METHODS BASICS

YEAR 11/12

SETS

RELATIONS

FUNCTIONS

SETS

A set is a collection of numbers.

The numbers in a set are called elements of the set.

Ways of writing sets

1. Listing
2. By rule
3. With a single capital letter
4. Interval notation

1. Listing a set

Listing is useful for discrete, finite sets.

eg1 $\{2, 7, -5, 14, 100\}$

eg2 $\{1, 2, 3, 4, \ldots 50\}$

Listing may also be used for infinite sets that follow a pattern

eg3 $\{0, 2, 4, 6, 8, 10, \ldots\}$

eg4 $\left\{-\frac{1}{2}, -\frac{1}{3}, -\frac{1}{4}, -\frac{1}{5}, -\frac{1}{6}, -\frac{1}{7}, \ldots\right\}$

Exercise 1

Use the method of *listing* to describe these sets.

a) All odd numbers between 3 and 11 inclusive.

b) All even numbers between 50 and 60, not including 50 and 60.

c) All solutions to the equation $x^2 + 1 = 10$.

d) The set of all odd numbers greater than 3.

e) The set of all prime numbers.

2. Defining a set by rule

This way of writing a set is mostly used for infinite sets where the elements follow some rule.

We write: $\{x : \underbrace{\qquad\qquad}_{\text{rule}}\}$

We say: "the set of *x*'s such that $\underbrace{\qquad\qquad}_{\text{rule}}$"

eg1 The set of all positive numbers.

OR

The set of all *x*'s such that *x* is greater than zero.

$\{x : x > 0\}$

eg2 The set of all numbers below or equal to -1.

OR

The set of all *x*'s such that *x* is less than or equal to negative one.

$\{x : x \le -1\}$

eg3 The set of all numbers between -5 and 4.

OR

The set of all *x*'s such that *x* is greater than or equal to -5 and less than or equal to 4.

$$\{x : -5 \le x \le 4\}$$

smallest number biggest number

$-5 \le x \le 4$

means the same as writing:

$x \ge -5$ and $x \le 4$,

without the need for the word "and".

In this notation, with the smallest number on the left of *x* and the largest on the right, why must both inequality signs ($\le$) point left?

eg4 The set of all numbers between 1 and 5, including 1 but *not* including 5.

OR

The set of all *x*'s such that *x* is greater than or equal to 1 and less than 5.

$$\{x : 1 \le x < 5\}$$

Exercise 2

Use the method of *defining by rule* to describe these sets.

a) The set of all negative numbers.

b) The set of all numbers strictly less than -2 i.e. less than -2 and *not* including -2.

c) The set of all numbers between -10 and 10 inclusive.

d) The set of all numbers between -1000.5 and 15.6, not including -1000.5 and not including 15.6.

3. Special sets- defined by a capital letter

Some sets are given their own special shorthand notation.

N = the set of natural numbers

$$=\{1,2,3,4,5,...\}$$

Z = the set of integers

$$=\{...-5,-4,-3,-2,-1,0,1,2,3,4,5,...\}$$

$\mathbf{Z}^{+}$ = the set of positive integers

$$=\{1,2,3,4,5,...\}$$

= **N**

$\mathbf{Z}^{-}$ = the set of negative integers

$$=\{-1,-2,-3,-4,-5,...\}$$

Q = the set of rational numbers

Ie the set of all numbers that can be written as fractions

Q' = the set of irrational numbers

Ie the set of all numbers that cannot be written as fractions

R = the set of real numbers

Ie the set of all numbers on the number line

Note: The number line is made up of all rational and irrational numbers.

Exercise 3

$\in$ means "belongs to" or "is an element of"

$\notin$ means "doesn't belong to" or "is not an element of"

We can write: "______ $\in$ *or* $\notin$ ______"

a number or variable a set

eg1 $1 \in$ **N**

eg2 $-2 \in$ **Z**

eg3 $\pi \notin$ **Z**

eg4 $\frac{1}{3} \in$ **Q**

eg5 $\pi \in$ **R**

1. Complete the following with $\in$ or $\notin$.

a) 3____ **N**

b) -5____ **N**

c) $\frac{1}{2}$____ **Q**

d) 4____ **Q**

e) π____ **Q**

f) π____ **Q'**

g) $-\frac{1}{2}$____ **Z**

h) 0____ **R**

i) $0.\dot{3}$____ **Q**

j) $3.\dot{3}$ ____ **R**

k) $\sqrt{17}$____ **R**

l) $\sqrt{-1}$____ **R**

The symbol $\subset$ means " is a subset of ".

The set on the left of this symbol must fit inside the set on the right.

eg $\mathbf{N} \subset \mathbf{Z}$

$\{1,2,3,4,5,...\}$ $\{...-5,-4,-3,-2,-1,0,1,2,3,4,5,...\}$

The symbol $\not\subset$ means " is *not* a subset of ".

eg $\mathbf{Z} \not\subset \mathrm{N}$

2. Complete the following using **N**, **Z**, **Q** and **R**

$\_\_\_ \subset \_\_\_ \subset \_\_\_ \subset \_\_\_$

The symbol $\cup$ means " combined with" and is called "union".

eg If **A** = $\{1,2,3\}$ and **B** = $\{-3,-2,-1,0,1\}$

then **A** $\cup$ **B** = $\{-3,-2,-1,0,1,2,3\}$

Note: The union of two sets contains all elements of both sets but elements must *not* be repeated.

The symbol $\cap$ means " overlapped with" and is called "intersection".

eg For the above sets **A** and **B**, the only element they have in common is 1, so we write:

A $\cap$ **B** = $\{1\}$

Note: The union or intersection of sets is also a set.

The symbol $\mid$ is a subtraction symbol for sets and is called "slash".

eg **N** $\mid$ **A** = $\{4,5,6,7,...\}$

3. Complete the following

a) **N** $\cup$ $\{0\}$ = $\{\_\_,\_\_,\_\_,\_\_,\_\_,...\}$

b) **N** $\cap$ **Z** = ___

c) $\mathbf{Z}|\ \ \mathbf{N} = \{\_\_,\_\_,\_\_,\_\_,\_\_,\ldots\}$

$= \mathbf{Z}^{-} \cup \{\_\_\}$

d) $\mathbf{Q} \cup \mathbf{Q}' =$

4. Interval notation for sets

This way of writing a set is useful for continuous infinite sets with a lower boundary or upper boundary or both.

eg1 The set of all numbers between 0 and 2, including 0 and 2 would be written:

$$\left[0,2\right]$$

smallest number   biggest number

Square brackets at both ends indicate 0 and 2 are both included.

eg2 The set of all numbers between 0 and 2, not including 0 and not including 2 would be written:

$$\left(0,2\right)$$

smallest number   biggest number

Rounded brackets at both ends indicate 0 and 2 are both excluded.

eg3 The set of all numbers between -5 and 3, including -5, not including 3 would be written:

$$[-5,3)$$

smallest number　　biggest number

Square bracket indicates -5 is included and rounded bracket indicates 3 is excluded.

A rounded bracket is also used when there is no upper boundary or no lower boundary.

Eg4 The set of all numbers greater than or equal to -2 would be written:

$$[-2,\infty)$$

Eg5 The set of all numbers smaller than 8 would be written:

$$(-\infty,8)$$

<u>**Exercise 4**</u>

1. Write each of the following sets in interval notation.

a) The set of all numbers between -100 and 47, including -100 and including 47.

b) The set of all numbers between 0 and 1, including 0 but not including 1.

c) The set of all numbers between 7 and -10, including 7 but not including -10.

d) The set of all numbers greater than 82.

e) The set of all numbers less than or equal to -5.8.

f) $\{x : 2 \leq x < 9\}$

g) $\{x : x > 0\}$

h) $\{x : x < 0\}$

2. Complete the following:

g) above could also be written as $\mathbf{R}^{+}$.

h) above could also be written as ___.

3. Is there a way to write **R** in interval notation?

4. Is there a way to write **Z** in interval notation?

DOMAIN AND RANGE

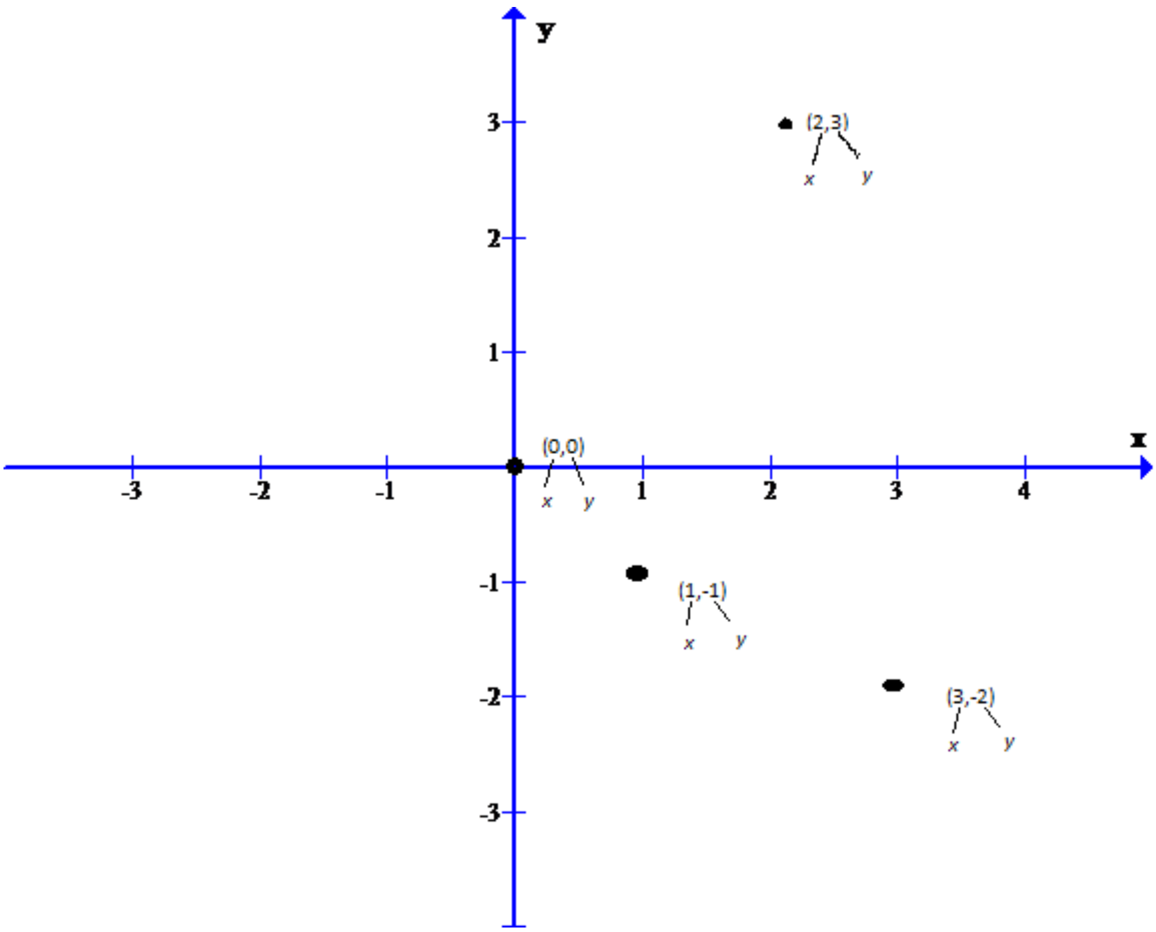
A relation is a set of coordinates that usually follow some rule. To graph the relation, plot these coordinates on a Cartesian Plane.

