

ACOUSTIC CHARACTERIZATION OF VIOLIN FAMILY SIGNATURE MODES BY INTERNAL CAVITY MEASUREMENTS

Colin Gough

School of Physics and Astronomy, University of Birmingham, UK
profgough@googlemail.com

ABSTRACT

The sound radiated by the signature modes of stringed instruments in their lowest two octaves is shown to be directly related to the sound pressure at the “acoustic centre” of their hollow bodies, where the nodal lines of the longitudinal $a1$ and transverse $a2$ internal dipole modes of the air cavity intersect. Pressure measurements inside the instrument itself can therefore be used to characterise the acoustic radiating properties of the signature modes of instruments of any size without contamination from resonances of the surrounding space or interference from external noise. This is illustrated by high resolution, acoustic, location-independent, measurements on violins, a viola and several double basses, which could all have been made in the luthier’s workshop without any sophisticated equipment.

1. INTRODUCTION

The sound of all instruments of the violin family in their lowest two octaves is strongly influenced by the low frequency $A0$, CBR , $B1$ - and $B1+$ signature modes, responsible for prominent resonances in both admittance and acoustic measurements. These modes are closely related to the f -hole Helmholtz resonance, centre bout rotation and a mixture of component bending and strongly volume-changing breathing mode vibrations (Gough [1]). The breathing mode component of the signature modes is responsible directly and indirectly (via excitation of the air bouncing in and out of the f -holes) for almost all the sound radiated by stringed instruments in their lowest two octaves, which is strongly correlated with the perceived quality of an instrument (Bissinger [2]).

Over the last few years, a large number of measurements of the radiated sound have been made on many violins including many Stradivari, Guarneri and outstanding modern instruments. A common method has been to measure the acoustic impulse response, with the radiated sound excited by a sharp tap at the bridge measured at typical distances between around 25 cm to 40 cm from the violin (Curtin [3]). This is already a compromise, as close to the violin the sound includes a strong, near-field component (instrument length $\sim$ 34-35 cm).

The near-field sound is important to the player, with their ears close to the instrument, but decays much faster than the radiated sound contributing very little sound to the distant listener. Considerably larger measurements distance would be needed for comparable quality measurements for larger members of the violin family like the cello and double bass. However, as the radiated sound intensity falls as $1/r^2$, contamination of the measurements from acoustical resonances of the surrounding space increases. As a result, very few measurements of the acoustic properties of comparable quality exist for larger instruments like the cello and double bass.

An alternative method of identifying the low frequency acoustic characteristics of even the largest of instruments is to measure the internal sound pressure, as this is directly related to the radiated sound in the monopole limit. This was first described and demonstrated by Jansson, Morsett and Guttler [4]. Somewhat surprisingly, this method has not been widely adopted as a characterization tool by makers or researchers, despite the simplicity of making such measurements.

The volume into which the internally radiated sound is confined is relatively small and extremely well defined. This results in easily reproducible high sound pressures, which are virtually free of any contamination from resonances of the surrounding acoustic space or extraneous noise. Such measurements can then be made on instruments of any size - even in a small luthier's workshop. Effectively, the hollow cavity of the instrument itself becomes the recording studio.

The relationship between the internal and radiated sound pressures is described by standard loudspeaker-enclosure theory originally used by Caldersmith [5] to describe the acoustical properties of the guitar and later by Schelleng [6] for the violin. A generalized model will be described, which accounts for both the sound radiated and the internal pressure excited by all four $A0$, CBR , $B1$ - and $B1+$ signature modes.

Following a brief description of the underlying theoretical model, examples of the radiated and internal sound pressures of the violin are described. Measurements on the violin, a viola and a number of double basses are then presented

The examples highlight the potential value of such simple measurements, especially for the larger stringed instruments, where the normal techniques used to measure their radiating properties at low frequencies are impracticable because of their large size.

2. MODEL

The standard loudspeaker enclosure model assumes a simple vibrating piston source of area S exciting pressure fluctuations $p = \gamma P_0 \Delta V / V$ in an enclosure of volume V at ambient pressure P_0 , where ΔV is the inward volume change and γ is the ratio of adiabatic and isothermal heat capacities for air. The pressure induces the Helmholtz resonance of a plug of air bouncing in and out of the cavity through one or more holes of total area A cut into the walls.

