HARVARD | BUSINESS | SCHOOL

When Smaller Menus Are Better: Variability in Menu-Setting Ability

David Goldreich
Hanna Hałaburda

Working Paper

11-086

When smaller menus are better: Variability in menu-setting ability*

David Goldreich
Rotman School of Management
University of Toronto

Hanna Hałaburda
Harvard Business School

April 18, 2011

Abstract

The economics literature on choice focuses on individuals' decisions when faced with a given menu. However, the menu itself is often the result of pre-selection by a menu setter. We develop a model to study the relation between the ability of the menu setter and the size and quality of the menu. We show that when the cost of increasing the size of the menu is sufficiently small, a lower-ability menu setter optimally offers more items in the menu than a higher-ability menu setter. Nevertheless, the menu optimally offered by a higher-ability menu setter remains superior to the menu optimally offered by a lower-ability menu setter. This results in a negative relation between menu size and menu quality, i.e., a smaller menu is better than a larger menu. We illustrate this result empirically in the context of 401(k) plans, where we show a negative relation between the number of investment choices in a 401(k) plan and the quality of the optimal portfolio achievable given those investment choices.

JEL: D01, G23

Keywords: menu, menu setting, choice, pension plans, 401(k)

*We thank Ramon Casadesus-Masanell, Andreas Park, Lukasz Pomorski, Julie Wulf, and seminar participants at the University of Toronto and the Interdisciplinary Center Herzliya for helpful comments and discussions. We thank Matt Levi for excellent research assistance.

1 Introduction

In many settings, people choose from a menu designed by someone else. This can be a literal menu at a restaurant, a choice of products on a supermarket shelf, or a list of assets available for investment in a 401(k) retirement fund. There is a growing literature showing how the choices of economic agents are influenced by the composition of the menu and more specifically by the number of items offered on the menu. However, the size and the composition of the menu are themselves the result of a prior selection by another agent which we refer to as the *menu setter*. There is often a large universe of potential choices from which the menu setter selects a subset as a menu. In this paper we take a different perspective from the existing literature, by focusing on the role of the menu setter. Specifically, we consider the menu setter's decision on what to include in the menu, and how large a menu to construct.

In our analysis, we recognize that menu setters differ in ability. We show that menu setters optimally offer different menu sizes, depending on their ability. In particular, if the cost of increasing the menu size is low, then a lower-ability menu setter offers more items on the menu than a higher-ability menu setter. However, we also show that the menu offered by a lower-ability menu setter is of lower quality. Combining the two results leads to a negative relation between menu size and menu quality: Larger menus are worse. This counterintuitive finding follows from the point that the smaller menu set by a high-ability menu setter is *not* a subset of the larger menu set by a lower-ability menu setter. Rather, a large menu offered by a lower-ability menu setter tends to include choices that are less attractive or even redundant, while a small menu is more likely to include higher quality items.

Although our arguments can be applied to many settings and we describe it generally, we illustrate the finding with an example of investment menus offered to employees in 401(k) pension plans. 401(k) plans include a menu of investment options — usually mutual funds, and employees allocate their pension money across the investment options in the plan. We measure the quality of a plan by the risk and return of the optimal portfolio achievable within the plan.

Given a menu of potential investments in a 401(k), the individual investor's goal is to choose a portfolio from among those assets that achieves a high expected return while at the same time having low risk. Low risk is achieved by diversification, i.e., by choosing assets that have low correlations with each other. Thus, a mutual fund on a menu that is highly correlated with other funds on the menu does not reduce the portfolio risk much, and is unlikely to significantly improve the quality of the portfolios achievable with the menu.

We measure the quality of each plan as the highest Sharpe ratio achievable from among the portfolios that can be constructed from the assets in that plan.[1] We find a negative relation between the number of investment choices and the plan quality. This negative relation can be explained by differing abilities of menu setters, as derived in our model, (although we do not directly observe those abilities in our empirical setting).

We model menu setters as selecting a menu of items from a large universe. Items may be of the same or different *types*. When evaluating a menu, each new type of item in the menu improves the quality of the menu. But as the menu size increases, the marginal benefit of adding another new type declines. Two items of the same type, however, are considered redundant, and adding a redundant item does not increase the quality of the menu at all.

High-ability (or *expert*) menu setters are those who can always identify the non-redundant items and for any menu size offer the highest quality menu. Lower-ability menu setters are less likely to identify non-redundant items, so the menu that they offer includes some redundant items.

In the absence of any cost of adding items to the menu, a menu setter would include the whole universe of items and individuals would be able to find their most favored choice regardless of the menu setter's ability.[2] However, if there is a cost of offering more items on the menu, for example, a fixed cost of stocking each item, then the menu setter would limit the number of choices offered.

When deciding to add one more item to the menu, the menu setter compares the marginal benefit of including an additional item to the cost of including that item. For an expert menu setter, the marginal benefit is the marginal value of the additional non-redundant item in

[1]The Sharpe ratio is a common measure of portfolio quality that incorporates both risk and return. In Section 4, we detail how we estimate the expected Sharpe ratio.

[2]We assume that individuals are fully rational and can costlessly identify the best choice from a menu.

the menu. Because the marginal benefit of adding a new type is declining, and the cost of including each additional item is assumed to be constant, the expert menu setter optimally offers a limited menu.

For a lower-ability menu setter, the marginal benefit of adding the n^{th} item to the menu may be higher or lower than the marginal benefit of adding the n^{th} item for the expert menu setter. On the one hand, for any additional item the expected marginal benefit to the lower-ability menu setter is reduced because there is a certain probability that it is a redundant item which does not improve menu quality at all. On the other hand, if previous items in the menu were redundant and the lower-ability menu setter has fewer non-redundant items in his menu, the marginal benefit of an additional non-redundant item is higher. We show that in general, when the cost of adding an item to the menu is small (and thus the menu size is relatively large), the lower-ability menu setter offers more items in his menu than the expert menu setter. Conversely, when cost is large (and the menu size is small) we obtain the opposite effect: the lower-ability menu setter offers fewer items. Nevertheless, for any cost the quality of the menu offered by the expert menu setter is superior to the quality of the menu offered by the lower-ability menu setter. Thus, for low costs, we obtain a seemingly paradoxical result that a smaller menu is of higher quality than a larger menu.

The literature on choice often focuses on how individuals select from a given menu, and often relies on behavioral explanations or bounded rationality to explain agents' preferences for different menus. In a well known study, Iyengar and Lepper (2000) find that shoppers are more likely to try a free sample of a product when a large menu is offered, but are more likely to buy when offered a smaller menu. Similarly, Huberman, Iyengar, and Jiang (2006) find that employees are less likely to participate in 401(k) plans when the menu of funds offered is larger. Iyengar and Kamenica (2007) further study choice overload and how individuals make suboptimal decisions when faced with too many options. Despite these findings, Scheibehenne, Greifeneder, and Todd (2010) argue that choice overload only occurs in limited contexts. A number of recent books have also argued that psychology (e.g. Schwartz, 2004, and Iyengar, 2010), and neurology (see Lehrer, 2009) can affect individual choices. In an experimental study, Salgado (2005) focuses on individual behavior when faced with different menus. She finds that individuals' preferences for smaller menus depend on their perceptions of the skill of a menu setter who limits the choice set to a subset of a larger menu.

Our study differs from that literature in that we do not focus on individual behavior or psychology, but instead, we focus on the menu setting decision. We show that optimization by menu setters results in a relation between menu size and menu quality. Most importantly, we do not rely on any behavioral explanations when arguing that smaller menus can be superior to larger menus.

The literature on 401(k) plans focuses on documenting the investment decisions made by employees, and often the behavioral biases driving those decisions. Huberman and Jiang (2006) document that employees typically only select a small subset of funds from the menu of investment options. Agnew (2003) and Agnew, Balduzzi, and Sunden (2003) also document apparently suboptimal choices by employees in 401(k) plans. Benartzi and Thaler (2001) show that pension plan investors follow naive diversification (i.e., with a $1/n$ heuristic) that could allow the menu size to affect investment choices. Tang, Mitchell, Mottola, and Utkus (2010) argue that poor decision making by participants in 401(k) plans is more harmful than inefficiencies in the menu of plan offerings. Mottola and Utkus (2003) apply the results of the choice literature to pension plans and draw implications for pension plan sponsors. Our paper differs from this literature, by focusing on the menus in 401(k) plans and showing that larger menus often have poorer risk/return possibilities than smaller menus.

The paper proceeds as follows. The model is developed and the theoretical results are given in Section 2. The empirical setting and data are described in Section 3. Empirical results are in Section 4. Section 5 concludes. All proofs are in the Appendix.

2 Model

Suppose that individuals must choose an item or a bundle of items from a menu that is comprised of a number of goods. The menu itself is pre-selected from a large universe of goods by a menu setter. The menu setter selects a subset of n items to be included in the menu offered to individuals.

