

Just Whole Numbers

Junior Mathematics
Study of Whole Numbers, Operations, Problems
and Patterns
(Grades 3,4 & 5)

LEARNING LOG



http://images.clipartpanda.com/math-rocks-clipart-6198852076_f881be1b4d_z.jpg

Name: _____

Junior Whole Numbers Learning Log

Copyright: Barbara J. Smith

First Edition, June 2016
3600 Yonge St.
Toronto, Ontario, Canada M4N3R8
Author: Barbara J. Smith

For other access and permission to use this resource and revise/customize it, please contact:
zpdschoolandcurriculumdesign@gmail.com

This document edition will be used as a pilot resource to support innovative schools. The intent of sharing this first version with students, staff and families, is so we can gather further input for future revisions of this living curriculum. All we ask is that if you use these materials that you give credit to the author(s) of this initial work, in your introduction.

Acknowledgement: Many thanks to Mark Brown for providing editorial support for this initial draft.

PURPOSE of LEARNING LOG RESOURCE:

1. To support the Ontario Mathematics Curriculum
2. To support independent and paired study during station work or during home study (holiday or at-home interest/extended homework activities)
3. To add support as an enrichment or remedial resource (students can work at their own pace)
4. To provide a learning log (evidence of learning) built in to student resources

What's in each lesson/unit?

- **Check In** – (diagnostic quiz – to reduce repeated teaching)
- **Essential Targets (ET)** – state or provincial expectations
- **Examples** (patterns for examining the math)
- **TECH CHECK** (more ways and examples for practicing the math)
- **REAL WORLD Problems** (context for math)
- **Fun and Games** (activities for making memories)
- **Reciprocal Teaching** (talking and demonstrating 'like a teacher')
- **GOT IT** (learning log/notebook evidence of learning)
- **Habit Check** (checking in on how students are doing the math)
- **Extensions** (students can extend where lesson leads next)
- **Master Quiz** (sample unit quiz)
- **Math Project** (culminating task revealing applied mastery of many ET's)



The MATH Challenge!

Trailblazer (Expert)	450 + points
Pathfinder (Apprentice)	400 - 449 points
Rookie (Novice)	< than 400 points

<u>Challenge</u>	<u>Maximum Points</u>
Comparing Numbers Quiz	50
Patterns Quiz	40
Addition and Subtraction Quiz	100
Multiplication Quiz	100
Division Quiz	100
Sign Makers Project	100
Classroom Work (Learning Log entries)	10
TOTAL	500

TABLE OF CONTENTS

PAGE	Essential Understandings to Master	Got it!	Ontario Ministry of Education Standards
Knowing & Comparing Numbers			
7	1. Getting to Millions		3A.1.1; 3A.1.2; 3A.1.3; 3A.1.4; 4A.1.1; 4A.1.11; 4A.1.3; 4A.3.1; 5A.1.9; 6A.1.6
16	2. The Bigger Number		
Problems & Patterns			
20	3. From Necklace to Number Patterns		3D.1.1; 3D.1.3; 3D.1.4; 4D.1.1; 4D.1.2; 4D.2.4; 5D.1.2; 6D.1.4
Operations: From Addition to Multiplication			
32	4. Tricky Adding and Subtracting		3A.3.1; 3A.3.2; 3D.2.2; 4A.2.1; 4A.2.2; 5A.3.1
36	5. Multiple Patterns 1-4		3A.2.1; 3A.3.5; 3D.1.2; 3D.1.6; 4D.1.3; 5D.1.3;
38	6. Fab Four Multiples		3A.3.5; 3D.1.2; 4D.1.3; 5D.1.3
41	7. Common Multiples		8A.1.4
43	8. 1 Digit Multiplication		3A.3.5; 3A.3.6; 3D.2.3; 4A.2.5; 4D.2.2; 5A.3.1
47	9. Multiplying by 10's		4A.2.6
49	10. Multiplying 2 or more Digit Numbers		4A.2.7; 4D.2.2; 5A.3.3; 6A.2.5
52	11. Factors		6A.1.7; 8A.1.4
Operations: Properties and Division			
62	12. Commutative Property		4D.2.3; 6A.2.1
68	13. Associative Law		3D.2.4; 6D.1.4
68	14. Division as Inverse Operation		3D.2.1; 4D.2.1; 6D.1.4
70	15. Division with Arrays and Number Lines		3A.3.5; 4D.2.3; 4D.2.3; 6D.1.4
73	16. Divide by Single Digit Numbers		3A.3.5; 3A.3.6; 4A.2.8
76	17. Dividing by 2 or more Digit Numbers		4A.2.8; 5A.3.1; 5A.3.4; 6A.2.5
78	18. Expanded Form by Adding		3A.1.3; 3A.1.10
81	19. Expanded Form with Addition and Multiplication		4A.2.2
83	20. Expanded Notation using Powers		8A.1.1
90	Appendix		

MATHEMATICIANS DO...

A Rookie or **novice** mathematician - needs the example to complete the math task.
An **expert** or 'trailblazer' mathematician - does not need the example to complete the math task. S/he can explain how to do it to others 😊

MATHEMATICIANS SAY...

The **expert** mathematician explains to self and others using math words how to complete the task. The use "*math detective language*"

"perseverance"

I can try many times
and many ways to
understand and solve
math problems.

<http://1.bp.blogspot.com/-N5fM3XYBmpY/UttGN44O51I/AAAAAAAAItc/Oi7EKOD0qgQ/s1600/Slide4.JPG>

Contract (Commit to Becoming a Mathematician)

- ◆ I will not give up learning the rules of math. _____
- ◆ I will re-read and check over my work. _____
- ◆ I will teach what I learn to someone else. _____



<http://www.business.techniquet.org/itemimages/1e5482f9e076709b5dac38506b32f786.jpg>



A. Knowing and Comparing Numbers

CHECK IN: Maybe you already know this?
Show your teacher what you know!

- Circle the place value of 3 in each number:

2,003	tens	ones	hundreds
161,304,775	thousands	millions	hundred thousands
3,488,228	millions	thousands	hundred thousands

- Circle the greatest number from the following list:

852,903 819,278 698,456

- Write the number you circled in words:

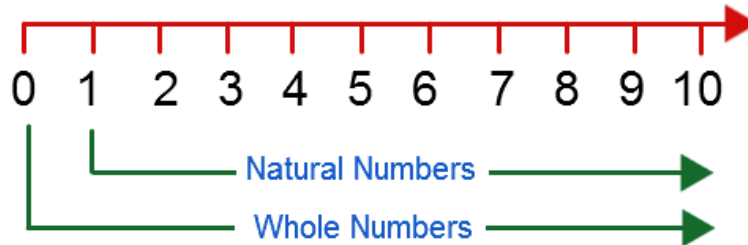
How well did I complete these tasks?	Like a Trailblazer (expert)	Like a Pathfinder (apprentice)	Like a rookie (need more help & practice)

1. Getting to Millions

RULE: You need seven digits to have a number that is at least 1 million (so long as the digit on the left is not zero).

EXAMPLES:

- Look at the image below and explain the difference between natural and whole numbers.



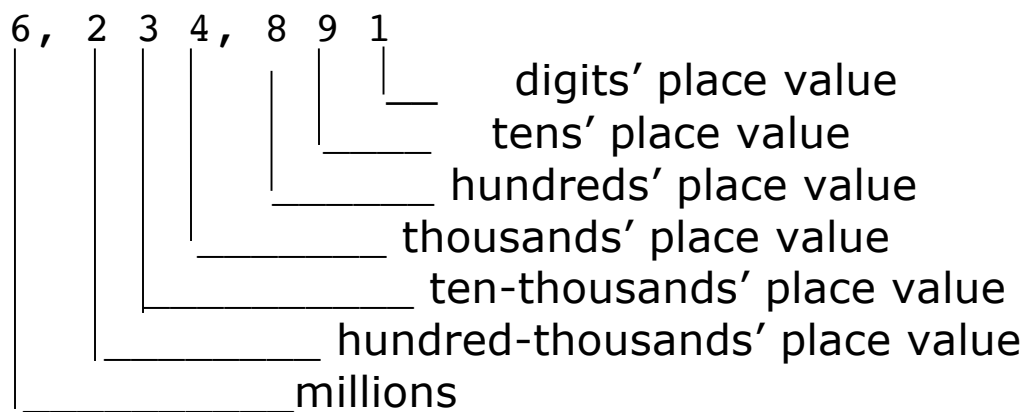
<http://images.tutorcircle.com/cms/images/tcimages/whole-and-natural-numbers.png>

- Look at how large numbers can be part of three groups.

MILLIONS			THOUSANDS			ONES		
hundred millions	ten millions	millions	hundred thousands	ten thousands	thousands	hundreds	tens	ones
7	4	5	3	0	9	2	8	1

https://www.eduplace.com/math/mw/background/4/01/graphics/ts_4_1_wi-2.gif

The number, 6,234,895, has a place value for each digit.



In this number there are:

- 6 sets of one million,
- 2 sets of one hundred thousand,
- 3 sets of ten thousand,
- 4 sets of one thousand,
- 8 sets of one hundred,
- 9 sets of ten, and 5 ones.

TECH CHECK!

- http://www.aaastudy.com/g5_41cx1.htm#section2
- http://www.softschools.com/quizzes/math/place_value_and_expanded_notation/quiz677.html
- <http://www.youtube.com/watch?v=e-pSz9p0Mvo&feature=fvwrel>

FUN & GAMES: Place Value Dirt Chart

- In pairs, make a place value dirt chart and use stones to make numbers.
- Go to each chart and read out the numbers your classmates have made on their place value charts.

Backward Blindfold Counting Game

- Each person wears a blindfold and counts backwards from 100 by:

2 _____ (your time)

5 _____ (your time)

10 _____ (your time)

- Count backward from 1000 by 10: _____ (your time)

RECIPROCAL TEACHING:

- "Like a teacher" explain how to identify place value with numbers between 0 and 1,000,000.
- Share examples and teach place value to a friend or family member.
- Use scrap paper or a notebook.
- Have your friend or family member change places and replay what you taught.



GOT IT! The 4 is in which place value?

(a) $5,784,223 =$ _____

(b) $4,689,315 =$ _____

(c) $9,573,428 =$ _____

- Put the following numbers in order from largest to smallest.

2,650,007

2,648,882

13,005,174

1. (largest): _____

2. _____

3. (smallest): _____

How well did you order numbers and use place value within 0 to 1 million?	Trailblazer (Expert)	Pathfinder (Apprentice)	Rookie (Not Yet)

RULE: You can convert numbers between written and standard form.

EXAMPLES:

Converting from Written and Standard Form

If you want to write a cheque, you will need to know how to write numbers in written form. Look at the samples below.

- Four hundred thousand one hundred and sixty six.

400 166

- Ninety one thousand six hundred and twenty one.

91 621

- Seventeen thousand and thirty five.

17 035

- Two million, three hundred thousand and seventy seven.

2 300 077

- Ten million, seventy one thousand four hundred and two.

10 071 402

http://images.slideplayer.com/16/4891081/slides/slide_8.jpg

The number in written form:

three million, forty-seven thousand, eight hundred and two
is 3,047,802.

Using place value, you can figure out where to put the '0' in as a place holder.

GOT IT!

- Write out the following numbers in written form.

(a) 494 = _____

(b) 103,780 = _____

(c) 21,552,704 = _____

(d) forty-five thousand and 3 = _____

(e) two million, 16 thousand and ten = _____

(f) seventeen million, seven hundred thousand, fourteen =

How well did you convert numbers between written and standard form?

Trailblazer (Expert)	Pathfinder (Apprentice)	Rookie (Not Yet)

Extension:

It doesn't stop with millions - there are billions and trillions and even more numbers out there!

8 , 165 , 432 , 098 , 710
trillions billions millions thousands ones

8 → Eight trillion,
165 → One hundred sixty-five billion,
432 → Four hundred thirty-two million,
098 → Ninety-eight thousand,
710 → Seven hundred ten

http://cnx.org/resources/30b147ad3d69711bd28cef174ae62cf550104bca/CNX_ElemAlg_Figure_01_01_005_img.jpg

- Teach a mini lesson using the following place value chart:

MathATube.com

Place Value Chart

Hundred-billions	Ten-billions	Billions	Hundred-millions	Ten-millions	Millions	Hundred-thousands	Ten-thousands	Thousands	Hundreds	Tens	Ones

<http://www.mathatube.com/sitebuilder/images/place-value-chart-53-609x431.jpg>

- Create a giant place value chart that orders numbers and place value and notation form from 0.01 to 1 billion.