The domain of a relation is the set containing all the x-values of the coordinates in the graph.

The range of a relation is the set containing all the y-values of the coordinates in the graph.

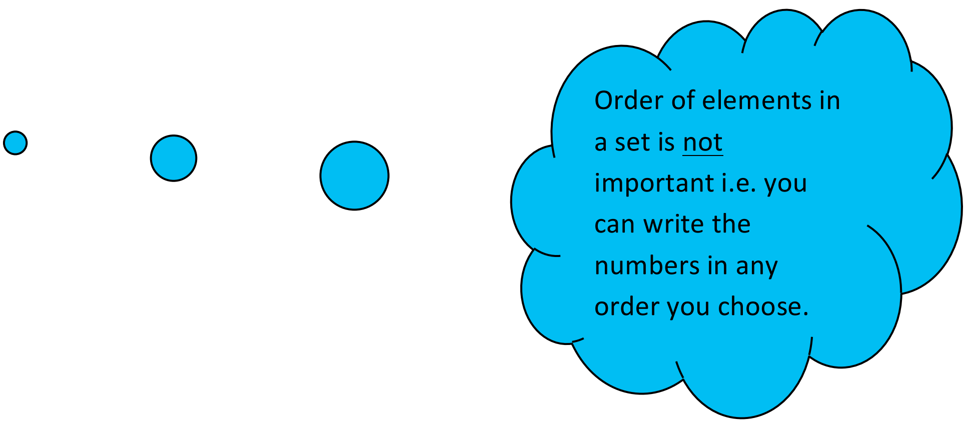
Exercise 5 – Class Activity

1.

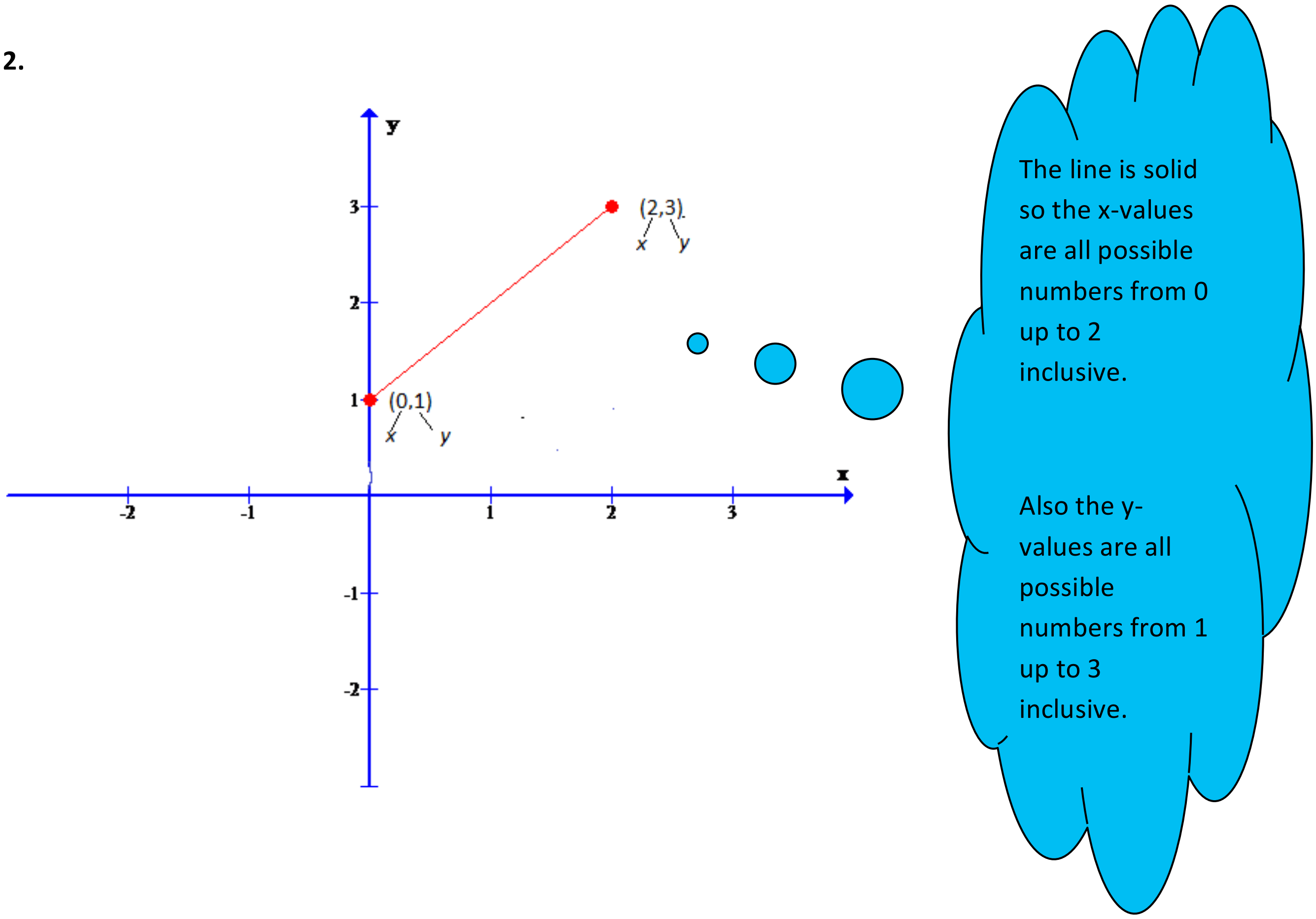


Domain = {__,__,__,__}

Range = {__,__,__,__}



2.



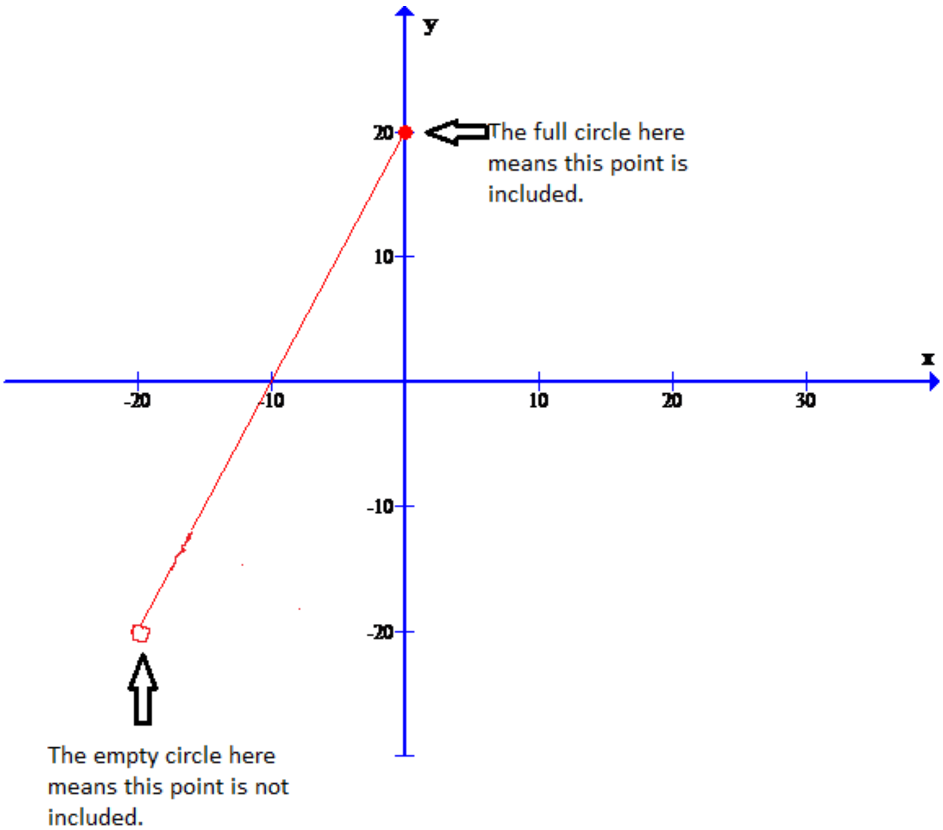
Domain = $\{x: \_\_\_\_ \leq x \leq \_\_\_\_\}$ Defining a set by rule

OR $[\_\_\_\_, \_\_\_\_]$ Interval notation

Range = $\{y: \_\_\_\_ \leq y \leq \_\_\_\_\}$ Defining a set by rule

OR $[\_\_\_\_, \_\_\_\_]$ Interval notation

3.