For the multi-resonant instruments of the violin family, the model can be generalised by the equivalent circuit shown in Figure 1. The currents represent the rate of volume flow u_{shell} induced by the flexural vibrations of the shell walls of thickness t , while $u_{\text{Helmholtz}}$ represents the induced flow of air in and out of the f -holes. The kinetic inductance of the m -th mode L_m is m_m / S_m^2 , where the effective mass at the driving point r_o is given by $m_m = M \psi_m(r_o)^2$, where the hollow shell mass $M = \int_S t \rho \psi_m(r)^2 dS$ is given by the mass-normalised flexural mode shape $\psi_m(r)$, with the integral is taken over the surface of the shell. The effective piston area $S_m = \int_S \hat{\psi}_m(r) \cdot \hat{n} dS$ describes the volume change associated with the inward flexural wave shell vibrations, where $\hat{n}$ is a unit vector perpendicular to the surface towards the inside of the hollow cavity of the instrument. The neck, fingerboard and all other attached components introduce weak perturbations, mostly affecting signature mode frequencies rather than mode shapes.

The cavity capacitance $C_{\text{cavity}} = V_{\text{res}} / \gamma P_0$, where the voltage across it represents the induced pressure p . The mode plate capacities $C_m = S_m^2 / L_m \omega_m^2$ and f -hole inductance $L_{f\text{-hole}} = 1 / C_{\text{cavity}} \omega_{\text{Helmholtz}}^2$, where ω_m and $\omega_{\text{Helmholtz}}$ are the uncoupled resonant angular frequencies of the plate and Helmholtz resonator (i.e. measured at zero ambient pressure and rigid cavity walls respectively). Damping is included by introducing mode specific Q -factors, such that $\omega_m^2 \rightarrow \omega_m^2 (1 + i/Q_m)$ for the individual resonant mode frequencies involved.

The above standard model assumes a uniform acoustic pressure p within the cavity. In practice, the Helmholtz mode acoustic pressure varies along the central length of the cavity as $p \phi_H(r)$, with the highest pressure at the upper bout end, the lowest opposite the f -holes and an intermediate value at the lower bout end, as described later. The coupling between the cavity wall modes and Helmholtz resonator then depends on the mode-dependent overlap integral $\int_S \hat{\psi}_m(r) \cdot \hat{n} \phi_H(r) dS$.

As described in an accompanying paper [1], the CBR, BI -, BI + normal modes involve combinations of component volume-changing breathing, centre bout rotation and bending vibrational modes. Computations suggest that only the volume-changing breathing mode component both radiates and couples strongly to the Helmholtz pressure fluctuations. This implies that each signature mode will contribute in equal measure to both the radiated and internal sound pressure, by an amount determined by the contribution of the component breathing mode vibrations in each mode. Nevertheless, the acoustic pressure driving

the plug of air through the f -holes will be reduced by the factor $S \phi_H(r_c)^2 / \int_S \phi_H(r)^2 dS$, where to a good approximation $\phi_H(r_c)$ is the normalized pressure at the acoustic centre of the shell driving the f -hole vibrations. This is equivalent to an increase in the effective cavity volume from V to V_r by the inverse factor.

The model describes the uncoupled flexural vibrations of the signature plate modes shell and Helmholtz air vibrations. It has four independent degrees of freedom, resulting in the four $A0$, CBR, BI - and BI + independent normal modes, describing the coupled motions of the previously uncoupled plate and Helmholtz component modes.

The monopole sound source generated by the net volume flows in and out of the cavity, $(u_{\text{plates}} - u_{\text{Helmholtz}})$, determines the radiated far field sound pressure proportional to $\omega (u_{\text{plates}} - u_{\text{Helmholtz}})$. In contrast, the internal cavity pressure p_{internal} is given by $(u_{\text{plates}} - u_{\text{Helmholtz}}) / i \omega C_V$. Hence, the ratio of radiated sound pressure to the internal pressure is proportional to ω^2 at all frequencies in the monopole signature mode regime (see also reference [4]).