Math Project: Sign Making



<https://s-media-cache-ak0.pinimg.com/564x/c0/ee/ca/c0eeca1f64ae2e94a843a8ed94dedf68.jpg>

1. Your teacher will assign partners for this project.

2. Select a list of 5 places around the world that you would like to feature on a sign.

3. List the city population facts and distances from your school in the table below.

4. Create another place value chart to illustrate the populations of each location.

5. Create a place value chart to illustrate the distances to each location.

- You might want to browse the web for some innovative and attractive highway signs for some ideas...
- Check out some highway sign art: <http://borisbally.com/about/>.

City	Population	Distance from School

Place Value Chart for City Populations

Place Value Chart for Distances from School

Self Check	Teacher Assigned POINTS for Sign Making	Teacher Score
	listed 5 names of real destinations on different continents.	/5
	recorded each population from each city in the Population Table below.	/5
	calculated the approximate distance between school and each city.	/10
	created a place value chart to illustrate the various populations.	/5
	created a place value chart to illustrate the various estimated distances from school to each city.	/5
	sign detailed population of each city on one side	/5
	sign detailed distance to school from 5 cities	/5
	used materials with care (used a ruler, cardboard/ wood)	/2
	sturdiness of sign	/2
	attractiveness of sign	/2
	creativity of sign	/2
	cooperated with partner	/2
TOTAL		/50

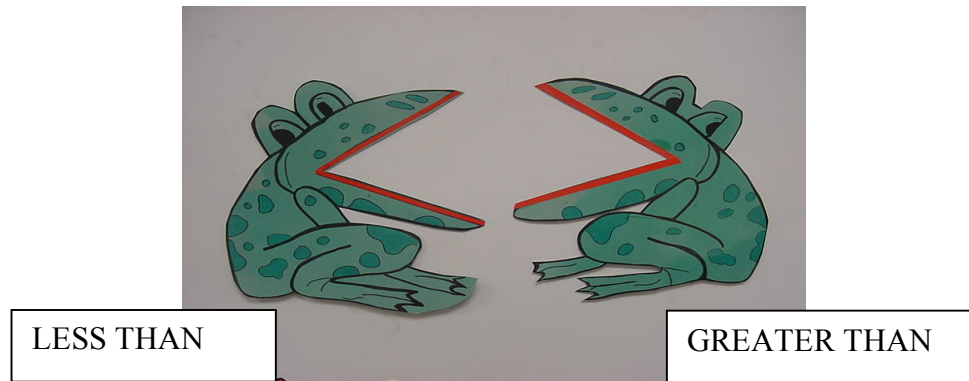


<http://www.freshdesignpedia.com/wp-content/uploads/cool-designer-furniture-from-old-road-signs/designer-furniture-colorful-and-creative.jpg>

2. The Bigger Number

Rule: Greater than 'opens up' to the bigger number. Less than points at the smaller number.

EXAMPLES:



<http://st024.k12.sd.us/MVC-053S.JPG>

$$4,376,849 > 4,373,875$$

$$7,531,684 < 7,536,327$$

TECH CHECK:

- http://www.aaastudy.com/g5_41bx1.htm
- http://www.innovationslearning.co.uk/subjects/maths/activities/year4/greater_than/start.htm
- <http://www.mathsisfun.com/equal-less-greater.html>
- <http://www.superteacherworksheets.com/place-value/greaterless-6digit-b.pdf>
- <http://www.superteacherworksheets.com/place-value/place-and-value.pdf>

GOT IT! (Fill in the sign below)

(a) 3,567,221 ____ 3,558,942

(b) 4,193,401 ____ 4,191,764

(c) 9,228,734 ____ 9,237,523

Extension:

- Take the Jack, King, and Queen out of a deck of cards.
- Deal out 7 cards to one team and seven cards to the other team.
- Make the largest number you can. (1 point)
- Write out a greater than/less than or equal to equation. (2 points)
- Read the equation out loud. (3 points)
- Repeat game. Different team members must read each time.
- Team members can help each other!

http://ayamcobek.files.wordpress.com/2008/09/ist2_493857-queen-of-hearts-playing-cards-background1.jpg



Daily Learning Log and Math Detective Habits:

___ stayed on task (worked well with partner and independently)

___ used math detective language to teach ideas

___ work is neat and easy to read

___ completed work by following instructions

___ work shows precision

QUIZ TIME - Can you complete without looking at examples?

- Circle the place value of 7 in each number:

2,073 tens ones hundreds

161,704,225 thousands millions hundred thousands

7, 488, 228 millions thousands hundred thousands

- Create a number; circle the ten-thousand's digit: _____

- Circle the lowest number from the following list:

852,903 698, 441 819, 278 698,456

- Write the number you circled in words:

- What is a digit? A digit is _____.

Questions I can now answer...

What is ten thousand?

What is one hundred thousand?

What is a digit?

How well did I complete	Like a Trailblazer (expert)	Like a Pathfinder (apprentice)	Like a rookie (need more)
-------------------------	-----------------------------	--------------------------------	---------------------------

these tasks?			help & practice)

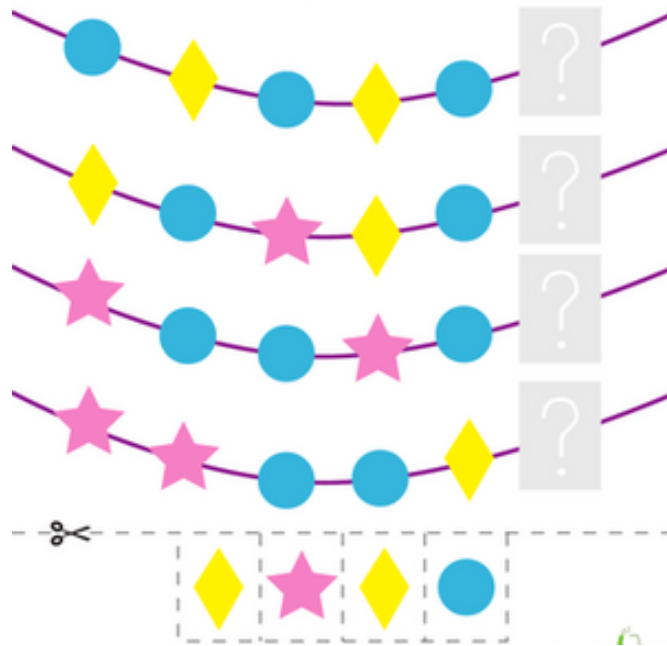
B. Problems and Patterns

CHECK IN: Maybe you already know all this? Show your teacher what you know!

- Complete the picture pattern:

Pearly Necklace

Which bead comes next? Help finish the pattern by cutting out the missing beads and gluing them on the correct necklace.



- Find a pattern in this sequence, and use that pattern to predict the next four numbers.

7, 10, 13, 16, 19, __, __, __, __.

What is the pattern of the numbers? _____

Solution: The next four numbers are _____

3. From Necklace to Number Patterns

Rule: How do we solve math problems?

- ✓ Read the question first to find out what you are trying to find.
- ✓ Re-read the problem and underline key words and numbers.
- ✓ Record the words or number in a list, a chart or a diagram.
- ✓ Focus on one solution at a time (Use your powers of elimination.)
- ✓ Separate rough work in a column on the left side of your working paper (use a ruler!)
- ✓ Use a table to find out what other numbers appear to be part of the pattern.

EXAMPLES:

- Find a pattern in this sequence, and use that pattern to predict the next four numbers.

7, 10, 13, 16, 19, __, __, __, __.

What is the pattern of the numbers?

Adding 3 to each number

Solution: The next four numbers are **22, 25, 28 and 31.**

TECH CHECK:

- <https://www.youtube.com/watch?v=d2Vq6wwJTDc>
- <https://www.youtube.com/watch?v=0TXxwNCRgKU>
- <http://www.ck12.org/book/CK-12-Algebra-I-Second-Edition/section/1.8/#sthash.iSx8NkYK.dpuf>

GOT IT!

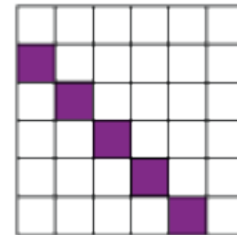
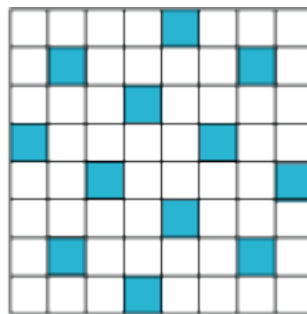
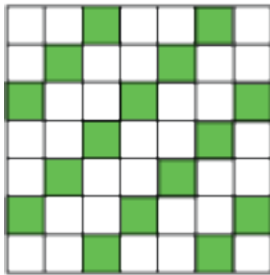
1) 9, 11, 13, 15, 17, 19, ___

2) 1, 7, 13, 19, 25, 31, ___

3) 8, 11, 14, 17, 20, 23, ___

4) 9, 16, 23, 30, 37, 44, ___

- What are the patterns on the following grids?

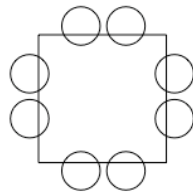


REAL WORLD PROBLEMS:

1. Josie takes up jogging. On the first week, she jogs for 10 minutes per day, on the second week she jogs for 12 minutes per day. Each week, she wants to increase her jogging time by 2 minutes per day. If she jogs six days each week, what will be her total jogging time on the sixth week?

ROUGH WORK 😊

2. You are at the party and sitting around the table with seven friends.



At the top left hand corner is the friend who is giving the party. S/he has a bag of treats and starts giving them out in a clockwise direction: one for her/himself, two for the next person and three for the next and so on. What are the total number of treats that children sitting opposite each other have?

ROUGH WORK 😊

--	--

3. Scientists are tracking two pods of whales during their migratory season. On the first day of June, one pod is 120 miles north of a certain group of islands, and every day thereafter it gets 15 miles closer to the islands. The second pod starts out 160 miles east of the islands on June 3, and heads toward the islands at a rate of 20 miles a day.

- a. Which pod will arrive at the islands first, and on what day?
- b. How long after that will it take the other pod to reach the islands?
- c. Suppose the pod that reaches the islands first immediately heads south from the islands at a rate of 15 miles a day, and the pod that gets there second also heads south from there at a rate of 25 miles a day. On what day will the second pod catch up with the first?
- d. How far will both pods be from the islands on that day?

	ROUGH WORK 😊
--	--------------

4. Kim owns a business that rents out tables and chairs for parties. For big events, she sets up tables end to end in long rows. Every individual table seats 2 people on each side. Two more people can sit on the ends of the rows. Kim would like to know how many chairs to get out of storage to set around rows of these tables. Make a picture and a chart to figure out how many people can sit around four tables.

ROUGH WORK 😊

Did you underline what you were trying to find out? Yes, No
 Did you circle key words and numbers in the problem? Yes, No?
 Did you try to do this in your head? Yes? No?

How well did I complete these tasks?	Like a Trailblazer (expert)	Like a Pathfinder (apprentice)	Like a rookie (need more help & practice)

STRATEGIES:

Create visual images. “Mind pictures” of a problem

Guesstimate. Trial-and-error approach for beginning data

Create a table. A table is an orderly arrangement of data.

Use manipulatives. By moving objects around on a table or desk, you can develop patterns

Work backward. Do calculations to arrive at the data presented at the beginning of the problem.

Look for a pattern. Looking for patterns to see if data falls into predictable patterns.

Create a systematic list. Record ideas in lists to determine patterns or similarities between problem elements.

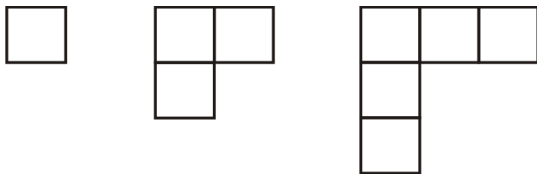
Simplifying. When a problem is too complex to solve in one step, it often helps to divide it into simpler problems and solve each one separately. Creating a simpler problem from a more complex one may involve rewording the problem; using smaller, simpler numbers; or using a more familiar scenario to understand the problem and find the solution.

Extension:

- Create an algebraic problem by letting a “letter” (a cursive lower case letter) represent the number you are trying to find.

QUIZ TIME: Complete without looking at examples.

1. Jeremy divides a 160-square-foot garden into plots that are either 10 or 12 square feet each. If there are 14 plots in all, how many plots are there of each size?
2. A pattern of squares is put together as shown. How many squares are in the 12th diagram?



3. In Harrisville, local housing laws specify how many people can live in a house or apartment: the maximum number of people allowed is twice the number of bedrooms, plus one. If Jan, Pat, and their four children want to rent a house, how many bedrooms must it have?

ROUGH WORK 😊

4. A new theme park opens in Milford. On opening day, the park has 120 visitors; on each of the next three days, the park has 10 more visitors than the day before; and on each of the three days after that, the park has 20 more visitors than the day before.

- a. How many visitors does the park have on the seventh day?
b. How many total visitors does the park have all week?