The universe of possible menu items includes both goods that are of different *types* as well as goods that are of the same type, i.e., redundant goods. We assume that the universe includes an infinite number of different types and an infinite number of redundant items. All else equal, the group of individuals that ultimately choose from among the menu items prefers a menu with a variety of types, either because each individual has a preference for a bundle with a variety of types (e.g., when choosing an investment portfolio), or because different individuals have different preferences across types (e.g., when choosing ice-cream flavors). However, the menu is not improved by including multiple items of the same type. Otherwise, for the group of individuals, each type is a priori the same, and the quality of the menu can be summarized by the number of distinct types included in the menu. Denote the quality of a menu with n distinct item types as $Q(n)$, which can be interpreted as the total utility achieved by the group of individuals optimally choosing from the menu. The marginal benefit of the n^{th} distinct type in the menu is the increase in the quality of the menu due to the n^{th} type, i.e., $q(n) = Q(n) - Q(n-1)$.[3]

Adding a new type of item to the menu always increases the quality of the menu, but not to the same degree. We assume that the quality improvement with each additional type is decreasing in n. That is, we assume that $Q(n)$ is strictly increasing but concave in n, or equivalently, that $q(n)$ is strictly positive for any n, and strictly decreasing in n. This assumption of declining marginal benefits captures a wide range of settings.

Assumption of Declining Marginal Benefit. *The marginal benefit of an additional item type in a menu is always positive, but strictly declining in the number of item types already in the menu, i.e.,* $q(n) > 0$ *and* $q(n+1) < q(n)$ *for all* n.

At this stage, the assumption of declining marginal benefit is very general. Later we propose a stronger assumption that imposes more structure on $q(n)$.

There are two types of menu setters: high-ability menu setters that we refer to as *experts*

[3]We do not require that the $n-1$ items are a subset of the n items. Instead, $Q(n-1)$ is defined as the maximal total utility achievable from a menu of $n-1$ item types, and $Q(n)$ is the maximal total utility achievable from a menu of n different item types.

and lower-ability menu setters. Expert menu setters can always identify items of different types. Thus, when an expert menu setter offers a menu with n items, they are all of different types and the quality of his menu is equal to $Q(n)$. Lower-ability menu setters recognize with error whether an item is of a new type. The first item offered by a lower-ability menu setter is always a new type, but after the first item, each additional item in the menu is of a new type with probability p, and is redundant with probability $1-p$. The ability of a menu setter is characterized by p. Expert menu setters are those with $p = 1$, while lower-ability menu setters are those with $p < 1$.

When a lower-ability menu setter selects n items into the menu, it is not certain how many distinct item types are included in the menu. Therefore, for any number of items n, the quality of the menu is a random variable, and its expectation depends on the menu setter's skill, p. Let $\mathbb{E}Q^p(n)$ denote the expected quality of a menu of size n chosen by a lower-ability menu setter with ability p.[4] When $p = 1$, $\mathbb{E}Q^p(n) = Q(n)$. Similar to $q(n)$, we define $\mathbb{E}q^p(n)$ as the expected marginal benefit of the n^{th} item in a menu chosen by a lower-ability menu setter, i.e., $\mathbb{E}q^p(n) = \mathbb{E}Q^p(n) - \mathbb{E}Q^p(n-1)$. All parties are risk neutral.

We assume that a menu setter earns a rent equal to a fraction $\alpha < 1$ of the quality of the menu offered. For an expert menu setter, the rent is $\alpha Q(n)$, and for an lower-ability menu setter, the expected rent is $\alpha\mathbb{E}Q^p(n)$.

For every item included in the menu, the menu setter bears a cost. We assume that the marginal cost of including an additional item in the menu (whether or not it is redundant) is a constant $c > 0$.[5] Because we investigate the effect of the menu setter's ability, we assume that both c and α are common for all menu setters.

The objective of a menu setter is to select a number of items from the universe into his menu, so that he maximizes his expected rent net of costs. Given p, the menu setter's problem is

$$\max_n \left\{\alpha\,\mathbb{E}\,Q^p(n) - c \cdot n\right\}. \tag{1}$$

Since the expected marginal benefit of the n^{th} item is $\mathbb{E}q^p(n)$ (or $q(n)$ for the expert menu setter), menu setters increase the size of the menu as long as α times this marginal

[4]$\mathbb{E}$ is the expected-value operator.

[5]If $c = 0$, all menu setters would offer menus of infinite size with all possible item types.

benefit exceeds the marginal cost, c. As menu setters differ in their ability to recognize non-redundant items, and thus differ in the marginal rent they receive, we expect menus to differ in quality and size.

2.1 Ability and menu size

Clearly, for a given menu size, n, an expert's menu is of higher quality than that of a lower-ability menu setter. On the face of it, it would appear that the expert menu setter should have a higher marginal benefit from increasing the menu size beyond n than would the lower-ability menu setter since the expert always identifies a new item type. However, in this section we present the conditions under which the lower-ability menu setter has a higher expected marginal benefit to increasing the menu size than does the expert menu setter.

The key point that we show below is that as n increases, the marginal benefit of the expert menu setter declines more steeply than that of the lower-ability menu setter. Moreover, there exists a number, n^*, such that the two marginal benefits are equal; for menu sizes larger than n^*, the expected marginal benefit achieved by the lower-ability menu setter is *larger* than the marginal benefit achieved by the expert menu setter. Thus, for low enough costs the lower-ability menu setter optimally offers a larger menu than the expert.

To formalize the argument, we must first prove a series of lemmas characterizing $\mathbb{E}q^p(n)$ and comparing it to $q(n)$. When determining the optimal menu size, menu setters increase the number of items offered, as long as α times the expected marginal benefit exceeds the marginal cost, c. For the expert menu setter, the optimal n is chosen such that $\alpha q(n) \geq c$ and $\alpha q(n+1) < c$. Similarly, for the lower-ability menu setter, the optimal n is chosen such that $\alpha \mathbb{E}q^p(n) \geq c$ and $\alpha \mathbb{E}q^p(n+1) < c$.

In Lemma 1 we show that $\mathbb{E}q^p(n)$ is decreasing in n. This result is obtained by directly applying the Assumption of Declining Marginal Benefit.

Lemma 1. *For any $0 < p \leq 1$, the expected marginal benefit of adding the n^{th} item to a menu, $\mathbb{E}q^p(n)$, is decreasing in n.*

Proof. See Appendix (page 28).

Lemma 1 states that like $q(n)$, $\mathbb{E}q^p(n)$ is decreasing in n. Although the menu of a lower-ability menu setter has an unknown number of distinct item types, when the menu setter adds the n^{th} item to the menu, he either successfully increases the number of distinct item types or he does not. If he successfully improves the menu, then the next improvement faces a declining marginal benefit regardless of the current number of item types because $q(n)$ is declining for all n. If he did not improve the menu, the marginal benefit of trying again is the same. So, on average, the $(n+1)^{st}$ item has a lower expected marginal benefit than the n^{th} item.

Now that we have established that the marginal benefit of increasing the menu size is declining for both types of menu setters, we must show that the expert menu setter has a higher marginal benefit when the menu is small, and that the lower-ability menu setter can have a higher marginal benefit when the menu is large.

To see this, first put aside the special case of a menu of size one, since it is always a new type and $\mathbb{E}q^p(1) = q(1)$. Consider the second item that is included on the menu. When an expert menu setter offers two items on a menu, the menu is of quality $Q(2) = q(1) + q(2)$. When a lower-ability menu setter offers two items, the expected marginal benefit of the second item must be lower, since it is only a new type with probability p, and the expected quality of the menu offered by the lower-ability menu setter is $\mathbb{E}Q^p(2) = q(1) + p \cdot q(2)$. Thus, the expected marginal benefit of increasing the menu size from one to two, $\mathbb{E}q^p(2)$ is equal to $p \cdot q(2)$, i.e., less than the marginal benefit for the expert menu setter.

However, for the third (and later) items, the comparison between marginal benefits of the two types of menu setters is not as straightforward. For the expert menu setter, the marginal benefit of the third item is $q(3)$. However, for the lower-ability menu setter, the marginal benefit of the third item depends on whether the second item was a new type or not. If the second item was a new type, then the expected marginal benefit of the third item would be $p \cdot q(3)$; but if the second item was redundant, then the expected marginal benefit of the third item would be $p \cdot q(2)$. Since we do not know if the second item is of a new type, the expected marginal benefit of the third item is

$$\mathbb{E}q^p(3) = p\left(p \cdot q(3) + (1-p) \cdot q(2)\right) > p \cdot q(3)\,. \tag{2}$$

Thus, $\mathbb{E}q^p(3)/q(3)$ is larger than $\mathbb{E}q^p(2)/q(2) = p$. In other words, when increasing the menu size from two to three, the marginal benefit curve for the expert menu setter is more steeply downward sloping than the expected marginal benefit curve for the lower-ability menu setter.