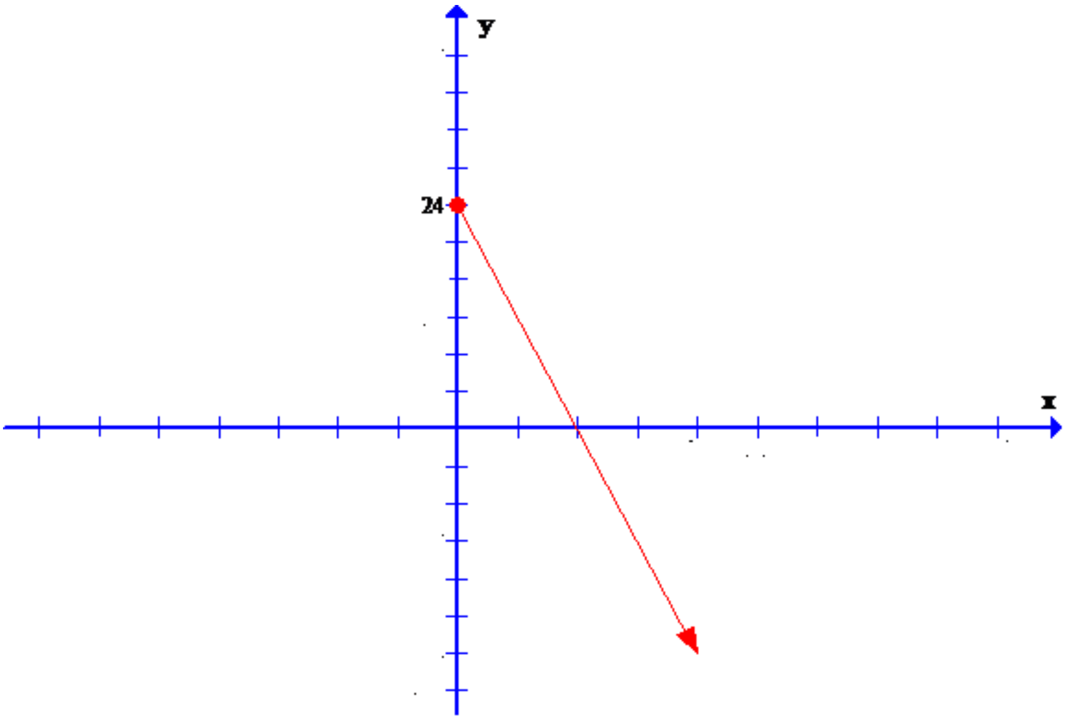
Domain = $\{x : \_\_\_\_\_\_\_\_\_\_\_\_\}$ Defining a set by rule

OR _________ Interval notation

Range = $\{y : \_\_\_\_\_\_\_\_\_\_\_\_\}$ Defining a set by rule

OR _________ Interval notation

4.



Domain = $\{x : \_\_\_\_\_\_\_\_\_\_\_\_\}$ Defining a set by rule

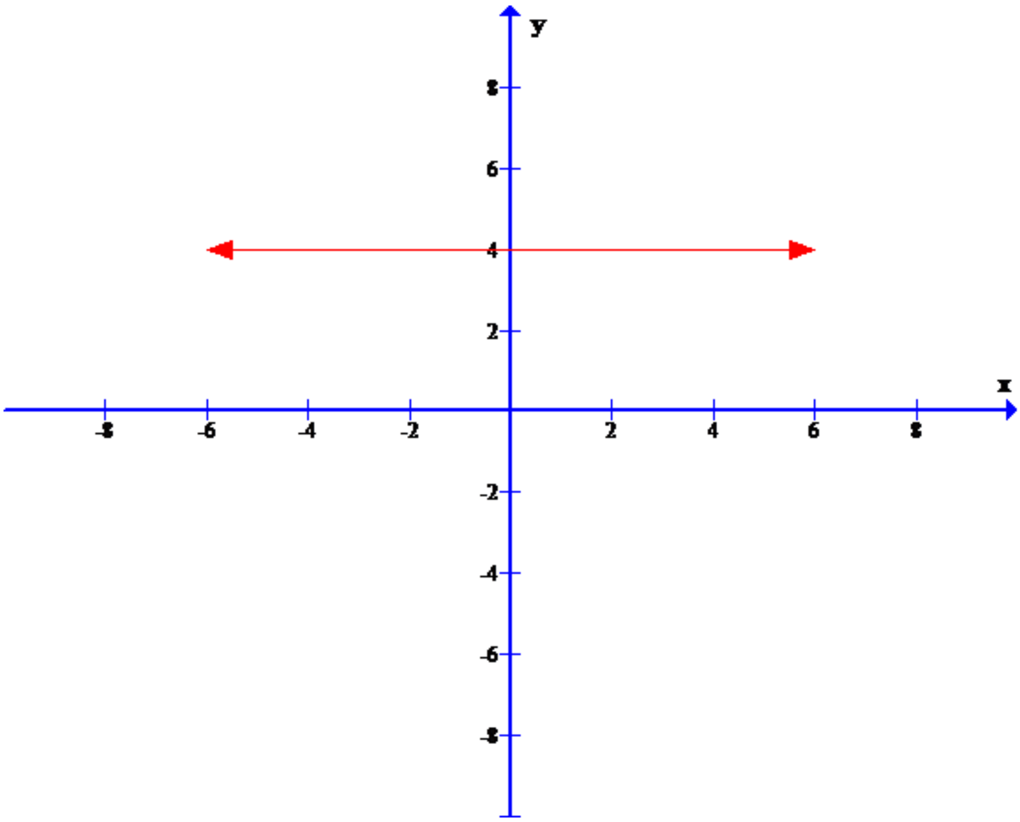
OR ________ Interval notation

OR $\mathbf{R}^{+} \cup \{\_\_\_\}$

Range = $\{y : \_\_\_\_\_\_\_\_\_\_\_\_\}$ Defining a set by rule

OR ________ Interval notation

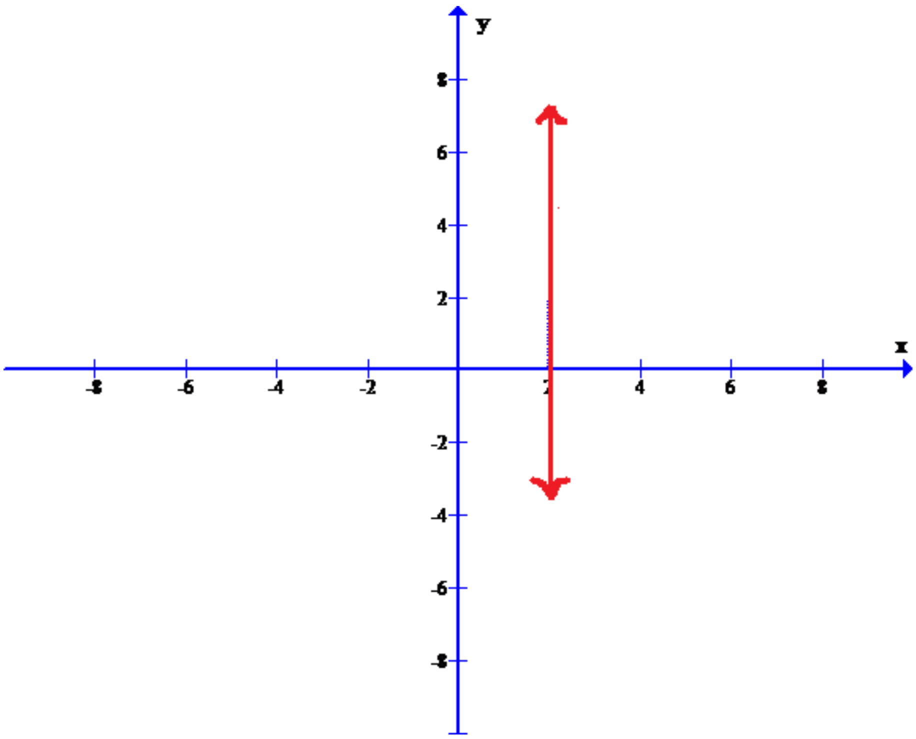
5.



Domain = _____

Range = _____

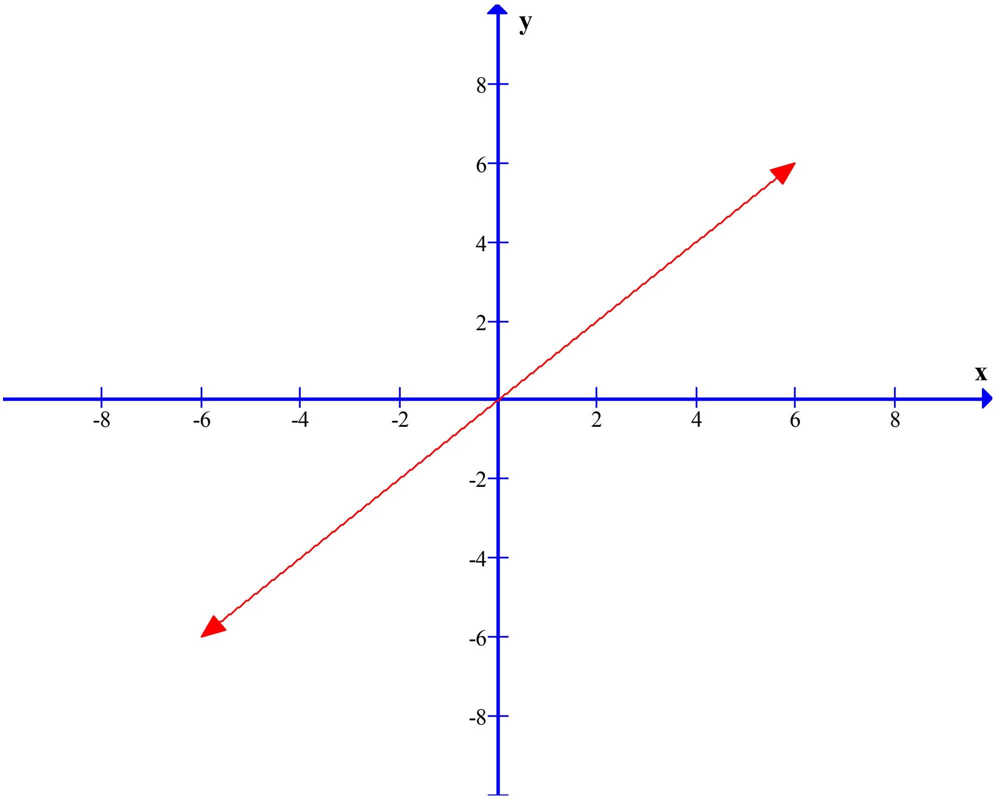
6.