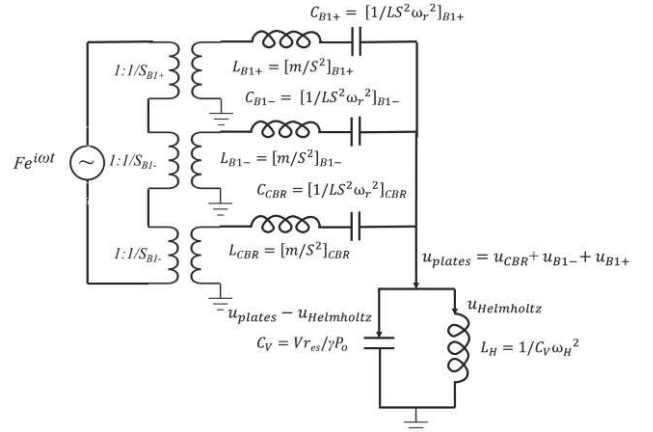


Figure 1. Equivalent circuit describing the coupling of the CBR, BI - and BI + signature modes measured in vacuum to the rigid-walled Helmholtz f -hole resonator.

At low frequencies, the outward flow of air out through the f -holes must match the inward flow from the flexural vibrations of the shell - the so-called *toothpaste* effect. Hence, the sound radiated by the CBR, BI - and BI + signature modes at low frequencies has to be matched in opposite polarity by the low frequency response of the $A0$ mode.

As a first approximation, we assume contributions from the higher order cavity modes can be neglected. It is then possible to describe the radiated sound over the whole of the monopole signature mode frequency range, simply from the CBR, BI - and BI + modes, with the addition of an $A0$ mode resonance of opposite polarity to cancel their low frequency response.

This is illustrated in figure 2 by Curtin-rig measurements [4] of the radiated sound from a violin, described over the whole signature mode region by first fitting resonances to the observed BI - and BI + peaks and then adding an $A0$ resonance of opposite polarity to give the required pressure cancellation at low frequencies. The only adjustable parameter required to fit the data below 400 Hz is then the $A0$ resonant frequency and its Q -value,

which is only important within a semi-tone or two of its resonance. Despite the neglect of the higher order cavity air modes excited, the fitted curves are in excellent agreement with the radiated pressure over the whole of the signature mode region below ~ 800 Hz. An additional small peak from the $A1$ cavity air resonance at ~ 500 Hz has also been added.

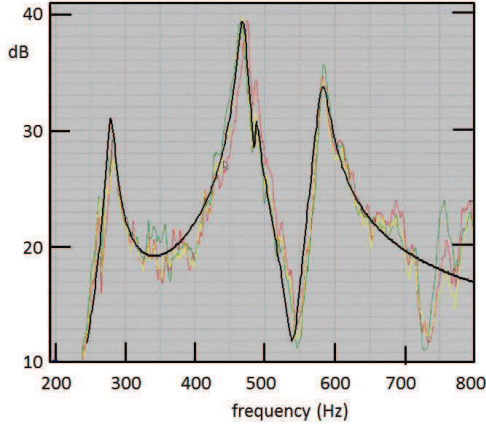


Figure 2. Radiated sound pressure at 12 azimuthal angles around the “Miles” violin fitted to the $A0$, $B1$ - and $B1+$ modes (and a small $A1$ internal cavity mode) using the generalized loudspeaker enclosure model (measurements courtesy of Borman).

Fits of similar quality have been made for several other instruments justifying the above approximation. For this particular violin, like many others, there was no significant contribution from the CBR mode. Because of the quality of the fit, one would expect the predicted frequency-squared dependence for the ratio of radiated to internal sound to be maintained throughout the isotropic, monopole radiation regime, for all instruments of the violin family.

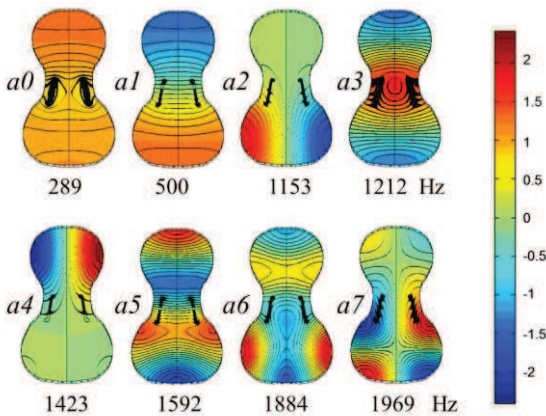


Figure 3. Computed central plane pressure of the first eight cavity air modes for a rigid, arched, violin cavity with f -holes. The pressure dependence on the central plane is only weakly perturbed by the f -holes.