Indeed, $\mathbb{E}q^p(3)$ may even be larger than $q(3)$. A tradeoff occurs because on the one hand, a lower-ability menu setter only adds a marginal benefit with a probability of p. But on the other hand, when he does add a marginal benefit, it may be $q(3)$ or it may be $q(2)$, which is larger than $q(3)$. Thus, conditional on the third item being of a new type, the expected marginal benefit of the lower-ability menu setter, i.e., $p \cdot q(3) + (1-p) \cdot q(2)$, is higher than the marginal benefit of the expert menu setter, $q(3)$. Unconditionally, the expected marginal benefit, $\mathbb{E}q^p(3)$, is larger than $q(3)$ when

$$\mathbb{E}q^p(3) > q(3) \quad \iff \quad q(3) < q(2)\frac{p}{1+p} \quad \iff \quad p > \frac{q(3)}{q(2)-q(3)} . \tag{3}$$

In other words, the marginal benefit of each new item type must be declining rapidly for this inequality to hold. If $q(3)$ is much smaller than $q(2)$, then the marginal benefit of the expert menu setter is very small; but the marginal benefit of the lower-ability menu setter is higher in expectation because he is possibly adding $q(2)$. Conversely, if $q(3) > \frac{1}{2}q(2)$ then there is no p that would satisfy inequality (3).[6]

For larger n, the condition under which $\mathbb{E}q^p(n) > q(n)$ is easier to satisfy. That is, the tradeoff tends to shift in favor of the expected marginal benefit of the lower-ability menu setter. Given any n, the lower-ability menu setter always finds a non-redundant item with probability p. But as n grows, it is more and more likely that the lower-ability menu setter's menu includes fewer than n distinct item types, and the marginal benefit conditional on the $(n+1)^{st}$ item being a new item type is likely to be substantially greater than the expert menu setter's marginal benefit of the next item, $q(n+1)$. If this difference is large enough, then the *unconditional* expected marginal benefit of the lower-ability menu setter is larger than $q(n+1)$.

[6]Notice that regardless of the difference between the marginal benefits, the total quality of a menu with three items is always greater for the expert menu setter than for the lower-ability one, i.e., $Q(3) > \mathbb{E}Q^p(3)$. We generalize this in Proposition 2.

Of course, the exact nature of the tradeoff between the lower probability of identifying a non-redundant item and the higher marginal benefit of a new item type for the lower-ability menu setter depends on the parameters, and especially on the marginal benefit function $q(n)$. Therefore, for the results in the remainder of this section we impose a specific structure on $q(n)$ with the stronger assumption that the marginal benefit of each new item type declines at a fixed rate, i.e. $q(n+1) = k \cdot q(n)$, where $0 < k < 1$.

Strong Assumption of Declining Marginal Benefit. *For any n, the marginal benefit of an additional item type in a menu is strictly positive ($q(n) > 0$), and declines at a rate such that $q(n+1) = k \cdot q(n)$ for some $k < 1$.*

Recall that the benefit of including the first item in the menu is the same regardless of the menu setter's ability, $q(1) = \mathbb{E}q^p(1)$, and so it is only interesting to consider the differences between marginal benefits for $n \geq 2$.

Lemmas 2 and 3 and Corollary 1 below establish a *single crossing* property between the marginal benefits of the two types of menu setters: For any n the expert menu setter's marginal benefit declines quicker than the marginal benefit of the lower-ability menu setter. There exists a unique number n^* where the two marginal benefits are equal. Thus, for all menu sizes larger than n^*, the marginal benefit of the lower-ability menu setter is larger than the marginal benefit of the expert menu setter.

Lemma 2. *The marginal benefit of an additional menu item for the expert menu setter decreases more quickly than the expected marginal benefit of an additional menu item for the lower-ability menu setter, i.e., $\frac{q(n+1)}{q(n)} = k < \frac{\mathbb{E}q^p(n+1)}{\mathbb{E}q^p(n)}$ for $n \geq 2$ and for any $0 < p < 1$.*

Proof. See Appendix (page 28).

Lemma 2 arises because the marginal benefit for the expert menu setter of adding an additional item to the menu declines at a rate $q(n+1)/q(n) = k$. In contrast, when the lower-ability menu setter adds the n^{th} item, he may or may not have been successful in adding a non-redundant item. If he was successful, then the expected marginal benefit for the $(n+1)^{st}$ item declines at a rate k — just like the expert menu setter; but if he was unsuccessful, then the expected marginal benefit remains unchanged. Thus, the expected marginal benefit declines with a factor $p \cdot k + (1-p) \cdot 1 > k$, and the overall expected decline in the marginal benefit is not as steep as it is for the expert menu setter.

Corollary 1 to Lemma 2 states that if for some n the marginal benefit for the lower-ability menu setter is higher than the marginal benefit for the expert menu setter, then it remains the case for any size of the menu larger than n.

Corollary 1. *For any $0 < p < 1$, if $q(n) < \mathbb{E}\, q^p(n)$, then $q(n+1) < \mathbb{E}\, q^p(n+1)$.*

Proof. See Appendix (page 29).

Lemma 3 completes the single-crossing property argument by proving that there always exists a finite menu size n such that $q(n) < \mathbb{E}\, q^p(n)$, i.e., the marginal benefit for the lower-ability menu setter is larger than the marginal benefit for the expert menu setter when the menu size is n. Moreover, Lemma 3 characterizes the crossing point.

Lemma 3. *For any $0 < p < 1$, let*

$$n^* = 2 + \frac{ln(p)}{ln(k) - ln\big((1-p) + pk\big)} \,. \tag{4}$$

(i) The expected marginal benefit of an additional menu item is the same for the expert menu setter and the lower-ability menu setter, i.e., $\mathbb{E}\, q^p(n^*) = q(n^*)$, if and only if $n = n^*$.

(ii) For n larger (smaller) than n^*, the expected marginal benefit of an additional menu item for the lower-ability menu setter is larger (smaller) than the marginal benefit for the expert menu setter, i.e., $n > n^* \Rightarrow \mathbb{E}\, q^p(n) > q(n)$, and $n < n^* \Rightarrow \mathbb{E}\, q^p(n) < q(n)$.

Proof. See Appendix (page 29).

Lemma 3 characterizes n^* as the point at which the marginal benefit for the expert menu setter and the marginal benefit for the lower-ability menu setters are the same. It should not be surprising that $n^* > 2$ as we have previously shown that $\mathbb{E}\, q^p(2) = p \cdot q(2)$. Of course, if n^* is not an integer, there is no menu size at which the two types of menu setters have exactly the same marginal benefit. Nevertheless, n^* delineates where the marginal benefit of an additional menu item is higher for one type of menu setter or the other: for menu sizes larger than n^*, the expected marginal benefit for the lower-ability menu setter is larger than the marginal benefit of the expert menu setter, and for menu sizes smaller than n^*, the marginal benefit for the expert is larger. Figure 1(a) displays an example of the comparison between the expert and lower-ability menu setters' marginal benefits.

Recall that menu setters increase the menu size as long as α times the expected marginal benefit exceeds the marginal cost of including an additional item. As we can see in Figure 1(b), for any cost above $\alpha\, q(n^*)$ the lower-ability menu setter includes fewer items in his menu than the expert menu setter.[7] However, for any cost below $\alpha\, q(n^*)$, the expert menu setter includes fewer items in his menu than the lower-ability menu setter. This property is formally stated in Proposition 1. For clarity, we abuse the notation somewhat by allowing n to be a continuous variable, and allowing $q(\cdot)$ to be defined over that continuous variable.[8]

Proposition 1. *The relative sizes of menus optimally offered by expert and lower-ability menu setters depend on the marginal cost of increasing the menu size, c.*

(i) If $c < \alpha q(n^*)$, the lower-ability menu setter optimally includes more items in his menu than the expert menu setter.

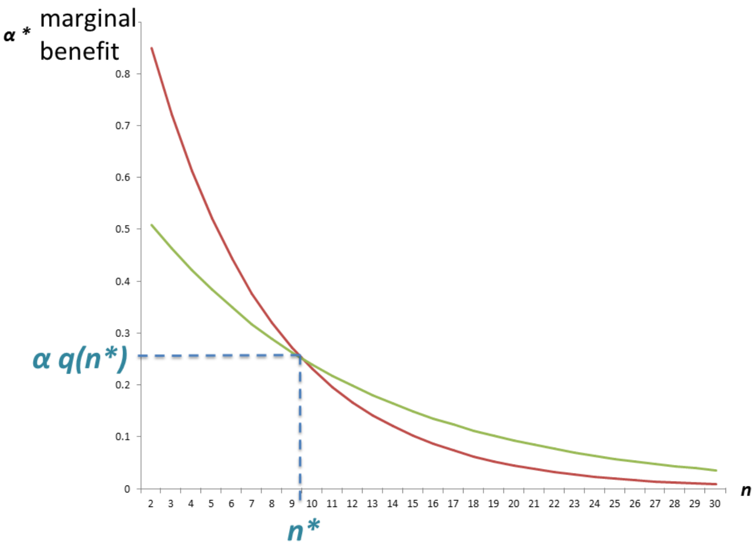
(ii) If $c > \alpha q(n^*)$, the expert menu setter optimally includes more items in his menu than the lower-ability menu setter.