Domain = _____

Range = _____

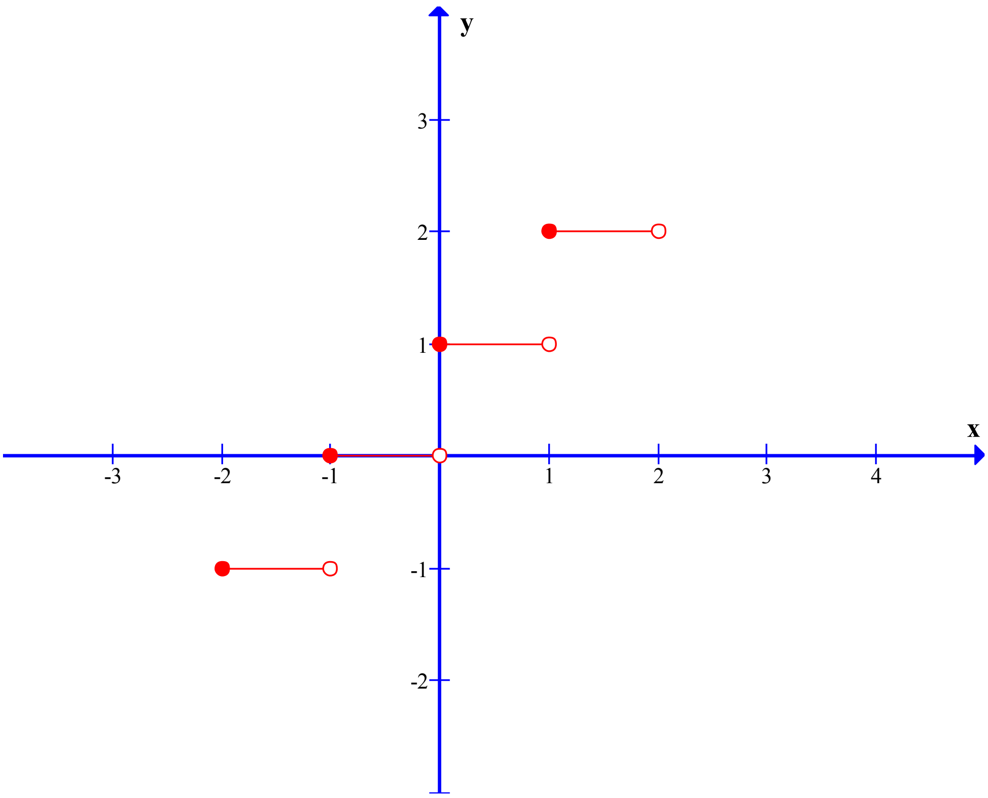
7.



Domain = _____

Range = _____

8.

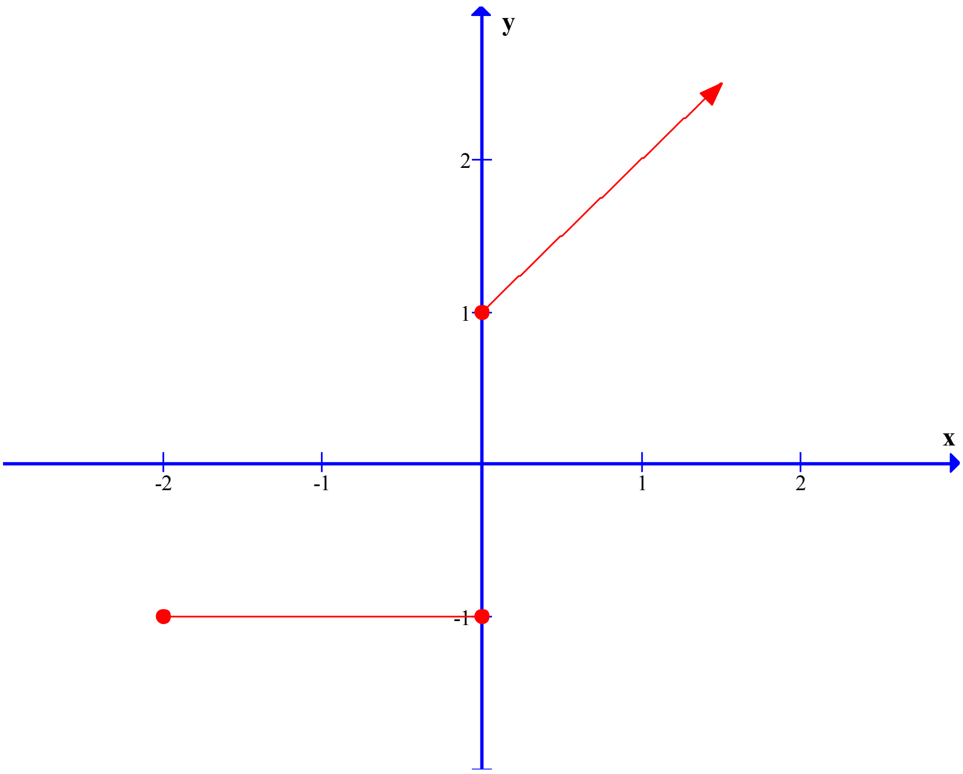


Domain = ____________________________

OR ____________

Range = ____________________________

9.

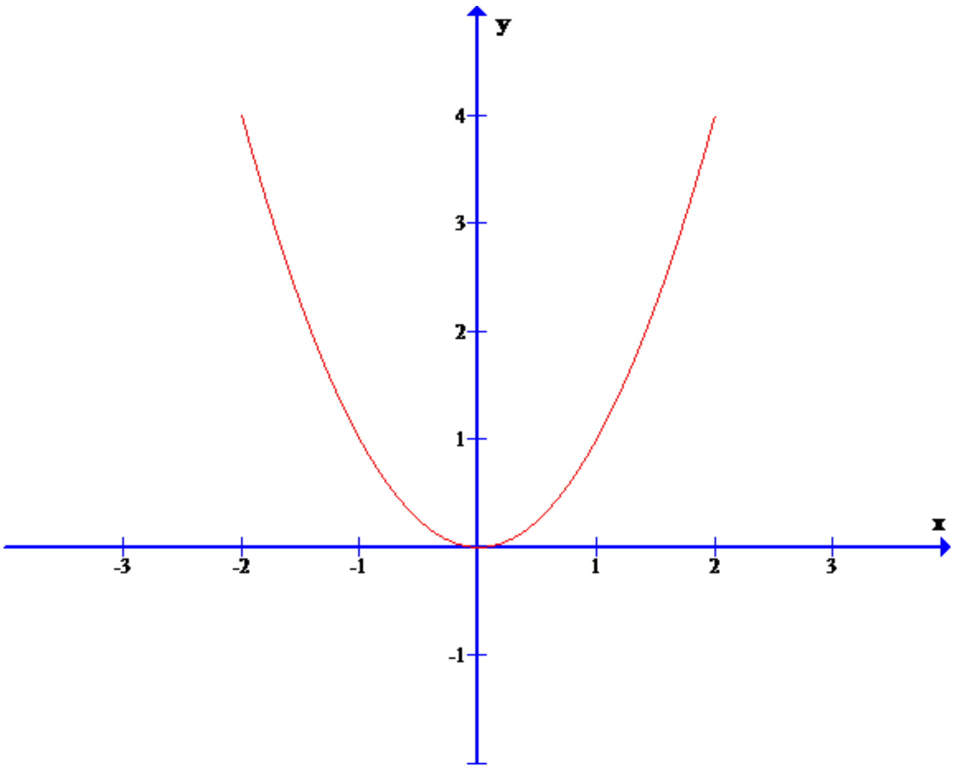


Domain = ____________________________

OR ____________

Range = ____________________________

10.

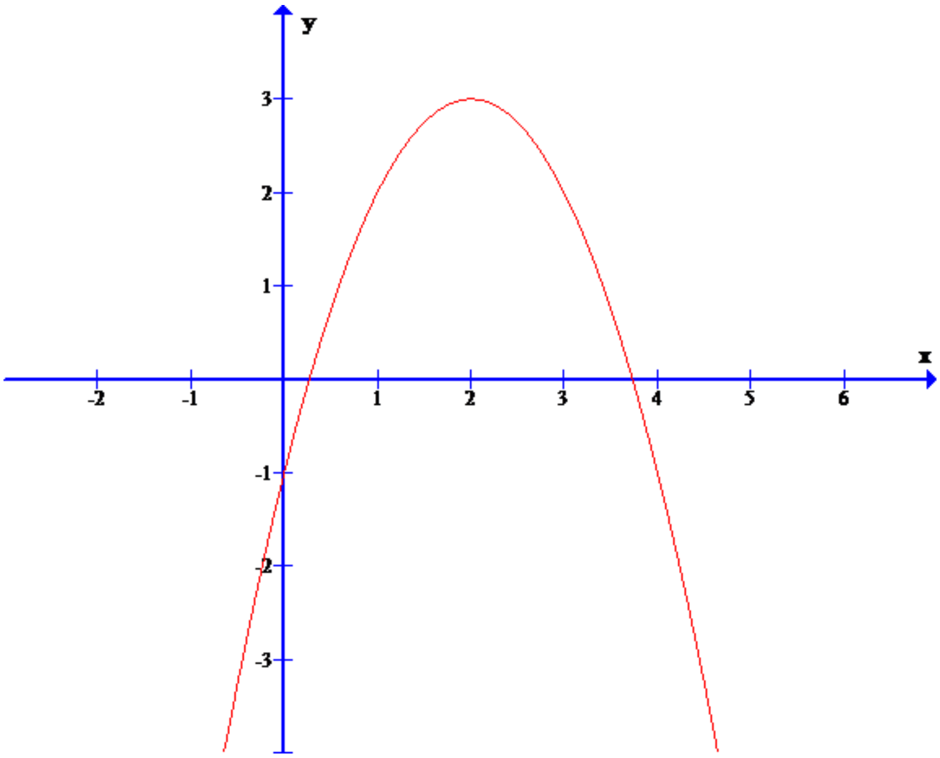


Domain = ______

Range = ______________________________

OR ____________

11.

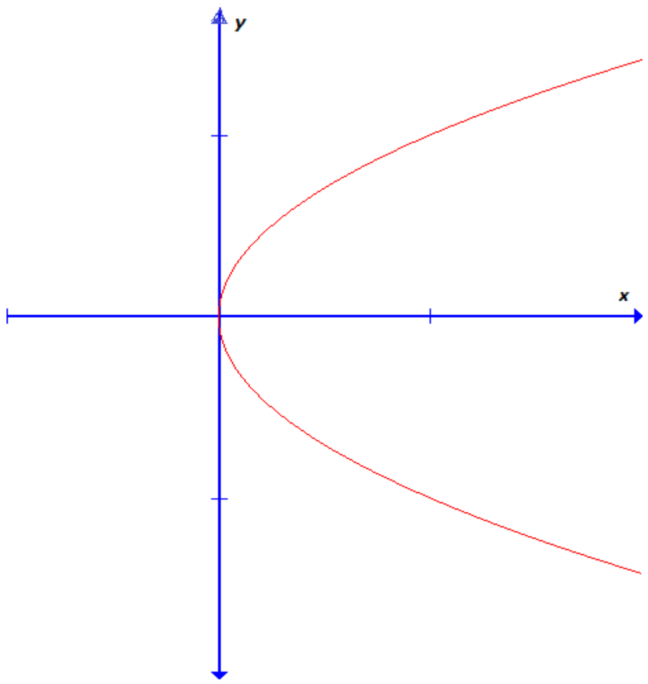


Domain = ______

Range = _____________________________

OR ____________

12.