Figures 3 and 4 illustrate the computed spatial variations of pressure of the first eight cavity air modes of a rigid walled, arched, violin shell with f -holes cut to give an $a0$ resonance at 289 Hz. Note the significant pressure changes along the length of the cavity for the $a0$ mode. This arises from the flow of air towards the f -holes from

both the upper and lower bouts. Because the $a0$ pressure is a minimum in line with the f -holes, there is little pressure difference along their length, in contrast to the assumptions made in the 2-degrees of freedom Shaw model [7] for the $a0$ and $a1$ air modes, which also neglects pressure variations in the lower bouts from flow towards the f -holes.

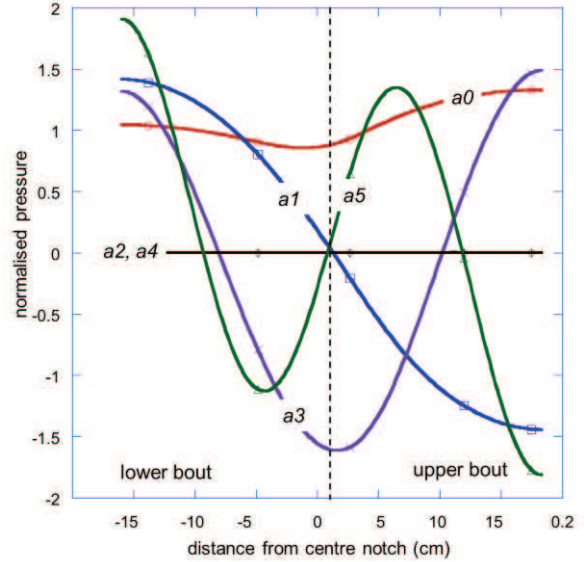


Figure 4. Spatial dependence of the internal pressure of the first six cavity air modes along the central axis of an arched violin shell with open f -holes measured from a position parallel to the central f -hole notches towards the upper bouts.

The $a0$, $a1$... cavity modes are the independent, non-interacting, normal modes of the rigid cavity body, which are perturbed to form the non-interacting $A0$, $A1$... normal modes, which includes their coupling to the flexural vibrations of the shell which excite them.

Our present interest is the acoustic pressure at the *acoustic centre* of the cavity, almost in-line with the f -hole notches. This is close to the nodes of the six lowest frequency cavity modes below ~ 1.6 kHz, other than $a0$ at ~ 290 Hz and $a3$ at ~ 1.2 kHz.

In the signature mode regime below 1 kHz, the sound pressure at the acoustic centre of the violin will therefore be dominated by the $a0$ mode. Because of the need for a uniform pressure within the cavity at low frequencies, additional contributions from the higher order $a1$ and $a3$ modes must also be excited. However, by definition, the $a1$ component will not contribute to the sound at the acoustic centre of the cavity, while the $a3$ contribution will be relatively weak because of its significantly higher frequency ~ 1.2 kHz. This justifies the assumptions made in fitting to the measurements in figure 2.

Around and above 1 kHz or so, the $a3$ mode, with one full wavelength along the length, will be coupled to any plate vibrations with nearby resonant frequencies having non-zero overlapping mode shapes. This will result in a set of normal modes describing their coupled vibrations, which will contribute to the acoustic pressure at the acoustic centre and hence also radiate through the f -holes.

Ideally, a calibrated microphone should be used to measure the internal sound pressure excited using a cali-

brated impact hammer – in just the same way that impulse measurements of the radiated sound are made.

As the measurements are made inside the cavity of the instrument itself, the sound pressure is high. This results in a large signal to noise ratio, free from external noise and complications from excited room acoustics. The measurements are highly reproducible when made at different locations, as the cavity volume is always the same, wherever the measurements are made.

Accurate comparisons of the acoustic properties of different instruments can therefore be made even in a noisy environment. In the measurements described here, a subminiature microphone was used, either freely supported or positioned at the acoustic centre on a very light curved wooden or plastic beam sprung between the outer edges of the f -holes. The acoustic centre can easily be determined by eliminating any weak $A1$ resonance, though this is easily recognizable and only weakly contaminates the measurements when present.