Proof. See Appendix (page 30).

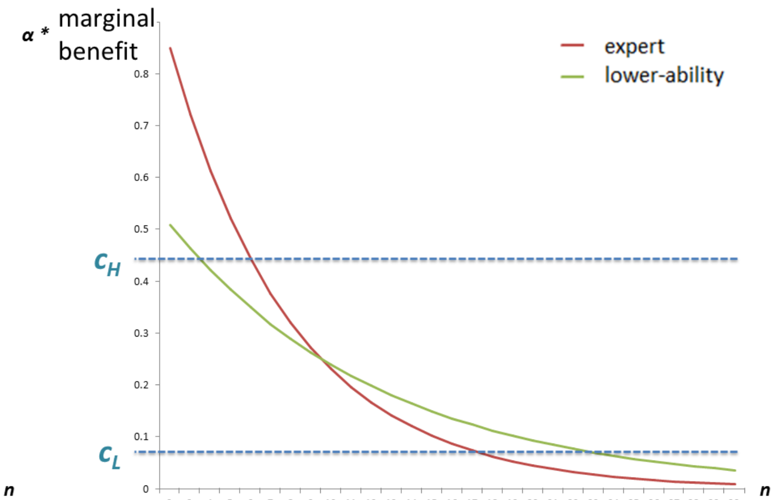
When the cost of including more items is high, we are in the conventional situation in which a higher-ability menu setter uses that ability to find a higher number of beneficial

[7]This inequality is strict when we ignore the complication that n must be an integer. However, if the menu size must be an integer, it becomes a weak inequality — both menu setters offer the same menu size when they both round down to the same integer. Menu setters never round up, since that would result in a marginal benefit less than the marginal cost.

[8]Ellison, Fudenberg, and Mobius (2004) take a similar approach in an auction problem, when they ignore the integer constraint on the number of participants and refer to their result as a quasi-equilibrium.



(a)



(b)

Figure 1. Marginal benefit as a function of the menu size for expert and lower-ability menu setters. Costs c_H and c_L are chosen so that $c_L < \alpha q(n^*) < c_H$. This example assumes $p = 0.6$, $k = 0.85$, and $\alpha q(1) = 1$.

items, while the lower-ability menu setter does not find it worthwhile to include as many items (see Figure 1(b) for cost c_H). In such a case, a larger menu would suggest an expert menu setter. Moreover, a larger menu would be associated with better performance — both because the larger menu is designed by a menu setter with higher ability, and more simply because a larger menu offers more choices to individuals.

However, Proposition 1 shows that when the cost of including more items is low, lower-ability menu setters offers larger menus than expert menu setters (as in Figure 1(b) for cost c_L). This is because when c is small, all menu setters include many items in their menus, and by the nature of the lower-ability menu setter, it is very likely that his menu has fewer non-redundant item types than the menu of the expert menu setter. Since the marginal benefit of each new item type is decreasing, the lower-ability menu setter has a higher expected marginal benefit (even after accounting for the probability that the new item will fail to be a new type).

Thus, when $c < \alpha q(n^*)$, the lower-ability menu setter optimally offers more items in his menu, leading to a negative relation between the number of items observed and the menu setter's ability.

When lower-ability menu setters offer larger menus, there is a tradeoff when comparing a large menu with a small menu. If the dominant factor is the number of items in the menu then there would still be a positive relation between the number of items and menu quality. This would lead to a paradoxical result that the menu offered by a lower-ability menu setter would be of higher quality than the menu offered by an expert menu setter.[9] However, if the dominant factor is the ability of the menu setter, smaller menus would be of higher quality than larger menus. There would be a negative relation between menu size and quality.

We explore the tradeoff in the following two sections.

2.2 Ability and menu quality

We now consider the relation between the ability of the menu setter and the quality of the menu. In Proposition 2, we establish that when both menu setters select their menu sizes optimally, the menu offered by the expert menu setter is always of higher total quality than the menu offered by the lower-ability menu setter, even when the lower-ability menu setter offers more items.

Proposition 2. *Suppose that for a given c, an expert menu setter ($p = 1$) and a lower-ability menu setter ($0 < p < 1$) set their optimal menu sizes. Then the quality of the menu offered by the expert menu setter is always higher than the expected quality of the menu offered by the lower-ability menu setter.*

Proof. See Appendix (page 31).

Proposition 2 is not immediately obvious, because if the cost c is large, from Proposition 1 we know that the lower-ability menu setter optimally offers more items in his menu. The tradeoff is then between a larger menu from a lower-ability menu setter, and a smaller but more carefully designed menu from the expert menu setter. Proposition 2 shows that given each menu setter's optimal menu size, the expert's menu is of higher quality. The driver

[9] In this model we do not allow investors to choose the menu setter (they are exogenously assigned), so the expert menu setter has no incentives to mimic the lower-ability menu setter.

of this result is Lemma 2, which states that the marginal benefit of an additional menu item declines more rapidly for the expert menu setter than for the lower-ability menu setter. The intuition is follows: When each type of menu setter chooses the optimal menu size, they are both at the same marginal benefit. If they were to increase the menu size by one more item (which, in the presence of costs they would not choose to do), from Lemma 2 the extra marginal benefit to the lower-ability manager would be greater than the extra marginal benefit for the expert menu setter. This inequality would continue for hypothetical increases in the menu size ad infinitum. Thus, the total foregone benefit is larger for the lower-ability menu setter than for the expert menu setter. Since the total menu quality in the absence of costs would be the same for the two menu setters (i.e., the quality of a menu with all possible item types), this implies that at the optimal menu in the presence of costs, the total quality of the menu set by the expert menu setter is higher than the total quality of the lower-ability menu setter's menu.

In Appendix B we show the condition that ensures that Proposition 2 holds when n is limited to being an integer.

2.3 Menu size and menu quality

Since lower-ability menu setters sometimes offer larger menus than the expert menu setters (Proposition 1) but at the same time the expert's menu is always of higher quality (Proposition 2), it follows that larger menus may be of lower quality. More specifically, if the menus are larger than n^* (i.e., if $c < \alpha q(n^*)$), then there is a negative relation between the menu size and quality. This seemingly counterintuitive result follows from a difference in composition: a smaller menu offered by an expert menu setter is not merely a subset of the larger menu offered by a lower-ability menu setter. Instead, the smaller menu is likely comprised of more item types, and consequently of higher quality in expectation.

The possibility of a negative relation between menu size and menu quality is driven by differences in the abilities of menu setters. Most importantly, this prediction would not hold in an alternative model with menu setters of the same ability, but who differ in the cost, c, each incurs when increasing the size of the menu. If the menu setters differ in cost but not in ability, we would only observe a positive relation between the menu size and menu ability.

Different costs would lead menu setters to offer menus of different sizes. Specifically, a menu setter with a low cost would offer a larger menu then a menu setter with a high cost. However, for a menu setter of a fixed ability (or two menu setters of the same ability), a larger menu always has higher expected quality. Since menu setters of the same ability identify non-redundant items with the same probability, in expectation a larger menu would include a larger number of non-redundant items than the smaller menu. The expected total benefit from the larger menu is higher than the total benefit from the smaller menu.

Conversely, if the menu setters differ in ability, but incur the same cost, c, the menu of the expert menu setter is likely to include a larger number of non-redundant items then the menu of the lower-ability menu setter. Therefore, when c is small, the lower-ability menu setter would offer a larger menu than the expert, and we would observe a negative relation between the menu size and menu quality.

Until now, we have been comparing expert menu setters who always successfully identify new menu item types with lower-ability menu setters. However, in most settings, it is unlikely that any menu setter will be perfect. Instead, it is more fitting to compare two non-expert menu setters, each with a different ability. All of the results in this section can be generalized, albeit with slightly more complicated notation, if we were to compare two non-expert menu setters that differ in their abilities.

If two menu setters had abilities p' and p, where $p' > p$, the crossing point of marginal benefits would be

$$n^{**} = 2 + \frac{ln(p) - ln(p')}{ln[(1-p') + p'k] - ln[(1-p) + pk]} \,.$$

When costs are sufficiently low, i.e., when $c < \alpha \mathbb{E} q^{p}(n^{**}) = \alpha \mathbb{E} q^{p'}(n^{**})$, the lower-ability menu setter offers more menu items than the higher-ability menu setter, and again there is a negative relation between menu size and menu quality.