ROUGH WORK 😊

BONUS:

- How can you describe the pattern below?



Questions I can now answer...

What is a pattern?

What is a table?

What are strategies I can use to solve pattern problems?

Daily Learning Log and Math Detective Habits:

___ stayed on task (worked well with partner and independently)

___ used math detective language to teach ideas

___ work is neat and easy to read

___ completed work by following instructions

___ work shows precision

How well did I complete these tasks?	Like a Trailblazer (expert)	Like a Pathfinder (apprentice)	Like a rookie (need more help & practice)

C. Operations - From Addition to Multiplication

CHECK IN: Maybe you already know all this? Show your teacher what you know!

Question	Rough Work
1) $\begin{array}{r} 1574 \\ + 6287 \\ \hline \end{array}$	<u>Show inverse operation to check</u>
2) $\begin{array}{r} 6287 \\ - 1574 \\ \hline \end{array}$	<u>Show inverse operation to check</u>
3) List the multiples of 7 (end at 70).	

4) List the Least Common Multiples (LCM) of 6 and 7.

ROUGH WORK 😊

5)
$$\begin{array}{r} 7 \\ \times 3 \\ \hline \end{array}$$

6)
$$\begin{array}{r} 11 \\ \times 11 \\ \hline \end{array}$$

7) Factor 48.

8) Find the Greatest Common Factor (GCF) for 32 and 48	
9) Solve this problem. David was asked to put 5 baskets of pencils together with 3 pencils in each (one for each member of the group). How many pencils did he have all together?	

How well did I complete these tasks?	Like a Trailblazer (expert)	Like a Pathfinder (apprentice)	Like a rookie (need more help & practice)

4. Tricky Adding and Subtracting

RULE: When you add part of a number and the answer is greater than 10, place the last number under the column and any remaining numbers above the column to the left (carry it over). When you subtract numbers where the digit is smaller on top, then you borrow from the left to make the digit larger. This reduces the value of the digit on the left by 1 (borrowing).

EXAMPLES:

$$\begin{array}{r}
 11 \\
 1574 \\
 + 6287 \\
 \hline
 7861
 \end{array}$$

$$\begin{array}{r}
 4\ 078 \\
 89 \\
 386 \\
 + 9\ 347 \\
 \hline
 13\ 900
 \end{array}$$

$\begin{array}{r} 15 \\ 765 \\ - 39 \\ \hline 726 \end{array}$ <p>I am adding a 10 I'm adding one here too.</p> <p>Borrow and Pay Back Method Or Equal Additions</p>	$\begin{array}{r} 515 \\ 765 \\ - 39 \\ \hline 726 \end{array}$ <p>Borrow and Regroup Method Or Decomposition</p>
---	--

<http://www.littlehouseinthevalley.com/wp-content/themes/images/subtraction.jpg>

TECH CHECK:

- <http://quizmoz.com/tests/Maths-Tests/a/Adding-and-subtracting-Test.asp>
- <http://www.mathsisfun.com/numbers/subtraction-regrouping.html>
- <http://www.myschoolhouse.com/courses/O/1/40.asp>
- http://www.mathsteacher.com.au/year7/ch01_whole/03_sub/sub.htm
- <http://www.youtube.com/watch?v=jPb2SDBUGns>

GOT IT! Show your rough work.

(a) $5479 + 456$

Rough Work/Check

(b) $951 - 86$

(c) 6801 - 4286	
REAL WORLD PROBLEMS: A new movie theater sells 6,783 tickets in the first year, 5,697 tickets in the second year, and in its third year, sells 634 fewer tickets than in its second year. How many tickets are sold in 3 years?	
	ROUGH WORK 😊
34	

STEP OUTSIDE:

- Create an addition or a subtraction problem using numbers and things outside.
- Solve your problem and then solve a problem a classmate made.



http://i.vimeocdn.com/video/465454232_1280x720.jpg

My Problem:

ROUGH WORK 😊

My Classmate's Problem:

ROUGH WORK 😊

How well did I complete these tasks?	Like a Trailblazer (expert)	Like a Pathfinder (apprentice)	Like a rookie (need more help & practice)

ET - Identify multiples of 1 through 5.

5. Multiple Patterns 1 to 4

RULE: Multiples are lists of products. To find multiples you just need to know how to multiply the number by 1, 2, 3, 4, 5, 6, 7, 8, and 9.

Why do I need to know multiples?

Multiples help you learn the pattern of "products" so you can multiply without having to do repeated addition!

EXAMPLES (View these examples of multiples of 1 and 3):

1, 2, 3, 4, 5, 6, 7..... (multiples of 1)
(1x1), (1x2), (1x3), (1x4), (1x5), (1 x6), (1 x 7)

3, 6, 9, 12, 15, 18, 21, 24, 27, 30.. (multiples of 3)
(3x1),(3x2),(3x3),(3x4),(3x5),(3x6),(3x7),(3x8),(3x9), (3x10)

FUN & GAMES:

Sing along with the following:

<https://www.youtube.com/watch?v=9ue9Kux95H0> (4)

RECIPROCAL TEACHING:

- "Like a teacher" explain how to find multiples of numbers.
- Share examples and teach multiples to a friend or family member.
- Use scrap paper.
- Have your friend or family member change places and replay what you taught.

HOME CHALLENGE - "Pillow Talk" - Do multiples of 3 to 30 and 4 to 40 in your head before you go to sleep. Count multiples instead of sheep!

TECH CHECK: (Prove it by doing a quiz or completing a tech task below!)

- http://www.sheppardsoftware.com/mathgames/multiple/multiple_frenzy.htm
- <http://www.homeschoolmath.net/worksheets/examples/multiples-skip-count-3.htm>
- <http://www.ezscool.com/Games/MultiMultiples.html>

GOT IT!

1) List the multiples of 2 up to 20.

2) List the multiples of 4 up to 40.

3) What kind of numbers are multiples of 2?

Odd numbers, Even numbers, Prime Numbers

4) Complete this chart ☺.

MULTIPLES CHART *Fill in the rest... CHUNK (Do 2 and 4)

	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10
1	1	2	3	4	5	6	7	8	9	10
2										
3	3	6	9	12	15	18	21	24	27	30
4										

How well did you identify multiples of 1 through 5?

Trailblazer (Expert)	Pathfinder (Apprentice)	Rookie (Not Yet)

6. Fab 4 Multiples (6,7,8 & 9)

RULE: To find the Fab 4 multiples, multiply each digit by 1, 2,3,4,5,6,7,8,9,10,11,12 to get the first 12 multiples. Knowing the pattern of multiples helps you memorize your speedy multiplication tables.

-
-
-
-
-
-
-

EXAMPLES:

7, 14, 21, 28, 35, 42, 49, 56, 63, 70, 77, 84... (multiples of 7)

9, 18, 27, 36, 45, 54, 63, 72, 81, 90, 99, 108... (multiples of 9)

FUN & GAMES:

- Sing along with the following videos:
 - https://www.youtube.com/watch?time_continue=178&v=9os1VUUUp5io (5 x 6)
 - <https://www.youtube.com/watch?v=9XzfQUXqiYY> (3 times)
 - <https://www.youtube.com/watch?v=VEnQbnxWtqM> (7)
 - <https://www.youtube.com/watch?v=0X620leUkYE> (8)
 - https://www.youtube.com/watch?v=q_zUEV5uK8Q (9) gentle song

TECH CHECK:

- <https://www.youtube.com/watch?v=VhpUt0vXI4w>
- <http://espanol.video.yahoo.com/watch/1380772/4752513>
- http://www.helpingwithmath.com/by_subject/factors_multiples/fac_multiples.htm
- <http://www.ezschoo.com/Games/MultiMultiples.html>
- <http://www.math-play.com/Factors-and-Multiples-Jeopardy/Factors-and-Multiples-Jeopardy.html>

GOT IT!

1) List the multiples of 6 up to 72.

2) List the multiples of 8 up to 96.

3) List multiples of 11 to 121.

4) Complete this chart ☺.

MULTIPLES CHART *Fill in the rest... CHUNK (Do 6,7,8 and 9)

	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6										
7										
8										
9										
10	10	20	30	40	50	60	70	80	90	100

Home Challenge - "Pillow Talk" - Do multiples of 6 to 60, 7 to 70, 8 to 80, and 9 to 90 in your head before you go to sleep. Count multiples instead of sheep!

Multiples (Lists of Product)	60 sec	30 sec
6, 12, 18, 24, 30, 36, 42, 48, 54, 60		
7, 14, 21, 28, 35, 42, 49, 56, 63, 70		
8, 16, 24, 32, 40, 48, 56, 64, 72, 80		
9, 18, 27, 36, 45, 54, 63, 72, 81, 90		

*Sign the Multiples Quilt (poster on wall):

--	--

<i>Sign if you can say the “6” multiples in under a minute!</i>	<i>Sign if you can say the “7” multiples in under a minute!</i>
<i>Sign if you can say the “8” multiples in under a minute!</i>	<i>Sign if you can say the “9” multiples in under a minute!</i>

STEP OUTSIDE: Skipping Multiples

- Using skipping ropes, jump and say out loud multiples of 6,7,8 and 9.

How well did I complete these tasks?	Like a Trailblazer (expert)	Like a Pathfinder (apprentice)	Like a rookie (need more help & practice)

Extension:

- Create a quilt for 11, 12, 13, 14 and see which ones you can do in under a minute!
- Calculate exponents and correctly use the order of operations.

ET – Factor & find Least Common Multiples (LCM) for simple numbers (1-100)

7. Common Multiples

Rule: To find common multiples between two numbers you compare multiples for each number and list which ones are common. To find the **LOWEST COMMON MULTIPLE (LCM)**, you list the lowest number that both groups have in common!

EXAMPLE:

- Find the Lowest or least common multiple for 3 and 9.

SHOW THE WORK.

3 - 3, 6, 9, 12, 15, 18, 21, 24, 27, 30...
9 - 9, 18, 27....

Common Multiples = 9, 18, 27

Lowest Common Multiple = 9

TECH CHECK:

- <http://www.factmonster.com/ipka/A0933352.html>
- http://www.mathsteacher.com.au/year7/ch03_prime/02_comm/comm.htm
- <http://www.mathsisfun.com/least-common-multiple.html>

RECIPROCAL TEACHING:

- "Like a teacher" explain how to find the Least Common Multiple (LCM) of a number.
- Share examples and teach LCM to a friend or family member.
- Use scrap paper.
- Have your friend or family member change places and replay what you taught.

GOT IT!

- 1) Find the common multiples and the LCM for 2 and 7.

2- _____ (end at 40)

7- _____ (end at 42)

- 2) Find the lowest common multiple for 4 and 6.

4 - _____

6- _____

3) Find the LCM for 5 and 8.

How well did you factor & find common multiples for simple numbers (1-100)?

Trailblazer (Expert)

Pathfinder
(Apprentice)

Rookie
(Not Yet)

ET - Multiply & divide within one digit numbers.

8. 1 Digit Multiplication

RULE: Memorize these facts. Multiplying saves time! Instead of adding 3 groups of 7 = $(7 + 7 + 7)$, you know that $7 \times 3 = 21$.

HERE ARE 12 EASY STEPS for Multiplying and Dividing

1. LEARN one MULTIPLE PATTERN at a time!
2. Chunk them - Memorize 1,2,3,4 AND 5 MULTIPLES FIRST!
3. Try to say them aloud with a partner in under 60 seconds; then try them in under 30 seconds.
4. Get the number patterns in your head, "pillow talk" before you go to sleep. Do not count sheep - count multiples!
5. THEN Memorize YOUR 6,7,8,9 MULTIPLES.
6. Then do Pillow Talk with 6,7,8 and 9 multiples.
7. PRACTICE WITH IXL OR OTHER MATH GAMES.
8. When multiples are solid, then work on single digit multiplication and single digit division.
9. Practice making questions with MULTIPLES and Factors.
10. Multiplication saves you time and helps you be ready to do all sorts of mathematics! *For fun sing the multiplier hint songs - or create your own!
11. REMEMBER "1" X any number = that number.
12. AND "0" X any number is always 0!!!

