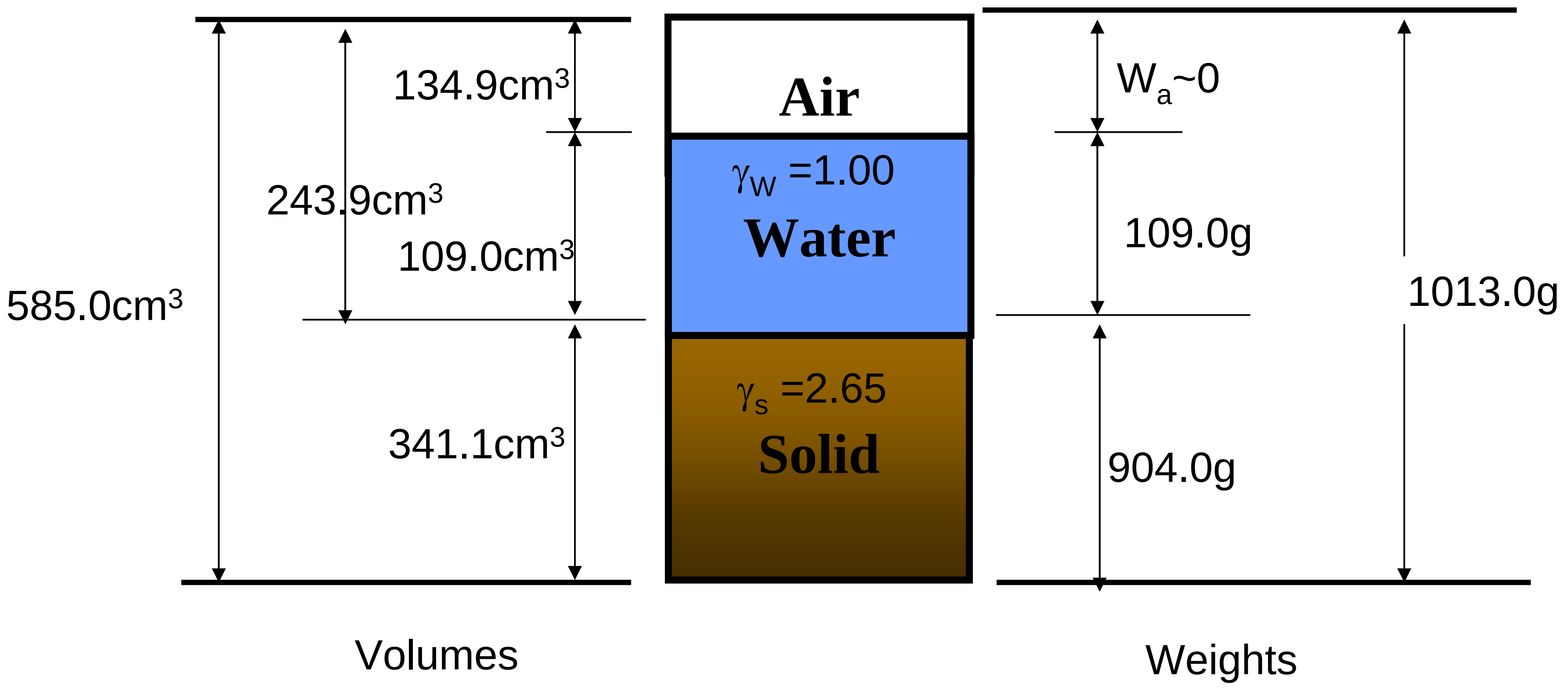
Example:

- Determine moisture content, void ratio, porosity and degree of saturation of a soil core sample. Also determine the dry unit weight, γ_d

Data:

- Weight of soil sample = 1013g
- Vol. of soil sample = 585.0cm^3
- Specific Gravity, G_s = 2.65
- Dry weight of soil = 904.0g