3. VIOLIN MEASUREMENTS

The measurements in Figure 5 illustrate strong contributions to the internal sound field from the $B1-$ and $B1+$ mode resonances, a small contribution from the CBR mode just below 400 Hz, and a splitting of the strong $A0$ mode resonance, almost certainly from coupling to the $B0$ longitudinal neck-body bending mode, with another much weaker resonance at a slightly lower frequency probably from the sideways-yaw or bending vibrations of the neck and body.

The assumption that the violin acts as a monopole source of sound in the signature mode region assumes that $(ka)^2 < 1$, where a is the effective size of the instrument. Around and above 800 Hz ($ka \sim 1$), the frequency

dependence will flatten off to a constant value. In this higher frequency range, the measurements reveal the anticipated strong group of resonances from normal modes involving the coupled $a3$ cavity air resonance at around 1.2 kHz and nearby in frequency shell vibrations. Such shell modes are likely to radiate significantly through the f -holes, which are close to the anti-node of the $a3$ mode.

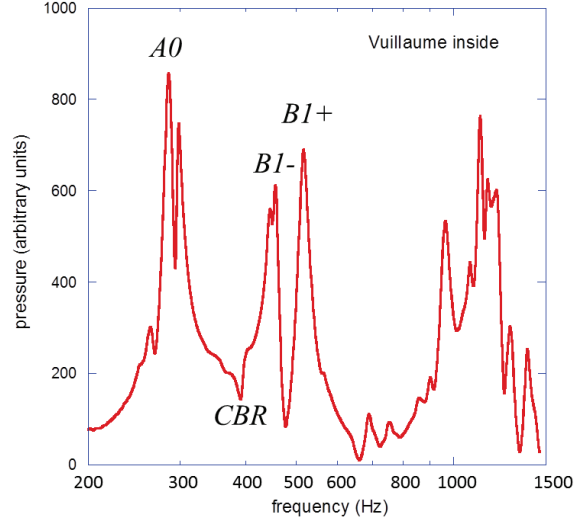


Figure 5. Typical measurements of the internal sound pressure spectrum at the acoustic centre of a Vuillaume violin with damped strings.

The measurements highlight the high quality of the internal sound measurements, which provide valuable information on the frequencies, damping and relative strengths of the four main signature modes – and a number of higher frequency modes as well.