- Read the names of the parts of the multiplication equation:

Multiplication:

The diagram shows the equation $6 \times 3 = 18$. A blue arrow points from the label "Factor (or Multiplier)" to the number 6. A red arrow points from the label "Factor (or Multiplicand)" to the number 3. A green arrow points from the label "Product" to the number 18.

$$6 \times 3 = 18$$

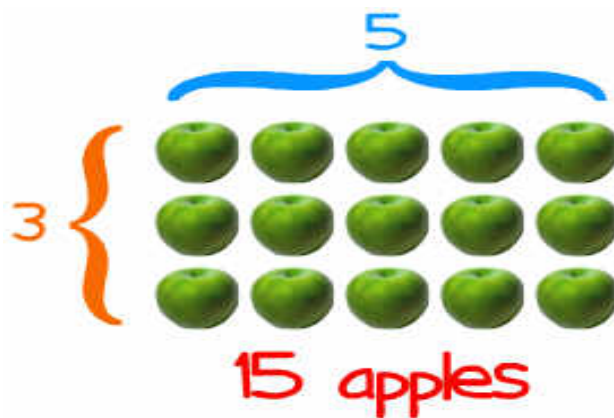
Factor (or Multiplier) Factor (or Multiplicand) Product

EXAMPLES:

0 X 1 = 0	1 X 1 = 1	2 X 1 = 2	3 X 1 = 3	4 X 1 = 4	5 X 1 = 5	6 X 1 = 6
0 X 2 = 0	1 X 2 = 2	2 X 2 = 4	3 X 2 = 6	4 X 2 = 8	5 X 2 = 10	6 X 2 = 12
0 X 3 = 0	1 X 3 = 3	2 X 3 = 6	3 X 3 = 9	4 X 3 = 12	5 X 3 = 15	6 X 3 = 18
0 X 4 = 0	1 X 4 = 4	2 X 4 = 8	3 X 4 = 12	4 X 4 = 16	5 X 4 = 20	6 X 4 = 24
0 X 5 = 0	1 X 5 = 5	2 X 5 = 10	3 X 5 = 15	4 X 5 = 20	5 X 5 = 25	6 X 5 = 30
0 X 6 = 0	1 X 6 = 6	2 X 6 = 12	3 X 6 = 18	4 X 6 = 24	5 X 6 = 30	6 X 6 = 36
0 X 7 = 0	1 X 7 = 7	2 X 7 = 14	3 X 7 = 21	4 X 7 = 28	5 X 7 = 35	6 X 7 = 42
0 X 8 = 0	1 X 8 = 8	2 X 8 = 16	3 X 8 = 24	4 X 8 = 32	5 X 8 = 40	6 X 8 = 48
0 X 9 = 0	1 X 9 = 9	2 X 9 = 18	3 X 9 = 27	4 X 9 = 36	5 X 9 = 45	6 X 9 = 54
0 X 10 = 0	1 X 10 = 10	2 X 10 = 20	3 X 10 = 30	4 X 10 = 40	5 X 10 = 50	6 X 10 = 60
0 X 11 = 0	1 X 11 = 11	2 X 11 = 22	3 X 11 = 33	4 X 11 = 44	5 X 11 = 55	6 X 11 = 66
0 X 12 = 0	1 X 12 = 12	2 X 12 = 24	3 X 12 = 36	4 X 12 = 48	5 X 12 = 60	6 X 12 = 72

7 X 1 = 7	8 X 1 = 8	9 X 1 = 9	10 X 1 = 10	11 X 1 = 11	12 X 1 = 12
7 X 2 = 14	8 X 2 = 16	9 X 2 = 18	10 X 2 = 20	11 X 2 = 22	12 X 2 = 24
7 X 3 = 21	8 X 3 = 24	9 X 3 = 27	10 X 3 = 30	11 X 3 = 33	12 X 3 = 36
7 X 4 = 28	8 X 4 = 32	9 X 4 = 36	10 X 4 = 40	11 X 4 = 44	12 X 4 = 48
7 X 5 = 35	8 X 5 = 40	9 X 5 = 45	10 X 5 = 50	11 X 5 = 55	12 X 5 = 60
7 X 6 = 42	8 X 6 = 48	9 X 6 = 54	10 X 6 = 60	11 X 6 = 66	12 X 6 = 72
7 X 7 = 49	8 X 7 = 56	9 X 7 = 63	10 X 7 = 70	11 X 7 = 77	12 X 7 = 84
7 X 8 = 56	8 X 8 = 64	9 X 8 = 72	10 X 8 = 80	11 X 8 = 88	12 X 8 = 96
7 X 9 = 63	8 X 9 = 72	9 X 9 = 81	10 X 9 = 90	11 X 9 = 99	12 X 9 = 108
7 X 10 = 70	8 X 10 = 80	9 X 10 = 90	10 X 10 = 100	11 X 10 = 110	12 X 10 = 120
7 X 11 = 77	8 X 11 = 88	9 X 11 = 99	10 X 11 = 110	11 X 11 = 121	12 X 11 = 132
7 X 12 = 84	8 X 12 = 96	9 X 12 = 108	10 X 12 = 120	11 X 12 = 132	12 X 12 = 144

<http://rightideaproductions.com/wp-content/uploads/2009/05/timestable.png>



TECH CHECK:

- <http://www.prongo.com/math/multiplication.html>
- <http://resources.oswego.org/games/SpeedGrid/Multiplication/urikamultires.html>

- <http://www.wmnet.org.uk/resources/gordon/Hit%20the%20button%20v9.swf>
- <http://www.factmonster.com/quizzes/multiply/1.html>
- <http://www.playkidsgames.com/games/Tunnel/multiplyFrame.htm>
- <http://www.amblesideprimary.com/ambleweb/mentalmaths/tabletrees.html>
- <http://www.multiplication.com/flashgames/GrandPrix.htm>
- <http://www.kwiznet.com/p/takeQuiz.php?ChapterID=1304&CurriculumID=3&Num=9.5>
- <http://www.superteacherworksheets.com/multiplication/multiply0-12.pdf>
- <http://www.superteacherworksheets.com/multiplication/scrambled-facts-basic2.pdf>
- <http://www.youtube.com/watch?v=MwTvKhyQ0uk>
- http://www.google.com/imgres?imgurl=http://www.kwiznet.com/images/questions/grade3/times_table1.gif&imgrefurl=http://www.kwiznet.com/p/takeQuiz.php%3FChapterID%3D200%26CurriculumID%3D3%26Num%3D1.2&h=268&w=370&sz=7&tbnid=FpzCJLcW-zuZIM:&tbnh=88&tbnw=122&prev=/images%3Fq%3D3%2Btimes%2Btable&zoom=1&q=3+times+table&usg=__egr91EkNKYx1EUM6QPBdmgkgzTA=&sa=X&ei=XrZ4TbWQA8XlrAGI5a2QBg&ved=0CCYQ9QEwAw
- <http://www.bbc.co.uk/skillswise/numbers/wholenumbers/multiplication/timestables/game.shtml>

FUN & GAMES:

- <http://www.multiplicationhiphopforkids.com/samples-1.htm>
- 3's - <https://www.youtube.com/watch?v=dzVyBQ5uTbo>
- <https://www.youtube.com/watch?v=L6yaevdRgC4>
- Look at how the number sentence names the following donut array:

$$3 \times 4 = 12 \text{ OR } 4 \times 3 = 12$$



RECIPROCAL TEACHING:

- "Like a teacher" explain how to do 1 digit multiplication.
- Share examples and teach 1 digit multiplication to a friend or family member.

- Use scrap paper.
- Have your friend or family member change places and replay what you taught.

GOT IT!

- Draw a diagram (array) to figure out these products?

1) $9 \times 6 = \underline{\hspace{2cm}}$

Diagrams 😊

2) $7 \times 8 = \underline{\hspace{2cm}}$

3) $4 \times 4 = \underline{\hspace{2cm}}$

How well did you use arrays to multiply one digit numbers?

Trailblazer
(Expert)

Pathfinder
(Apprentice)

Rookie
(Not Yet)

9) Multiplying by 10's

RULE: Multiply by 10 - just move the numbers one place to the left and add one zero. Multiply by 100 - just move numbers two places to left and add two zeros. Quick Way: count zeros and put them to right of number being multiplied.

EXAMPLES:

$36 \times 10 = 360$	Add on one 0.
$36 \times 100 = 3600$	Add on two 0's.
$36 \times 1000 = 36,000$	Add on three 0's.

TECH TIME:

- https://www.khanacademy.org/math/arithmetic/multiplication-division/multi_digit_multiplication/v/multiplying-by-multiples-of-10
- <http://www.toonuniversity.com/flash.asp?err=517&engine=13>
- http://www.primaryresources.co.uk/maths/pdfs/robot_multiplication.pdf
- <https://www.youtube.com/watch?v=jPO5zAwfRSc>
- https://www.youtube.com/watch?v=20aHQOG__8M
- <https://www.youtube.com/watch?v=3huvvxUHDmM>
- <http://www.gscdn.org/library/cms/11/13411.pdf>

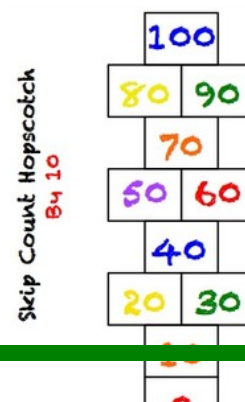
STEP OUTSIDE: Hopscotch

- Using sticks create your own hopscotch design with at least 5 places to hop to.
- The roll a die to get a number.
- Multiply that number by ten with each jump you take in your hopscotch design.

Example: Mr. Rogers rolls a "6", so when he jumps, he calls out loud:

6, 60, 600, 6000, 60,000 - as he jumps spot.

through each



Extension:

Create a hopscotch with 7 or more spaces to hop to!

GOT IT!_(Complete questions below.)

1) 19×10 _____

Rough Work 😊

2) $741 \times 100 =$ _____

3) $4 \times 10000 =$ _____

How well did you multiply by 10's?

Trailblazer
(Expert)

Pathfinder
(Apprentice)

Rookie
(Not Yet)

10. Multiplying 2 or More Digit Numbers

RULE: Know all 7 steps. You multiply two numbers and then you add them together. Before you multiply the left digit on the bottom, you need to put a zero in the one's columns.

It is much easier to explain the steps with an example:

http://www.ehow.com/how_6193967_multiply-2-digit-numbers.html

$$\begin{array}{r} 1 \\ 34 \\ 546 \\ \times 27 \\ \hline 3822 \leftarrow 546 \times 7 \\ 10920 \leftarrow 546 \times 20 \\ \hline 14,742 \end{array}$$

http://www.eduplace.com/math/mw/models/graphics/5_9a.gif

TECH TIME:

- <http://www.prongo.com/math/multiplication.html>
- <http://www.quia.com/mathjourney.cgi>
- http://www.numbernut.com/basic/activities/mult_quiz_2x1nocarry-v.shtmlhttp://www.myquizzes.ca/viewquiz.asp?quiz_id=8683&quizname=Multiply%20%20digit%20X%20%20digit
- <http://www.thatquiz.org/tq/previewtest?XVCX6129>

RECIPROCAL TEACHING:

- "Like a teacher" explain how to do 2-digit multiplication.
- Share examples and teach 2-digit multiplication to a friend or family member.
- Use scrap paper.
- Have your friend or family member change places and replay what you taught.

FUN & GAMES:

- Find at least 9 stones and place numbers on them.
- Create 2 and 3-digit multiplication questions using these stones.

- Place your multiplication questions here to complete your calculations.

Q: _____

ROUGH WORK 😊

Q: _____

How well did I complete these tasks?	Like a Trailblazer (expert)	Like a Pathfinder (apprentice)	Like a rookie (need more help & practice)



GOT IT!

(a) 32×47

ROUGH WORK 😊

(b) 25×148

ROUGH WORK 😊

(c) 457×26

How well did you multiply 2-digit numbers?

Trailblazer
(Expert)

Pathfinder
(Apprentice)

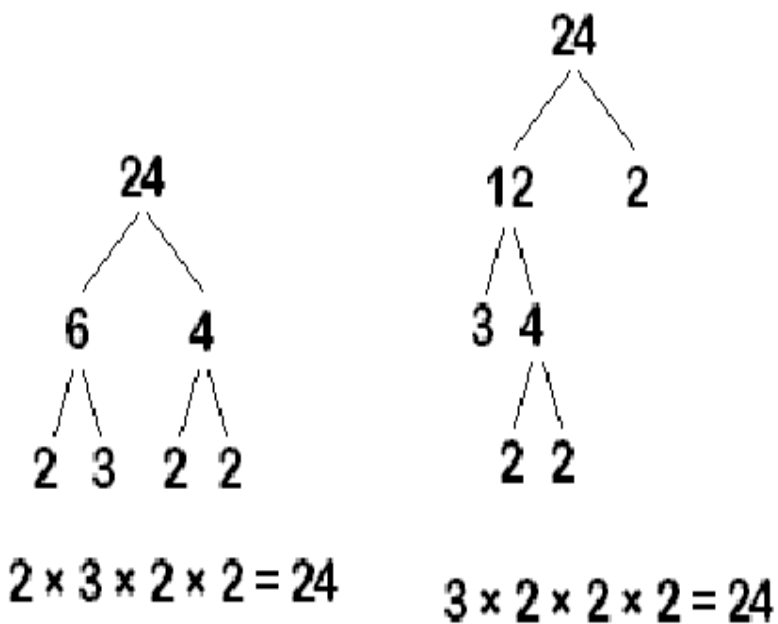
Rookie
(Not Yet)

ET - Use multiples of 6,7,8 & 9 to find greatest common factor.