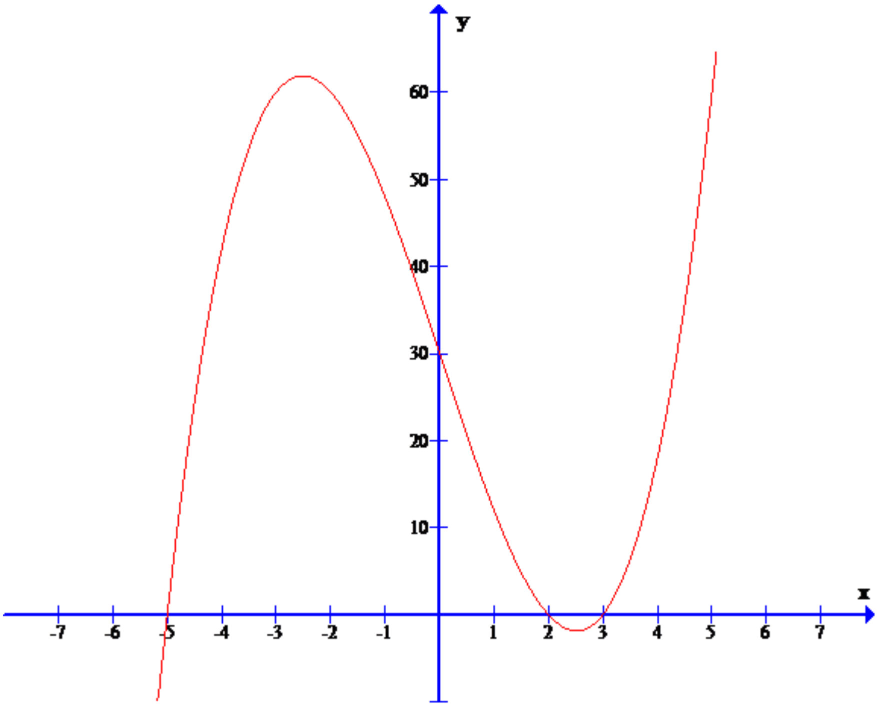
Domain = ___________________________

OR ___________

OR ___________

Range = _____

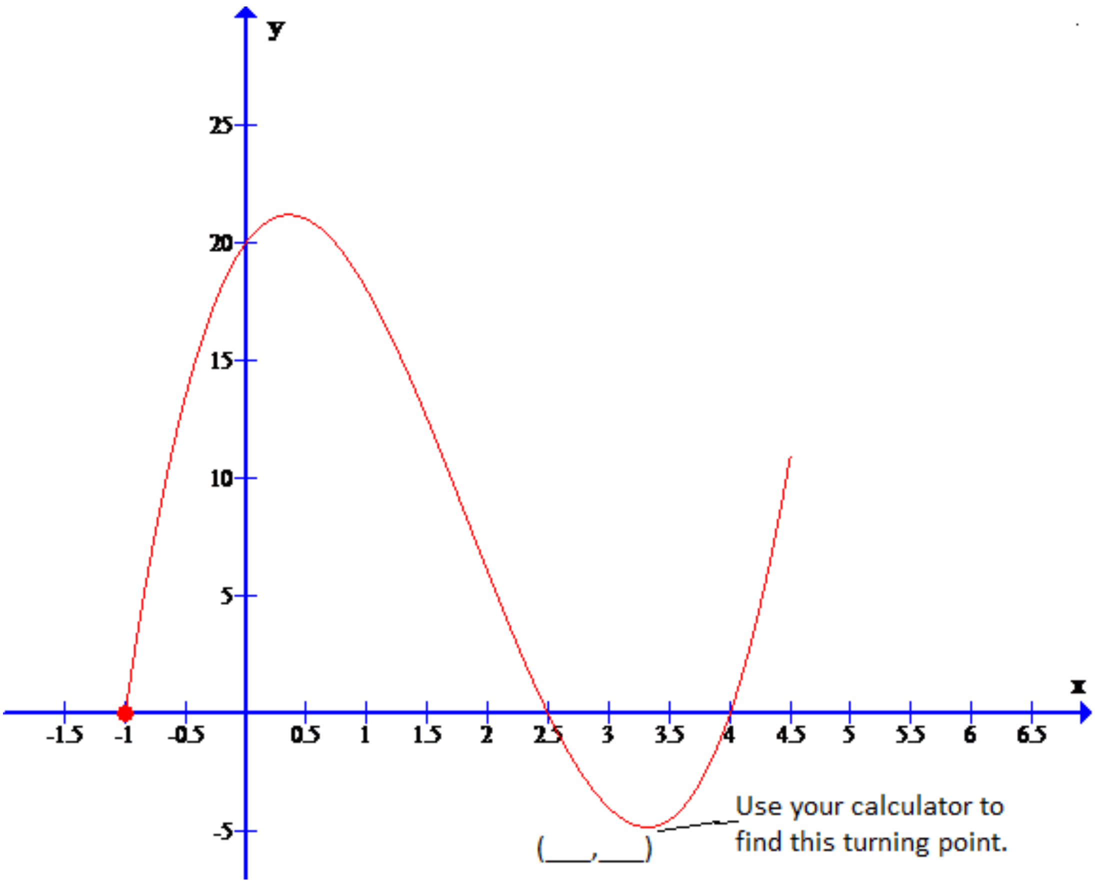
13.



Domain = _____

Range = _____

14.



The rule for the above curve is $y=2x^3-11x^2+7x+20$

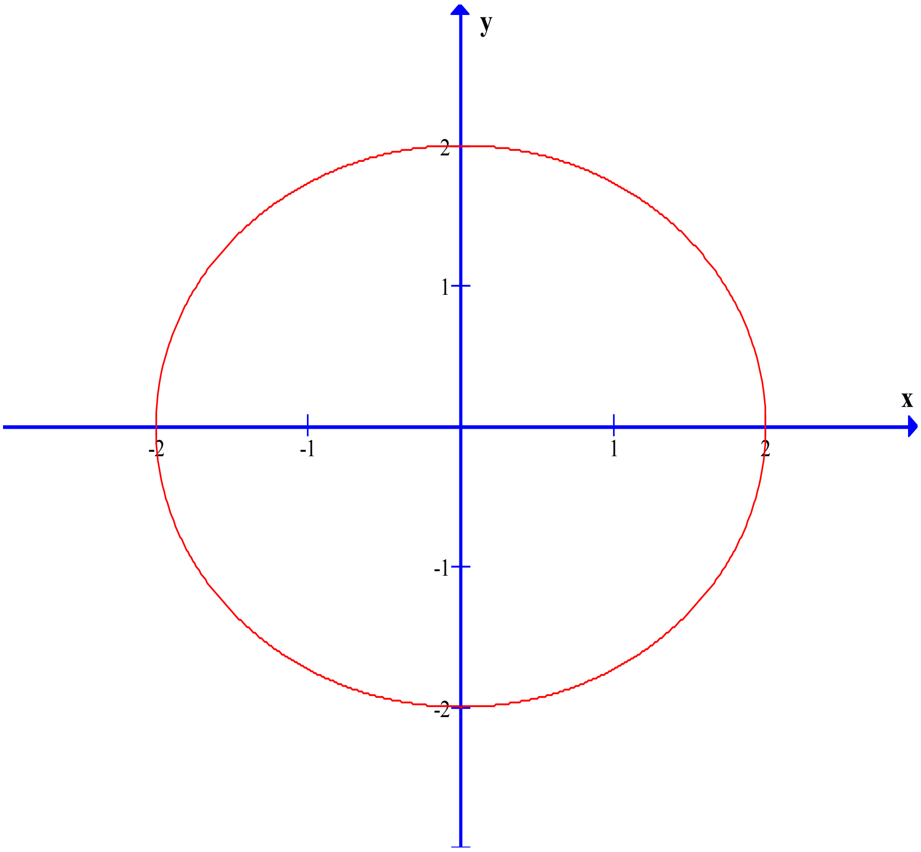
Domain = ______________________________

OR ____________

Range = ______________________________

OR ____________

15.



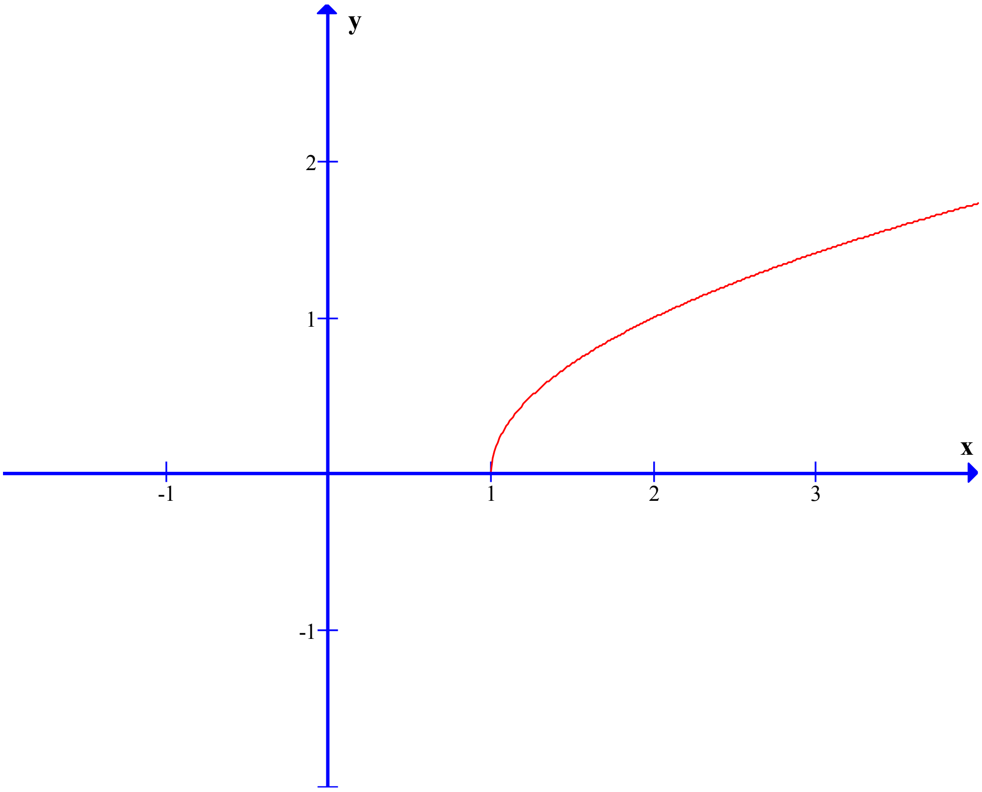
Domain = ____________________________

OR ____________

Range = ____________________________

OR ____________

16.