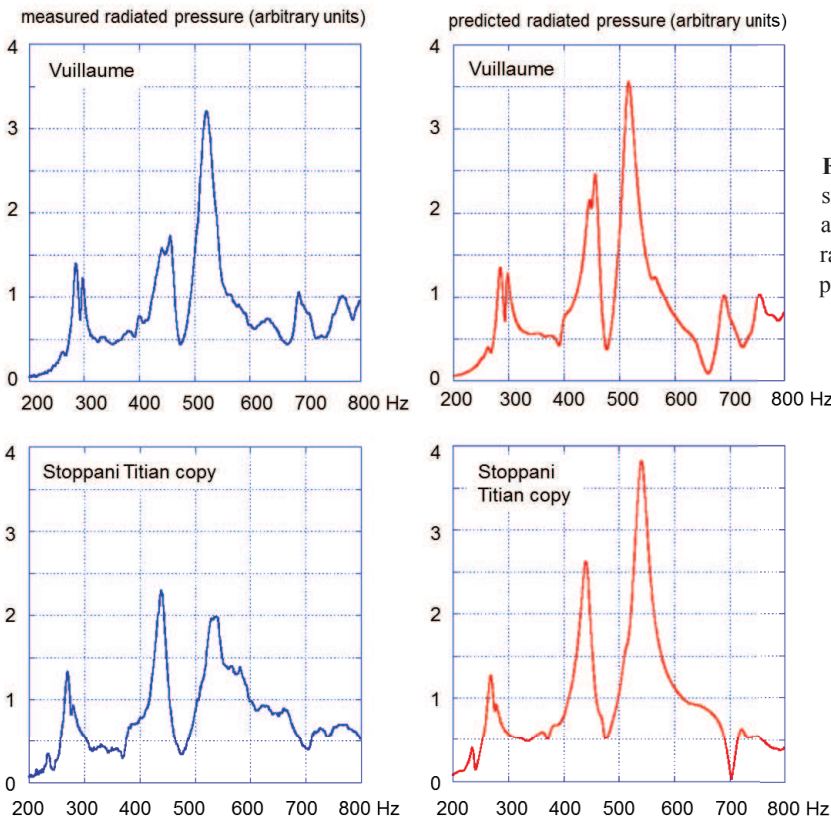


Figure 6. A comparison of the radiated sound pressure (left hand side) for two violins at a distance of ~ 30 cm and the predicted radiated pressure proportional to internal pressure $p \times \omega^2$.

Figure 6 compares the directionally averaged radiated sound pressure, measured at around 30 cm from two violins, with the radiated pressure derived from the predicted ω^2 - dependence of the measured sound at their acoustic centres. The radiation pressure was measured in the open air to eliminate contamination from room resonances – equivalent to making measurements in an anechoic chamber with a reflecting floor.

Despite the qualitative good agreement between the measured and derived frequency dependences, there are significant mode-dependent departures from the predicted ω^2 dependence, even allowing for the expected departures as ka approaches unity. Similar unexplained differences were observed by Jansson et al [4]. These could arise from systematic errors in the measurements themselves (near/far-field corrections, for example).

Such departures could also be explained, if the component bending modes of the $B1-$ and $B1+$ signature modes, vibrating in and out of phase with the breathing mode, contribute significantly to the radiated sound, despite the small volume changes and coupling to the internal air pressure predicted by shell model computations (Gough [1]). If so, the relative coupling strengths of the bending and breathing modes to the Helmholtz mode pressure fluctuation are likely to be very different from their coupling to the monopole radiation field modes. This would then account for the difference in relative heights of the $B1-$ and $B1+$ modes in the radiated and internal pressure measurements. Further investigations are required to clarify the origin of such differences.

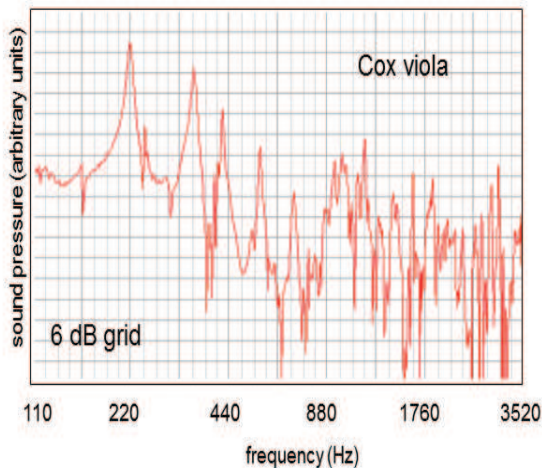


Figure 7. Acoustic centre pressure for a modern viola by Douglas Cox illustrating the high resolution, high signal to noise ratio and absence of contamination from room acoustics and external noise.

As an example of the use of internal sound measurements to characterize the acoustically important vibrational modes of instruments other than the violin, Figure 7 plots the acoustic centre pressure of a modern viola by Douglass Cox. This instrument has a number of extremely well defined modes in the signature mode region below

around 800 Hz and a clutch of strongly excited modes around 1 kHz and above.

Quite apart from their acoustic interest, the data illustrates the very high signal to noise ratio of such measurements, achieved using an inexpensive (~\$3) sub-miniature microphone. The data also illustrates the lack of any spurious resonances from room acoustics or extraneous noise from the relatively noisy environment of an Oberlin Acoustic Workshop. Reproducible, well-defined modal resonances are observed almost 50 dB below the dominant $A0$ resonance.

Interestingly, the measurements reveal four well-defined signature modes above the $A0$ resonance at 220 Hz. The shell model for string instrument presented separately (Gough [1]) suggests these are likely to be the CBR , $B1-$, $B1+$ and LD (longitudinal dipole/upper bout breathing) modes of the instrument.

4. DOUBLE BASS MEASUREMENTS

As examples of the use of internal air measurements to characterize the acoustic properties of very large instruments and as a personal tribute to the late Knut Guettler – an esteemed colleague, innovative acoustician and virtuoso bass player – some preliminary internal cavity measurements on a number of double basses will be described.