Example

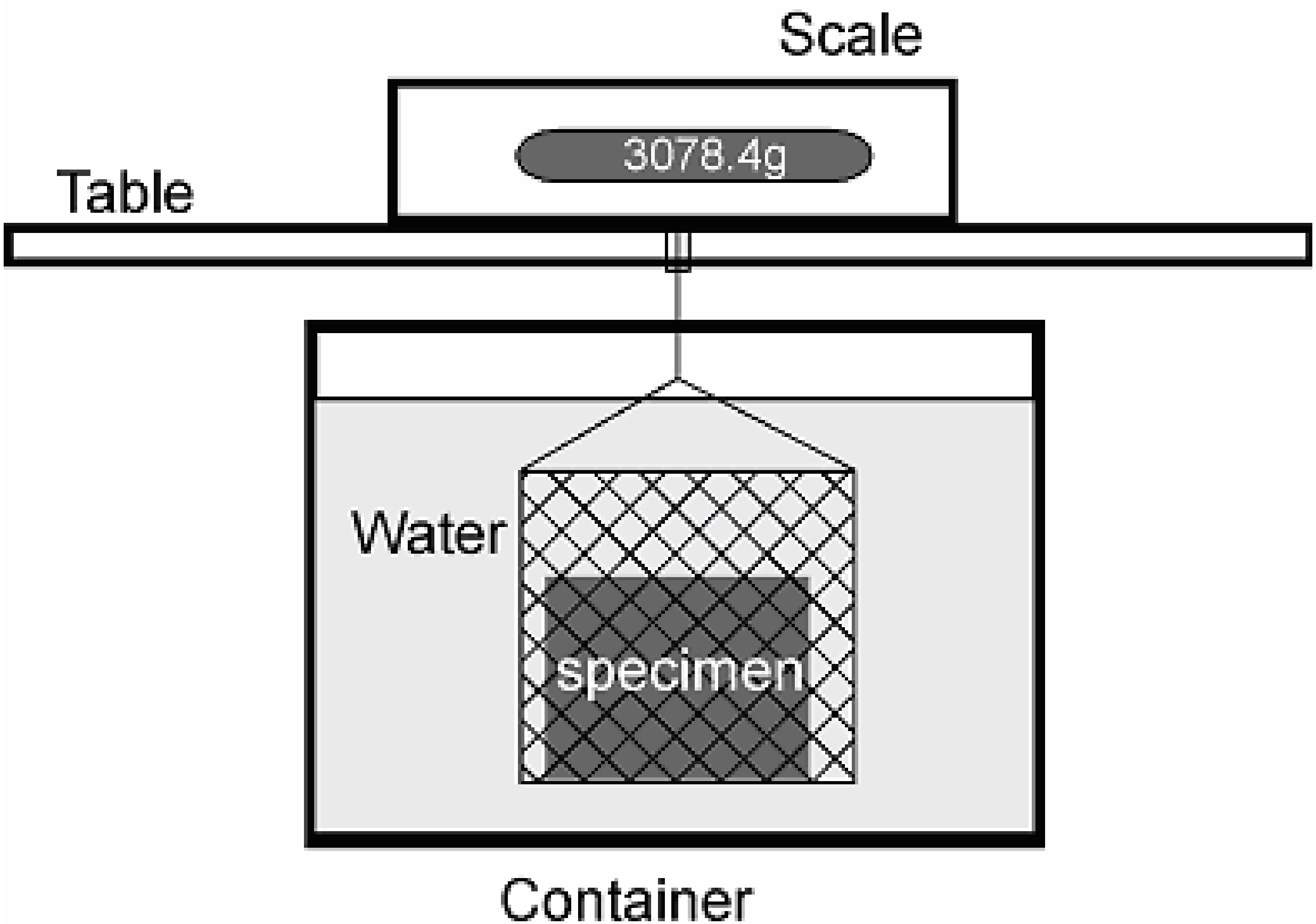


134.9cm3
243.9cm3
109.0cm3
585.0cm3
341.1cm3
Air
γW =1.00
Water
γs =2.65
Solid
Wa~0
109.0g
1013.0g
904.0g
Volumes
Weights

Results

- From the three phase diagram we can find:
  - Moisture content, *w* $$w = \frac{W_w}{W_s} = \frac{109\,(g)}{904\,(g)} \times 100 = 12.1\%$$
  - Void ratio, *e* $$e = \frac{V_v}{V_s} = \frac{243.9cm^3}{341.1cm^3} = 0.715$$
  - Porosity, *n* $$n = \frac{V_v}{V_T} = \frac{243.9\ (cm^3)}{585.0\ (cm^3)} \times 100 = 41.7\%$$
  - Degree of saturation, *S* $$S = \frac{V_w}{V_v} = \frac{109}{243.9} \times 100 = 44.7\%$$
  - Dry unit weight, γ_d $$\gamma_d = \frac{W_s}{V_T} = \frac{904}{585} = 1.55\ \frac{g}{cm^3}$$

Measurement of the submerged density (or unit weight)



Now consider the submerged case, i.e., the two-phase system has been put into the water:

$$M_w - buoyancy = e\rho_w - e\rho_w = 0 \quad and$$
$$M_s - buoyancy = G_s\rho_w - 1\rho_w = (G_s - 1)\rho_w$$

Thus, the submerged density is

$$\rho_{submerg} = \rho' = \frac{M_t'}{V_t} = \frac{M_w + M_s}{V_t} = \frac{0 + (G_s - 1)\rho_w}{1+e} = \frac{(G_s - 1)\rho_w}{1+e}$$

In a two-phase system, i.e., if S=100%, and we let V_s=1, we then have:

$$V_t = 1 + e, \quad \text{since} \quad V_w = V_v = e$$

Consider the not-submerged case, i.e., the two-phase system has been just put in the air:

$$M_w = \rho_w V_w = e\rho_w \qquad \textit{and}$$

$$M_s = \rho_s V_s = G_s \rho_w V_s = G_s \rho_w$$

Thus, the saturated density is

$$\rho_{sat} = \frac{M_w + M_s}{V_t} = \frac{e\rho_w + G_s\rho_w}{1+e} = \frac{\rho_w(e+G_s)}{1+e}$$

and the dry density is

$$\rho_d = \frac{M_s}{V_t} = \frac{G_s\rho_w}{1+e}$$

Recall that the saturated density is

$$\rho_{sat} = \frac{M_w + M_s}{V_t} = \frac{\rho_w (e + G_s)}{1 + e}$$

If we do the following

$$\rho_{sat} - \rho_w = \frac{(e + G_s)\rho_w}{1 + e} - \rho_w = \frac{(e + G_s)\rho_w - (1 + e)\rho_w}{1 + e}$$

$$= \frac{(e + G_s - 1 - e)\rho_w}{1 + e} = \frac{(G_s - 1)\rho_w}{1 + e} = \rho'$$

$$i.e., \quad \rho' = \rho_{sat} - \rho_w$$

If you have got the submerged density ρ' and sure you know water density ρ_w

$$\rho_{sat} = \frac{M_w + M_s}{V_t} = \frac{\rho_w (e + G_s)}{1 + e}$$

You can calculate the saturated density ρ_{sat}.If you know ρ_w then you can calculate the void ratio e. if you think you can know e from the dry density ρ_d

$$\rho_d = \frac{G_s \rho_w}{1 + e}$$

You can also calculate the submerged density ρ' when the sample is not 100% saturated.

$$\rho' = \frac{(G_s - 1)\rho_w + e(S - 1)}{1 + e}$$