3 Empirical setting and data

While the theory above is written to be general and applies to many settings, as an empirical illustration we consider the menu of investment choices (i.e., menu items) offered to employees in companies' 401(k) pension plans. Specifically, we analyze the relation between the number of investment choices offered in a plan and the quality of the plan.

When a company provides a 401(k) plan for its employees, it typically hires an outside firm to design and manage the plan. Each plan is a menu of investment choices for employees' retirement savings, typically mutual funds or similar investments, as well as a money market fund. Frequently the company's own stock is one of the possible investments. Each employee allocates his 401(k) money across the various choices in the plan.

Our data is comprised of 401(k) plans of companies that file SEC form 11-K. This form must be filed if the company offers its own stock as one of the choices in the 401(k) plan, and includes the full menu of investment choices offered to employees. We use the 11-K data of 131 randomly selected company plans offered in 2007 to examine how the menu setters affect the portfolios that investors can achieve. Among the plans in our data set, the number of funds offered (aside from a company's own stock and any money market funds) ranges from 4 to 28.

We study how the number of fund choices in a plan menu affects the quality of the plan. We define a plan's quality as the highest expected Sharpe ratio achievable within the plan. The portfolio of investment choices that achieves this maximum expected Sharpe ratio is referred to as the optimal portfolio of the plan.

The expected Sharpe ratio of a portfolio is the expected return of the portfolio above the risk-free rate divided by the standard deviation of the portfolio's returns. Thus, the Sharpe ratio increases in the expected return and decreases in the risk of the portfolio. The idea is that in the absence of any differences in menu setters' abilities, larger menus are more likely to include high expected return funds and/or provide more scope for diversification of risk.

In Section 3.1 below, we provide further details on determining the number of choices in each plan. In Section 3.2 we explain the methodology for estimating the maximum expected Sharpe ratio.

Table 1 summarizes the number of choices and Sharpe ratios for the plans in our data set. Figure 2 shows the average optimal Sharpe ratio in the plans broken up into three groups. The groups are constructed based on the number of fund choices available in each plan. The plans with the largest number of fund choices have a smaller average optimal Sharpe ratio

Table 1. Summary statistics. The optimal Sharpe ratio is based on the World CAPM, assuming an equity risk premium of 0.05.

	average	std dev	minimum	maximum
# of funds	13.2	3.97	4	28
# of stocks	1.02	0.15	1	2
# money market	1.35	0.62	0	4
optimal Sharpe ratio	0.303	0.064	0.101	0.432

than the plans with a medium or small number of fund choices. This difference is statistically significant at the 10% and 5% levels, respectively. However, there is no statistically significant difference between the averages of the optimal Sharpe ratio across the groups with the large and medium number of choices. In Section 4, we further consider the relation between the number of choices and the optimal Sharpe ratio.

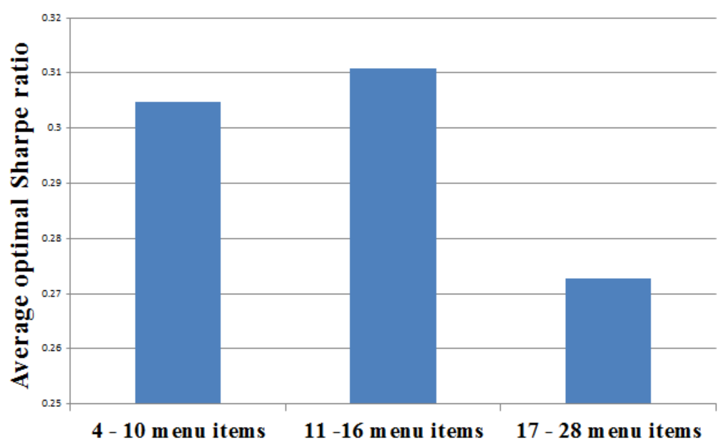


Figure 2. Average optimal Sharpe ratio in plans, broken up into three ranges based on the number of fund choices in each plan. The three ranges are comprised of 31, 78, and 22 plans, respectively.

3.1 Number of funds

Our data set includes the information from 11-K forms for 131 401(k) plans. We originally collected 200 randomly selected 11-K forms. The 11-K forms include a list of all investment choices available for employees' pension savings, and the total amount invested in each.[10] Most of the investment choices in each plan are mutual funds. In addition, every plan in our sample includes the company's own stock, since offering the company's own stock to employees is the trigger for the requirement to file an 11-K. Each plan menu also includes one or more money market funds. Finally, some of the funds offer nonstandard investment choices, such as insurance contracts, warrants, as well as self-directed accounts. Whenever possible, we obtained historical fund prices for the mutual funds in the data from CRSP. However, for a significant subset of the funds, we were unable to identify the CRSP identifier. We exclude plans if more than 6% of the amount invested is in funds for which we were unable to obtain price data. This reduced our sample size from 200 pension plans to 131.

Within the 131 plans, there are a total of 806 unique mutual funds. Most funds (almost 60%) are offered by only one of the 401(k) plans in our sample. More than 90% of the funds are offered by five or fewer plans. A small number of funds are very common: Six funds are offered by at least 20 plans, and the most common fund is offered by 53 different 401(k) plans in our sample.

For each plan, we counted the number of investment choices available to participants. We exclude nonstandard investments, as well as any choices that are not open to new investments. For example, after a merger, there may be investments that remain in funds that were previously available to the employees of the acquired company, but are no longer open to new investments. Our count of investment choices does not include money market funds or the company's own stock. However, excluding money market funds and own stock from the count makes little difference since there is very little variation in this across plans. A number of plans include a set of "lifecycle" funds. Each lifecycle fund targets a subset of employees based on their expected retirement date. For example, a plan may include a set of lifecyle funds aimed at employees who will retire in the years 2020, 2030, 2040, and 2050.

[10]The total amount invested in each asset is given at the aggregate level for the plan, thus it is not appropriate for studying individual investors' choices.

In such cases, since each employee is targeted by one of these funds, we count the entire set of lifecycle funds as one choice.

In order to find the optimal Sharpe ratio, we included all mutual funds available for investment, as well as the company's own stock. The exclusion of money market funds does not affect the Sharpe ratio since it affects the numerator and denominator in the same way.

3.2 Expected Sharpe ratios

We determine the expected return of each fund using the world capital asset pricing model (CAPM).[11] To obtain expected returns, we regress the returns of each investment choice against the MSCI World Index returns using weekly data over the five year period 2003-2007. The estimated coefficient on the world index, i.e., the *beta*, is used within the CAPM to estimate the expected return.[12] The expected return of a portfolio of funds within a plan is the weighted average of the expected returns of the component funds. The standard deviation of returns for a portfolio of funds is estimated based on the historical variance-covariance matrix of weekly returns of the funds within each plan. For each 401(k) plan, we identify the optimal portfolio as the weights on each fund within the plan that give the highest expected Sharpe ratio. Since short sales are not possible in 401(k) accounts, the optimization routine assumes that the amount of money invested in any choice is non-negative.

We use the world index for the beta estimation, as opposed to a U.S. index such as the S&P 500, because international equity funds are an important area of potential diversification. We use weekly data, as opposed to daily data, to minimize the lead-lag effects in international beta estimation that can be caused by time zone differences. While some lead-lag issues surely remain in our data, we find that weekly data is an effective compromise with the need for a larger sample. Note that we use a model-based estimate of expected returns rather than historical returns in the numerator of the Sharpe ratio to avoid the well-known problems associated with backward-looking returns. In contrast, since variances and

[11] We assume that in expectation, fund managers do not earn any excess return (often referred to as *alpha*) above that predicted by the CAPM.

[12] We assume an equity risk premium of 5%, but because the expected Sharpe ratio is proportional to the equity risk premium, it does not have any effect on the statistical significance in our cross-sectional analysis.

covariances tend to be persistent, we use historical data to estimate the variance-covariance matrix.

Moreover, by using a CAPM-based estimate of expected returns, we are abstracting away from any differences in fees charged by each fund.

When estimating variances and covariances, we run into the problem that newer funds do not have a full five year return history. In these cases we necessarily estimate the variance based on the shorter time series and we estimate the covariance based on the shorter of the two time series' for each pair of funds.[13]

4 Empirical analysis

The main result from the theory is that when the cost of including an additional menu item is low, there is a negative relation between the number of choices and the menu quality. Now, in the context of 401(k) plans, we compare the optimal (annualized) Sharpe ratio — a measure of plan quality — and the number of funds available for investment in the plan. The regression is

$$\text{optimal Sharpe ratio}_{\,i} = a + b \cdot \text{number of funds}_{\,i} \ ,$$

where the optimal Sharpe ratio for a plan and the number of funds in the plan menu are defined in the previous section. The results are displayed in Table 2. Regressions (1) and (2) use the number funds in the plan as an explanatory variable, while Regressions (3) and (4) use the natural logarithm of the number of funds. For readability, the coefficients in the table are multiplied by 100.