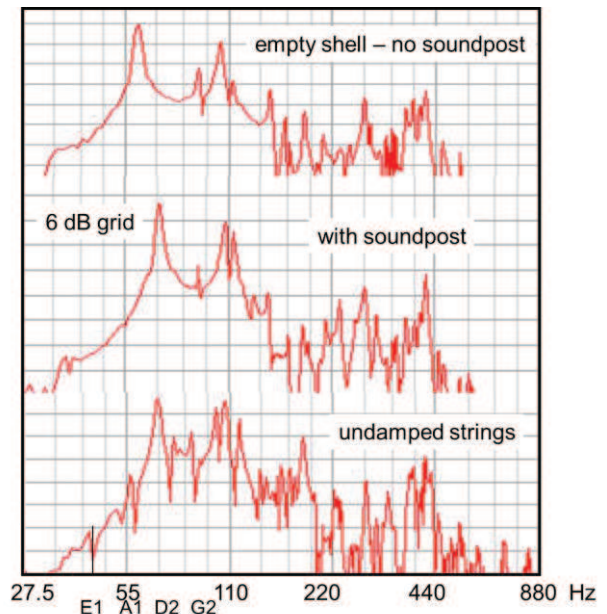


Figure 8. Internal sound measurements at the acoustic centre of modern Robbie McIntosh double bass without, with soundpost and undamped strings. The frequencies of the open strings are also indicated

The measurements were made at the 2012 Bass Oberlin Workshop in collaboration with the Violin Acoustics Workshop, where a dozen or so double basses were being temporarily stored in a single room, in various states of assembly. With invaluable assistance from several young makers, the following measurements were made in a single evening. The measurements should simply be seen

as an initial foray into the world of double bass acoustics. Future investigations including modal analysis measurements need to be made to confidently identify the principal modes observed.

Figure 8 shows internal pressure measurements for an outstanding modern bass by Robbie McIntosh, as judged by all the bass players and makers present. Measurements were made on this and other instruments without a sound post, with a soundpost, and set-up with a bridge and both damped and undamped strings.

The main features of the measurements on this instrument were reproduced on almost all dozen or so instruments, illustrated by some additional examples in Figure 9.

With the notable exception of the old European flat-back bass, the internal sound pressure, hence radiated sound at low frequencies, was dominated by only two strong resonances, rather than the three dominant *A0*, *B1*- and *B1* modes of the violin. The positions and relative strengths of the two prominent acoustic modes were similar for almost all the empty and set-up instruments. Additional analysis could extract comparative mode strengths, resonant frequencies and *Q*-values for all the observed resonant modes.

The lower mode of the empty McIntosh bass shell was raised on insertion of the soundpost from 60 Hz (*C2*) to 69 Hz (*C#2*) and the upper mode from 105 Hz to 109 Hz, with two relatively small resonances of unidentified origin on either side.

Earlier admittance and laser modal analysis measurements by Askenfelt [7] and Brown [8] exhibited very similar peaks in a number of arched-back double basses and identified them as *A0* resonances around 65 Hz and “top plate resonances” (equivalent to a *B1*-breathing mode [1]) around 115 Hz, consistent with the above measurements. In making acoustic radiation measurements in an anechoic chamber, Brown noted that ‘disturbances from standing waves and other acoustic problems were problematic...especially below 100 Hz where location dependent deviations in response of up to 10 dB were observed’. Furthermore, he points out that the wavelength of the lowest note on a double bass is around 8m, while measurements had to be made at around 1m. The measured acoustic pressure at such frequencies were therefore strongly influenced by the non-radiating near field with pressure varying as $1/r^2$ rather than the $1/r$ dependence of the radiating sound. By deducing the radiating sound from internal cavity measurements one avoids all such problems.

In all the measurements, the bass was hand-held resting on a soft support in an upright position. Because of the existence of only a single mode above the Helmholtz *A0* resonance and the relatively insensitive of any of the low frequency modes to the way the bass was held or supported, it seems likely that the component bending mode of the bass plays a less important acoustic role than for the violin and viola.