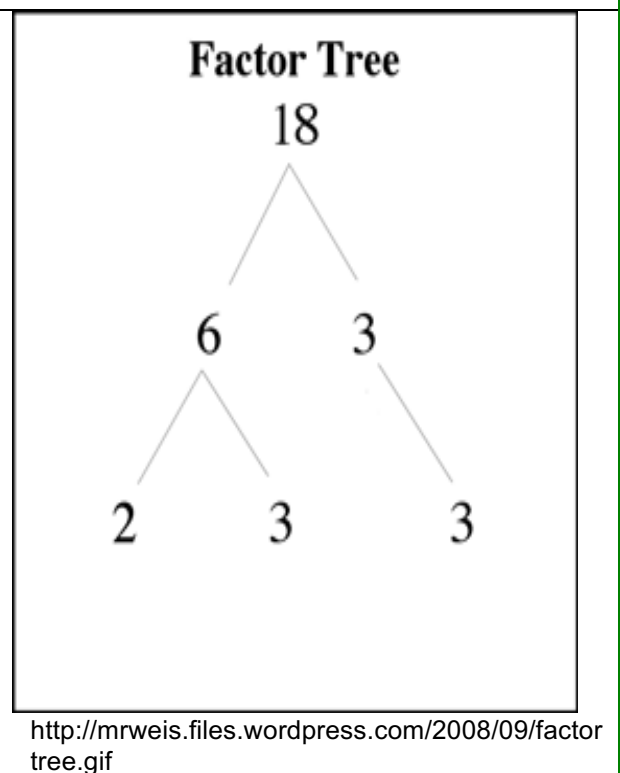
11. Factors

RULE: You factor numbers to get down to the prime numbers (the numbers that can only be divided evenly by 1 or itself). You can find the *GCF* (Greatest Common Factor) when you compare the factors of two numbers. Note: composite numbers can be divided evenly by numbers other than 1 and itself!

EXAMPLES:



http://www.mathscitutor.com/articles_imgs/867/prime-12.gif



- Find the **GCF** (Greatest Common Factor) of 36 and 54.

The factors of 36 are 1, 2, 3, 4, 6, 9, 12, 18, and 36.

The factors of 54 are 1, 2, 3, 6, 9, 18, 27, and 54.

The **common factors** of 36 and 54 are 1, 2, 3, 6, 9, 18

Although the numbers in **bold** are all common factors of both 36 and 54, **18** is the **greatest common factor**. <http://www.helpwithfractions.com/greatest-common-factor.html>

TECH TIME:

- http://www.mathgoodies.com/factors/factor_tree.asp
- <http://www.aaamath.com/g72b-grt-com-fac.html#section2>
- http://www1.teachertube.com/viewVideo.php?video_id=9793
- <http://www.schooltube.com/video/7d1a4d1a539e42c9aff8/Greatest-Common-Factor-GCF-2>

RECIPROCAL TEACHING:

- "Like a teacher" explain how to find the *Greatest Common Factor (GCF)* of numbers.
- Share examples and teach *GCF* to a friend or family member.
- Use scrap paper.
- Have your friend or family member change places and replay what you taught.

STEP OUTSIDE:

- In pairs, take your whiteboards outside and count your steps to the end of the driveway. This will be your first number.
- Then count your steps back, but stop $\frac{1}{2}$ way.
- Use these two numbers to figure out the greatest common factors of both numbers.
- Place your work on the next page:



<p>_____</p>		<p>ROUGH WORK 😊</p>
--------------	--	---------------------

Therefore, the greatest common factor is: _____.

- Share your findings with the class.

GOT IT!

- Find the factors of the following pairs and figure out their GCF.

1) 21

35

ROUGH WORK 😊

Therefore, the greatest common factor is: _____.

2) 48

72

ROUGH WORK 😊

Therefore, the greatest common factor is: _____.

3) 93, 39

ROUGH WORK 😊

Therefore, the greatest common factor is: _____.

How well did you use multiples of 6,7,8 & 9 to find the Greatest Common Factor (GCF)?

Trailblazer (Expert)

Pathfinder
(Apprentice)

Rookie
(Not Yet)

Daily Learning Log and Math Detective Habits:

___ stayed on task (worked well with partner and independently)

___ used math detective language to teach ideas

___ work is neat and easy to read

___ completed work by following instructions

___ work shows precision

QUIZ TIME: Complete these questions without looking at the examples.

Question	Rough Work
1) 1364 + <u>6287</u>	<u>Show inverse operation to check</u>
2) 6287 - <u>1364</u>	<u>Show inverse operation to check</u>
3) List the multiples of 6 (end at 72).	
4) List the Least Common Multiples (LCM) of 3 and 4.	
5) 7 <u> X 6</u>	
6). 11 <u> X 11</u>	
7) Factors of 36:	

8) Find the <i>Greatest Common Factor (GCF)</i> for 36 and 32.	

9). Solve this problem.

David was asked to put 5 baskets of pencils together with 3 each (one for each member of the group). How many pencils did he have all together?

How well did I complete these tasks?	Like a Trailblazer (expert)	Like a Pathfinder (apprentice)	Like a rookie (need more help & practice)

Operations - Properties and Division

CHECK-IN: Maybe you already know all this? Show your teacher what you know!

Question

Rough Work 😊

1) Prove $9 \times 7 = 7 \times 9$

2) Prove $(3 \times 2) \times 5 = 3 \times (2 \times 5)$

3) Show, using counters, how to find the solution for $27/9$

4) Show using a number line how to find $18/3$

5) Divide 965 by 5 (show long division)

Rough Work 😊

6) Divide 360 by 12 (show long division)

7) Expand the following number - 501,765

(a) with addition

(b) with addition and multiplication

(c) using powers with addition and multiplication

How well did I complete these tasks?	Like a Trailblazer (expert)	Like a Pathfinder (apprentice)	Like a rookie (need more help & practice)

12. Properties (Commutative, Associative and Distributive)

RULE: The commutative property for addition and multiplication is that you can change the order of the numbers being added or multiplied, and the sum or product remains the same.

EXAMPLES:

$$\begin{array}{c} \text{6 blue circles} \\ + \text{3 orange circles} \\ \hline 6 + 3 \end{array} = \begin{array}{c} \text{3 orange circles} \\ + \text{6 blue circles} \\ \hline 3 + 6 \end{array}$$

... or when you **multiply**:

$$\begin{array}{c} \text{2 rows of 4 blue circles} \\ \hline 2 \times 4 \end{array} = \begin{array}{c} \text{4 columns of 2 blue circles} \\ \hline 4 \times 2 \end{array}$$

GOT IT!

- Create 2 examples for addition and 2 examples for multiplication.

Addition

Multiplication

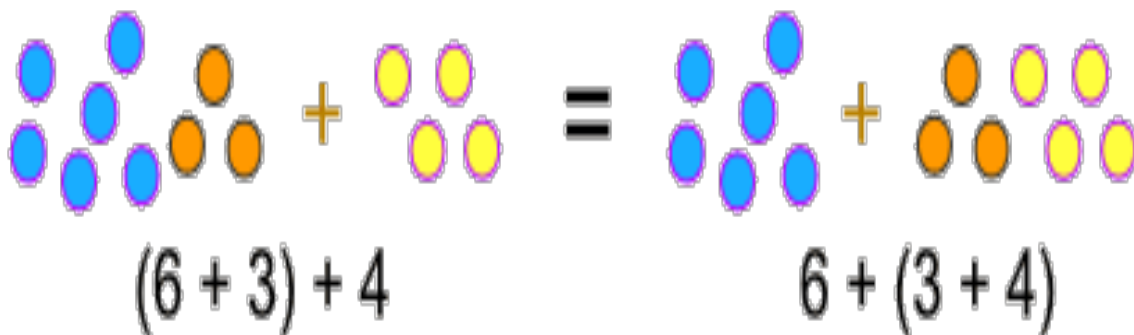
Addition**Multiplication**

How well did I complete these tasks?	Like a Trailblazer (expert)	Like a Pathfinder (apprentice)	Like a rookie (need more help & practice)

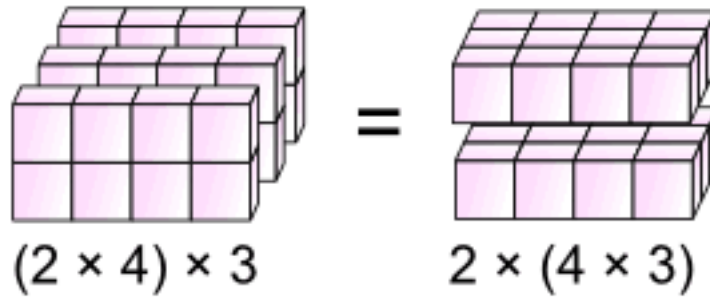
RULE: The associative "law" says that it doesn't matter how we group the numbers for addition and multiplication, the results will be the same. You can change the order of the numbers being added or multiplied and the sum or product remains the same. (Note: Always do parentheses/brackets first).

EXAMPLES: $7 + 4 = 11$ and $4 + 7 = 11$ (addition)
 $(2 + 5) + 4 = 11$ and $2 + (5 + 4) = 11$

$7 \times 4 = 28$ and $4 \times 7 = 28$ (multiplication)
 $7 \times (2 \times 2) = 28$ and $(7 \times 2) \times 2 = 28$



... or when we multiply:



GOT IT!

*Prove these by doing the rough work in the box.

- Prove these equations are correct:
 $(9 + 3) + 4 = 16$ and $9 + (3 + 4) = 16$

$(3 \times 2) \times 4 = 24$ and $3 \times (2 \times 4) = 24$

How well did I complete these tasks?	Like a Trailblazer (expert)	Like a Pathfinder (apprentice)	Like a rookie (need more help & practice)

Extension:

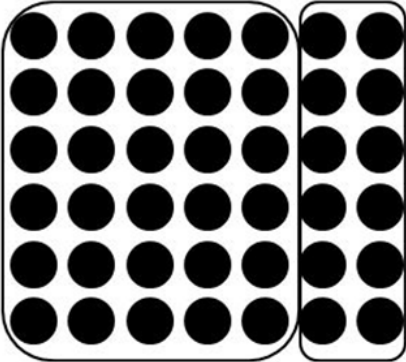
- Step outside and look for patterns in nature.
- Instead of counting trees, stones, or bushes, how can you create equations to show numbers?
- Example: 36 bushes = 3 rows of 10 + 6 random bushes
 $= (3 \times 10) + 6 = 36$ (equation)
- Place your equations here:

Rule: We use the distributive property to break numbers into smaller parts.

Distributive Property

- Look at the three posters below and talk about how you can distribute numbers to make them easier to add, multiply, subtract or divide.

Distributive Property = Fact Fluency




$6 \times 7 = \square$
 $6 \times (5 + 2) = \square$
 $(6 \times 5) + (6 \times 2) = \square$

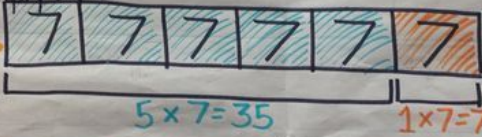
<http://www.mathcoachscorner.com/wp-content/uploads/2013/01/Distributive-Property.png>

Distributive Property

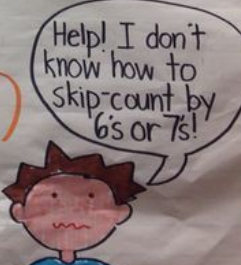
A multiplication fact broken into 2 smaller \times facts.

 We anchor on to 5 because it's easy to multiply by 5!

$6 \times 7 = 42$



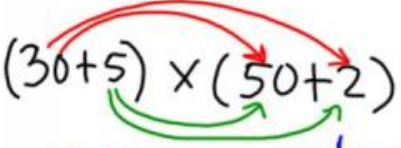
$6 \times 7 = (5 + 1) \times 7$
 $= (5 \times 7) + (1 \times 7)$
 $= 35 + 7$
 $= 42$



<https://s-media-cache-ak0.pinimg.com/736x/63/25/46/6325467e7e1d23f95dff81e6214201cd--math-anchor-charts-math-charts.jpg>

35×52

$(30+5) \times (50+2)$



$30 \times 50 =$	1500
$30 \times 2 =$	60
$5 \times 50 =$	250
$5 \times 2 =$	10
	$+$
	1820

Steps:

1. Make numbers into more friendly numbers. (ie. tens + ones)
2. Multiply the first friendly number on the left with the friendly numbers on the right.
 Multiply the second friendly number on the left with the friendly numbers on the right.
3. Add the partial products.

http://wecliptart.com/gimg/BE1C0DBED3BEDD93/Distributive%20Property_8.jpeg

- Experiment with dividing numbers.
- Keep track of which numbers wind up giving you no remainders, and which ones have remainders.
- Talk about patterns and ways to predict how certain numbers may leave no remainders and others do.