When the entire data set is included (Regression (1)), we indeed find a negative relation between the number of choices and the optimal Sharpe ratio, but the relation is not statistically significant ($t = 1.55$). However, examination of the data shows that there is a large amount of variability in the Sharpe ratio for plans with the fewest number of funds. Thus, in Regression (2) we exclude those funds with fewer than ten menu items, and run the regression on the remaining 86% of the data. The results are starker when the small plans

[13] We exclude a very small number of funds for which we have less than 20 weekly observations.

Table 2. Regression of optimal Sharpe ratio on number of choices. For readability, coefficients are multiplied by 100 and intercepts are suppressed. The t-statistics are in parentheses. Triple asterisk, ***, denotes statistical significance at the 1% level.

	(1) all data	(2) $n \geq 10$	(3) all data	(4) $n \geq 10$
# of menu items	-0.219	-0.515^{***}		
	(-1.55)	*(-2.98)*		
ln(# of menu items)			-1.784	-8.159^{***}
			(-1.00)	*(-3.11)*
R^2	1.84%	7.43%	0.77%	8.01%
N	131	113	131	113

are excluded. There is a negative relation between the number of funds and the optimal Sharpe ratio, and this relation is significant at the 1% level ($t = -2.98$). For plans with at least 10 choices, the coefficient on the number of funds in a plans is -0.00515. When compared to the average annual optimal Sharpe ratio of 0.303, each additional investment item corresponds to a 1.7% decline in the plan's quality. A one standard deviation change in the number of menu items (i.e., a change of 3.97 menu item) corresponds to a 6.7% change in plan quality.[14]

The results are very similar when we regress the optimal Sharpe ratio on the logarithm of the number of menu items (Regressions (3) to (4)). When considering only the plans with at least ten investment funds, there is a strongly significant negative relation between the number of menu items and plan quality. Doubling the number of menu items corresponds to a reduction of the optimal Sharpe ratio by $ln(2) * .08159/.303 = 18.7\%$.

When limited to plans with at least ten funds, the R^2 of the regression ranges from 7.43% to 8.01%. Thus, there are other important factors explaining the variation in quality across 401(k) plans. Nevertheless, these results indicate that the number of funds plays an

[14]For robustness, we also calculate Sharpe ratios using less than the full five years of historical data. When we estimate the Sharpe ratio using weekly data for the period 2005 to 2007, we obtain qualitatively similar results. When we use just 2007 data, the estimates become noisier and we lose statistical significance.

economically significant role.

While we can not explain why the plans with fewest number of menu items do not display the downward relation, the fact that the bulk of the plans have a strongly significant negative relation can be explained by our theory: expert menu setters offer smaller but better menus, while lower-ability menu setters offer larger but worse menus. Moreover, whether or not we exclude any plans, there is certainly not a positive relation between the number of choices and plan quality as would be predicted by the alternative theory with homogenous menu-setter ability and varying costs.

As is generally well known, employees who invest in 401(k) plans tend to hold a portion of their assets in the company's own stock. In our data, the percentage of assets held by employees in company stock within the 401(k) plan varies, but averages 17.0% of assets.[15] Not surprisingly, even though company stock is one of the menu items, the optimal portfolio rarely includes company stock. Nevertheless, employees hold company stock, whether due to restrictions, explicit incentives, implicit incentives, or behavioral biases. As such, for robustness, we consider an alternative measure of menu quality in which we assume that employees hold company stock, and we calculate the Sharpe ratio from an optimization constrained to having a portion of assets in the company stock.

In Table 3 we display the results of the regressions of the optimal Sharpe ratio, in which the optimization is constrained to have a weight on the company stock equal to the actual weight on company stock held in aggregate by employees. We find qualitatively similar results. When considering plans with at least 10 funds in the menu, there is a negative relation between the number of menu items and the constrained-optimal Sharpe ratio. However, the statistical significance is weaker and the negative relation is only significant at the 10% level.

Because the value of assets in each plan varies widely,[16] Regressions (1) and (2) of Table 4 include the logarithm of the total assets as a control variable. In Regressions (3) and (4) we also include the interaction term between the number of menu items and the logarithm of

[15]Our data includes the aggregate amount held by employees in each investment. Unfortunately, we do not observe the selections of individual employees.

[16]The plan sizes range from less than $2 million to over $4 billion, with a mean of $308 million and a median of $60 million.

Table 3. Regression of constrained optimal Sharpe ratio on number of choices. For these regressions, the optimal Sharpe ratio is constrained to fix the weight on the company stock equal to the observed weight. (Average = 17.0%, stdev = 14.6%) For readability, coefficients are multiplied by 100 and intercepts are suppressed. The t-statistics are in parentheses. Single asterisk, *, denotes statistical significance at the 10% level.

	(1) all data	(2) $n \geq 10$	(3) all data	(4) $n \geq 10$
# of menu items	-0.171	-0.513^*		
	(-0.77)	*(-1.87)*		
ln(# of menu items)			-0.773	-7.682^*
			(-0.28)	*(-1.83)*
R^2	0.46%	3.05%	0.06%	2.94%
N	131	113	131	113

total assets.[17]

As above, the results indicate that the negative relation between the number of funds in a pension plan and the quality of the plan remains statistically significant and robust as long as the the plans with the fewest menu items are excluded. Moreover, even when all plans are included, the negative relation has some, albeit weak, statistical significance. Interestingly, we find evidence of mixed statistical significance that beyond the number of menu items, the size of the 401(k) plans, as measured by log(total assets), is positively related to the optimal Sharpe ratio. Moreover, the interaction term is positive and is statistically significant when we restrict the analysis to plans with at least ten funds. This means that the negative relation between the number of menu items and plan quality is more pronounced for plans with less assets than it is for plans with more invested. Nonetheless, the results confirm the robustness of the central result.

[17] The number of menu items and the logarithm of total assets have each been demeaned.

Table 4. Regression of optimal Sharpe ratio on the number of choices controlling for plan assets. This table includes robustness checks for the logarithm of total assets, as well as an interaction term. The variables *# of menu items* and *ln(total assets)* are demeaned. For readability, coefficients are multiplied by 100 and intercepts are suppressed. The t-statistics are in parentheses. Triple, double, and single asterisks denote statistical significance at the 1%, 5%, and 10% levels, respectively.

	(1) all data	(2) $n \geq 10$	(3) all data	(4) $n \geq 10$	(5) all data	(6) $n \geq 10$
# of menu items	-0.251^{*}	-0.536^{***}	-0.230	-0.612^{***}	-0.260^{*}	-0.621^{***}
	(-1.80)	*(-3.14)*	*(-1.62)*	*(-3.45)*	*(-1.85)*	*(-3.54)*
ln(total assets)	0.715^{**}	0.685^{**}			0.705^{**}	0.621^{*}
	(2.25)	*(2.01)*			*(2.21)*	*(1.83)*
# of menu items × ln(total assets)			0.051 *(0.67)*	0.196^{**} *(1.98)*	0.042 *(0.56)*	0.177^{*} *(1.80)*
R^2	5.58%	10.70%	2.18%	10.63%	5.81%	13.29%
N	131	113	131	113	131	113

5 Conclusion

In this paper we study the relation between menu size and menu quality. There exists a growing literature showing that individuals often prefer to choose from a smaller set. This preference is usually ascribed to behavioral biases. We take a different approach, and show that larger menu may be objectively worse. We recognize that menus are generally pre-selected by menu setters from a larger universe of items, and that these menu setters may differ in their ability to construct menus. Our theoretical results show that when the marginal cost of increasing a menu size is low, menu setters with lower ability offer larger menus than expert menu setters. At the same time, the higher-ability menu setter's menu is of higher quality in the sense that it offers a larger number of distinct item types. Together, these two results lead to a prediction that smaller menus can be of higher quality than larger menus.

Empirically, we study the relation between the number of investment choices offered by 401(k) plans and the quality of those plans. We measure plan quality by the maximum expected Sharpe ratio achievable given the investment choices in the plan. Excluding the funds with the fewest investment choices, we find a statistically significant negative relation between the number of investment choices and plan quality.

While the empirical application in our paper addresses only investment portfolios, the central insight that menus are pre-selected by a menu setter is applicable to many scenarios. For example, one ice cream store might offer ten flavors of ice cream, while another store may offer only three flavors. It should be immediately apparent that the three flavors are unlikely to be a random subset of the ten flavors, but are chosen with forethought to appeal to as many customers as possible.