Also note the importance of the coupled string vibrations on the acoustical properties of the bass. The strings produce strong, narrow, resonances in the internal and

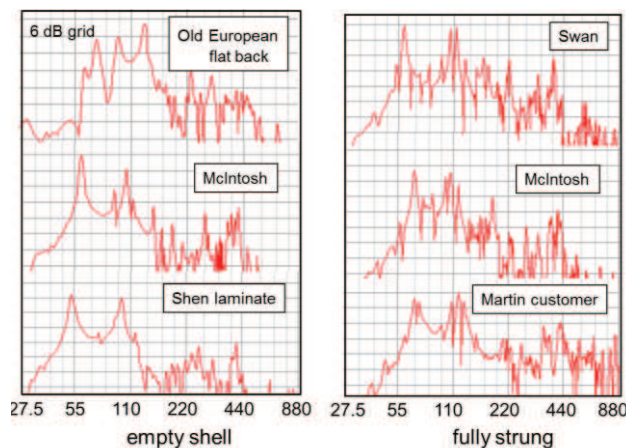


Figure 9. Internal sound measurements for a number of empty and fully-strung double basses

radiated sound. These will not only contribute to the liveliness of the bowed instrument when played with vibrato, but will also ring for much longer when plucked, as well as exciting slowly decaying transients at the start and end of any bowed note.

For all instruments of the violin family, measurements with damped strings help to identify the acoustical properties of the instrument itself, without the added complexity of the vibrating strings. However, if one wants to compare the sound quality of an instrument with physical measurements, the strings should arguably be left undamped - as they would be under normal playing conditions. For the player and listener, the transient response of an instrument associated with the undamped strings is important at the start and end of bowed notes of any length and also notes played with vibrato. This is particularly so for the bass, where the quality of plucked notes is often just as important as bowed notes.

5. CONCLUSIONS

Measurements of the sound pressure at the acoustic centre of the violin and related instruments are shown to characterise the acoustic modes of stringed instruments of any size at low frequencies, free of contamination from room acoustics and external noise.

A generalized model relating the internal to radiated sound pressures has been described. Demonstrations of the value of such measurements have been presented for the violin, viola and double bass. Using an inexpensive sub-miniature microphone with a laptop and soundcard, measurements are shown to be easily made in any environment including the luthier’s workshop.

Acknowledgments

I am particularly grateful to Anders Buen for drawing attention to the earlier double bass measurements of Andrew Brown, to all the students and bass makers at the 2012 Oberlin Bass Workshop who assisted with the bass measurements and set-ups, and to George Stoppani and Terry Borman for providing some of the experimental data.

6. REFERENCES

- [1] C. Gough, Vibrational modes of the violin family, in *Proc. SMAC2013*, Stockholm, 2013.
- [2] G. Bissinger, Structural Acoustics of good and bad violins, *J. Acoust. Soc. Am.*, vol. 124, no.3, 2008, pp. 1764-1773.
- [3] J. Curtin, Measuring Violin Sound radiation using an Impact Hammer, *J. Violin Soc. Am.: VSA Papers*, vol.22, no.1, 2009, pp. 186-209.
- [4] E.V. Jansson, L.A. Morset and Knut Guettler, Prediction of violin radiation properties in the 200-700 Hz range, *Proc. ISMA2001, Perugia*, pp 123-126, 2001.
- [5] G. Caldersmith, Guitar as a reflex enclosure, *J. Acoust. Soc. Am.*, vol. 63, 1978, pp. 1566-1575.
- [6] J. Schelleng, The violin as a circuit. *J. Acoust. Soc. Am.*, vol. 35, no.3, 1963, pp. 326-338.
- [7] E.A.G. Shaw, Cavity resonance in the violin: Network representation and the effect of damped and undamped rib holes, *J. Acoust. Soc. Am.*, vol. 7, no.1, 1990, pp. 398-410.
- [8] A. Askenfelt, Eigen modes and tone quality of the double bass, *STL-QPSR*, Vol 23, no.4, 1982, pp.19-174, with data reproduced in *The Science of String Instruments* (ed. T.D. Rossing), Springer, Chpt. Double Bass (Askenfelt), pp 259-278, 2010.
- [9] A.W. Brown, *Acoustical studies of flat-backed double bass*, PhD Thesis, Vienna, April 2004, and Documentation of double bass plate modes using the laser vibrometer, *Proc ISMA 2004*, Nara, 344-347, 2004.