GOT IT!

- Use the distributive property to find these products.

(a) 4×9

(b) 15×6

(c) 16×4

(d) 72×56

How well did I complete these tasks?	Like a Trailblazer (expert)	Like a Pathfinder (apprentice)	Like a rookie (need more help & practice)

14. Division as Inverse Operation

RULE: Division questions are the inverse of multiplication question.

EXAMPLE:

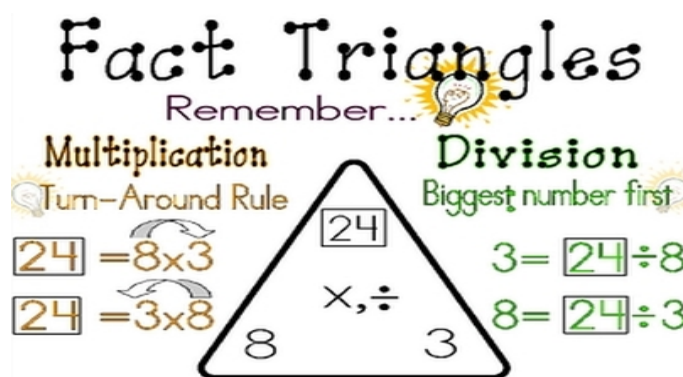
The equation $3 \times 7 = 21$ has the inverse relationships:

$$21 \div 3 = 7 \quad 21 \div 7 = 3$$

Similar relationships exist for division.

The equation $45 \div 5 = 9$ has the inverse relationships:

$$5 \times 9 = 45 \quad 9 \times 5 = 45$$



<http://missdupont.weebly.com/uploads/2/4/4/2/24421791/3909085.jpg?432>

- Make fact triangles using numbers that multiply and divide with 6, 7, 8 and 9.

Extension:

- Make fact triangles for large numbers that multiply and divide with two digit numbers.

GOT IT!

- Figure out the following blanks.

1) If $5 * 7 = 35$, then $35 / \underline{\quad} = 7$

2)..... If $10 * 4 = 40$, then $40 / \underline{\quad} = 4$

3) Make Fact Triangles for $6 * 7$ and $8 * 9$:

4)

5)

6)

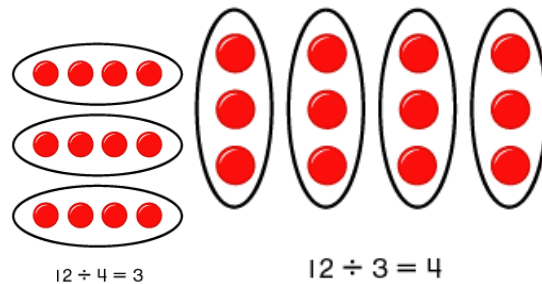
How well did I complete these tasks?	Like a Trailblazer (expert)	Like a Pathfinder (apprentice)	Like a rookie (need more help & practice)

ET - Use arrays and number lines to solve math problems.

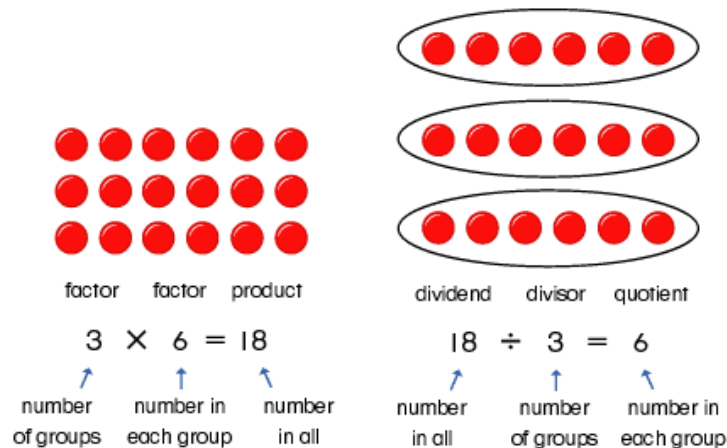
15. Division with Arrays and Number Lines

RULE: Since division is the inverse, or opposite, of multiplication, you can use arrays to help understand how multiplication and division are related. If in multiplication we find the product of two factors, in division we find the missing factor if the other factor and the product are known. Division “undoes” multiplication and multiplication “undoes” division. So, when multiplying or dividing, you can use a fact from the inverse operation.

EXAMPLES:



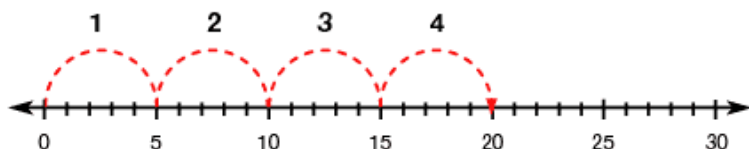
Since you know that $4 \times 5 = 20$, you also know the related division fact $20 \div 4 = 5$ or $20 \div 5 = 4$. You can also check their work by using the inverse operation.



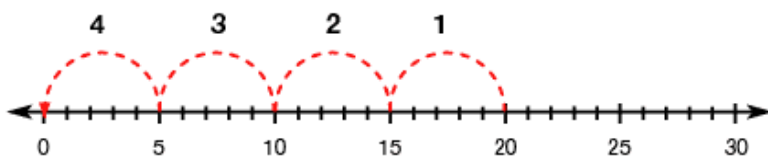
In multiplication the numbers you multiply are called **factors**; the answer is called

the **product**. In division the number being divided is the **dividend**, the number that divides it is the **divisor**, and the answer is the **quotient**.

Here is an example using a number line.



$$\begin{array}{ccccc} \text{Factor} & & \text{Factor} & & \text{product} \\ 4 & \times & 5 & = & 20 \end{array}$$



$$\begin{array}{ccccc} \text{Dividend} & & \text{Divisor} & & \text{quotient} \\ 20 & \div & 5 & = & 4 \end{array}$$

RECIPROCAL TEACHING:

- "Like a teacher" explain how to use arrays and number lines to divide numbers.
- Share examples and teach division using arrays and number lines to a friend or family member.
- Use scrap paper.
- Have your friend or family member change places and replay what you taught.

GOT IT!

- Create a diagram with counters and a number line to show the following division questions:

$$18 \div 6 = \underline{\quad}$$

Show using counters

Number Line

$27/9 = \underline{\quad}$

Show using counters

Number Line

How well did you use arrays and number lines to solve math problems?

Trailblazer (Expert)

Pathfinder
(Apprentice)

Rookie
(Not Yet)

16. Divide by Single Digit Numbers

RULE: Put the number being divided (the "dividend" under the bracket; put the number doing the dividing "the divisor" on the left of the bracket. Leave room for the quotient (the answer) to go above the bracket!

EXAMPLES:

$$\begin{array}{r} 15 \\ 5 \overline{)75} \\ \underline{-5} \\ 25 \\ \underline{-25} \\ 0 \end{array}$$

$3 \times 5 = 15$

<http://www.coolmath4kids.com/long-division/images/long-division-31.gif>

$$5 \overline{)75}$$

Rewrite the equation using the division bracket.

$$\begin{array}{r} 10 \\ 5 \overline{)75} \end{array}$$

Make a guess at the value by noting that one 5 is contained in 75 at least 10 times, but less than 20 times, so we use 10 as a first guess.

$$\begin{array}{r} 10 \\ 5 \overline{)75} \\ \underline{-50} \\ 25 \end{array}$$

Determine how close the estimate is by multiplying 5 by 10 and then subtracting the 50 from 75, resulting in a remainder of 25.

Next estimate the number of fives that go into 25.

$$\begin{array}{r} 5 \\ 10 \\ 5 \overline{)75} \\ \underline{-50} \\ 25 \\ \underline{-25} \\ 0 \end{array}$$

Our estimate is 5.

We add the original 10 to the 5 we just calculated, which results in 15.

You can check your work with multiplication:
 $15 \times 5 = 75$.

<http://cnx.org/content/m26903/latest/graphics1.png>

TECH TIME:

- <http://www.youtube.com/watch?v=yUp8biwdvA4&feature=relmfu>
- <http://www.quia.com/mathjourney.cgi>
- http://www.numbernut.com/basic/activities/div_quiz_2x1rem.shtml
- <http://www.glencoe.com/sec/math/studytools/cgi-bin/msgQuiz.php4?isbn=0-02-105733-8&chapter=8&lesson=2&&headerFile=0>

RECIPROCAL TEACHING:

- "Like a teacher" explain how to use arrays to do simple long division.
- Share examples and teach simple long division to a friend or family member.
- Use scrap paper.
- Have your friend or family member change places and replay what you taught.

GOT IT!

Question	Rough Work
1) $252/6$	
2) $104/8$	

3) 414/9

How well did I complete these tasks?	Like a Trailblazer (expert)	Like a Pathfinder (apprentice)	Like a rookie (need more help & practice)

ET - Multiply and divide two digit numbers without remainders. (distributive property).

17) Dividing by 2 or More Digit Numbers

Rule: Multiply to guess the first number to place in quotient. Then subtract what you multiply from dividend. REPEAT until you get to 0 remainder. In whole numbers - there are no remainders. If you get one, you need to go back and check for careless errors.

EXAMPLE:

A handwritten long division problem showing 30 dividing 360. The quotient 12 is written above the 360. A horizontal line is drawn under 360. Below the line, 30 is written and a red arrow points down to the 0 in 360. Below that, 60 is written in red, and another red arrow points down to the 0 in 360. Below that, -60 is written in red, and a horizontal line is drawn under it. The final result is 0.

<http://www.tutornext.com/system/files/u27/fig4.JPG>

A handwritten long division problem showing 45 dividing 630. The quotient 14 is written above the 630. A horizontal line is drawn under 630. Below the line, 45 is written and a red arrow points down to the 0 in 630. Below that, 180 is written, and another red arrow points down to the 0 in 630. Below that, 180 is written, and a horizontal line is drawn under it. The final result is 0.

<http://www.kwiznet.com/px/homes/i/math/G5/di45.gif>

TECH TIME:

- <http://www.quia.com/mathjourney.cgi>
- http://www.eduplace.com/kids/hmcam/practice/templates/rules.jsp?ID=hmcam_ep/gr5/15&GRADE=5&UNIT=6&CHAPTER=15&UNIT_TITLE=Operations%20with%20Decimals&CHAPTER_TITLE=Divide%20by%202-Digit%20Divisors
- <http://www.kwiznet.com/p/takeQuiz.php?ChapterID=2357&CurriculumID=5&Num=3.9>

RECIPROCAL TEACHING:

- "Like a teacher" explain how to divide a number by 2 digits.
- Share examples and teach dividing by 2 digits to a friend or family member.
- Use scrap paper.
- Have your friend or family member change places and replay what you taught.

GOT IT!

(a) $1,034/47$

ROUGH WORK 😊

(b) $9,828/78$

(c) $24,986/62$

How well did you multiply and divide with two digit numbers without

Trailblazer (Expert)

Pathfinder
(Apprentice)

Rookie
(Not Yet)

remainders?

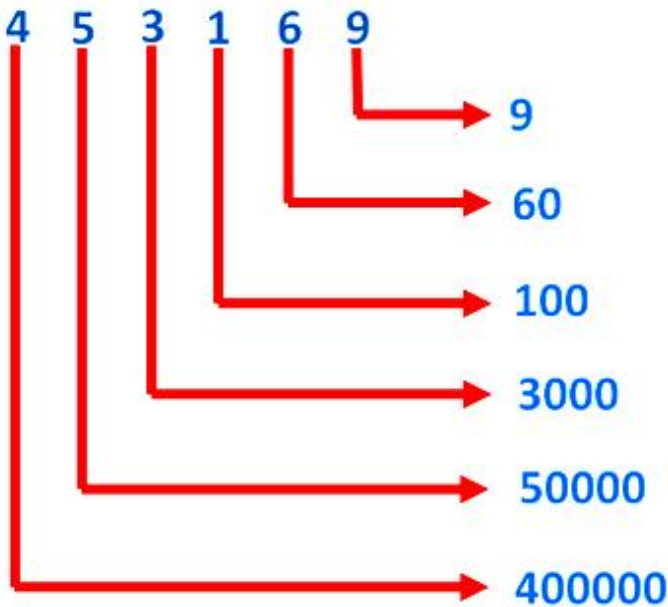
--	--	--

18. Expanded Form by Adding

Rule: To expand a number just using addition, you must break down a number into place value parts.