Following the logic in our theory, if a skilled ice cream purveyor offers the flavors favored by almost all customers, the marginal benefit of adding one more flavor may be small (and in particular, may be less than the cost of stocking the additional flavor); whereas an unskilled vendor who does not know the preferences of customers may not carry the most desired flavors, and thus may find it profitable at the margin to offer more flavors.

Of course, other forces come into play when evaluating menus. Certainly the behavioral effects recently addressed in the literature affect choices made by agents facing menus of different sizes. Nevertheless, we argue that one must be aware of the role played by menu setters in designing the menu offered to individuals.

A Appendix: Proofs

Proof of Lemma 1 (page 8)

Proof. For $p = 1$, $\mathbb{E}q^p(n) = q(n)$, and thus is decreasing by the assumption on $q(n)$.

Fix arbitrary $p \in (0, 1)$. The general formula for $\mathbb{E}q^p(n)$ is

$$\mathbb{E}q^p(n) = \sum_{i=0}^{n-2} \frac{(n-2)!}{(n-2-i)!i!} p^{n-2-i} (1-p)^i \, q(n-i) \, . \tag{5}$$

We must show that $\mathbb{E}q^p(n+1) < \mathbb{E}q^p(n)$. First, by applying formula (5) to $n+1$

$$\mathbb{E}q^p(n+1) = \sum_{i=0}^{n-1} \frac{(n-1)!}{(n-1-i)!i!} p^{n-1-i} (1-p)^i \, q(n+1-i) \, . \tag{6}$$

It is straightforward (although tedious) to show that (6) is equivalent to

$$(1-p) \sum_{i=0}^{n-2} \frac{(n-2)!}{(n-2-i)!i!} p^{n-2-i} (1-p)^i \, q(n-i) + p \sum_{i=0}^{n-2} \frac{(n-2)!}{(n-2-i)!i!} p^{n-2-i} (1-p)^i \, q(n+1-i) \, . \tag{7}$$

By the Assumption of Declining Marginal Benefit, $q(n)$ is decreasing, i.e., $q(n+1) < q(n)$. Therefore,

$$\mathbb{E}q^p(n+1) = (1-p) \sum_{i=0}^{n-2} \frac{(n-2)!}{(n-2-i)!i!} p^{n-2-i} (1-p)^i \, q(n-i) + p \sum_{i=0}^{n-2} \frac{(n-2)!}{(n-2-i)!i!} p^{n-2-i} (1-p)^i \, q(n+1-i) <$$

$$< \sum_{i=0}^{n-2} \frac{(n-2)!}{(n-2-i)!i!} p^{n-2-i} (1-p)^i \, q(n-i) = \mathbb{E}q^p(n) \, .$$

This completes the proof of Lemma 1. □

Proof of Lemma 2 (page 11)

Proof. By the Strong Assumption of Declining Marginal Benefit, $\frac{q(n+1)}{q(n)} = k < 1$. $\mathbb{E}q^p(n)$ decreases more slowly when $k < \frac{\mathbb{E}q^p(n+1)}{\mathbb{E}q^p(n)} < 1$. We can write $\mathbb{E}q^p(n)$ as

$$\mathbb{E}q^p(n) = p \cdot \mathbb{E}(q| \, \textit{success at } n) \, ,$$

where $\mathbb{E}(q| \, \textit{success at } n)$ is the expected marginal benefit *conditional* on getting a non-redundant item (depending on previous successes, this success may bring the second, third, etc up to n^{th} new, non-redundant item).[18]

[18]Since $\mathbb{E}q^p(n)$ is given by (5) and $q(n) = k^{n-2} \, q(2)$, then

$$\mathbb{E}(q| \, \textit{success at } n) = \sum_{i=0}^{n-2} \frac{(n-2)!}{(n-2-i)!i!} p^{n-3-i} (1-p)^i \, k^{n-i-2} q(2) \, .$$

Then $\mathbb{E}q^p(n+1)$ can be further written as[19]

$$\mathbb{E}q^p(n+1) = p\left[(1-p)\mathbb{E}(q|\, success\ at\ n) + p\,k\,\mathbb{E}(q|\, success\ at\ n)\right] =$$
$$= \left[(1-p) + p\,k\right] p\,\mathbb{E}(q|\, success\ at\ n) = \left[(1-p) + p\,k\right] \mathbb{E}q^p(n)\,.$$

Therefore,

$$\frac{\mathbb{E}q^p(n+1)}{\mathbb{E}q^p(n)} = (1-p) + p\,k\,.$$

For $p \in (0,1)$, this ratio is a convex combination between k and 1. Therefore, $k < \frac{Eq^p(n+1)}{Eq^p(n)} < 1$. This completes the proof of Lemma 2. □

Proof of Corollary 1 (page 12)

Proof. By the Strong Assumption of Declining Marginal Benefit, $q(n+1) = k\,q(n)$ for some $k < 1$. Suppose also that $q(n) < \mathbb{E}q^p(n)$.

From the proof of Lemma 2, $\mathbb{E}q^p(n+1) = \left[(1-p) + p\,k\right]\mathbb{E}q^p(n) > k\,\mathbb{E}q^p(n)$. Then, since $\mathbb{E}q^p(n) > q(n)$,

$$\mathbb{E}q^p(n+1) > k\,\mathbb{E}q^p(n) > k\,q(n) = q(n+1) \iff \mathbb{E}q^p(n+1) > q(n+1)\,.$$

This completes the proof of Corollary 1. □

Proof of Lemma 3 (page 12)

Proof. By construction, $Eq^p(2) = p\,q(2)$. And by the Strong Assumption of Declining Marginal Benefit, $q(2) = k\,q(1)$, and $q(n) = k^{n-2}q(2) = k^{n-1}q(1)$. Moreover, $Eq^p(2) = p\,q(2) = p\,k\,q(1)$. By Lemma 2, for $n \geq 2$, $Eq^p(n) = Eq^p(2)[(1-p) + p \cdot k]^{n-2}$. Let $n^* = 2 + \frac{ln(p)}{ln(k) - ln\left((1-p)+p\cdot k\right)}$.

We first prove part (ii) of the Lemma, then part (i).

[19]By substituting $q(n) = k^{n-2}\,q(2)$ into (5), (6) and (7), we also obtain $\mathbb{E}q^p(n+1) = [(1-p)+p\,k]\,\mathbb{E}q^p(n)$.

(ii)

$$
\begin{aligned}
&\mathbb{E}q^p(n) > q(n) \iff \\
\iff & p\,\cancel{q(2)}[(1-p)+p\cdot k]^{n-2} > k^{n-2}\cancel{q(2)} \iff \\
\iff & p > \left[\frac{k}{(1-p)+p\cdot k}\right]^{n-2} \iff \\
\iff & p^{\frac{1}{n-2}} > \frac{k}{(1-p)+p\cdot k} \iff \\
\iff & \frac{1}{n-2} ln(p) > \underbrace{ln(k) - ln\big((1-p)+p\cdot k\big)}_{-} \iff \\
\iff & \frac{1}{n-2}\frac{ln(p)}{ln(k) - ln\big((1-p)+p\cdot k\big)} < 1 \iff \\
\iff & n > 2 + \frac{ln(p)}{ln(k) - ln\big((1-p)+p\cdot k\big)} = n^* \,.
\end{aligned}
$$

Therefore, $\mathbb{E}q^p(n) > q(n) \iff n > n^*$. By similar calculations, we obtain $\mathbb{E}q^p(n) < q(n) \iff n < n^*$.

(i) By similar calculations as above (with "=" substituted for ">"), we obtain $\mathbb{E} = q(n) \iff n = n^*$.

This completes the proof of Lemma 3. $\square$

Proof of Proposition 1 (page 13)

Proof. The objective of the menu setter is to

$$\max_n \left\{\alpha\,\mathbb{E}\,Q^p(n) - c\cdot n\right\}.$$

By the first order condition, the optimal menu size for the lower-ability menu setter, n^i, is characterized by $\alpha\,\mathbb{E}q^p(n^i) = c$. And the optimal menu size for the the expert menu setter, n^e, is characterized by $\alpha\,q(n^e) = c$.

(i) If $c < \alpha q(n^*)$ then $c = \alpha\,q(n^e)$ for $n^e > n^*$ (by decreasing $q(n)$). Then, by Lemma 3(ii), $\mathbb{E}q^p(n^e) > q(n^e)$. So, $c < \alpha\,\mathbb{E}q^p(n^e)$. Since $\mathbb{E}q^p(n)$ is also decreasing, $c = \alpha\mathbb{E}q^p(n^i)$ for $n^i > n^e$. Therefore, the menu of the lower-ability menu setter is larger than the menu of the expert menu setter.

(ii) By analogous reasoning, we obtain that for $c > \alpha q(n^*)$ the expert offers larger menu than the lower-ability menu setter.