Domain = ____________________________

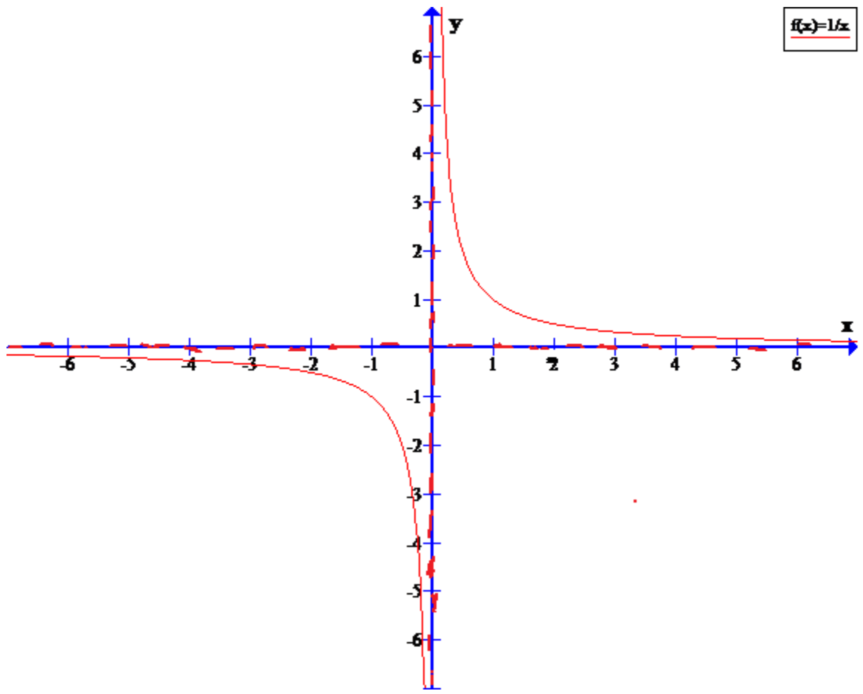
OR ____________

Range = ____________________________

OR ____________

OR ____________

17.



Using the rule for this relation $y = \frac{1}{x}$, explain why the graph of the relation gets closer and closer to the Y-axis but never touches it.

____

____

In such cases, the Y-axis is called a vertical asymptote and denoted by a dashed line along the axis.

In this case, the X-axis is also an asymptote, a horizontal asymptote ie the graph gets closer and closer to the X-axis but never touches it. Using the rule explain why.

____

____

Domain = ______________________________

OR ____________

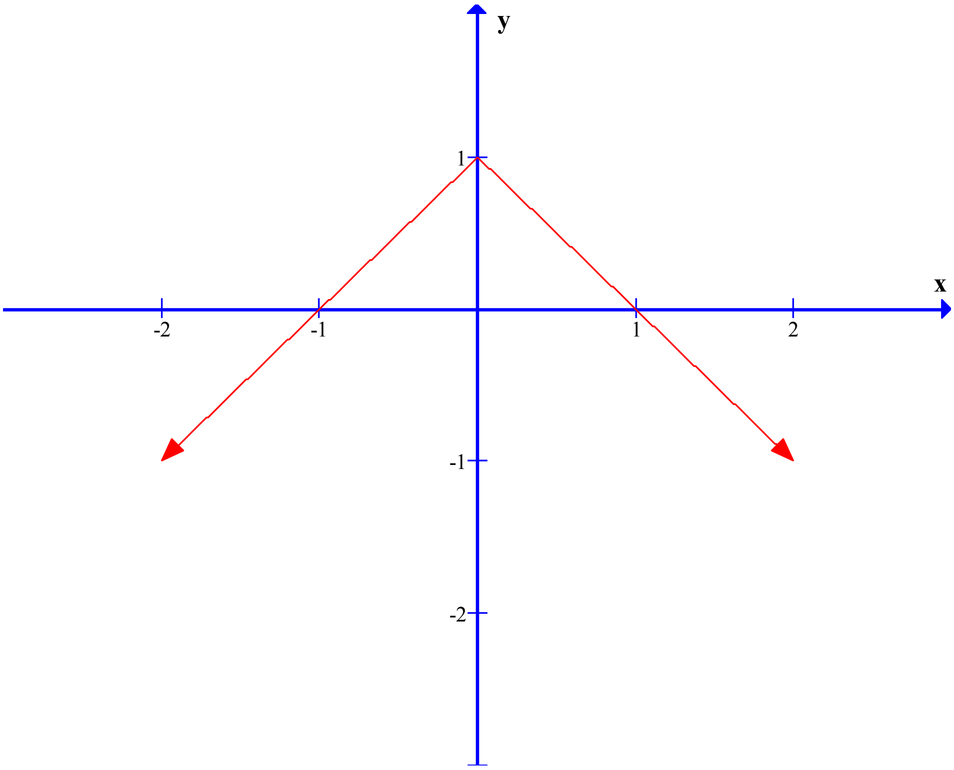
OR ____________

Range = ______________________________

OR ___________

OR ____________

18.

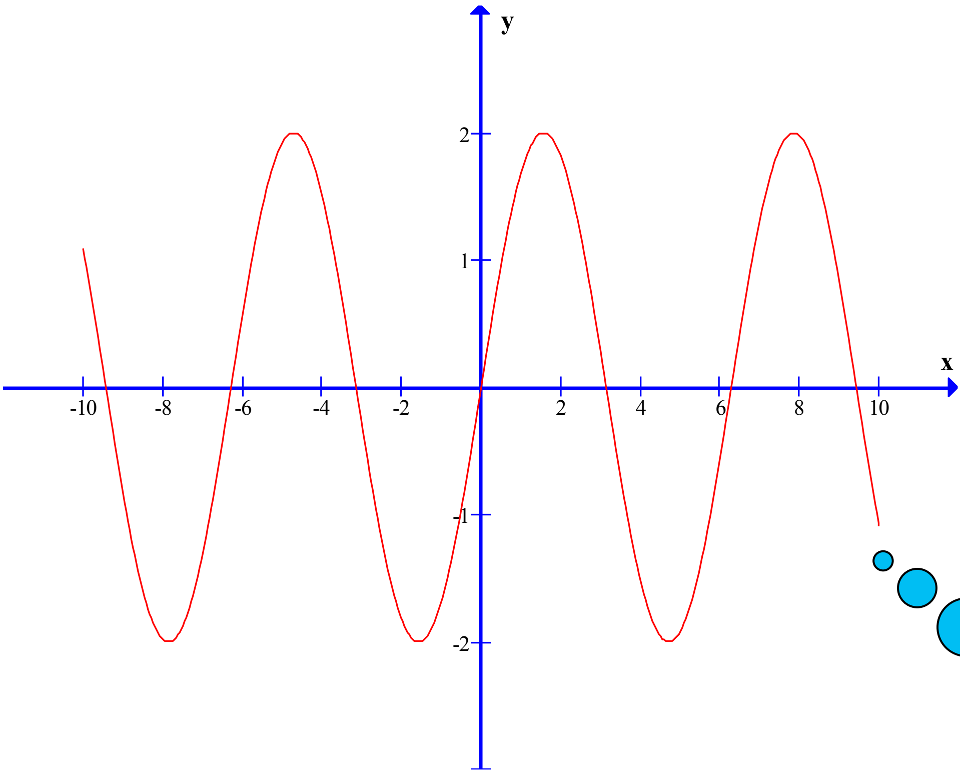


Domain = _____

Range = _____________________________

OR ___________

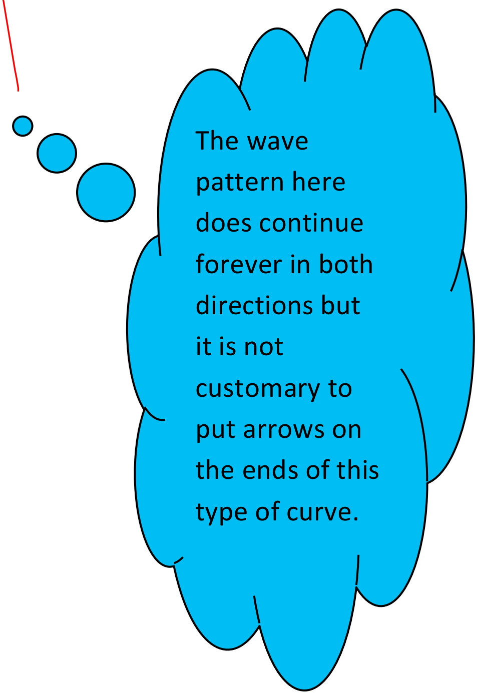
19.



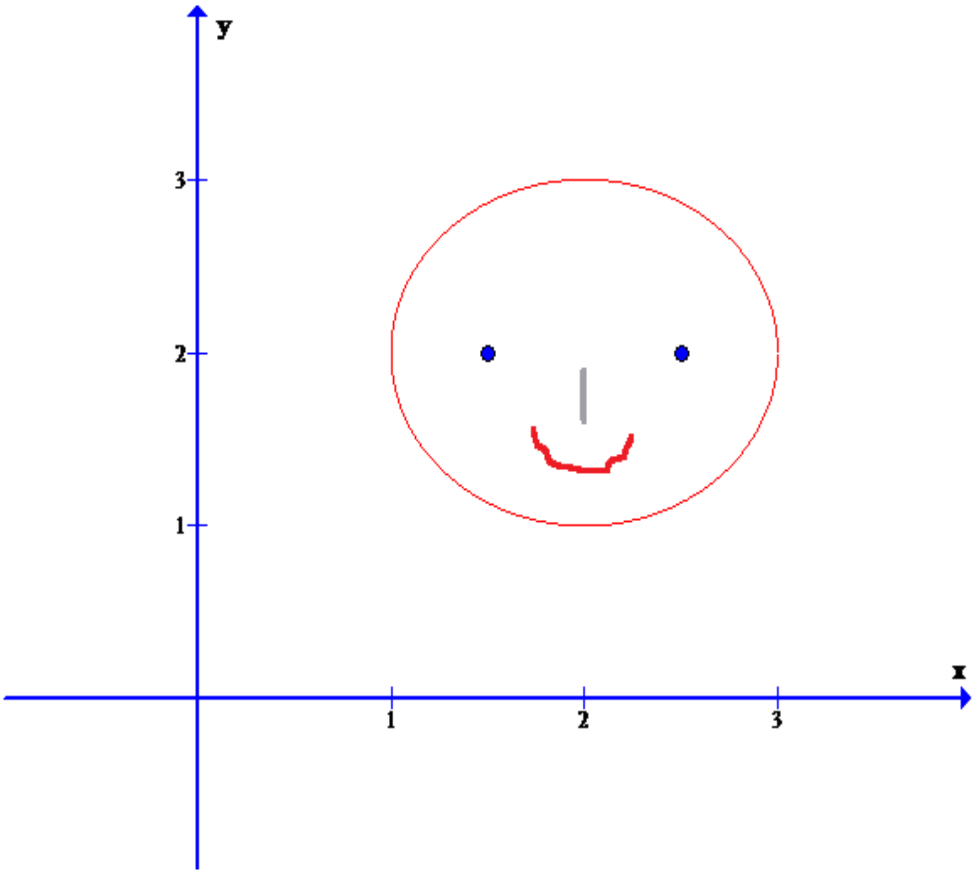
Domain = _____

Range = ______________________________

OR ___________



20.