EXAMPLES:

Another way of looking at numbers is by seeing what adds up to the number.



<http://www.math-only-math.com/images/find-the-expanded-form.jpg>

- Check out how to expand 4,985,346, vertically and horizontally.

$$\begin{aligned} &4,000,000 + \\ &900,000 + \\ &80,000 + \\ &5,000 + \\ &300 + \\ &40 + \\ &6 + \end{aligned}$$

$$4,985,346 = 4,000,000 + 900,000 + 80,000 + 5,000 + 300 + 40 + 6$$

TECH CHECK:

- <http://www.youtube.com/watch?v=4LI0Rh32PZo&feature=related>
- <http://www.kwiznet.com/p/takeQuiz.php?ChapterID=800&CurriculumID=5&Num=1.28>
- <http://www.quia.com/cc/279741.html>
- <http://www.proprofs.com/quiz-school/story.php?title=expanded-notation-quiz>
- <http://www.superteacherworksheets.com/place-value/expanded-form-millions.pdf>

RECIPROCAL TEACHING:

- "Like a teacher" explain how to expand numbers with addition.
- Share examples and teach how to expand numbers with addition to a friend or family member.
- Use scrap paper.
- Have your friend or family member change places and replay what you taught.

FUN & GAMES:

- Find out three addresses including the school - and then expand these number.
- Show work here:

GOT IT!

- Expand the following using addition.

$2,985 = \underline{\hspace{15em}}$

$7,628,406 = \underline{\hspace{15em}}$

$3,500,650 = \underline{\hspace{15em}}$

-
-
-
-
-

How well did I complete	Like a Trailblazer (expert)	Like a Pathfinder (apprentice)	Like a rookie (need more help & practice)
-------------------------	-----------------------------	--------------------------------	---

these tasks?			
--------------	--	--	--

19. Expanded Form with Addition and Multiplication

RULE: To expand a number using addition and multiplication, you need to multiply the digit by a factor of 10 and add the number in the 1's place. Use brackets to keep the multiplied numbers together!

EXAMPLES:

4,985, 346 =

(4 × 1,000,000) +

(9 × 100,000) +

(8 × 10,000) +

(5 × 1,000) +

(3 × 100) +

(4 × 10) +

6

Or 4,985, 346 = (4 × 1,000,000) + (9 × 100,000) + (8 × 10,000) + (5 × 1,000)
+ (3 × 100) + (4 × 10) + 6

TECH CHECK:

- http://www.dadsworksheets.com/v1/Worksheets/Numbers%20in%20Standard,%20Expanded%20and%20Word%20Form/Seven_Digit_Number_With_Gaps_To_Word_Form_V1.html
- <http://www.youtube.com/watch?v=whPuseMqs2s&feature=related>

RECIPROCAL TEACHING:

- "Like a teacher" explain how to expand numbers with addition and multiplication.

- Share examples and teach how to expand numbers with addition and multiplication to a friend or family member.
- Use scrap paper.
- Have your friend or family member change places and replay what you taught.

GOT IT!

- Expand the following using addition and multiplication

(a) 5,079,541

(b) 1,772,005

(c) 6,800,540

-
-

How well did I complete these tasks?	Like a Trailblazer (expert)	Like a Pathfinder (apprentice)	Like a rookie (need more help & practice)

ET - Use expanded form with whole numbers & base 10.

20. Expanded Notation using POWERS

RULE: To expand a number using addition and multiplication of powers, you need to multiply the digit by a power of 10 and add the number in the 1st place value. Use brackets to keep the multiplied numbers together!

EXAMPLES:

$$4,985,346 =$$

$$(4 \times 10^6)+$$

$$(9 \times 10^5)+$$

$$(8 \times 10^4)+$$

$$(5 \times 10^3)+$$

$$(3 \times 10^2)+$$

$$(4 \times 10^1)+$$

6

$$4,985,346 = (4 \times 10^6) + (9 \times 10^5) + (8 \times 10^4) + (5 \times 10^3) + (3 \times 10^2) + (4 \times 10^1) + 6$$

- Why do you think scientists need to learn about notation?

TECH CHECK:

- http://www1.teachertube.com/viewVideo.php?title=Power_of_Ten_Expanded_Notations&video_id=49690

RECIPROCAL TEACHING:

- "Like a teacher" explain how to expand numbers using scientific notation.
- Share examples and teach how to expand numbers using scientific notation to a friend or family member.
- Use scrap paper.
- Have your friend or family member change places and replay what you taught.

GOT IT!

- Expand the following using powers:

(a) 6,800,540

=

(b) 3,578,644

=

•

How well did you use expanded form with whole numbers & base 10?	Trailblazer (Expert)	Pathfinder (Apprentice)	Rookie (Not Yet)

Extension:

- Use scientific notation with whole numbers.

Quiz Time: Complete these questions without looking at the examples.

Question	Rough Work 😊
1) Prove $6 \times 7 = 7 \times 6$	
2) Prove $(3 \times 4) \times 5 = 3 \times (4 \times 5)$	
3) Show, using counters, how to find the solution for $32/8$	

4) Show using a number line how to find $28/4$

5) Divide 955 by 5 (show long division)

Rough Work 😊

6) Divide 480 by 12 (show long division)

7) Expand the following number - 5210765

(a) with addition

(b) with addition and multiplication

(c) using powers with addition and multiplication

How well did I complete these tasks?	Like a Trailblazer (expert)	Like a Pathfinder (apprentice)	Like a rookie (need more help & practice)

Checking Your Understanding

Date: _____

Dear students,

We have completed Just Whole Numbers.

Did I learn from making mistakes? _____

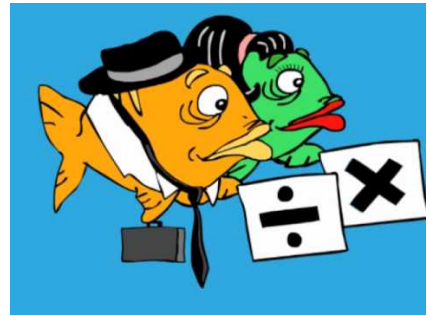
I am proudest about the work we did on page ____ because....

We think the trickiest part of this Math was....

because _____

I enjoyed working (with others or on my own) because....

Sincerely,



https://i.ytimg.com/vi/0uZiqk_ZdcA/hqdefault.jpg

Daily Learning Log and Math Detective Habits:

___ stayed on task (worked well with partner and independently)

___ used math detective language to teach ideas

___ work is neat and easy to read

___ completed work by following instructions

___ work shows precision

Appendix A: Ontario Ministry of Education and Training Expectations

3A.NUMBER SENSE AND NUMERATION

3A1.1 represent, compare, and order whole numbers to 1000, using a variety of tools (e.g., base ten materials or drawings of them, number lines with increments of 100 or other appropriate amounts)

3A.1.2 read and print in words whole numbers to one hundred, using meaningful contexts (e.g., books, speed limit signs)

3A.1.3 identify and represent the value of a digit in a number according to its position in the number

3A.1.4 compose and decompose three-digit numbers into hundreds, tens, and ones in a variety of ways, using concrete materials

3A.1.10 solve problems that arise from real-life situations and that relate to the magnitude of whole numbers up to 1000

3A.2.1 count forward by 1's, 2's, 5's, 10's, and 100's to 1000 from various starting points, and by 25's to 1000 starting from multiples of 25, using a variety of tools and strategies (e.g., skip count with and without the aid of a calculator; skip count by 10's...)

3A.2.2 count backwards by 2's, 5's, and 10's from 100 using multiples of 2, 5, and 10 as starting points, and count backwards by 100's from 1000 and any number less than 1000, using a variety of tools (e.g., number lines, calculators, coins) and strategies.

3A.3.1 solve problems involving the addition and subtraction of two-digit numbers, using a variety of mental strategies (e.g., to add $37 + 26$, add the tens, add the ones, then combine the tens and ones, like this: $30 + 20 = 50$, $7 + 6 = 13$, $50 + 13 = 63$)

3A.3.2 add and subtract three-digit numbers, using concrete materials, student generated algorithms, and standard algorithms

3A.3.5 relate multiplication of one-digit numbers and division by one-digit divisors to real life situations, using a variety of tools and strategies (e.g., place objects in equal groups, use arrays, write repeated addition or subtraction sentences)

3A.3.6 multiply to 7×7 and divide to $49 \div 7$, using a variety of mental strategies (e.g., doubles, doubles plus another set, skip counting).

3D. PATTERNING AND ALGEBRA

3D.1.1 identify, extend, and create a repeating pattern involving two attributes (e.g., size, colour, orientation, number), using a variety of tools (e.g., pattern blocks, attribute blocks, drawings).

3D.1.2 identify and describe, through investigation, number patterns involving addition, subtraction, and multiplication, represented on a number line, on a calendar, and on a hundreds chart (e.g., the multiples of 9 appear diagonally in a hundreds chart)

3D.1.3 extend repeating, growing, and shrinking number patterns (Sample problem: Write the next three terms in the pattern 4, 8, 12, 16, ...)

3D.1.4 create a number pattern involving addition or subtraction, given a pattern represented on a number line or a pattern rule expressed in words.

3D.1.6 demonstrate, through investigation, an understanding that a pattern results from repeating an action (e.g., clapping, taking a step forward every second), repeating an operation (e.g., addition, subtraction), using a transformation (e.g., slide, flip, turn), or making some other repeated change to an attribute (e.g., colour, orientation).

3D.2.1 determine, through investigation, the inverse relationship between addition and subtraction (e.g., since $4 + 5 = 9$, then $9 - 5 = 4$; since $16 - 9 = 7$, then $7 + 9 = 16$)

3D.2.2 determine, the missing number in equations involving addition and subtraction of one- and two-digit numbers, using a variety of tools and strategies (e.g., modelling with concrete materials, using guess and check with and without the aid of a calculator) (Sample problem: What is the missing number in the equation $25 - 4 = 15 + ?$)

3D.2.3 identify, through investigation, the properties of zero and one in multiplication (i.e., any number multiplied by zero equals zero; any number multiplied by 1 equals the original number) (Sample problem: Use tiles to create arrays that represent 3×3 , 3×2 , 3×1 , and 3×0 . Explain what you think will happen when you multiply any number by 1, and when you multiply any number by 0.)

3D.2.4 identify, through investigation, and use the associative property of addition to facilitate computation with whole numbers (e.g., "I know that $17 + 16$ equals $17 + 3 + 13$. This is easier to add in my head because I get $20 + 13 = 33$.").

4A. NUMBER SENSE AND NUMERATION

4A.1.1 represent, compare, and order whole numbers to 10 000, using a variety of tools (e.g., drawings of base ten materials, number lines with increments of 100 or other appropriate amounts)

4A.1.3 read and print in words whole numbers to one thousand, using meaningful contexts (e.g., books, highway distance signs)

4A.1.11 solve problems that arise from real-life situations and that relate to the magnitude of whole numbers up to 10 000.

4A.2.1 add and subtract two-digit numbers, using a variety of mental strategies (e.g., one way to calculate $73 - 39$ is to subtract 40 from 73 to get 33, and then add 1 back to get 34)

4A.2.2 solve problems involving the addition and subtraction of four-digit numbers, using student-generated algorithms and standard algorithms (e.g., "I added $4217 + 1914$ using $5000 + 1100 + 20 + 11$.")

4A.2.5 multiply to 9×9 and divide to $81 \div 9$, using a variety of mental strategies (e.g., doubles, doubles plus another set, skip counting); – solve problems involving the multiplication of one-digit whole numbers, using a variety of mental strategies (e.g., 6×8 can be thought of as $5 \times 8 + 1 \times 8$)

4A.2.6 multiply whole numbers by 10, 100, and 1000, and divide whole numbers by 10 and 100, using mental strategies (e.g., use a calculator to look for patterns and generalize to develop a rule)

4A.2.7 multiply two-digit whole numbers by one-digit whole numbers, using a variety of tools (e.g., base ten materials or drawings of them, arrays), student-generated algorithms, and standard algorithms

4A.2.8 divide two-digit whole numbers by one digit whole numbers, using a variety of tools (e.g., concrete materials, drawings) and student-generated algorithms

4A.3.1 describe relationships that involve simple whole-number multiplication (e.g., "If you have 2 marbles and I have 6 marbles, I can say that I have three times the number of marbles you have.")

4D. PATTERNING AND ALGEBRA

4D.1.1 extend, describe, and create repeating, growing, and shrinking number patterns (e.g., "I created the pattern 1, 3, 4, 6, 7, 9, ... I started at 1, then added 2, then added 1, then added 2, then added 1, and I kept repeating this.")