This completes the proof of Proposition 1. □

Proof of Proposition 2 (page 15)

Proof. Suppose that for a given c, $\widehat{x}$ is the optimal size of a menu selected by the expert menu setter, i.e., $\alpha\, q(\hat{x}) = c$. And $\widehat{m}$ is the optimal size of a menu selected by the lower-ability menu setter, i.e., $\alpha\, \mathbb{E}q^p(\widehat{m}) = c$. Since the cost is the same for both menu setters, it must be that the respective optimal menu sizes satisfy $q(\widehat{x}) = \mathbb{E}q^p(\widehat{m})$.

By the Strong Assumption of Declining Marginal Benefit, $q(\widehat{x}) = k^{\widehat{x}-1}q(1)$. By Lemma 2, for $\widehat{m} \geq 2$, $\mathbb{E}q^p(\widehat{m}) = \mathbb{E}q^p(2)\left[(1-p)+pk\right]^{\widehat{m}-2}$. Moreover, $\mathbb{E}q^p(2) = pkq(1)$. Therefore,

$$q(\widehat{x}) = \mathbb{E}q^p(\widehat{m}) \iff k^{\widehat{x}-1}\, q(1) = pk\, q(1)\left[(1-p)+pk\right]^{\widehat{m}-2} \iff k^{\widehat{x}-2} = p\left[(1-p)+pk\right]^{\widehat{m}-2}. \quad (8)$$

The respective qualities of the menus are given by

$$\begin{aligned} Q(\widehat{x}) &= \sum_{l=1}^{\widehat{x}} q(l) = q(1) + q(1)\sum_{l=1}^{\widehat{x}-1} k^l = q(1) + q(1)\, k \sum_{l=0}^{\widehat{x}-2} k^l \\ \mathbb{E}Q^p(\widehat{m}) &= \sum_{l=1}^{\widehat{m}} \mathbb{E}q^p(l) = \mathbb{E}q^p(1) + \sum_{l=2}^{\widehat{m}} \mathbb{E}q^p(l) = q(1) + pk\, q(1)\sum_{l=0}^{\widehat{m}-2}\left[(1-p)+pk\right]^l. \end{aligned}$$

The finite sums of the geometric series can be expressed in the following way[20]

$$\begin{aligned} \sum_{l=0}^{\widehat{x}-2} k^l &= \frac{k^{\widehat{x}-1}-1}{k-1} \\ \sum_{l=0}^{\widehat{m}-2}\left[(1-p)+pk\right]^l &= \frac{\left[(1-p)+pk\right]^{\widehat{m}-1}-1}{(\cancel{1}-p)+pk-\cancel{1}} = \frac{\left[(1-p)+pk\right]^{\widehat{m}-1}-1}{p(k-1)}. \end{aligned}$$

Then,

$$\begin{aligned} Q(\widehat{x}) =& q(1) + q(1)\, k\, \frac{k^{\widehat{x}-1}-1}{k-1} = q(1) + q(1)\frac{k}{1-k}\left(1-k^{\widehat{x}-1}\right) \\ \mathbb{E}q^p(\widehat{m}) =& q(1) + \cancel{p}k\, q(1)\, \frac{\left[(1-p)+pk\right]^{\widehat{m}-1}-1}{\cancel{p}(k-1)} = q(1) + q(1)\frac{k}{1-k}\left(1-\left[(1-p)+pk\right]^{\widehat{m}-1}\right). \end{aligned}$$

[20] It follows directly from the formula for the finite geometric series: $\sum_{K=0}^{N-1} g^K = \frac{g^N-1}{g-1}$.

And

$$Q(\widehat{x}) > \mathbb{E}q^p(\widehat{m}) \iff k^{\widehat{x}-1} < \left[(1-p)+pk\right]^{\widehat{m}-1} \iff k \cdot k^{\widehat{x}-2} < [(1-p)+pk] \cdot \left[(1-p)+pk\right]^{\widehat{m}-2} \iff$$

$$\iff_{by(8)} kp \cancel{\left[(1-p)+pk\right]^{\widehat{m}-2}} < [(1-p)+pk] \cdot \cancel{\left[(1-p)+pk\right]^{\widehat{m}-2}} \iff p < 1\,.$$

Therefore, for all $p < 1$, $Q(\widehat{x}) > \mathbb{E}q^p(\widehat{m})$. This completes the proof of Proposition 2. $\square$

B Appendix: Proposition 2 with integer constraint

The proof of Proposition 2 above ignores the constraint that the menu size, n, must be an integer. With the integer constraint, both the lower-ability and the expert menu setters round down the number of menu items offered. For the purpose of Proposition 2 the concern is for situations in which the expert menu setter rounds down the number of menu items more so than the lower-ability menu setter, and the resulting possibility that the lower-ability menu setter's menu would be of higher quality. Below we derive the condition that ensures that even in the most severe case of rounding, the expert menu setter always offers the superior menu.

We approach the problem while accounting for the integer constraint as follows: Suppose that x' and m' are the largest integers less than or equal to $\widehat{x}$ and $\widehat{m}$. (Note that (8) still holds for $\widehat{x}$ and $\widehat{m}$.) The *worst case scenario* is when $\widehat{m}$ is an integer and $\widehat{x}$ is just below an integer. Then $m' = \widehat{m}$ and we can approximate $x' = \widehat{x} - 1$. Then, to assure that the expert offers always a better menu than the lower-ability menu setter, it must also hold under this worst case scenario, i.e., $Q(\widehat{x} - 1) > \mathbb{E}Q^p(\widehat{m})$.

By similar calculations as above, we find that

$$Q(\widehat{x} - 1) = q(1) + q(1)k \sum_{l=0}^{\widehat{x}-3} k^l = q(1) + q(1)\frac{k}{1-k}\left(1 - k^{\widehat{x}-2}\right)\,.$$

Then,

$$Q(\widehat{x} - 1) > \mathbb{E}Q^p(\widehat{m}) \iff k^{\widehat{x}-2} < \left[(1-p)+pk\right]^{\widehat{m}-1} \iff p < 1 - p + pk \iff p(2-k) < 1\,.$$

That is, for sufficiently small p and large k the condition will be satisfied.

Notice that this is a strong sufficient condition designed to hold under the worst case scenario for x' and m'. In most cases, the inequality $Q(x') - \mathbb{E}Q^p(m') > 0$ is satisfied for weaker condition.

References

[1] Agnew, J., 2003, "Inefficient choices in 401(k) plans: Evidence from individual level data," Working paper, College of William and Mary.

[2] Agnew, J., P. Balduzzi, and A. Sunden, 2003, "Portfolio choice and trading in a large 401(k) plan," *American Economic Review*, 93, 193-215.

[3] Benartzi S. and R. Thaler, 2010, "Naıve diversification strategies in defined contribution saving plans," *American Economic Review*, 91, 79-98.

[4] Ellison, G., D. Fudenberg, and M. Mobius, 2004, "Competing auctions," *Journal of the European Economic Association*, 2, 30-66.

[5] Huberman G. and W. Jiang, 2006, "Offering versus choice in 401(k) plans: equity exposure and the number of funds," *Journal of Finance* 61, 763-801.

[6] Iyengar, S., 2010, *The Art of Choosing*, New York: Twelve.

[7] Iyengar, S., G. Huberman, and W. Jiang, 2004, "How much choice is too much: determinants of individual contributions in 401(k) retirement plans." In: O.S. Mitchell and S. Utkus, Editors, Pension Design and Structure: New Lessons from Behavioral Finance, Oxford University Press, Oxford, pp. 83-95.

[8] Iyengar, S. and E. Kamenica, 2010, "Choice proliferation, simplicity seeking, and asset allocation," *Journal of Public Economics*, 94, 530-539.

[9] Iyengar, S., and M. Lepper, 2000, "When choice is demotivating: Can one desire too much of a good thing?" *Journal of Personality and Social Psychology*, 76, 995-1006.

[10] Lehrer, J., 2009, *How We Decide*, New York: Houghton Mifflin.

[11] Mottola, G.R. and S. Utkus, 2003, "Can there be too much choice in a retirement savings plan," Vanguard Center for Retirement Research.

[12] Salgado, M., 2006, "Choosing to have less choice," mimeo, Northwestern University.

[13] Scheibehenne, B., R. Greifeneder, and P.M. Todd, 2010, "Can there ever be too many options? A meta-analytic review of choice overload," *Journal of Consumer Research*, 37, 409-25.

[14] Schwartz, B., 2004, *The Paradox of Choice*, New York: Ecco.

[15] Tang N., O.S. Mitchell, G.R. Mottola, and S.P. Utkus, 2010, "The efficiency of sponsor and participant portfolio choices in 401(k) plans," *Journal of Public Economics*, 94, 1073-1085.