Domain = ____________________________

OR ____________

Range = ____________________________

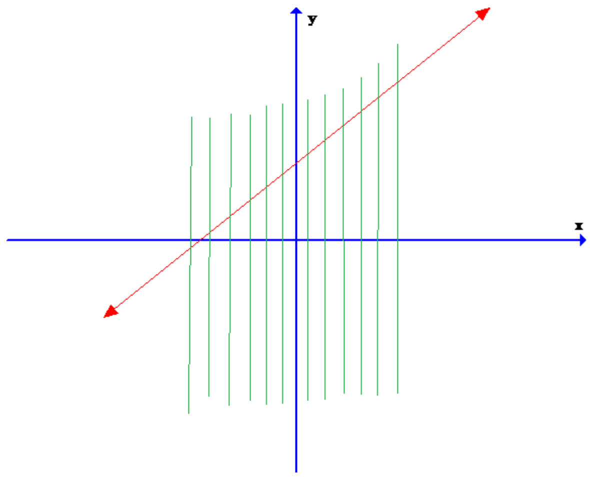
OR ____________

FUNCTIONS

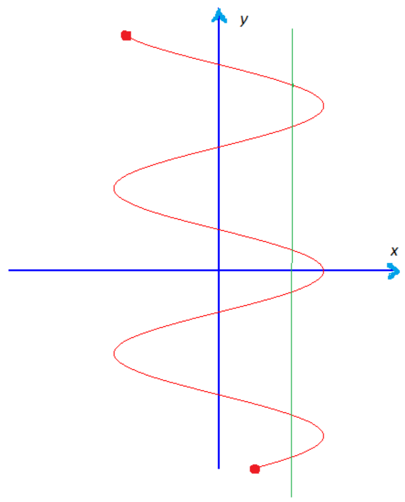
A function is a special type of relation.

To decide if a relation is a function, you can look at the graph.

If you are able to draw a *vertical line* anywhere that *crosses the graph more than once* then the relation is *not a function*. This is called the vertical line test.



All vertical lines touch only once.
This *is a function*.



We have a vertical line that touches the graph in 6 places.
This *is not a function*.

Exercise 6

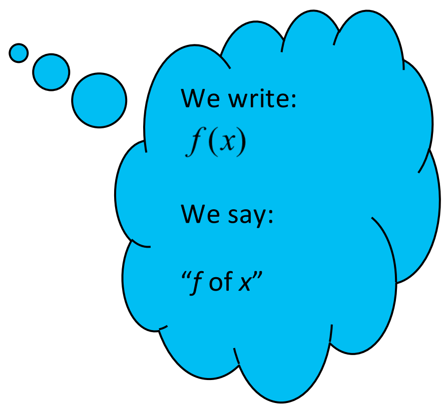
Go through the relations in exercise 5 and identify which of these relations are also functions.

Function Notation

When a relation is known to be a function, we can replace "y" in the rule with "*f(x)*" or "*g(x)*" or "*h(x)*".

eg1 $y = x + 1$ is a function so we can write the rule as: $f(x) = x + 1$.

Eg2 $y = x^2$ is a function so we can write the rule as: $g(x) = x^2$.



What is *f(2)*?

f(2) is the value of *y* when *x* takes the value 2.

eg1 If $f(x) = x + 1$

then $f(2) = 2 + 1 = 3$

f(2)=3 is just another way of writing:

when *x* = 2, *y*=3

eg2 If $g(x) = x^2$

then $g\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

$g\left(\frac{1}{2}\right) = \frac{1}{4}$ is just another way of writing:

when $x = \frac{1}{2}$, $y = \frac{1}{4}$

Exercise 7

1. $f(x) = 2x^2 - 1$, find

a. $f(0)$ **b.** $f(1)$ **c.** $f(-2)$

2. $g(x) = 2 - \frac{1}{x}$, find

a. $g(-1)$ **b.** $g\left(\frac{1}{2}\right)$ **c.** $g(3)$

3. $h(x) = 2x^3 + x^2 - 1$, find

a. $h(0)$ **b.** $h(-1)$ **c.** $h(a)$ where $a \in R$

Recall: $a \in R$ means "*a*" is some particular, fixed real number.

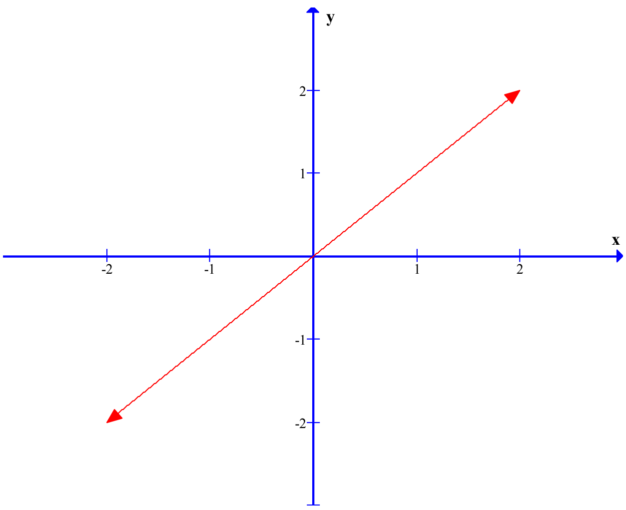
A fixed number is also called a constant.

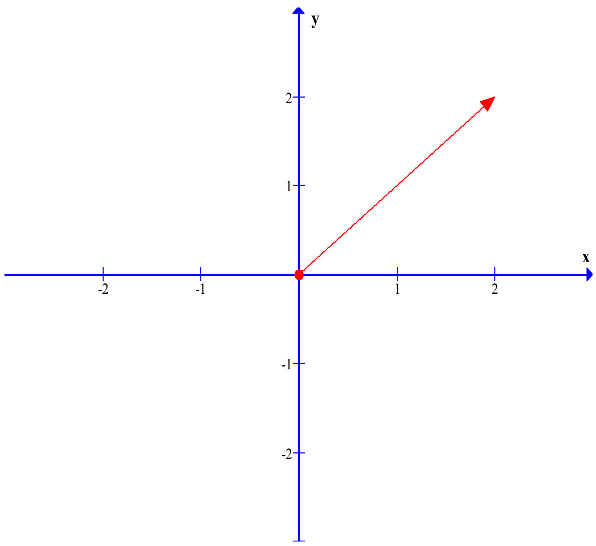
x and *y* on the other hand are variables and can assume a variety of values.

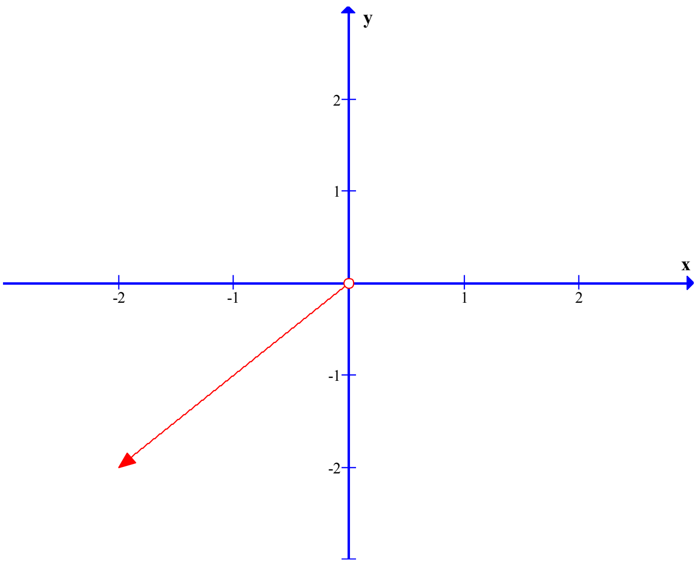
Rule and domain

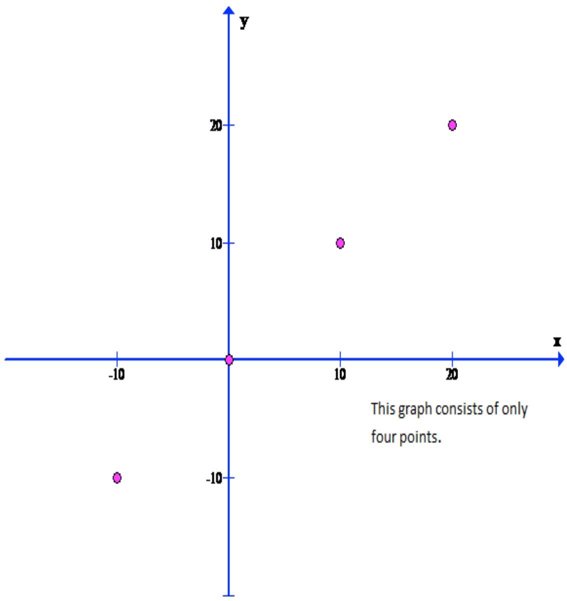
The rule for a relation does not tell me everything about that relation.

eg $y = x$ is the rule for all of the following graphs:









The thing that distinguishes (tells apart) these graphs is the domain.

If you know the rule for a relation and you know the domain then you know everything about that relation.

Class Discussion

Why don't we also need to state what the range is?

More Function Notation

The domain and rule for a function are conventionally written in one statement.

We write:

$$\boxed{f:}\ \boxed{}\ \boxed{\to}\ \boxed{R}\ \boxed{,}\ \boxed{f(x) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_}$$

Write the domain in here.

Write the rule last.

Write the co-domain in here. The co-domain can be the range or a bigger set than the range.

$$range \subseteq codomain$$

For our purposes, we can always put R in this spot.

We say:

"***f*** maps __________ into ***R*** where ***f*** of ***x*** equals____________________."

domain

rule

eg $y = x + 1$ with domain R^{+} is a function so we can write:

$f : R^{+} \to R, f(x) = x + 1$

We say:

"*f maps R plus into R where f of x equals x plus one*."

OR "*f maps positive real numbers into R where f of x equals x plus one*."

Exercise 8

Go back to the activity on "domain and range" and rewrite 2,3,5,7,10,11,13,14,16,17 in full function notation.