4D.1.2 connect each term in a growing or shrinking pattern with its term number (e.g., in the sequence 1, 4, 7, 10, ..., the first term is 1, the second term is 4, the third term is 7, and so on), and record the patterns in a table of values that shows the term number and the term

4D.1.3 create a number pattern involving addition, subtraction, or multiplication, given a pattern rule expressed in words (e.g., the pattern rule "start at 1 and multiply each term by 2 to get the next term" generates the sequence 1, 2, 4, 8, 16, 32, 64, ...)

4D.2.1 determine, through investigation, the inverse relationship between multiplication and division (e.g., since $4 \times 5 = 20$, then $20 \div 5 = 4$; since $35 \div 5 = 7$, then $7 \times 5 = 35$)

4D.2.2 determine the missing number in equations involving multiplication of one- and two-digit numbers, using a variety of tools and strategies (e.g., modelling with concrete materials, using guess and check with and without the aid of a calculator)

4D.2.3 identify, through investigation (e.g., by using sets of objects in arrays, by drawing area models), and use the commutative property of multiplication to facilitate

computation with whole numbers (e.g., "I know that $15 \times 7 \times 2$ equals $15 \times 2 \times 7$. This is easier to multiply in my head because I get $30 \times 7 = 210$."

4D.2.4 identify, through investigation (e.g., by using sets of objects in arrays, by drawing area models), and use the Distributive property of multiplication over addition to facilitate computation with whole numbers (e.g., "I know that 9×52 equals $9 \times 50 + 9 \times 2$. This is easier to calculate in my head because I get $450 + 18 = 468$."

5A. NUMBER SENSE AND NUMERATION

5A.1.9 solve problems that arise from real-life situations and that relate to the magnitude of whole numbers up to 100 000

5A.3.1 solve problems involving the addition, subtraction, and multiplication of whole numbers, using a variety of mental strategies (e.g., use the commutative property: $5 \times 18 \times 2 = 5 \times 2 \times 18$, which gives $10 \times 18 = 180$)

5A.3.3 multiply two-digit whole numbers by two-digit whole numbers, using estimation, student-generated algorithms, and standard algorithms

5A.3.4 divide three-digit whole numbers by one-digit whole numbers, using concrete materials, estimation, student-generated algorithms, and standard algorithms

5D. PATTERNING AND ALGEBRA

5D.1.2 build a model to represent a number pattern presented in a table of values that shows the term number and the term

5D.1.3 make a table of values for a pattern that is generated by adding or subtracting a number (i.e., a constant) to get the next term, or by multiplying or dividing by a constant to get the next term, given either the sequence (e.g., 12, 17, 22, 27, 32, ...) or the pattern rule in words (e.g., start with 12 and add 5 to each term to get the next term)

Extension:

6A. NUMBER SENSE AND NUMERATION

6A.1.6 solve problems that arise from real-life situations and that relate to the magnitude of whole numbers up to 1 000 000 (Sample problem: How would you determine if a person could live to be 1 000 000 hours old? Show your work.)

6A.1.7 identify composite numbers and prime numbers, and explain the relationship between them (i.e., any composite number can be factored into prime factors) (e.g., $42 = 2 \times 3 \times 7$).

6A.2.1 use a variety of mental strategies to solve addition, subtraction, multiplication, and division problems involving whole numbers (e.g., use the commutative property: $4 \times 16 \times 5 = 4 \times 5 \times 16$, which gives $20 \times 16 = 320$; use the distributive property: $(500 + 15) \div 5 = 500 \div 5 + 15 \div 5$, which gives $100 + 3 = 103$)

6A.2.2 solve problems involving the multiplication and division of whole numbers (four-digit by two-digit), using a variety of tools (e.g., concrete materials, drawings, calculators) and strategies (e.g., estimation, algorithms)

6D. PATTERNING AND ALGEBRA

6D.1.4 describe pattern rules (in words) that generate patterns by adding or subtracting a constant, or multiplying or dividing by a constant, to get the next term (e.g., for 1,

3, 5, 7, 9, ..., the pattern rule is "start with 1 and add 2 to each term to get the next term"), then distinguish such pattern rules from pattern rules, given in words, that describe the general term by referring to the term number (e.g., for 2, 4, 6, 8, ..., the pattern rule for the general term is "double the term number")

8A.1.1 express repeated multiplication using exponential notation (e.g., $2 \times 2 \times 2 \times 2 = 24$)

8A.1.4 determine common factors and common multiples using the prime factorization of numbers (e.g., the prime factorization of 12 is $2 \times 2 \times 3$; the prime factorization of 18 is $2 \times 3 \times 3$; the greatest common factor of 12 and 18 is 2×3 or 6; the least common multiple of 12 and 18 is $2 \times 2 \times 3 \times 3$ or 36).

Appendix B: Alberta Whole Numbers Mathematics Standards (Grades 3-6)

Grade 3

NUMBER

3A.1. Say the number sequence 0 to 1000 forward and backward by:

- 5s, 10s or 100s, using any starting point
- 3s, using starting points that are multiples of 3
- 4s, using starting points that are multiples of 4
- 25s, using starting points that are multiples of 25.

3A.2. Represent and describe numbers to 1000, concretely, pictorially and symbolically.

3A.3. Compare and order numbers to 1000.

3A.5. Illustrate, concretely and pictorially, the meaning of place value for numerals to 1000.

3A.6. Describe and apply mental mathematics strategies for adding two 2-digit numerals, such as:

- adding from left to right
- taking one addend to the nearest multiple of ten and then compensating using doubles.

3A.7. Describe and apply mental mathematics strategies for subtracting two 2-digit numerals, such as:

- taking the subtrahend to the nearest multiple of ten and then compensating
- thinking of addition using doubles.

3A.9. Demonstrate an understanding of addition and subtraction of numbers with answers to 1000 (limited to 1-, 2- and 3-digit numerals), concretely, pictorially and symbolically, by:

- using personal strategies for adding and subtracting with and without the support of manipulatives
- creating and solving problems in context that involve addition and subtraction of number than a given number.

3A.10. Apply mental mathematics strategies and number properties, such as:

- using doubles
- making 10
- using the commutative property
- using the property of zero
- thinking addition for subtraction in order to understand and recall basic addition facts and related subtraction facts to 18.

Understand, recall and apply addition and related subtraction facts to 18.

3A.11. Demonstrate an understanding of multiplication to 5×5 by:

- representing and explaining multiplication using equal grouping and arrays
- creating and solving problems in context that involve multiplication
- modelling multiplication using concrete and visual representations, and recording the process symbolically

- relating multiplication to repeated addition
- relating multiplication to division.

Understand and recall multiplication facts to 5×5 .

3A.12. Demonstrate an understanding of division (limited to division related to multiplication facts up to 5×5) by:

- representing and explaining division using equal sharing and equal grouping
- creating and solving problems in context that involve equal sharing and equal grouping
- modelling equal sharing and equal grouping using concrete and visual representations, and recording the process symbolically
- relating division to repeated subtraction
- relating division to multiplication.

Understand and recall division facts related to multiplication facts to 5×5 .

PATTERNS AND RELATIONS (Patterns)

3B.1. Demonstrate an understanding of increasing patterns by:

- describing
- extending
- comparing
- creating

numerical (numbers to 1000) and non-numerical patterns using manipulatives, diagrams, sounds and actions.

3B.2. Demonstrate an understanding of decreasing patterns by:

- describing
- extending
- comparing
- creating numerical (numbers to 1000) and non-numerical patterns using manipulatives, diagrams, sounds and actions.

3B.3. Sort objects or numbers, using one or more than one attribute.

Grade 4

NUMBER

4A.1. Represent and describe whole numbers to 10 000, pictorially and symbolically.

4A.2. Compare and order numbers to 10 000.

4A.3. Demonstrate an understanding of addition of numbers with answers to 10 000 and their corresponding subtractions (limited to 3- and 4-digit numerals) by:

- using personal strategies for adding and subtracting
- estimating sums and differences
- solving problems involving addition and subtraction.

4A.4. Apply the properties of 0 and 1 for multiplication and the property of 1 for division.

4A.5. Describe and apply mental mathematics strategies, such as:

- skip counting from a known fact
- using doubling or halving
- using doubling or halving and adding or subtracting one more group
- using patterns in the 9s facts
- using repeated doubling to determine basic multiplication facts to 9×9 and related division facts.

Understand and apply strategies for multiplication and related division facts to 9×9 .

Recall multiplication and related division facts to 7×7

4A.6. Demonstrate an understanding of multiplication (2- or 3-digit by 1-digit) to solve problems by:

- using personal strategies for multiplication with and without concrete materials
- using arrays to represent multiplication

- connecting concrete representations to symbolic representations
- estimating products
- applying the distributive property.

4A.7. Demonstrate an understanding of division (1-digit divisor and up to 2-digit dividend) to solve problems by:

- using personal strategies for dividing with and without concrete materials
- estimating quotients
- relating division to multiplication.

PATTERNS AND RELATIONS (Patterns)

4B.1. Identify and describe patterns found in tables and charts.

4B.2. Translate among different representations of a pattern, such as a table, a chart or concrete materials.

4B.3. Represent, describe and extend patterns and relationships, using charts and tables, to solve problems.

4B.4. Identify and explain mathematical relationships, using charts and diagrams, to solve problems.

Grade 5

5A.1. Represent and describe whole numbers to 1 000 000.

5A.3. Apply mental mathematics strategies and number properties, such as:

- skip counting from a known fact
 - using doubling or halving
 - using patterns in the 9s facts
 - using repeated doubling or halving in order to understand and recall basic multiplication facts (multiplication tables) to 81 and related division facts.
- Understand, recall and apply multiplication and related division facts to 9×9***

5A.4. Apply mental mathematics strategies for multiplication, such as:

- annexing then adding zero
- halving and doubling
- using the distributive property.

5A.6. Demonstrate, with and without concrete materials, an understanding of division (3-digit by 1-digit)

PATTERNS AND RELATIONS (Variables and Equations)

5B.2. Express a given problem as an equation in which a letter variable is used to represent an unknown number (limited to whole numbers).

5B.3. Solve problems involving single-variable, one-step equations with whole number coefficients and whole number solutions.

Grade 6

6A.2. Solve problems involving whole numbers .

6A.3. Demonstrate an understanding of factors and multiples by:

- _determining multiples and factors of numbers less than 100
- _identifying prime and composite numbers
- _solving problems using multiples and factors.

6A.7. Demonstrate an understanding of integers, concretely, pictorially and symbolically

6A.9. Explain and apply the order of operations, excluding exponents, with and without technology (limited to whole numbers).

PATTERNS AND RELATIONS (Patterns)

6B.1. Represent and describe patterns and relationships, using graphs and tables.

6B.2. Demonstrate an understanding of the relationships within tables of values to solve problems.

Appendix C: Common Core State Standards (Whole numbers Grades 3-5)

Grade 3

3.0A.A1 Interpret products of whole numbers, e.g., interpret 5×7 as the total number of objects in 5 groups of 7 objects each.

3.0A.A2 Interpret whole-number quotients of whole numbers, e.g., interpret $56 \div 8$ as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each.

3.0A.A4 Determine the unknown whole number in a multiplication or division equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations $8 \times ? = 48$, $5 = _ \div 3$, $6 \times 6 = ?$

3.0A.B5 Apply properties of operations as strategies to multiply and divide... (Commutative property of multiplication; Associative property of multiplication; (Distributive property.)

3.0A.B.6 Understand division as an unknown-factor problem. For example, find $32 \div 8$ by finding the number that makes 32 when multiplied by 8.

3.0A.C.7 Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division...By the end of Grade 3, know from memory all products of two one-digit numbers.

3.0A.D.9 Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.

3.NBT.A.1 Use place value understanding to round whole numbers to the nearest 10 or 100.

3.NBT.A.2 Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.

3.NBT.A.3 Multiply one-digit whole numbers by multiples of 10 in the range 10-90 (e.g., 9×80 , 5×60) using strategies based on place value and properties of operations.

Grade 4

4.NBT.B.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

4.NBT.B.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division

4.NBT.B.4 Fluently add and subtract multi-digit whole numbers using the standard algorithm.

4.NBT.B.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations

4.OA.A.1 Interpret a multiplication equation as a comparison, e.g., interpret $35 = 5 \times 7$ as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.

4.OA.A.2 Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.¹

4.OA.A.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

4.OA.B.4 Find all factor pairs for a whole number in the range 1-100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1-100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1-100 is prime or composite.

4.NBT.A.1 Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. For example, recognize that $700 \div 70 = 10$ by applying concepts of place value and division.

4.NBT.A.2 Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using $>$, $=$, and $<$ symbols to record the results of comparisons.

4.NBT.B.4 Fluently add and subtract multi-digit whole numbers using the standard algorithm.

4.NBT.B.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

4.NBT.B.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Grade 5

5.OA.A.2 Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them

5.OA.B.3 Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms

5.NBT.A.1 Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and $1/10$ of what it represents in the place to its left