ANSWERS

Exercise 1

a) $\{3,5,7,9,11\}$

b) $\{52,54,56,58\}$

c) $\{-3,3\}$

d) $\{5,7,9,11,13,15,17,19...\}$

e) $\{2,3,5,7,11,13,17,19,23...\}$

Exercise 2

a) $\{x : x < 0\}$

b) $\{x : x < -2\}$

c) $\{x : -10 \leq x \leq 10\}$

d) $\{x : -1000.5 < x < 15.6\}$

Exercise 3

1.

a) $3 \in$ **N**

b) $-5 \notin$ **N**

c) $\frac{1}{2} \in$ **Q**

d) $4 \in$ **Q**

g) $-\frac{1}{2} \notin$ **Z**

h) $0 \in$ **R**

i) $0.\dot{3} \in$ **Q**

j) $3.\dot{3} \in$ **R**

e) $\pi \notin$ **Q**

f) $\pi \in$ **Q'**

k) $\sqrt{17} \in$ **R**

l) $\sqrt{-1} \notin$ **R**

2. $N \subset Z \subset Q \subset R$

3.

a) $\{0,1,2,3,4,...\}$

b) N

c) $\{0,-1,-2,-3,-4,...\} = Z^{-} \cup \{0\}$

d) R

Exercise 4

1.

a) $[-100,47]$

b) $[0,1)$

c) $(-10,7]$

d) $(82,\infty)$

e) $(-\infty,-5.8]$

f) $[2,9)$

g) $(0,\infty)$

h) $(-\infty,0)$

2. R^{-}

3. R could be written in interval notation as $(-\infty, \infty)$

4. No - Interval notation is for continuous sets not discrete sets.

Exercise 5

1.

Domain = $\{0,1,2,3\}$

Range = $\{0,-1,-2,3\}$

2.

Domain = $\{x: 0 \leq x \leq 2\}$ Defining a set by rule

OR $[0,2]$ Interval notation

Range = $\{y: 1 \leq y \leq 3\}$ Defining a set by rule

OR $[1,3]$ Interval notation

3.

Domain = $\{x: -20 < x \leq 0\}$ Defining a set by rule

OR $(-20,0]$ Interval notation

Range = $\{y: -20 < y \leq 20\}$ Defining a set by rule

OR $(-20,20]$ Interval notation

4.

Domain = $\{x : x \geq 0\}$ Defining a set by rule

OR $[0,\infty)$ Interval notation

OR $R^{+} \cup \{0\}$

Range = $\{y : y \leq 24\}$ Defining a set by rule

OR $(-\infty,24]$ Interval notation

5.

Domain = R

Range = $\{4\}$

6.

Domain = $\{2\}$

Range = R

7.

Domain = R

Range = R

8.

Domain = $\{x : -2 \leq x < 2\}$

OR $[-2,2)$

Range = $\{-1,0,1,2\}$

9.

Domain = $\{x : x \geq -2\}$

OR $[-2, \infty)$

Range = $\{-1\} \cup \{y : y \geq 1\}$

10.

Domain = R

Range = $\{y : y \geq 0\}$

OR $[0, \infty)$

11.

Domain = R

Range = $\{y : y \leq 3\}$

OR $(-\infty, 3]$

12.

Domain = $\{x : x \geq 0\}$

OR $[0, \infty)$

OR $R^{+} \cup \{0\}$

Range = R

13.

Domain = R

Range = R

14.

Domain = $\{x : x \geq -1\}$

OR $[-1, \infty)$

Range = $\{y : y \geq -4.8^{*}\}$ * rounded to 1 decimal place

OR $[-4.8^{*}, \infty)$

15.

Domain = $\{x : -2 \leq x \leq 2\}$

OR $[-2, 2]$

Range = $\{y : -2 \leq y \leq 2\}$

OR $[-2, 2]$

16.

Domain = $\{x : x \geq 1\}$

OR $[1, \infty)$

Range = $\{y : y \geq 0\}$

OR $[0, \infty)$

OR $R^{+} \cup \{0\}$

17.

Using the rule for this relation $y=\frac{1}{x}$, explain why the graph of the relation gets closer and closer to the Y-axis but never touches it.

When $x=1$, $y=\frac{1}{1}=1$ so $(1,1)$ is on the graph.

When $x=\frac{1}{2}$, $y=\frac{1}{1/2}=2$ so $\left(\frac{1}{2},2\right)$ is on the graph.

When $x=\frac{1}{10}$, $y=\frac{1}{1/10}=10$ so $\left(\frac{1}{10},10\right)$ is on the graph.

When $x=\frac{1}{100}$, $y=\frac{1}{1/100}=100$ so $\left(\frac{1}{100},100\right)$ is on the graph.

When $x=\frac{1}{5000}$, $y=\frac{1}{1/5000}=5000$ so $\left(\frac{1}{5000},5000\right)$ is on the graph.

When $x=\frac{1}{1000000}$, $y=\frac{1}{1/1000000}=1000000$ so $\left(\frac{1}{1000000},1000000\right)$ is on the graph.

.... and so on

The points $(1,1)$, $\left(\frac{1}{2},2\right)$, $\left(\frac{1}{10},10\right)$, $\left(\frac{1}{100},100\right)$, $\left(\frac{1}{5000},5000\right)$, $\left(\frac{1}{1000000},1000000\right)$,........are getting closer and closer to the y axis.

At the y intercept, x would take the value zero and $y=\frac{1}{0}$, but $\frac{1}{0}$ is undefined so there is no y intercept. The graph will never reach the y axis.

In such cases, the Y-axis is called a vertical asymptote and denoted by a dashed line along the axis.

In this case, the X-axis is also an asymptote, a horizontal asymptote ie the graph gets closer and closer to the X-axis but never touches it. Using the rule explain why.

When $x = 1,\qquad y = \frac{1}{1} = 1$ so $(1,1)$ is on the graph.

When $x = 2,\qquad y = \frac{1}{2}$ so $\left(2, \frac{1}{2}\right)$ is on the graph.

When $x = 10,\qquad y = \frac{1}{10}$ so $\left(10, \frac{1}{10}\right)$ is on the graph.

When $x = 100,\qquad y = \frac{1}{100}$ so $\left(100, \frac{1}{100}\right)$ is on the graph.

When $x = 5000,\qquad y = \frac{1}{5000}$ so $\left(5000, \frac{1}{5000}\right)$ is on the graph.

When $x = 1000000,\qquad y = \frac{1}{1000000}$ so $\left(1000000, \frac{1}{1000000}\right)$ is on the graph.

.... and so on

The points $(1,1)$, $\left(2, \frac{1}{2}\right)$, $\left(10, \frac{1}{10}\right)$, $\left(100, \frac{1}{100}\right)$, $\left(5000, \frac{1}{5000}\right)$, $\left(1000000, \frac{1}{1000000}\right)$,........are getting closer and closer to the x axis.

At the x intercept, y would take the value zero so then

$$0 = \frac{1}{x}$$

multiplying both sides by x, we get

$$0 = 1$$

but this is impossible so y can't take the value zero i.e. there is no x intercept. The graph will never reach the x axis.

Domain = $\{x : x < 0\} \cup \{x : x > 0\}$

OR $(-\infty, 0) \cup (0, \infty)$

OR $R \mid \{0\}$

Range = $\{y : y < 0\} \cup \{y : y > 0\}$

OR $(-\infty, 0) \cup (0, \infty)$

OR $R \mid \{0\}$

18.

Domain = R

Range = $\{y : y \leq 1\}$

OR $(-\infty, 1]$

19.

Domain = R

Range = $\{y : -2 \leq y \leq 2\}$

OR $[-2.2]$

20.

Domain = $\{x : 1 \leq x \leq 3\}$

OR $[1, 3]$

Range = $\{y : 1 \leq y \leq 3\}$

OR $[1, 3]$

Exercise 6

1,2,3,4,5,7,8,10,11,13,14,16,17,18,19 are functions

Exercise 7

1.

a. $f(0) = -1$ **b.** $f(1) = 1$ **c.** $f(-2) = 7$

2.

a. $g(-1) = 3$ **b.** $g\left(\frac{1}{2}\right) = 0$ **c.** $g(3) = 1\frac{2}{3}$

3.

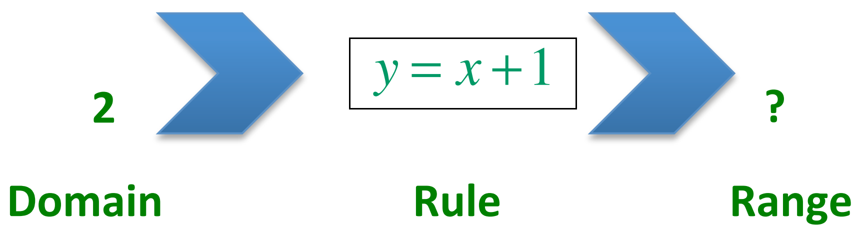
a. $h(0) = -1$ **b.** $h(-1) = -2$ **c.** $h(a) = 2a^3 + a^2 - 1 \qquad a \in R$

Class Discussion

The rule and the domain together determine the range.

Take each element of the domain and using the rule, you can calculate all the elements of the range.

e.g. Consider $y = x + 1$ with domain R i.e. $x \in R$. One of the elements of the domain is 2. Using the rule and $x = 2$ the corresponding element of the range, in this case 3, can be calculated.



Only the domain and rule are required to completely define a relation.

Note: When a domain is not stated, then the domain is taken to be the largest possible domain that the rule will allow.

e.g. Consider $y = \frac{1}{x}$

x can be any real number here except for 0 because $y=\frac{1}{0}$ is undefined, so if the domain is not stated, then it is taken to be $R\setminus\{0\}$.

Exercise 8

2. $f:[0,2]\to R,\quad f(x)=x+1$

3. $f:(-20,0]\to R,\quad f(x)=2x+20$

5. $f:R\to R,\quad f(x)=4$

7. $f:R\to R,\quad f(x)=x$

10. $f:R\to R,\quad f(x)=x^2$

11. $f:R\to R,\quad f(x)=-x^2+4x-1$

or

$f:R\to R,\quad f(x)=-(x-2)^2+3$

13. $f:R\to R,\quad f(x)=(x-2)(x-3)(x+5)$

or

$f:R\to R,\quad f(x)=x^3-19x+30$

14. $f:[-1,\infty)\to R,\quad f(x)=2x^3-11x^2+7x+20$

16. $f:[1,\infty)\to R,\quad f(x)=\sqrt{x-1}$

17. $f:R\setminus\{0\}\to R,\quad f(x)=\frac{1}{x}$